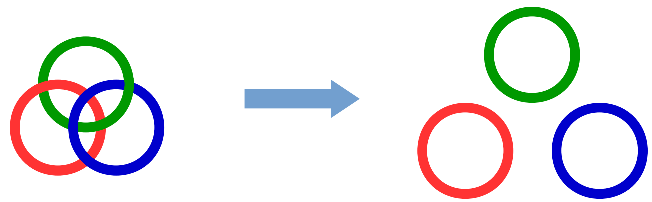
Color unwound: Glauber gluons and factorization in Drell-Yan azimuthal asymmetries



Jonathan Gaunt (CERN)

Based on SciPost Phys. 3, 040 (2017)
(Boer, van Daal, JG, Kasemets, Mulders)

QCD Evolution 2018, Santa Fe, NM, USA
21st May 2018

Factorisation Formulae

Factorisation formulae are essential to make predictions at colliders involving p/A.

Separate out short distance interaction of interest from long-distance QCD-dominated interactions. Low-momentum part of long-distance piece will not be calculable perturbatively, but is (hopefully) universal

Examples of factorisation formulae for pp collisions:

Collinear factorisation for pp → V + X **inclusive total** cross section, V **colourless**

PDFs (long distance physics, universal)

$$\sigma = \int dx_A dx_B \hat{\sigma}_{ij\to X}(\hat{s} = x_A x_B s) f_i(x_A) f_j(x_B) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^4}\right)$$

Parton-level cross section/coefficient function (short distance physics)

Corrections suppressed by Λ^2/Q^2

Factorisation Formulae

TMD factorisation for pp → V + X cross section **differential in p_T**, $p_T \ll Q$, **V colourless**

Hadronic tensor

Approximated momenta (transverse momenta → 0)

Rapidity regulator

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f C_f^{\mu\nu}\left(\hat{k}_A, \hat{k}_B\right) \int d^2\mathbf{b}_T e^{i\mathbf{p}_T\cdot\mathbf{b}_T} \tilde{f}_f\left(x_A, \mathbf{b}_T; \zeta_A\right) \tilde{f}_{\bar{f}}\left(x_B, \mathbf{b}_T; \zeta_B\right) + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$$

Transverse momentum dependent PDFs (long distance physics)

Coefficient function (short distance physics)

Corrections suppressed by p_T^2/Q^2 (can be augmented to Λ^2/Q^2 by adding matching to fixed order)

Both formula proved to leading power by Collins, Soper and Sterman

Bodwin Phys. Rev. 31 (1985) 2616, Collins, Soper, Sterman Nucl. Phys. B261 (1985) 104, Nucl. Phys. B308 (1988) 833, Collins, pQCD book
See also Diehl, JG, Ostermeier, Plößl, Schäfer, JHEP 1601 (2016) 076

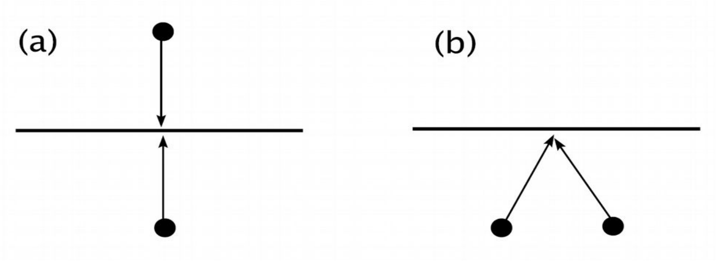
CSS Factorisation Analysis

How do we establish a leading power factorisation for a given observable?

I will review here the original **Collins-Soper-Sterman (CSS) method**

Want to identify IR leading power regions of Feynman graphs – i.e. small regions around the points at which certain particles go on shell, which despite being small are leading due to propagator denominators blowing up.

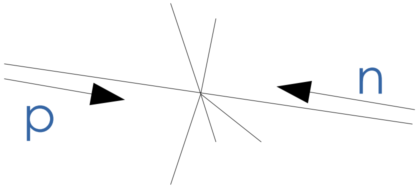
More precisely, need to find regions around **pinch singularities** – these are points where propagator denominators pinch the contour of the Feynman integral.



Pinched Non-pinched

Momentum regions

Once one has determined **where** the singularities are, need to determine their **strength**. Supplement pinch finding with a power counting analysis to determine if region around singularity gives a leading contribution, and what the **shape** of this region is.



In general, relevant regions in QCD are:

1) **Hard region** – momentum with **large virtuality** (order Q)

p n T

$$k \sim Q\,(1, 1, 1)$$

2) **Collinear region** – momentum **close to some beam/jet direction**

$$k \sim Q\,(1, \lambda^2, \lambda)$$ (for example)

3) **(Central) soft region** – all momentum components **small and of same order**

$$k \sim Q\,(\lambda^n, \lambda^n, \lambda^n)$$

$$\lambda \ll 1$$

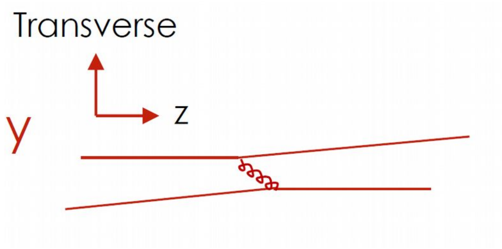
AND..

Momentum regions

4) **Glauber region**

Loosely speaking, the Glauber momentum region is characterised by **all components being small**, but the **transverse components being much larger than the longitudinal ones.**

Glauber exchanges mediate **low-angle 'forward' scattering.**



Technical definition: Momentum *k* of a Glauber particle satisfies:

$$|k^+ k^-| \ll \mathbf{k}_T^2 \ll Q^2$$

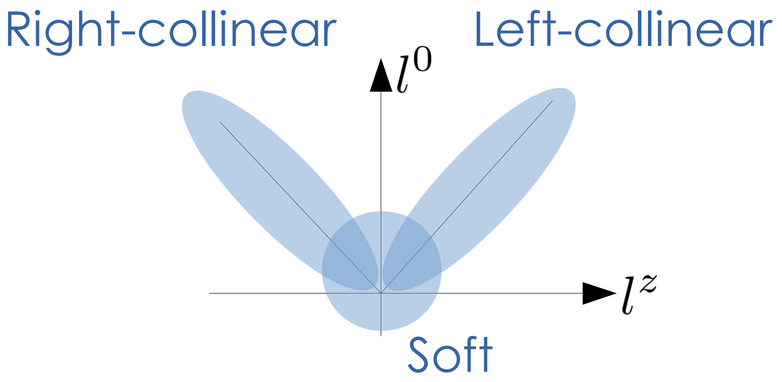
Canonical example: $k \sim Q\left(\lambda^2, \lambda^2, \lambda\right)$

Subtractions

For each region **approximations** are applied that are valid to leading power in that region: $\Gamma \to T_R \Gamma$

In the computation of each region contribution, one actually **integrates over all values of the loop momenta** rather than just in the region of interest (to avoid hard cutoffs etc.)

If one then just simply added up the region contributions there would be very significant **overcounting** (and many of the overcounted contributions would be **wrong**)



Subtractions

To avoid this one implements **subtraction terms** for smaller regions in each region:

$$C_R\Gamma \equiv T_R\Gamma - \sum_{R'<R} T_R C_{R'}\Gamma$$

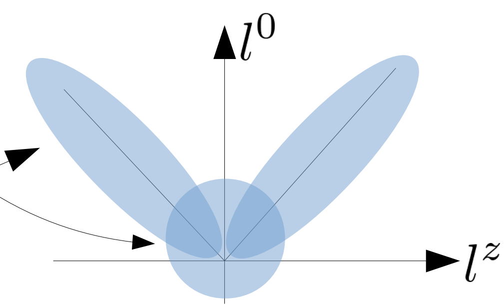
Collins, pQCD book

'Naive graph term' 'Subtraction terms'

Example:

$$C_S\Gamma = T_S\Gamma$$

$$C_C\Gamma = T_C\Gamma - C_S\Gamma = T_C\Gamma - T_S\Gamma$$

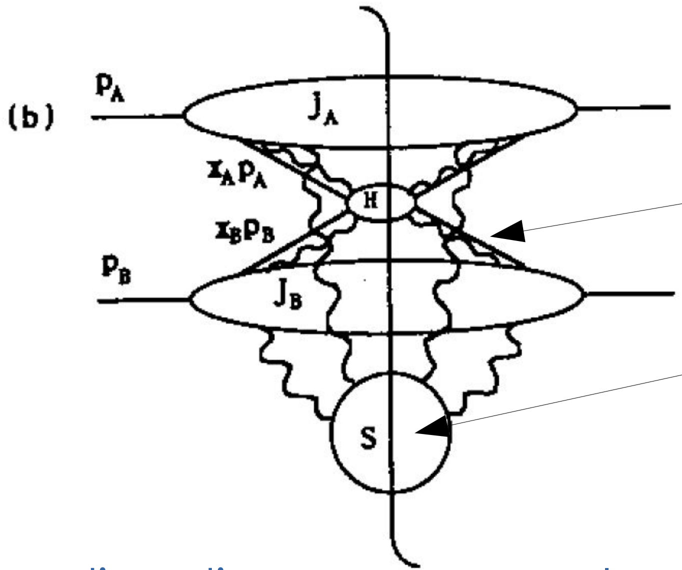


Then: $\Gamma = \sum_R C_R\Gamma$ +power suppressed corrections

In their factorisation proof, **CSS did not treat the S and G regions distinctly** – common approximation for both!

Obtaining a factorisation formula

Leading region for pp → V +X:

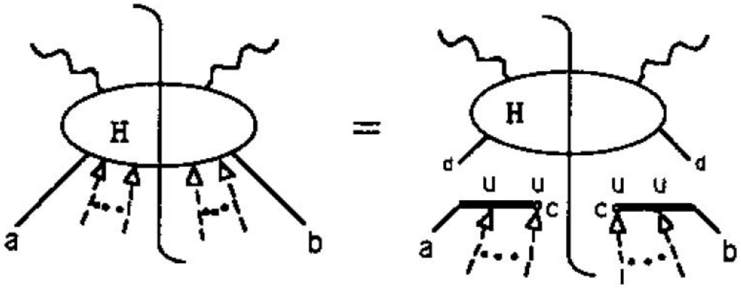


One physically polarised parton + multiple scalar polarised gluons

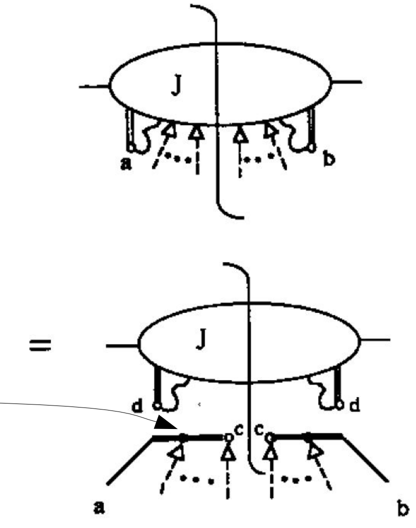
Soft + Glauber particles

Already with this leading diagram we are close to some kind of a factorisation formula, but must separate different pieces – **too many connections** between H, J , S at the moment!

Collinear scalar polarised gluons can be stripped from hard by using **Ward identities**, physically polarised parton detached using some projector.



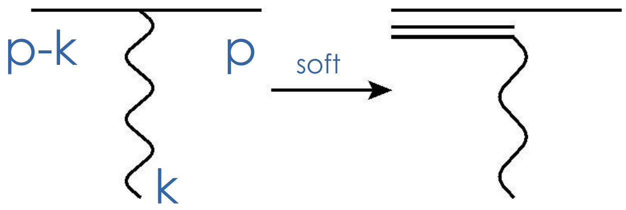
Obtaining a factorisation formula



If blob S only contained **central soft**, then we could strip soft attachments to collinear J blobs using **Ward identities** too.

Wilson line in direction of J

Simple example:



Propagator denominator:

$$(p-k)^2 = -2p\cdot k + k^2 \overset{\text{soft}}{\rightarrow} -2p\cdot k$$

Eikonal piece

Glauber Gluons and Factorisation

The manipulation used for soft gluons is **NOT POSSIBLE** for Glauber gluons

Propagator denominator:

$$(p-k)^2 = -2p \cdot k + k^2 \not\to -2p \cdot k$$

Two terms in denominator are of same order in Glauber region

How do we get around this problem?

CSS proof of Glauber cancellation

CSS showed that, for total and TMD colour singlet cross section, **loop integration contour can always be deformed into complex plane away from Glauber regions into central soft and/or collinear regions.**

This occurs only after one sums over possible final states, or more precisely after one **sums over possible cut s of each graph**.

Then can apply approximations that allow Ward identities and obtain factorisation.

Deformation is consistent with the formation of an initial-state Wilson line. **The final-state poles are removed after the sum over cuts.**

Fundamental principle underying cancellation is **unitarity** – loosely speaking, as long as the observable is insensitive to the effects of `final-state' interactions,the sum over all such possible interactions gives unity, and the associated final-state poles disappear.

Azimuthal asymmetries in Drell-Yan

This proof is supposed to extend in a straightforward way to spin-dependent observables
– e.g. TMD cross-section for DY with full dependence on angles of produced leptons, where factorised prediction is:

Boer, Brodsky, Huang, Phys. Rev. D67 (2003) 054003,
Boer, Phys. Rev. D60 (1999) 014012

$$\frac{d^6\sigma}{d\Omega\, dx_1 dx_2\, d^2\boldsymbol{q}} = \frac{\alpha^2}{N_c\, q^2} \sum_q e_q^2 \left\{ A(\theta)\, \mathcal{F}\left[f_1 \bar{f}_1\right] + B(\theta)\, \cos(2\phi)\, \mathcal{F}\left[w(\boldsymbol{k}_1, \boldsymbol{k_2})\, h_1^\perp \bar{h}_1^\perp\right] \right\},$$

Unpolarised TMD

Collins-Soper angles

Boer-Mulders TMD (measures correlation between quark transverse spin and transverse mtm)

$$\mathcal{F}\left[f_1 \bar{f}_1\right] \equiv \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, \delta^{(2)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{q})\, f_{1,q}(x_1, \boldsymbol{k}_1^2)\, \bar{f}_{1,q}(x_2, \boldsymbol{k}_2^2)$$

Note that the colour structure for the unpolarised and double BM piece is the same!

Colour entanglement?

However, in PRL 112 (2014), 092002 (Buffing, Mulders), it was suggested that there was **colour entanglement** in the φ dependent piece: factorised prediction **not correct for this.**

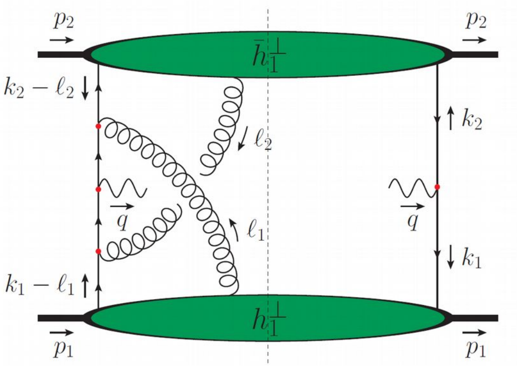
(also for double Sivers effect)

$$
\begin{aligned}
d\sigma_{\mathrm{DY}} &= \mathrm{Tr}_c \Big[U_-^\dagger[p_2]\Phi(x_1, p_{1T})U_-[p_2]\Gamma^* \\
&\quad \times U_-^\dagger[p_1]\overline{\Phi}(x_2, p_{2T})U_-[p_1]\Gamma \Big] \\
&\neq \frac{1}{N_c}\Phi^{[-]}(x_1, p_{1T})\Gamma^*\overline{\Phi}^{[-^\dagger]}(x_2, p_{2T})\Gamma,
\end{aligned}
$$

'Factorisation-breaking' effects start with this 'crossed gluon' diagram. Prediction that there is an extra colour factor of

$$-1/(N_C^2 - 1)$$

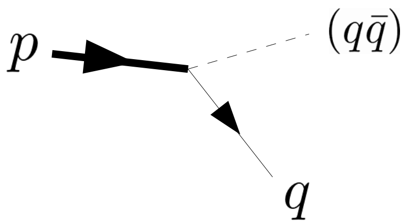
(change in sign!)



This would have important implications for experimental measurements! Also would indicate a loophole in the general CSS factorisation proof. Important to verify if it exists or not.

Model calculation

Want to do an **full explicit calculation** → use a model for the proton. In our model, each hadron is a massive spin ½ particle than can split into a massless spin ½ quark and a massive scalar 'diquark' via a Yukawa-type interaction.



Vector boson V produced via $q\bar{q}$ fusion.

Colour entanglement effect first appears at two gluon exchange level - compute **all diagrams** up to the two-gluon exchange level.

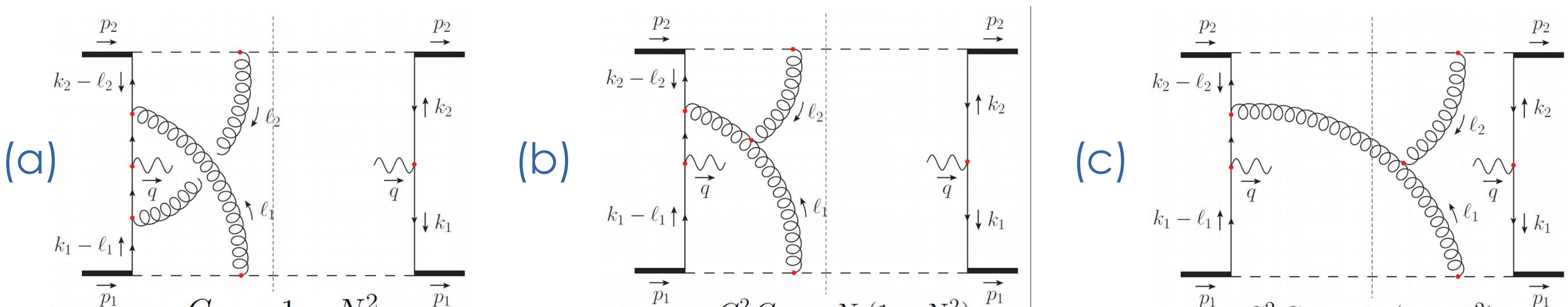
Couplings of gluons to quarks and scalars are given by **standard** (fermion or scalar) **QCD Feynman rules**

In this way we ensure the model obeys **key physical principles** also obeyed by QCD, such as **unitarity**.

Diagrams

Straightforward to show that parton-level and one-gluon exchange diagrams do not give any contribution to the ϕ dependent piece (as predicted by factorisation) – here focus on two-gluon exchange diagrams

Important (colour entangled) diagrams for ϕ dependence at two-gluon exchange level:



(a) $$-\frac{C_F}{2} = \frac{1 - N_c^2}{4N_c}$$

(b) $$-\frac{C_A^2 C_F}{2} = \frac{N_c(1 - N_c^2)}{4}$$

(c) $$-\frac{C_A^2 C_F}{2} = \frac{N_c(1 - N_c^2)}{4}$$

Remaining diagrams either give no leading-power φ dependence straight away, or cancel straightforwardly after the sum over cuts.

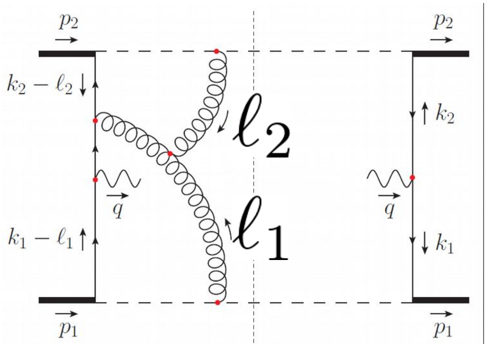
Regions

Diagrams (a)-(c) have four loops – impossible to fully evaluate directly (at present).

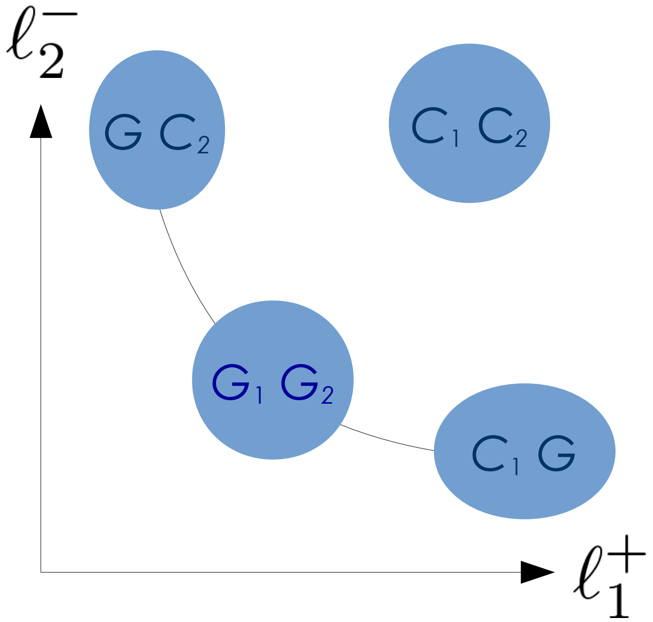
Instead we split calculation of each graph into leading momentum regions. Apply **approximations** valid for those regions, and **subtractions** for smaller regions, a la CSS.

Unlike CSS, we will **consider explicitly the Glauber region on its own,** with an associated approximation. We will also **not do any contour deformations** – just directly compute contributions from regions.

Regions



For (a) – (c), **four important momentum regions**:



$G : (\lambda^2, \lambda^2, \lambda)Q$ Central Glauber

$G_1 : (\lambda, \lambda^2, \lambda)Q$ Left-moving Glauber

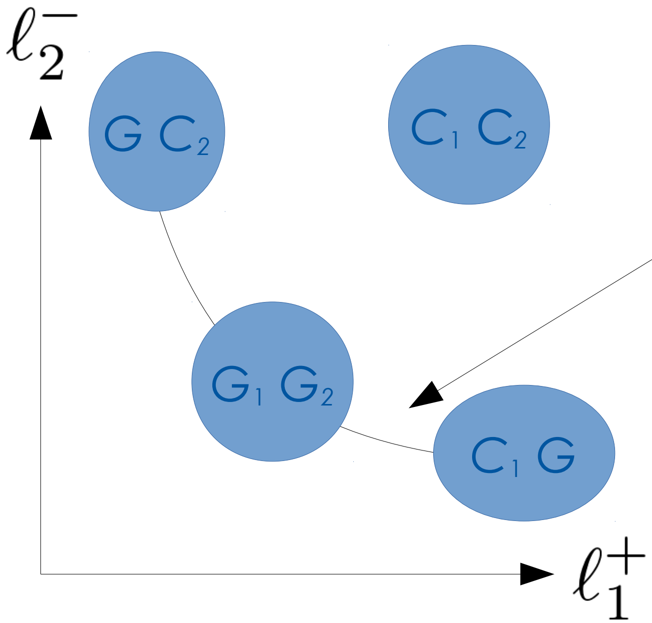
$G_2 : (\lambda^2, \lambda, \lambda)Q$ Right-moving Glauber

$C_1 : (1, \lambda^2, \lambda)Q$ Left-moving collinear

$C_2 : (\lambda^2, 1, \lambda)Q$ Right-moving collinear

Other leading-power scalings possible but these can be trivially 'absorbed into regions above'

Rapidity regulator



We have regions of the same virtuality smoothly connected in rapidity – leads to **rapidity divergences.**

Must insert a **rapidity regulator** into the computation of each region to get a well-defined result. The form we use is inspired by the 'CMU' or 'η' regulator $|\ell_1^+/\nu|^{-\eta_1}|\ell_2^-/\nu|^{-\eta_2}$

Chiu, Jain, Neill, Rothstein, Phys.Rev.Lett. 108 (2012) 151601, JHEP 1205 (2012) 084

Actually, must be careful with relative sizes of η_1, η_2 to ensure well-defined results. Different regulators for (a-c) – this is OK, as full graphs do not have rapidity divergences

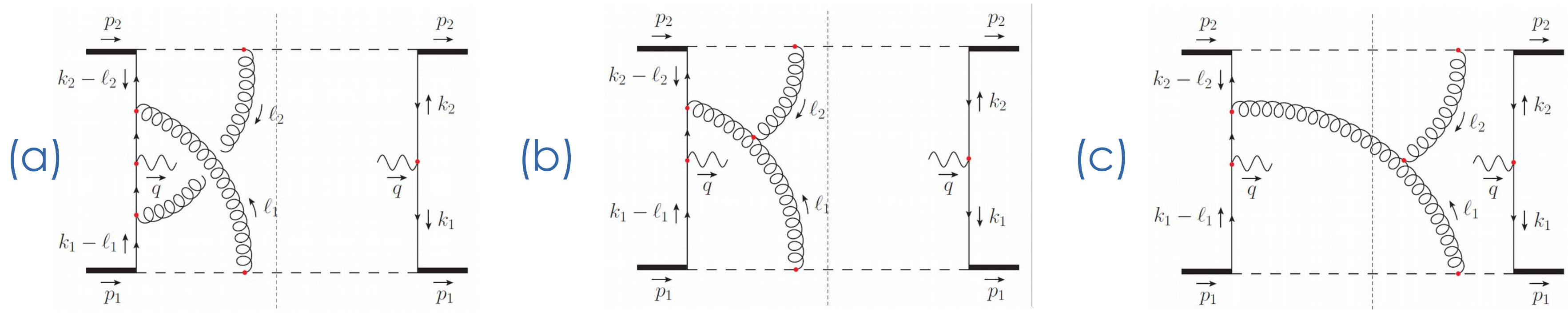
(a) $\frac{1}{2}\left(\left|\frac{\ell_1^+}{\nu}\right|^{-\eta_1}\left|\frac{\ell_2^-}{\nu}\right|^{-\eta_2}+\left|\frac{\ell_1^+}{\nu}\right|^{-\eta_{\bar{1}}}\left|\frac{\ell_2^-}{\nu}\right|^{-\eta_{\bar{2}}}\right)$

(b-c) $\left|\frac{\ell_1^+}{\nu}\right|^{-\eta_1}\left|\frac{\ell_2^-}{\nu}\right|^{-\eta_2}$

$$\eta_1 \gg \eta_2, \eta_{\bar{1}} \ll \eta_{\bar{2}}$$

Computation of the diagrams

Let's look in detail at G_1G_2 region calculation



$$d\sigma_{\mathrm{BM}} \propto C_{(a)}(I_{(a)} - N_C^2(I_{(b)} + I_{(c)}))$$

Where $I_{(n)}$ are the integrals over larger (λ scaling) lightcone components :

$$I_{(\mathrm{a})} \equiv \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{\nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{\ell_2^- + i\epsilon}.$$

Just initial-state poles in ℓ_1^+, ℓ_2^-

$$I_{(\mathrm{b})} \equiv \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{2\ell_1^+\nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2 + i\epsilon}.$$

Initial- and final-state poles

$$I_{(\mathrm{c})} \equiv 4\pi i \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi}\, \theta(-\ell_1^+)\, \ell_1^+\, \delta[2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2]\, \nu^{\eta_2}|\ell_2^-|^{-\eta_2}.$$

Computation of the diagrams

Now:

$$I_{(\mathrm{b})} \equiv \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{2\ell_1^+ \nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2 + i\epsilon}.$$

$$I_{(\mathrm{c})} \equiv 4\pi i \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \theta(-\ell_1^+)\, \ell_1^+\, \delta[2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2]\, \nu^{\eta_2}|\ell_2^-|^{-\eta_2}.$$

Final-state poles cancel **(ensured by unitarity!)**

$$\int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{\nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{\ell_2^- + i\epsilon} = I_{(a)}$$

$$d\sigma_{\mathrm{BM}} \propto C_{(a)}(1 - N_C^2) I_{(a)}$$

Restores colour factor predicted by factorisation! **(ensured by Ward identities!)**

Computation of the diagrams

We also showed that the colour entangled parts cancel region-by-region for the C_1G, GC_2 and C_1C_2 regions.

In these cases we could also see that the cancellation of the colour entanglement was being driven, behind the scenes, by **unitarity cancellations and Ward identities**.

Contribution from C_1G, GC_2 and C_1C_2 regions actually ends up being zero, and **full contribution to ϕ dependent piece comes from G_1G_2 region** – agrees with factorisation formula.

Region-by-region cancellation of colour entanglement is not guaranteed and depends on rapidity regulators used (although final result does not). If you use: $|\ell_1^+/\nu|^{-\eta_1}|\ell_2^-/\nu|^{-\eta_2}$ for (a)

$|(\ell_1^+ - \ell_2^-)/\nu|^{-\eta}$ for (b), (c)

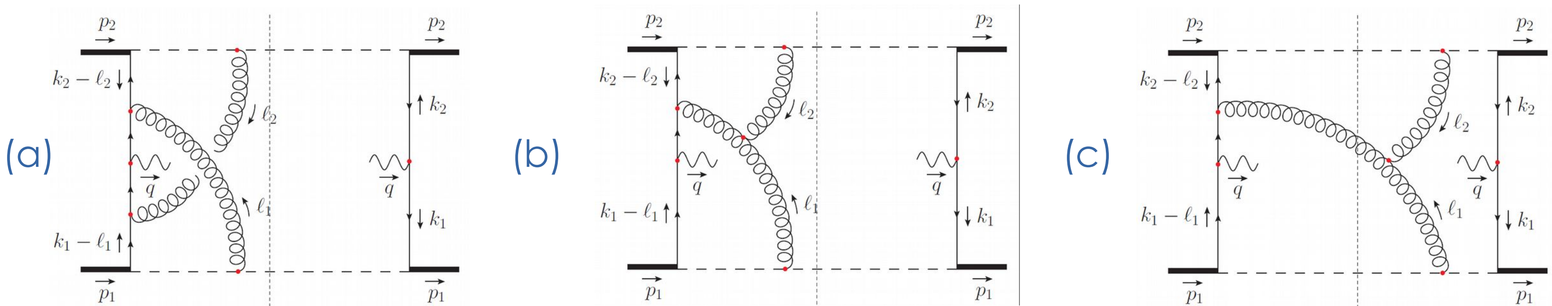
Then (b)+(c)=0, (a)≠0 for G_1G_2 and there is colour entanglement in this region. Colour entanglement must cancel between regions!

Other rapidity regulators

Another possibility for the rapidity regulator is a **theta function regulator**: $\theta\left[\min(k_1^+, p_1^+ - k_1^+) - |\ell_1^+|\right]$, $\theta\left[\min(k_2^-, p_2^- - k_2^-) - |\ell_2^-|\right]$

Then for $G_1 G_2$:

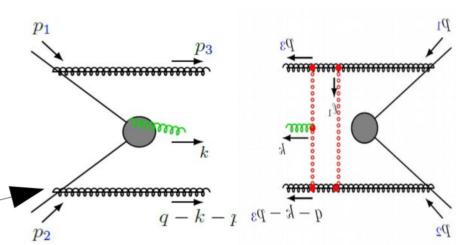
Most 'physical' regulator



REAL ◄— cancel —► REAL + IMAGINARY ◄— cancel —► IMAGINARY

But also cancel against c.c.

(a) and (b) **have same final state** – no factorisation breaking in $G_1 G_2$ for any observable (even e.g. **beam thrust**). **Should appear at next order** with the inclusion of another Glauber exchange. See also Schwartz, Yan, Zhu, 1801.01138



Should compute all diagrams and regions to make a definitive statement about factorisation (violation) at given order.

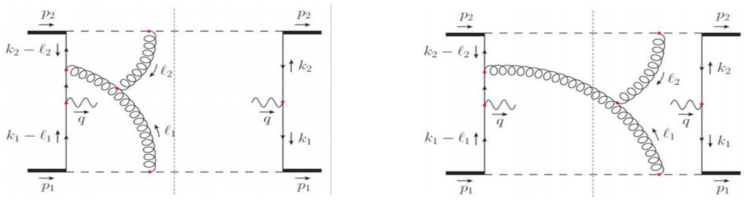
Glauber physics in a wider context

Our calculation is of interest outside the scope of spin physics as it **illustrates how the Glauber cancellation mechanism works explicitly at the two loop level**, along with extending CSS technique to study Glauber exchanges

– should be useful when studying Glauber exchanges and factoristation (violation) in **more complex scenarios**.

Also highlights **some points where the CSS proof could be improved.** CSS proof is not prescriptive about which Glauber momenta can be deformed into soft, and which into collinear. Naively one might think all can be deformed into soft, but our calculation provides explicit examples where this is not the case.

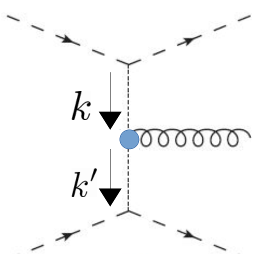
Glauber physics in a wider context



e.g. even summing (b) and (c), ℓ_1^-, ℓ_2^+ **are still trapped at values of order λ^2**

$$I_{(\mathrm{b})} \equiv \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{2\ell_1^+ \nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{2\ell_1^+\ell_2^- - (\ell_1+\ell_2)^2 + i\epsilon}.$$

Culprit seems to be this numerator factor. Part of the Lipatov vertex



(b)

$$\mathcal{A}_{\mathrm{L}} = -2\,g^2 \frac{1}{\vec{k}_\perp^2}\frac{1}{\vec{k}_\perp'^2} \bar{\xi}_n T^a \frac{\bar{\not{n}}}{2}\xi_n \bar{\xi}_{\bar{n}} T^b \frac{\not{n}}{2}\xi_{\bar{n}} \times (igf^{abc})\left(k_\perp^\alpha + k_\perp'^\alpha - \frac{1}{2}\bar{n}^\alpha n\cdot k' - \frac{1}{2}n^\alpha \bar{n}\cdot k - \frac{n^\alpha}{n\cdot k'}\vec{k}_\perp^2 - \frac{\bar{n}^\alpha}{\bar{n}\cdot k}\vec{k}_\perp'^2\right)$$

Taken from Fleming, Phys.Lett. B735 (2014) 266-271

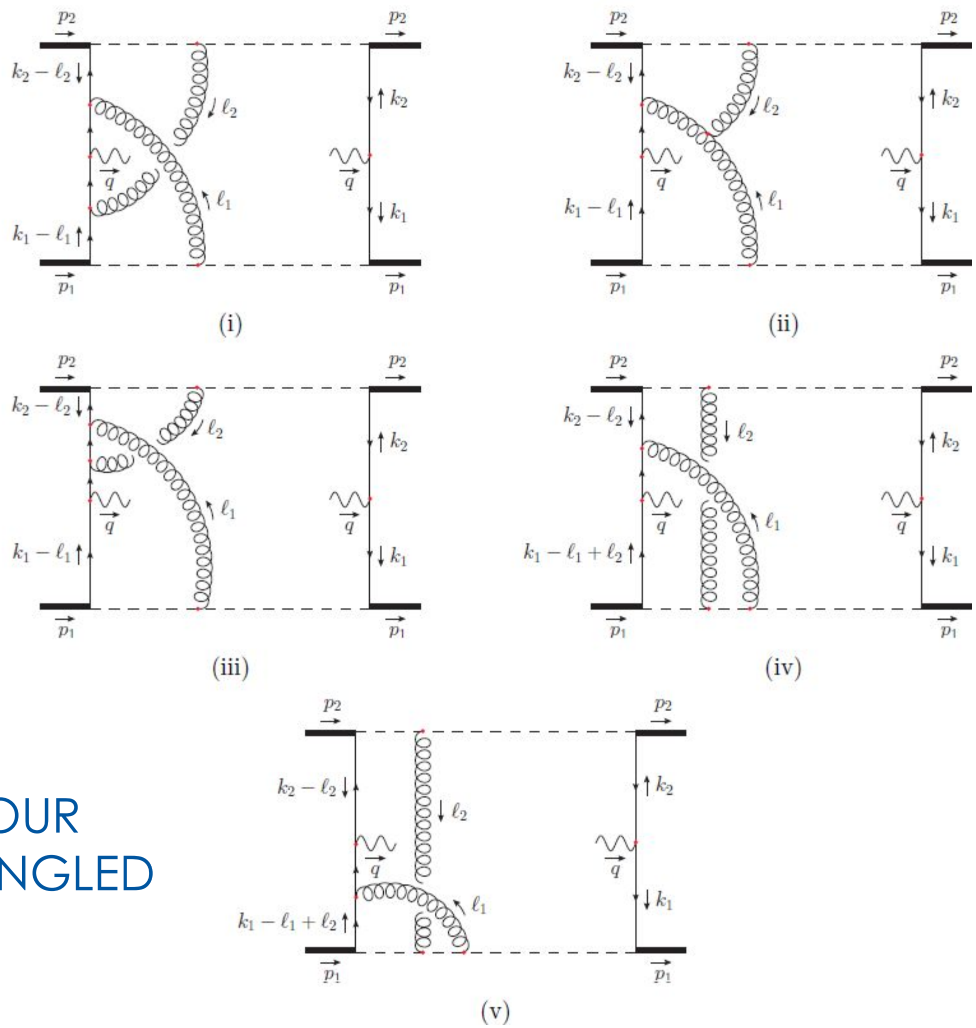
N.B. **CSS Glauber cancellation proof does not consider numerator explicitly**

Summary

- We performed an **explicit model calculation** at the **two gluon exchange level**, which indicates that **colour is not entangled for the spin-dependent TMD Drell-Yan cross section,** and that this obeys factorisation.

- Computation involves **extension of the CSS region+subtraction technique to explicitly consider Glauber region** – may be useful in further studies.

All non-zero diagrams

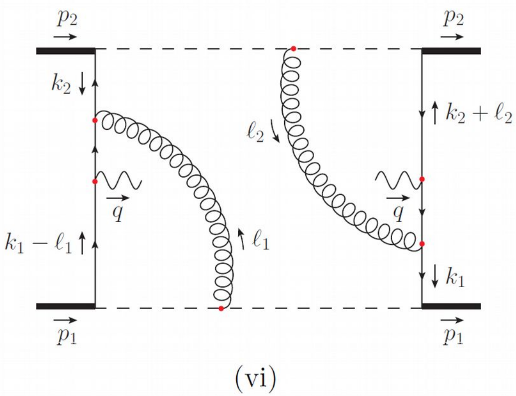


+ seagulls
+ h.c.
+ proton↔antiproton

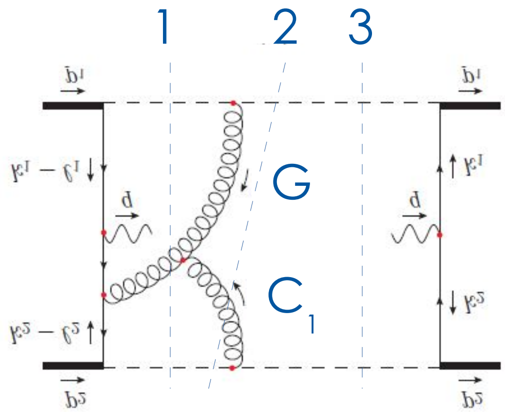
+ NON COLOUR ENTANGLED

COLOUR ENTANGLED

e.g.



C_1G calculation



Write numerator factor as

$$A(\ell_1^+, k_1, k_2, T) \cdot (\ell_1 + \ell_2)^2 + B(\ell_1^+, \ell_1^-, k_1, k_2, T),$$

Only cuts 2,3 possible, don't cancel. Have to add similar contributions from other diagrams

Cuts 1,2,3 cancel (unitarity cancellation) Similar to one-Glauber exchange