Progress on the Calculation of Pion Parton Distribution Functions from Current-Current Correlators

David Richards (Jefferson Lab)

with

J. Karpie, C. Egerer, J.W. Qiu, B. Chakraborty, R. Edwards, K. Orginos, R. Sufian

QCD Evolution 2018

Outline

- Introduction - Light-Cone Distributions from Euclidean-space Lattice QCD
- Lattice Cross Sections
- Why the Pion?
- Preliminary Results
- Future Plans

Introduction

- Euclidean lattice precludes the calculation of light-cone correlation functions
  - So… …Use *Operator-Product-Expansion* to formulate in terms of *Mellin Moments*

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P\rangle$$

$$\downarrow$$

$$\langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\ldots D_{\mu_n}\psi \mid P\rangle \to P_{\mu_1}\ldots P_{\mu_n} a^{(n)}$$

- Moment Methods
  - Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
  - Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

KF Liu, SJ Dong, PRL72, 1790 (1994)

- Hadronic Tensor (HT) $W_{\mu\nu} = \frac{1}{4\pi}\int d^4z\, e^{iq.z} \langle p \mid J_\mu(z)^\dagger J_\nu(0) \mid p\rangle$

$$C_4(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_f} e^{-i\vec{p}.\vec{x}_f} \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{q}.(\vec{x}_2-\vec{x}_1)} \langle N(\vec{x}_f,t_f) J_\mu(\vec{x}_2,t_2) J_\nu(\vec{x}_1,t_1) \bar{N}(\vec{0},t_0)\rangle$$

This is a ***four-point*** function.

Introduction - II

- Quasi-PDF (qPDF) interpreted in LaMET (Large Momentum Effective Theory) was proposed by X.Ji **X. Ji, Phys. Rev. Lett. 110 (2013) 262002**

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz' A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

Quasi distributions approach light-cone distributions in limit of large *P^z*

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

A. Radyushkin, PLB767 (2017)

$$\mathcal{M}^\alpha(z,p) = \langle p \mid \bar{\psi}(z)\gamma^\alpha \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0) \mid p\rangle$$

A.Radyushkin, J.Karpie, Tuesday

Lattice Cross Sections

- Good "Lattice Cross Sections" (LCS) Ma and Qiu, Phys. Rev. Lett. 120 022003

$$\sigma_n(\omega, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P \rangle$$

Expressed in coordinate space

Short distance scale

where

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda^2_{\rm QCD})$$

Calculated in LQCD

Structure function

Calculated in perturbation theory

Factorize in $\omega = P \cdot \xi, \; \xi^2 P^2$ providing $\xi << \dfrac{1}{\Lambda_{\rm QCD}}$

Momentum space

$$\tilde{\sigma}(\tilde{\omega}, q^2 P^2) \equiv \int \frac{d^4\xi}{\xi^4} \sigma(P \cdot \xi, \xi^2, P^2)$$

$$\tilde{\omega} = 1/x_B$$

Lattice Cross Sections - II

- Quasi- and Pseudo-distributions particular case

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

Wilson Line

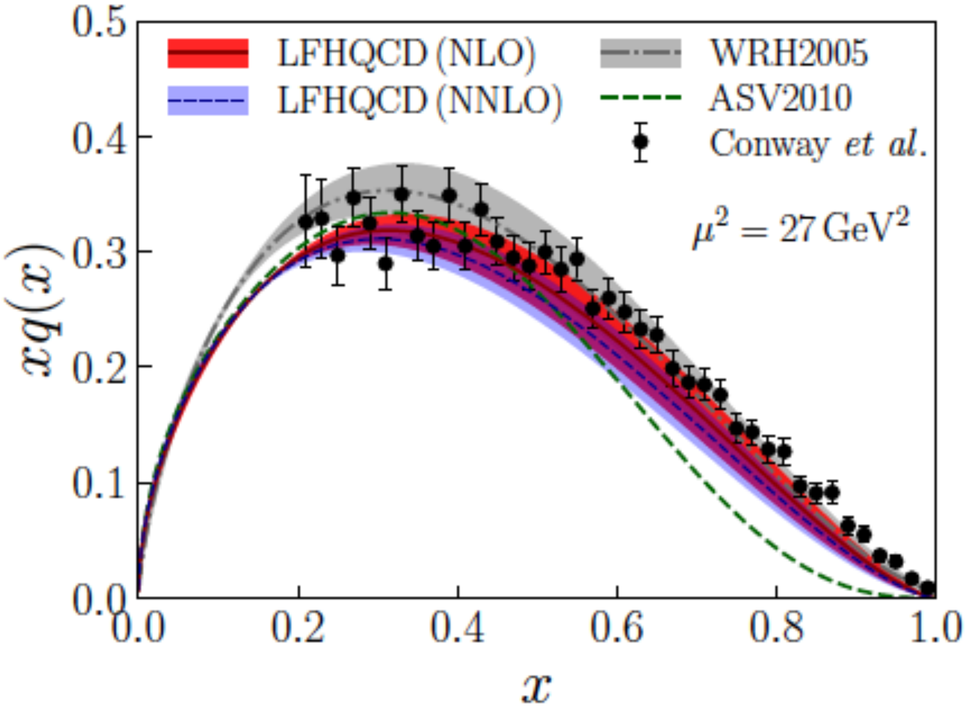
- Current-current correlators, e.g.

$$\begin{aligned}
\mathcal{O}_S(\xi) &= \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0) \\
\mathcal{O}_{V'}(\xi) &= \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi\cdot\gamma\psi_{q'}](\xi)[\bar{\psi}_{q'} \xi\cdot\gamma\psi](0) \\
& \quad F_{\mu\rho}F_{\nu}^{\rho}
\end{aligned}$$

- Gauge-invariant
- Renormalization straightforward

Why the Pion?

- u distribution of FNAL E615 to leading order
- C12-15-006 at Hall A will look at structure of pion
- C12-15-006A at Hall A will look at structure of Kaon
- No free pion target



de Teramond, liu, Sufian, Dosch, Brodsky, Deur, PRL (2018)

Discrepancy in large-x behavior of pion distribution

Why the Pion - II?

- Pion less computationally demanding that nucleon
  - *Larger signal-to-noise ratio*

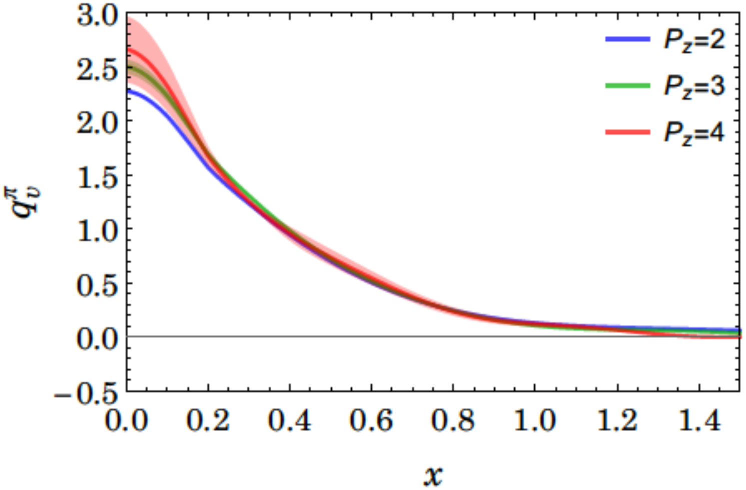
$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^\dagger(0,0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}} \to e^{-E(\vec{p})t}$$

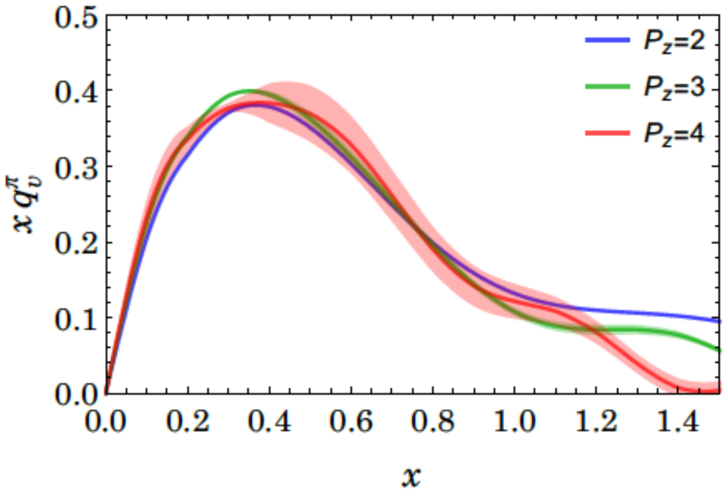
$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \to \begin{cases} e^{-m_\pi t} & \text{Mesons} \\ e^{-(3m_\pi/2)t} & \text{Baryons} \end{cases}$$

- Important constraint on systematic uncertainty is understanding operator renormalization
  - *Operator renormalization "independent" of external states*
- Admits simple computational methodology
  - *Coordinate-space currents computationally demanding in lattice QCD*

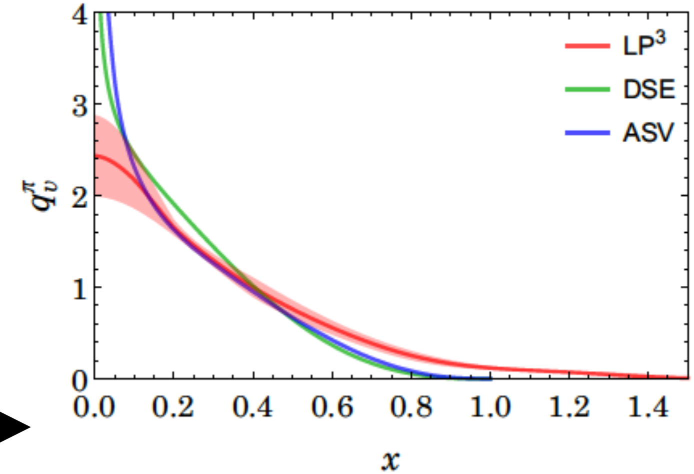
Quasi-Distribution of Pion

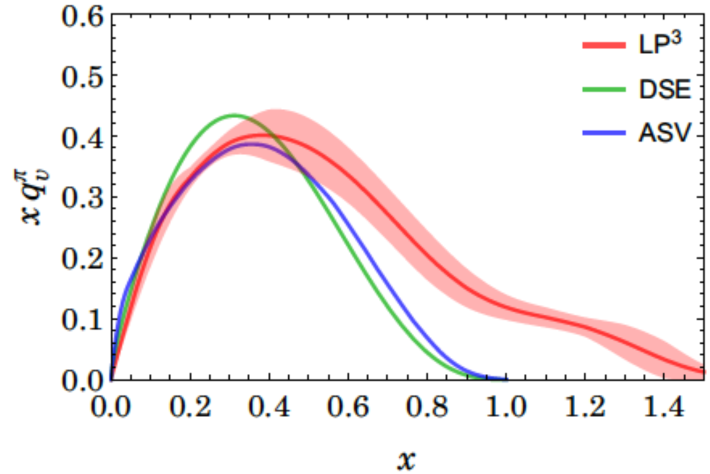
$m_\pi \simeq 300\,\mathrm{MeV}$



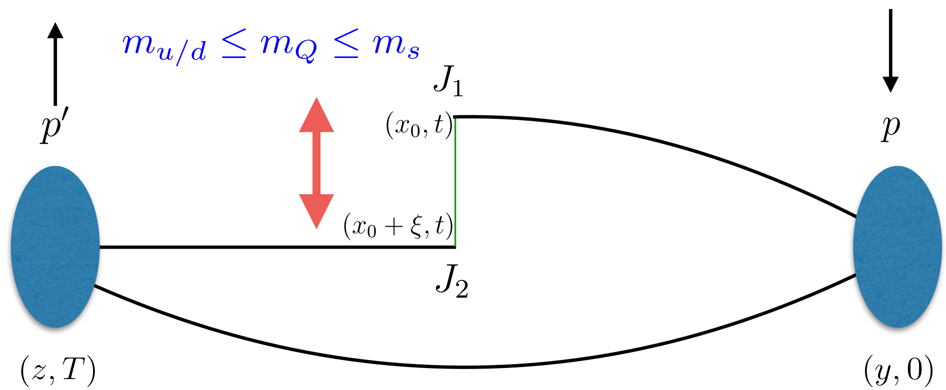


LP3, arXiv:1804.01483





Computational Setup



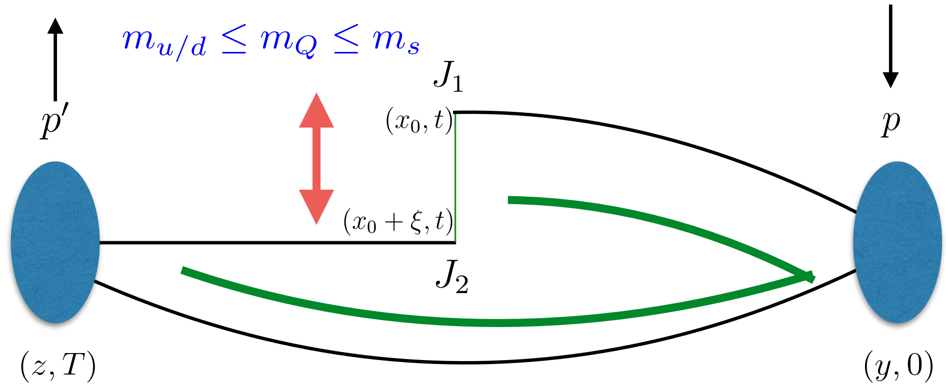
$$\langle \Pi(-p')|\mathcal{O}_{J_1}(x_0)\mathcal{O}_{J_2}(\xi)|\Pi(-p')\rangle =$$

$$= \sum_{y,z} e^{i(p'.z-p.y)} \langle \bar{q}\,\Gamma_\Pi\, q(z,T)\, \bar{Q}\, J_2\, Q(x_0+\xi,t)\, \bar{q}\, J_1\, q(x_0,t)\, \bar{q}\,\Gamma_\Pi\, q(y,0)\rangle$$

$$= \sum_{y,z} e^{i(p'.z-p.y)} \mathrm{Tr}[J_2\, G_Q(x_0+\xi,t;x_0,t)\, J_1\, G(x_0,t;y,0)\,\Gamma_\Pi G(y,0;z,T)\Gamma_\Pi\, G(z,T;x_0+\xi,t)]$$

Straightforward computational setup using sequential-source method:

Computational Setup



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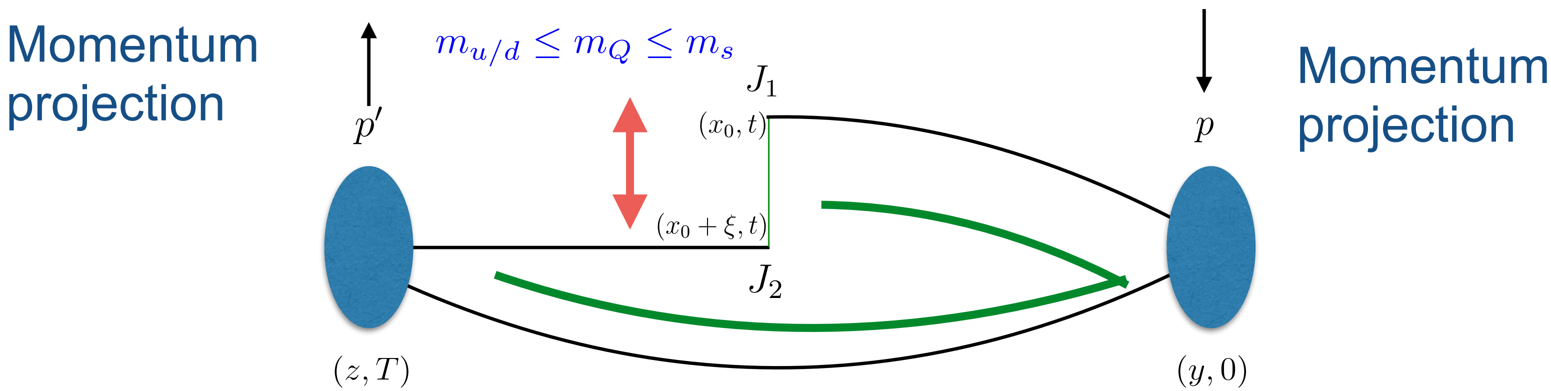
Straightforward computational setup using sequential-source method:

$$D(Z,T;w)H(w;x_0,t) = \sum_y e^{-ip\cdot y}\Gamma_\Pi G(y,0;x_0,t)$$

Computational Setup



Momentum *conservation*

$$\langle \Pi(-p')|\mathcal{O}_{J_1}(x_0)\mathcal{O}_{J_2}(\xi)|\Pi(-p')\rangle =$$

$$= \sum_{y,z} e^{i(p'.z-p.y)} \langle \bar{q}\,\Gamma_\Pi\, q(z,T)\, \bar{Q}\, J_2\, Q(x_0+\xi,t)\, \bar{q}\, J_1\, q(x_0,t)\, \bar{q}\,\Gamma_\Pi\, q(y,0)\rangle$$

$$= \sum_{y,z} e^{i(p'.z-p.y)} \mathrm{Tr}[J_2\, G_Q(x_0+\xi,t;x_0,t)\, J_1\, G(x_0,t;y,0)\,\Gamma_\Pi G(y,0;z,T)\Gamma_\Pi\, G(z,T;x_0+\xi,t)]$$

Straightforward computational setup using sequential-source method:

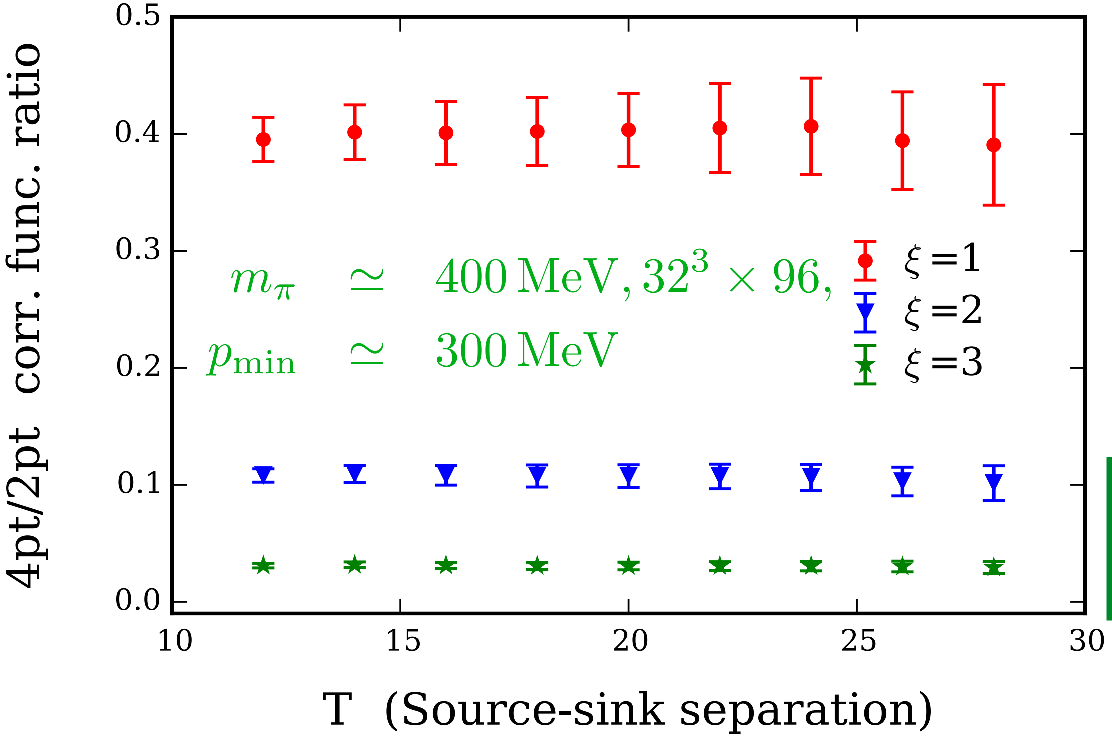
$$D(Z,T;w)H(w;x_0,t) = \sum_y e^{-ip\cdot y}\Gamma_\Pi G(y,0;x_0,t)$$

$$D(s;w)\tilde{H}(w;x_0,t) = \sum_z e^{ip\cdot z}\Gamma_\Pi H(z,T;x_0,t)$$

Preliminary Results

- 2+1 Flavor clover-fermion action

$$a \simeq 0.12, 0.09\,\mathrm{fm}$$
$$m_\pi \simeq 400, 440\,\mathrm{MeV}$$

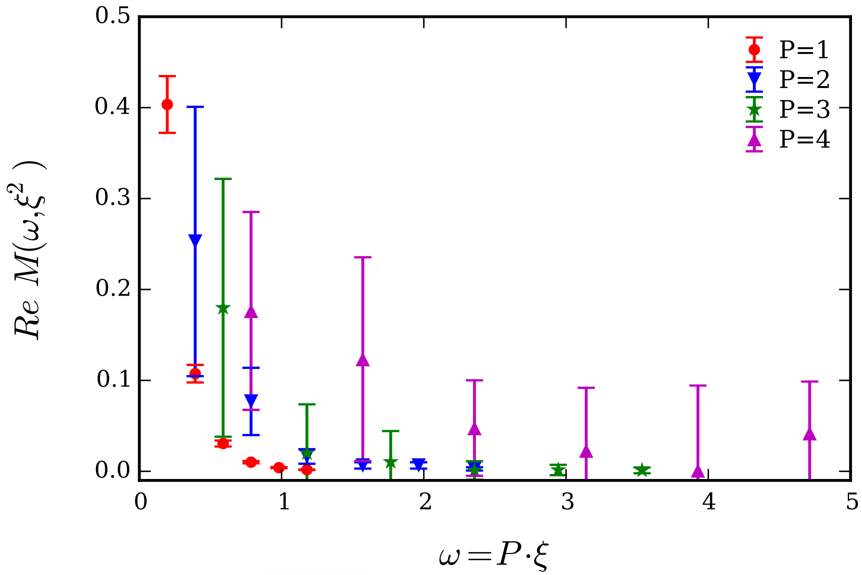


$$J_1 = V, J_2 = A$$

- 110 configurations
- Single source point for current J_1

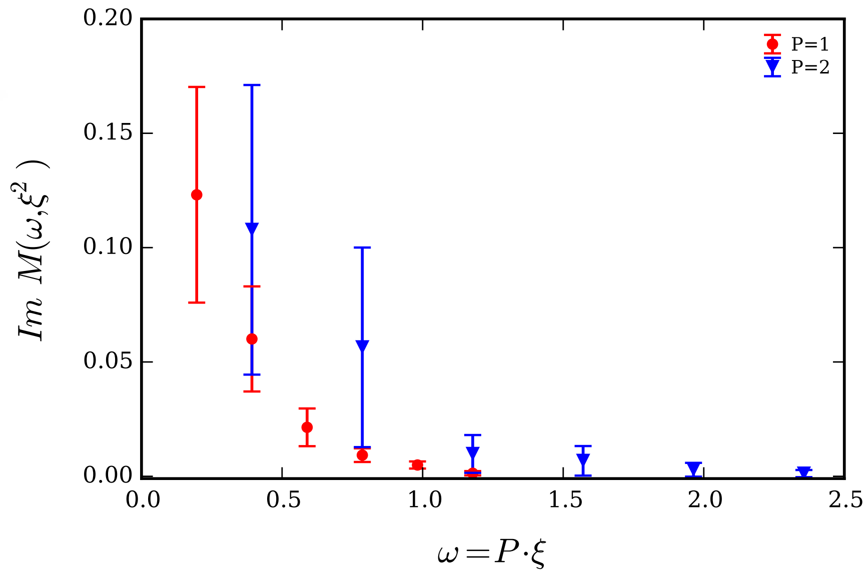
Clear isolation of pion matrix element

Preliminary Results - II



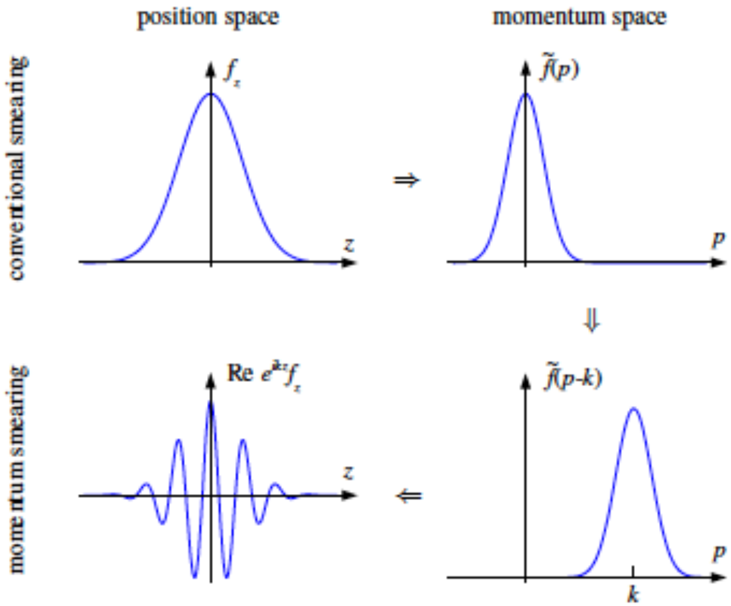
Clear signal in real part to p around 1 GeV

Imaginary part only to small values of p



Challenges/Questions

High spatial momentum and lattice systematics



Boosted interpolating operators

Bali *et al.*, Phys. Rev. D 93, 094515 (2016)

Inverse Problem - *common to all approaches*

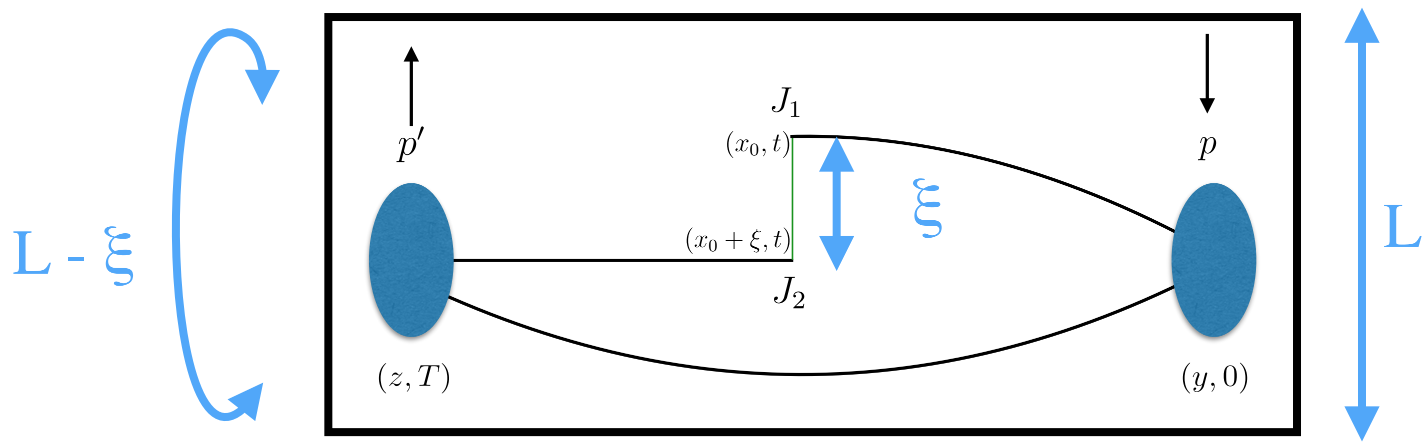
$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

Extract PDF?

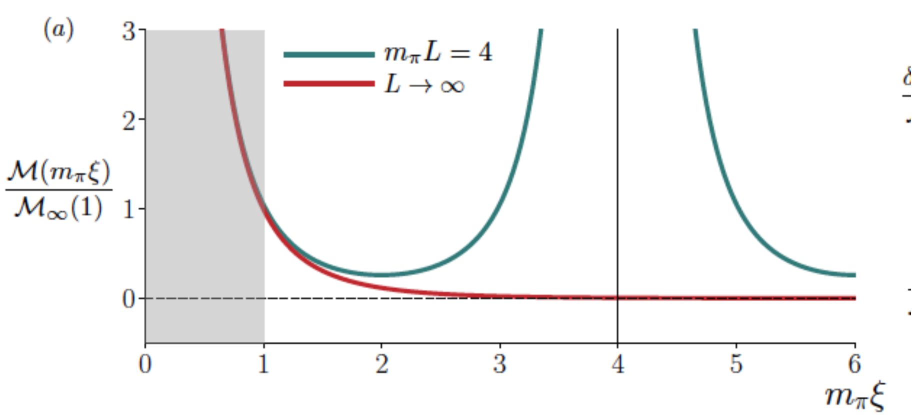
Calculate on Lattice

Calculate in PQCD

Finite Volume Effects



Briceno, Guerrero, Hansen and Monahan, arXiv:1805.01304



Typically $m_\pi L \simeq 4$

Future? $\begin{cases} \xi \text{ shortdistance} \\ m_\pi \to m_\pi^{\text{phys}} \end{cases}$

Summary

- Calculation of current-current correlators for pion *and kaon* in progress for variety of local operators
- Important to understand finite-volume effects
- Extending calculation to close-to-physical

$$m_\pi \simeq 170 \text{ MeV}$$
$$64^3 \times 128 \text{ Lattices}$$

- Application to nucleon
  – No straight-forward application of "sequential-source" method
  – Alternative approaches in progress
- Variety of lattice cross sections - including pseudo PDFs - on same ensemble of lattices.