Polarized TMDs, twist-3 functions, and the CSS formalism

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supported by

TMD Topical Collaboration

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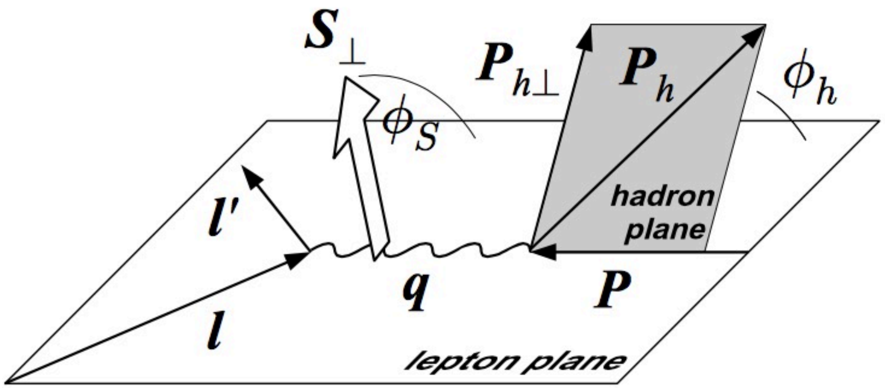
Outline

- Background
  - Transverse single-spin asymmetries (TSSAs)
  - TMD and collinear twist-3 (CT3) functions
- TMD and CT3 observables
  - Sivers and Collins effects
  - A_N in $pp \rightarrow \{\gamma\ ,\pi\}\ X$
- Relations between TMD and CT3 functions
  - Using CSS operators
  - Physical interpretation using "naïve" operators
- Summary

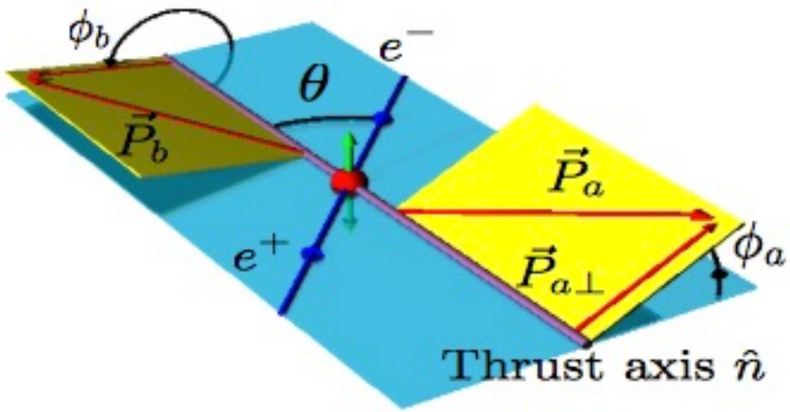
Background

$e\,N \to e'\,h\,X$



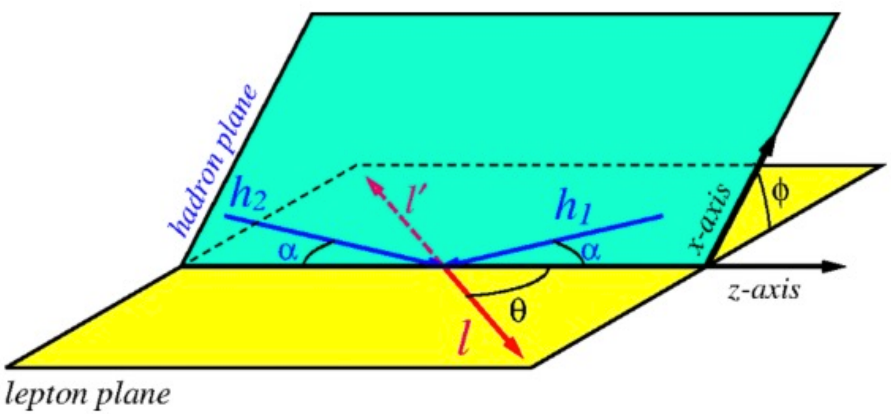
Sivers ~ *sin(*$\phi_h - \phi_s$*)*, Collins ~ *sin(*$\phi_h + \phi_s$*)*, ...

$e^+e^- \to h_a h_b X$



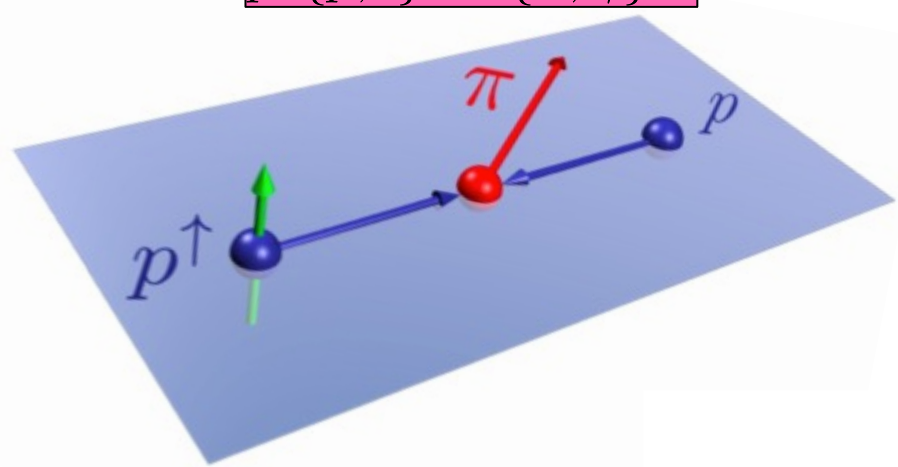
Collins ~ *cos(*$\phi_a + \phi_b$*)*, ...

$p^\uparrow\{p,\pi\} \to \{l^+l^-, W/Z\}X$



Sivers ~ *sin(*ϕ_s*)* (lepton pair) / Sivers ~ *cos(*$\phi_{W/Z}$*)* (boson)

$p^\uparrow\{p,l\} \to \{\pi,\gamma\}X$



$A_N \sim d\sigma_L - d\sigma_R$

TMD PDFs (x, k_T)

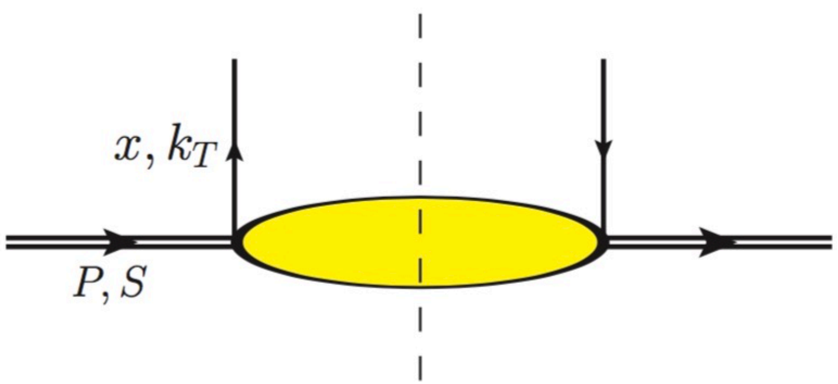
q pol. / H pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

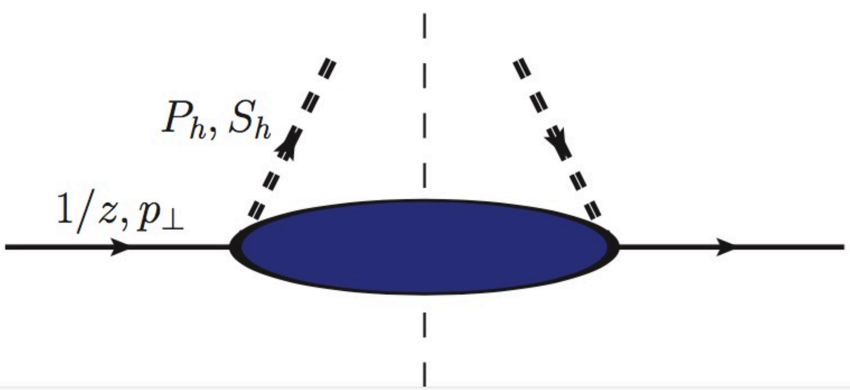
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

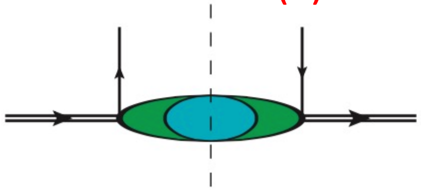
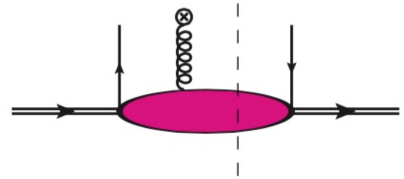
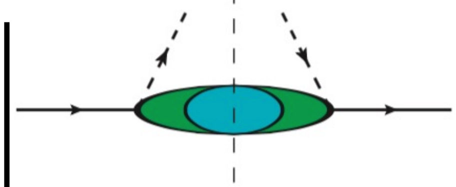
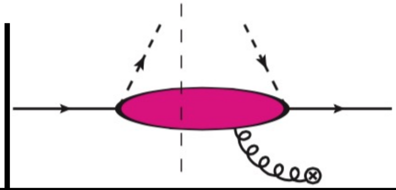
TMD FFs $(z, p_\perp)$

q pol. / H pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))



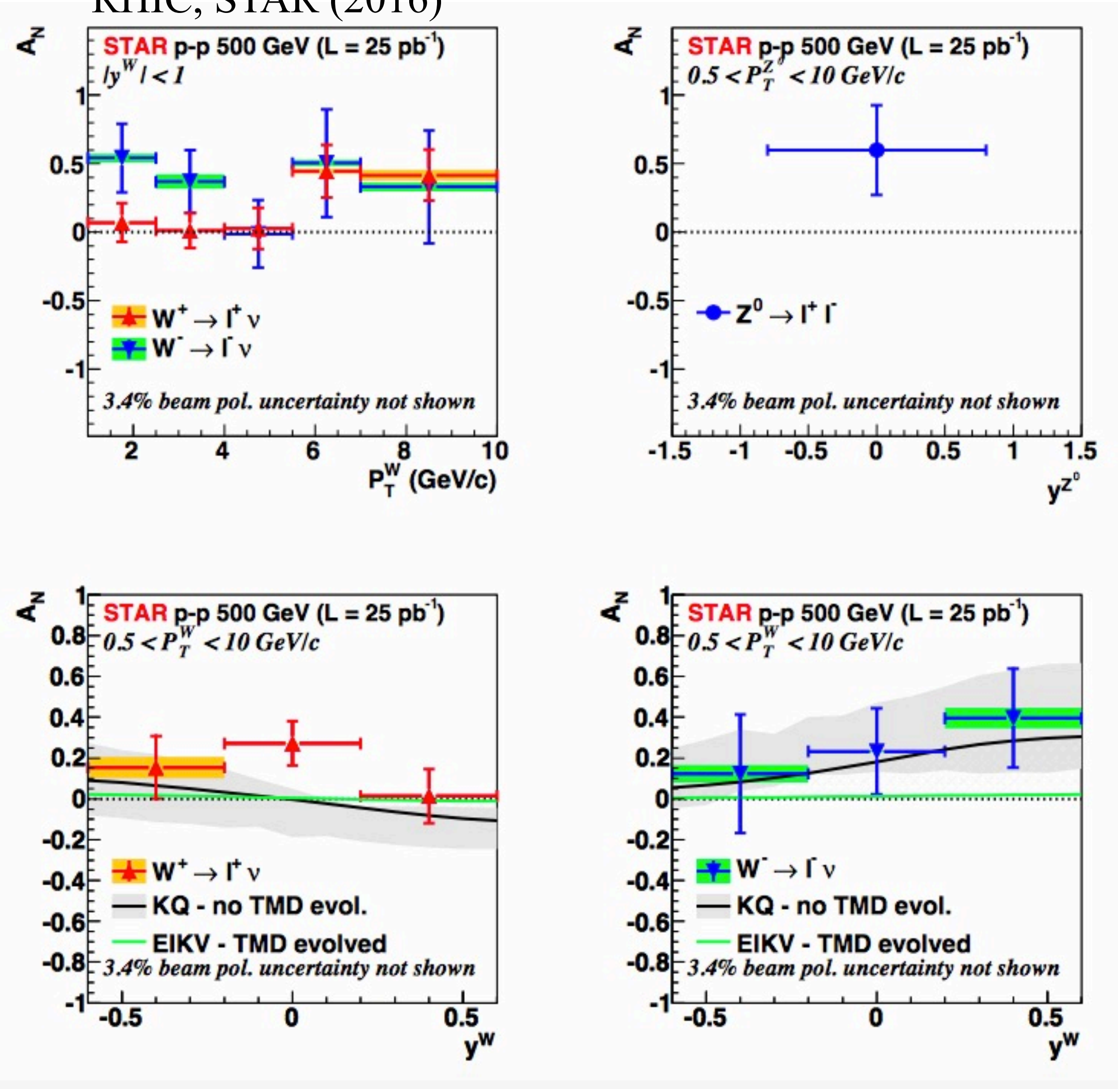


Hadron Pol.	CT3 PDF (x)		CT3 PDF (x, x_1)	CT3 FF (z)		CT3 FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

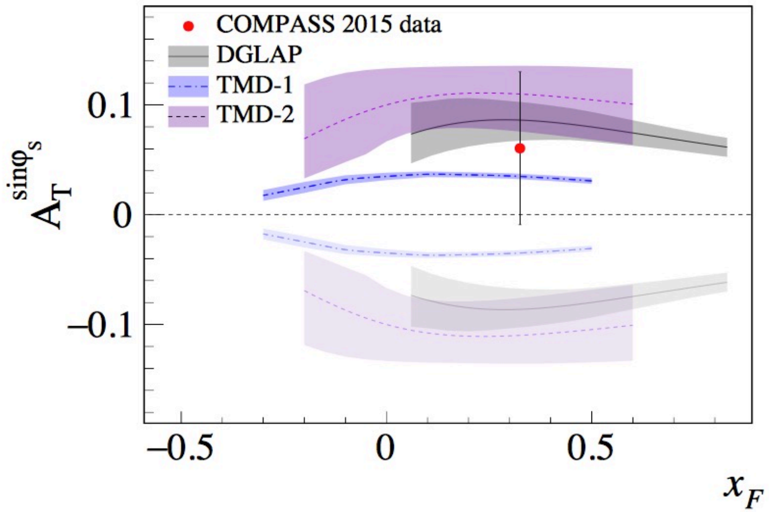
TMD and CT3 Observables

Drell-Yan Sivers effect

RHIC, STAR (2016)

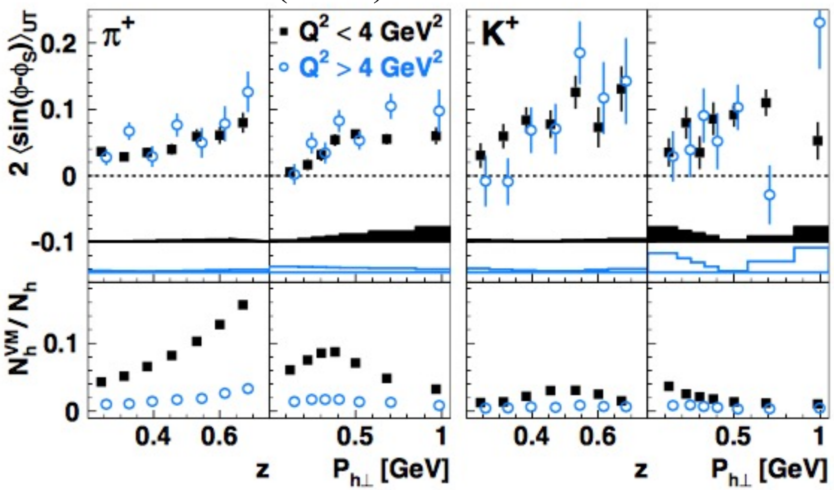


COMPASS (2017)

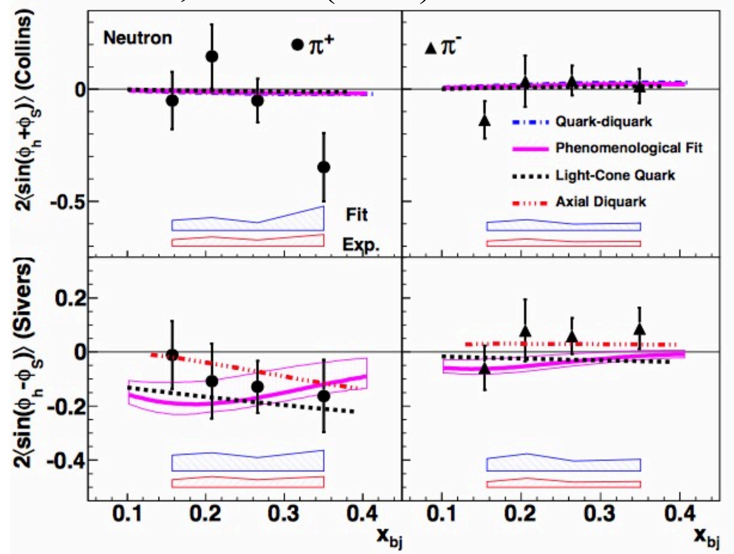


SIDIS Sivers effect ($sin(\phi_h - \phi_s)$)

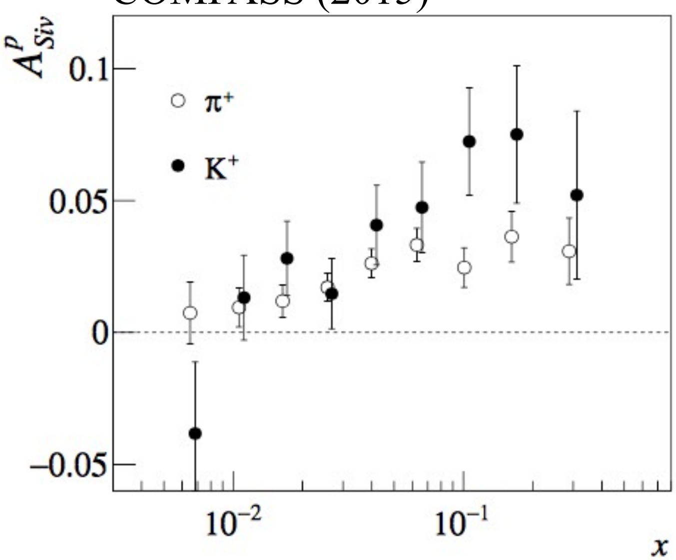
HERMES (2009)



JLab, Hall A (2011)

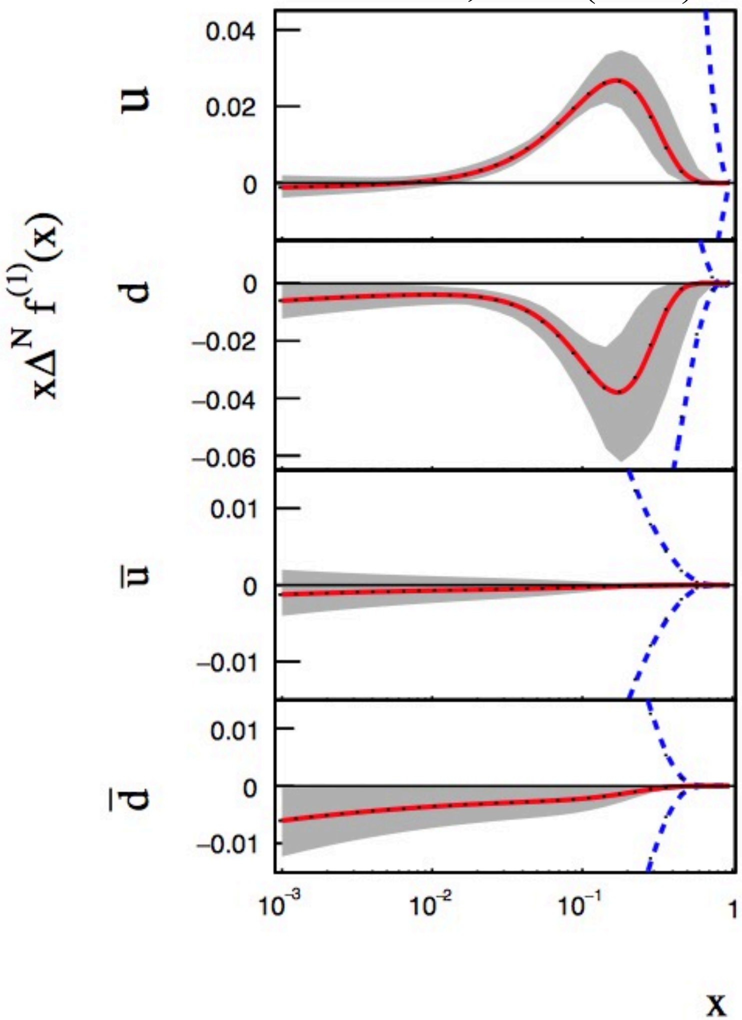


COMPASS (2015)

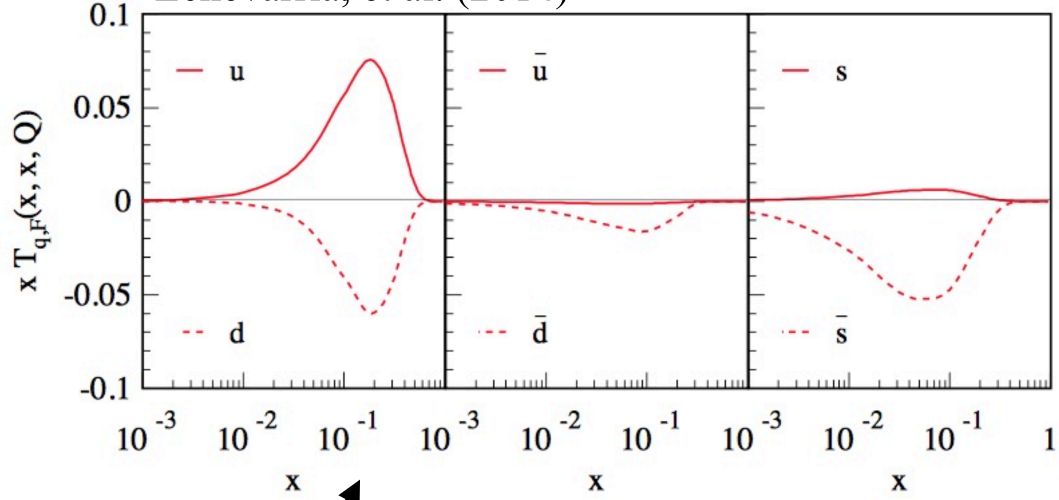


$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M}\, f_{1T}^{\perp} D_1\right]$$

Anselmino, et al. (2017)



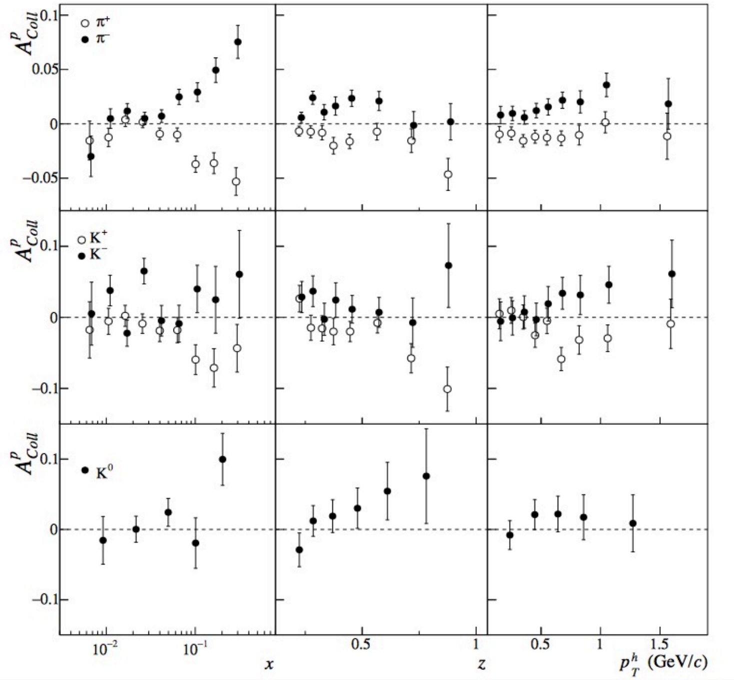
Echevarria, et al. (2014)



TMDs in CSS formalism

SIDIS Collins effect ($\sin(\phi_h + \phi_s)$)

COMPASS (2015)

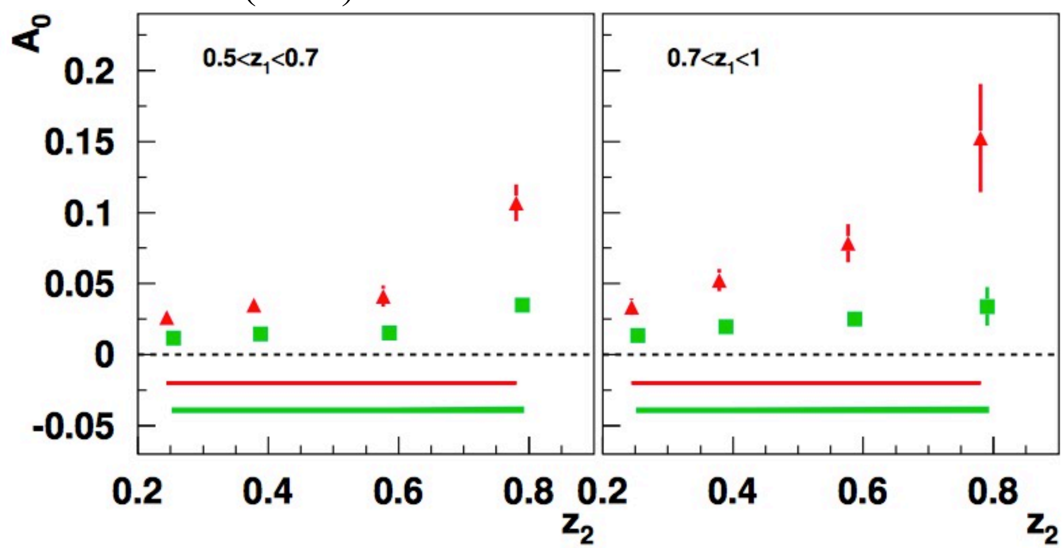


Also data from JLab Hall A (2011, 2014) and HERMES

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\,\boldsymbol{H_1^\perp}\right]$$

e^+e^- Collins effect ($\cos(2\phi_0)$)

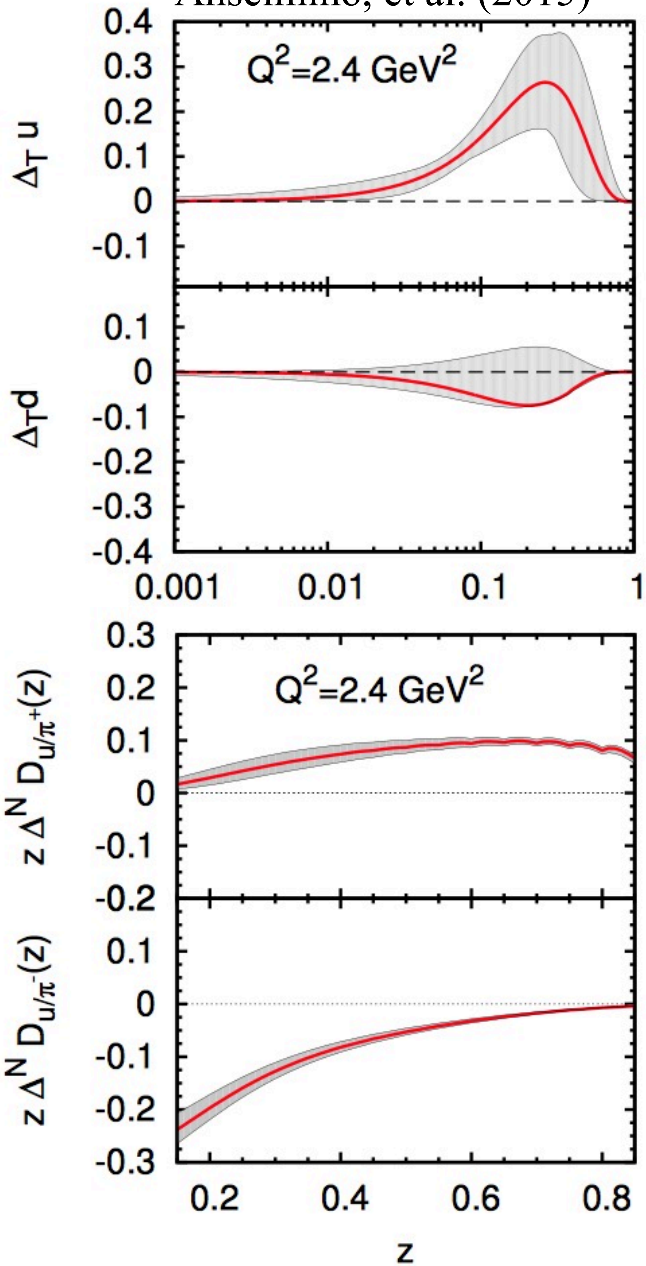
Belle (2008)



Also data from BaBar (2014) and BESIII (2016)

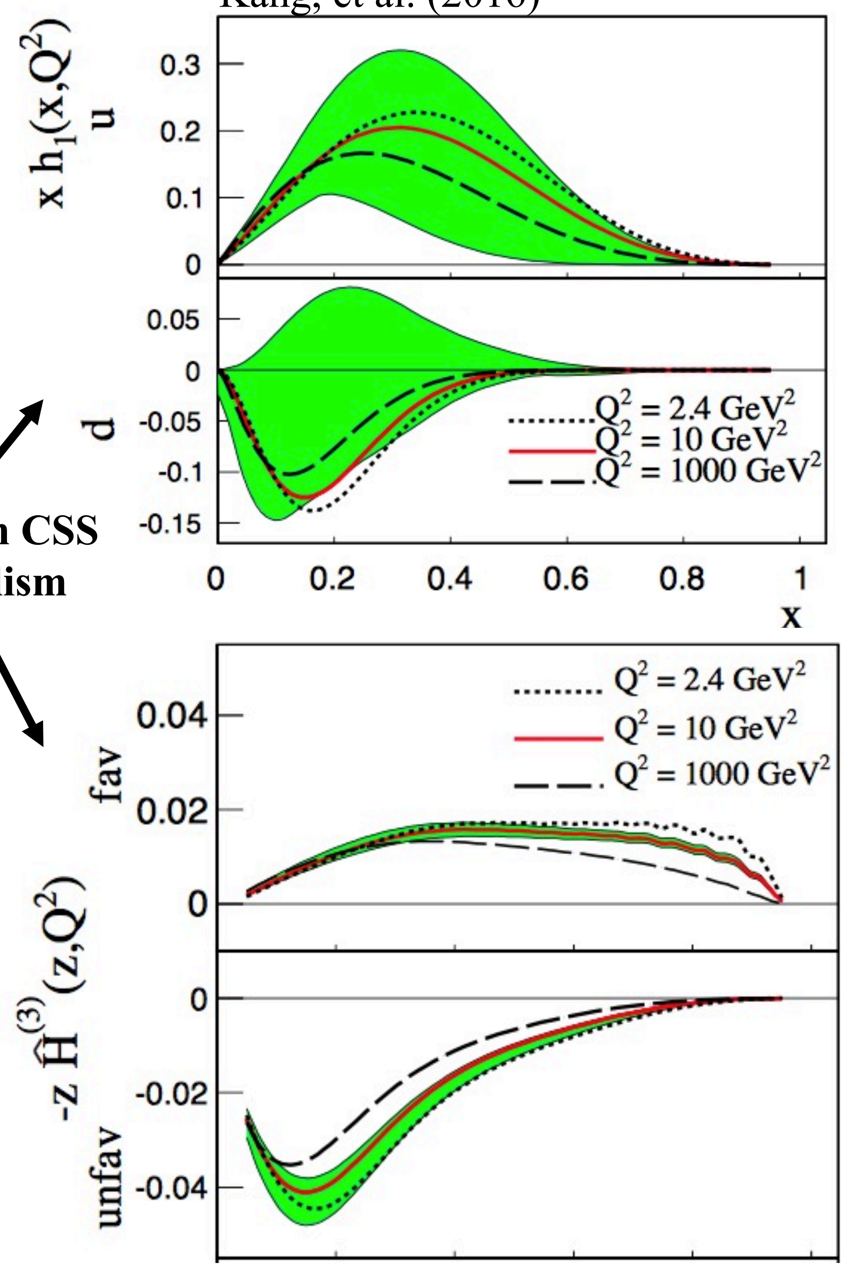
$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\,\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

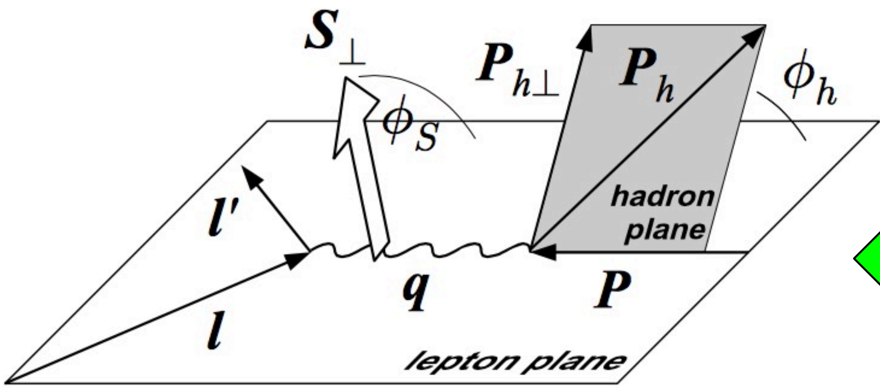
Anselmino, et al. (2015)

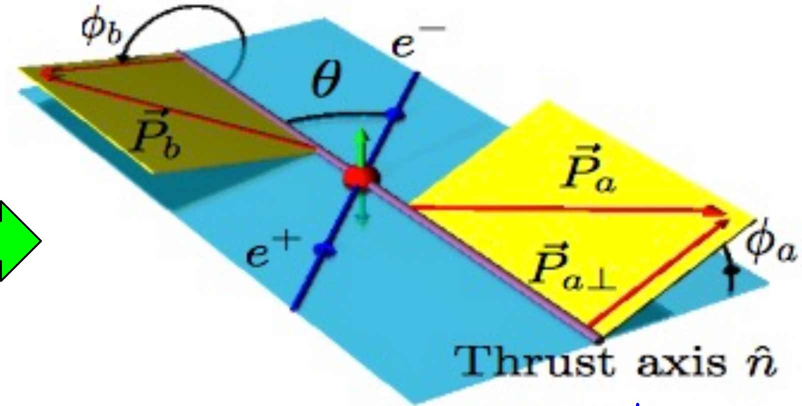


TMDs in CSS formalism

Kang, et al. (2016)

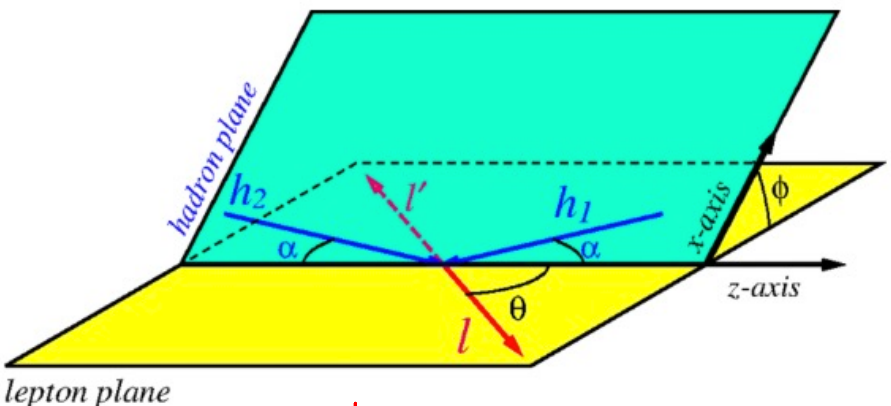






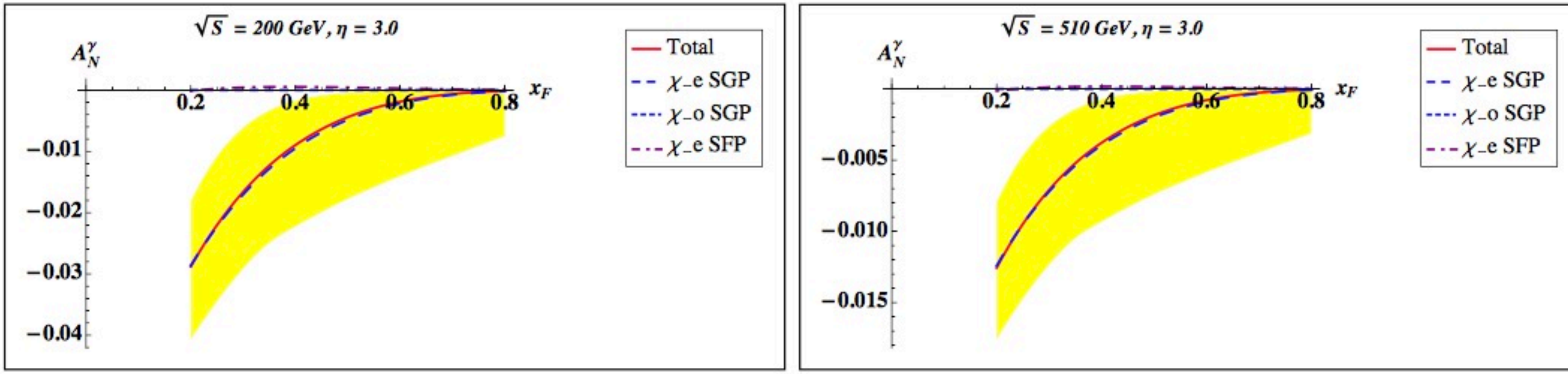
$h_1,\ f_{1T}^{\perp},\ H_1^{\perp}$

$H_1^{\perp}$



$f_{1T}^{\perp}$

A_N in $pp \to \gamma X$



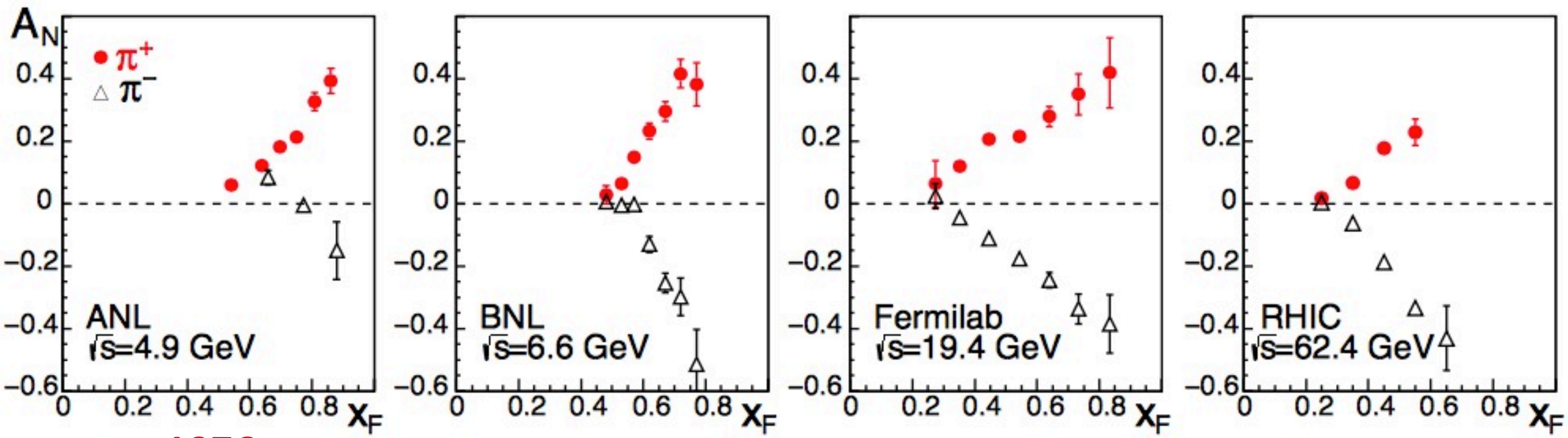
(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
(See also Gamberg, Kang, Prokudin (2013))

Qiu-Sterman term is the main cause of A_N in $pp \to \gamma X$

$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

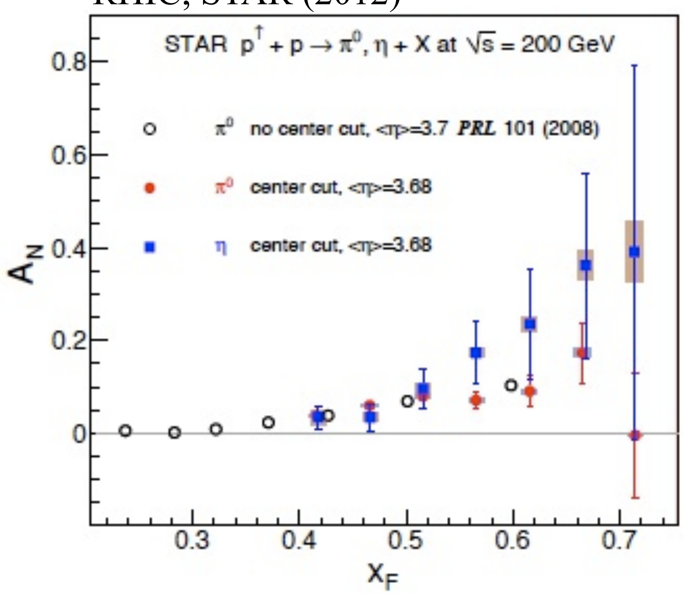
Qiu-Sterman function

A_N in *pp -> π X* – PUZZLE FOR 40+ YEARS!

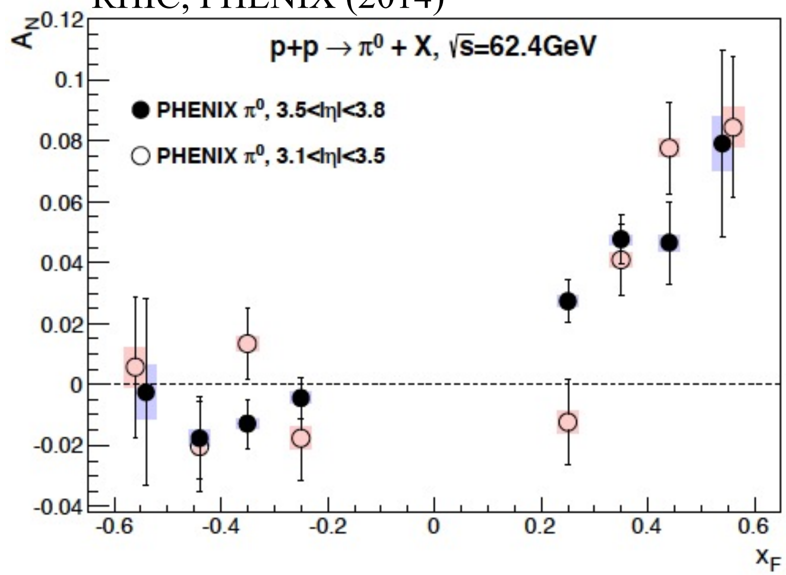


1976 →

RHIC, STAR (2012)



RHIC, PHENIX (2014)



$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^{1} \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^{1} \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x')$$
$$\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}}\right) \frac{1}{x} \left[T_{a,F}(x,x) - x\left(\frac{d}{dx} T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

$$\boxed{F_{FT} \sim T_F}$$

(Qiu and Sterman (1999), Kouvaris, et al. (2006))

For many years the Qiu-Sterman/Sivers-type contribution was thought to be the dominant source of TSSAs in $\boldsymbol{p^{\uparrow}p \longrightarrow \pi\, X}$

$$\cancel{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

$$\cancel{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\,\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz} \right] S^i_{H_1^\perp} + \frac{1}{z} H^c(z)\, S^i_H \right.$$
$$\left. +\; \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\, \hat{H}_{FU}^{c,\Im}(z,z_1)\, S^i_{\hat{H}_{FU}} \right\}$$

(Metz and DP - PLB **723** (2013))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$H^q(z) \;=\; -2z\, H_1^{\perp(1),q}(z) + \boxed{2z \int_z^{\infty} \frac{dz_1}{z_1^2} \, \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)}$$

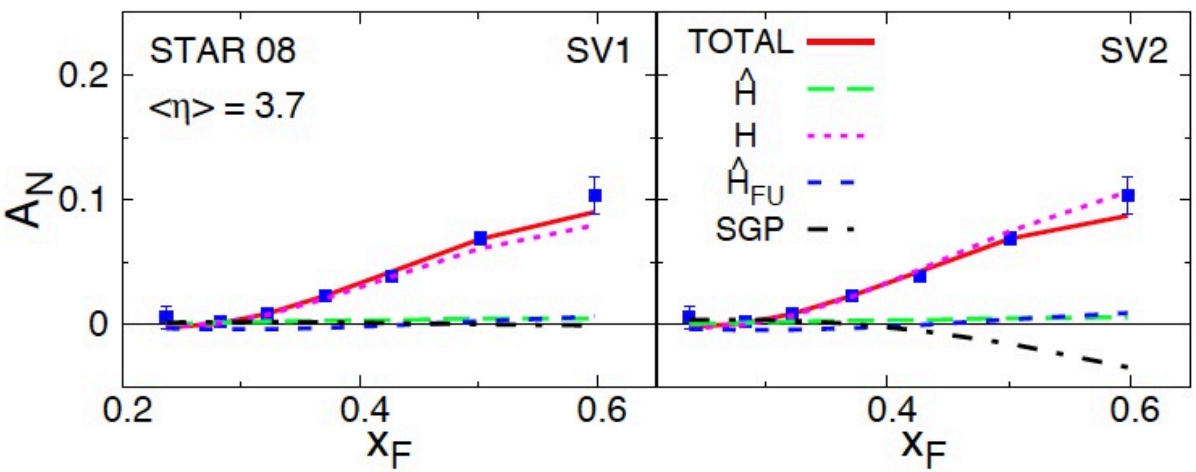
QCD e.o.m. relation (EOMR)

$$\equiv \tilde{H}^q(z)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

Also included the Qiu-Sterman term $\pi\, \boldsymbol{F_{FT}(x,x)} = \boldsymbol{f_{1T}^{\perp(1)}(x)}$



(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014))

Fragmentation term is the main cause of A_N in *pp -> π X*

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \tilde{\boldsymbol{H}}, \int \frac{dz_1}{z_1^2} \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^{\infty} \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$

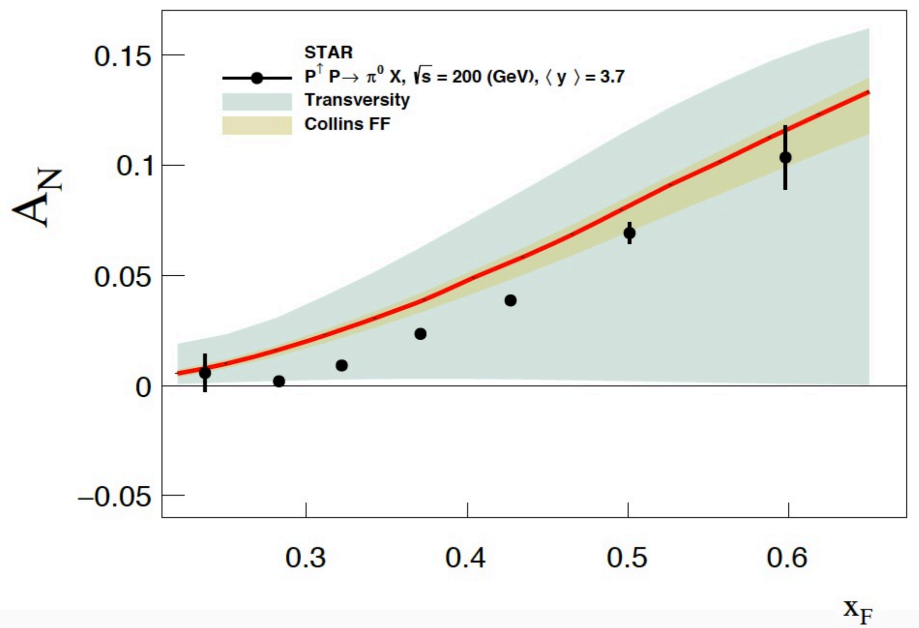
$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S} \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] \tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),c}(z) + \frac{1}{z}\tilde{H}^c(z) \right] \tilde{S}^i_H \right\}$$

where $\tilde{S}^i_{H_1^\perp} \equiv \dfrac{S^i_{H_1^\perp} - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}^i_H \equiv \dfrac{S^i_H - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

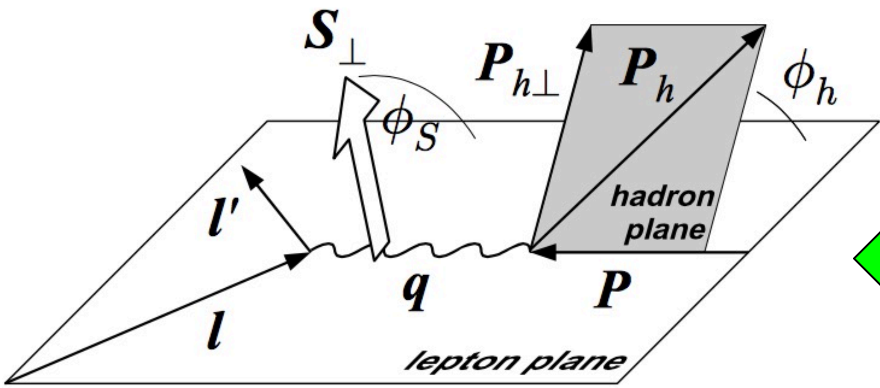
$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$



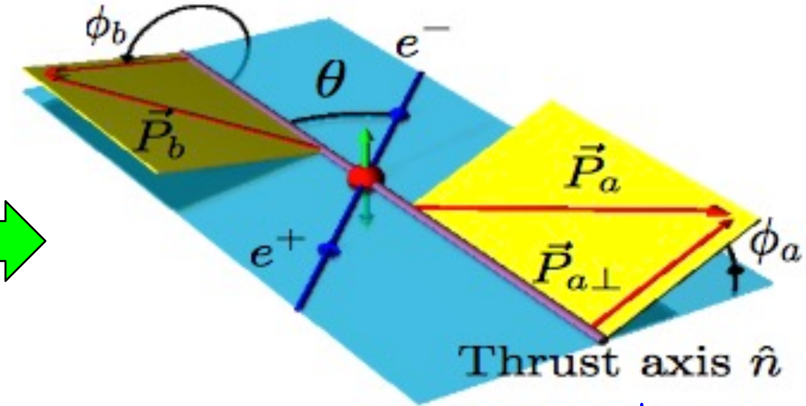
Fragmentation term is the main cause of A_N in *pp -> π X*

The A_N data from RHIC can be used along with measurements from SoLID to constrain ***transversity at large x***

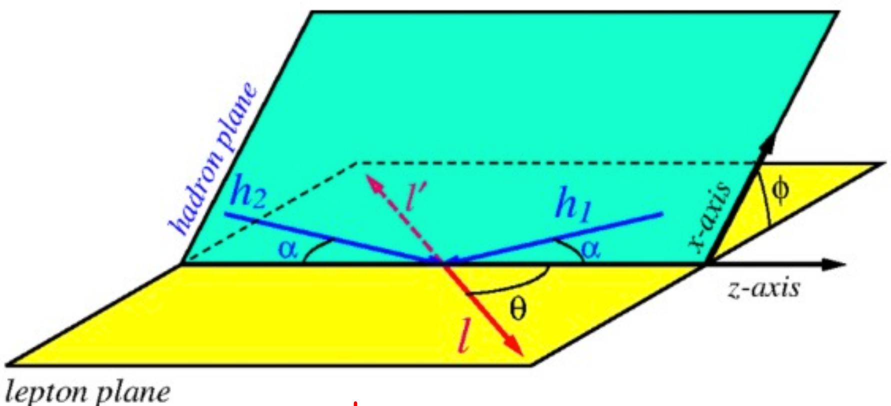
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



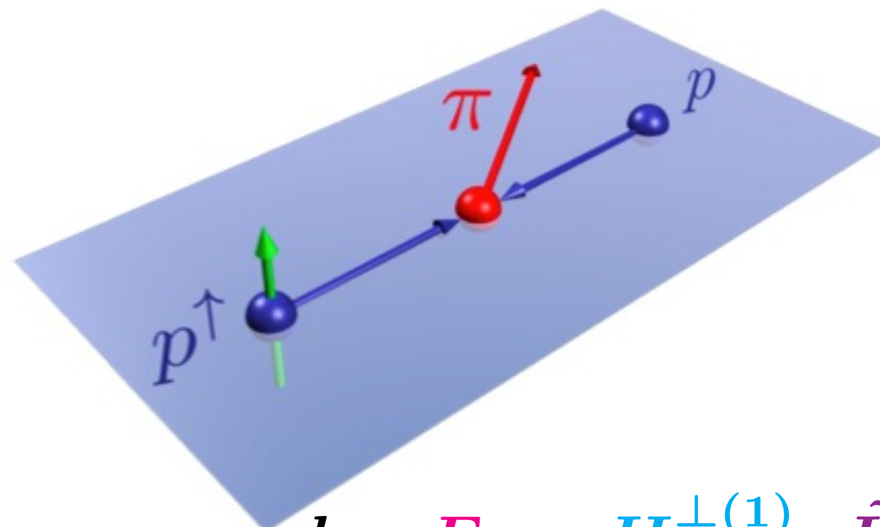
h_1, $f_{1T}^{\perp}$, $H_1^{\perp}$



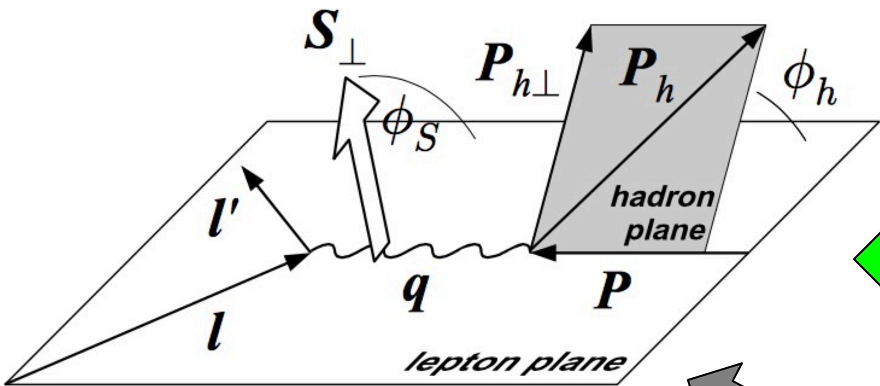
$H_1^{\perp}$

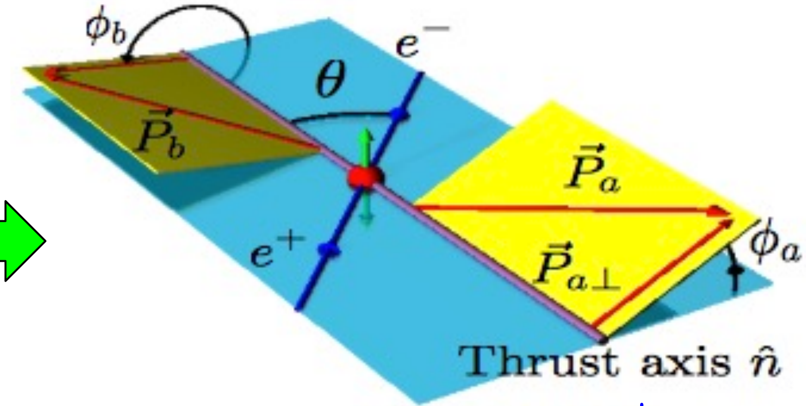


$f_{1T}^{\perp}$



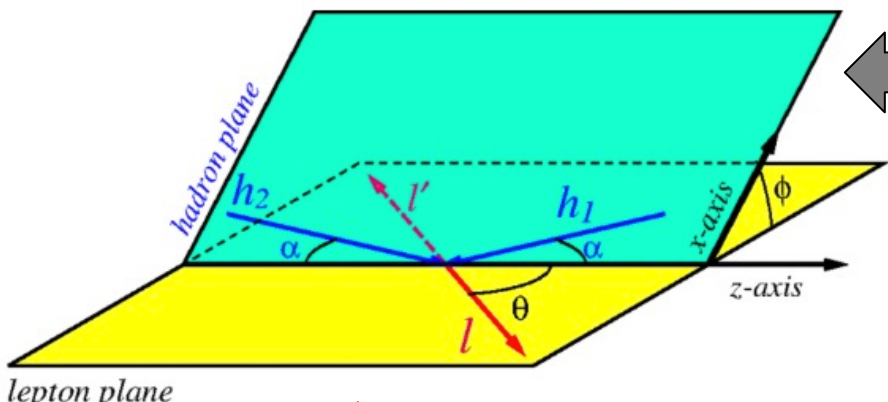
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

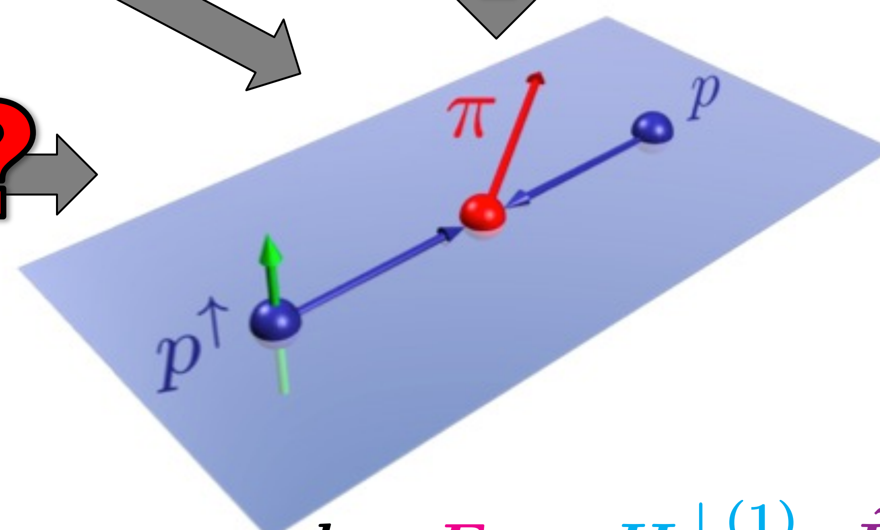




h_1, $f_{1T}^\perp$, $H_1^\perp$

$H_1^\perp$



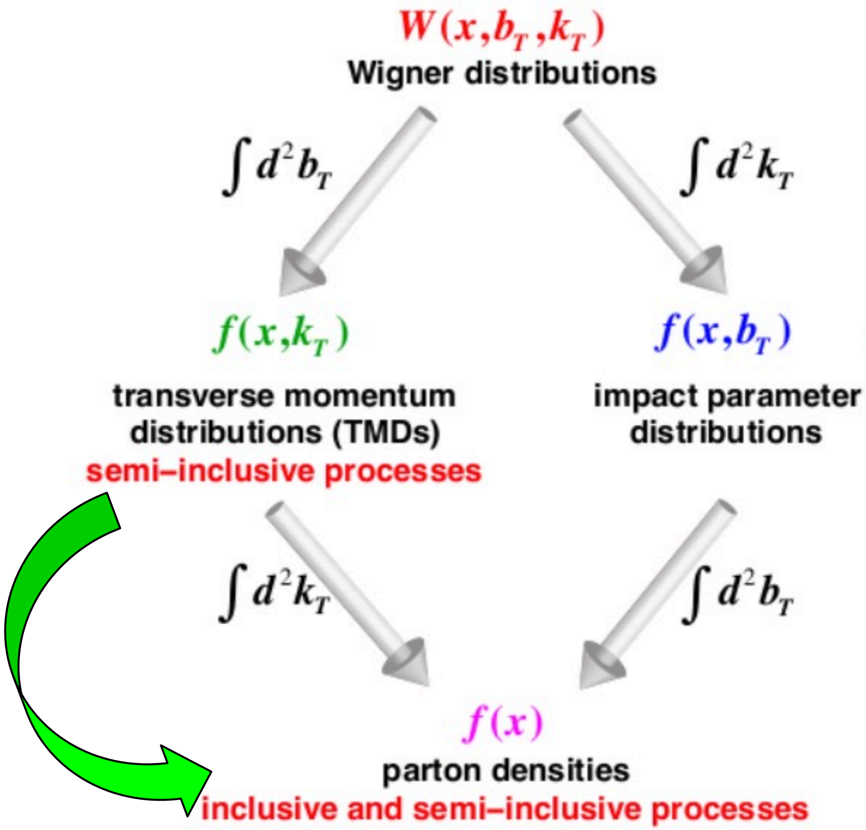


$f_{1T}^\perp$

h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Relations between TMD and CT3 Functions

Figure from EIC Whitepaper



One naively expects that we can obtain collinear functions by integrating TMDs over k_T

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

Takes into account "complications" of QCD (e.g., parton re-scattering and gluon radiation)

“Original CSS” (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

“b-space” correlator

$$\tilde{\Phi}^{[\gamma^+]}(x,\vec{b}_T;Q^2,\mu_Q) = \boldsymbol{\tilde{f}_1(x,b_T;Q^2,\mu_Q)} - iM\epsilon^{ij}b_T^i S_T^j\left[\boldsymbol{-\frac{1}{M^2}\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T;Q^2,\mu_Q)}\right]$$

Boer, Gamberg, Musch, Prokudin (2011)

$$\equiv \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x,b_T;Q^2,\mu_Q)}$$

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Collins (2011); ...

$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_1}(b_T,Q)\right]$$

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Collins (2011); ...

$$\tilde{f}_{1T}^{\perp(1)}(x,b_T;Q^2,\mu_Q) \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1,\hat{x}_2,b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes F_{FT}(\hat{x}_1,\hat{x}_2;\mu_{b_*})\right)$$
$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T,Q)\right]$$

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

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$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes f_1(\hat{x}; \mu_{b_*})\right)$$

$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

<u>perturbative Sudakov factor</u>

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

<u>non-perturbative Sudakov factor</u>

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

different for each TMD

universal

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\tilde{\boldsymbol{f}}_{\boldsymbol{1}}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1}(\boldsymbol{\hat{x}}; \boldsymbol{\mu_{b_*}})\right)$$

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Note: $b_*(0) = 0$ and $(\mu_{b_*})_{b_* \to 0} = \infty$ ➡ problematic large logarithms in S_{pert}

(Bozzi, Catani, de Florian, Grazzini (2006); Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

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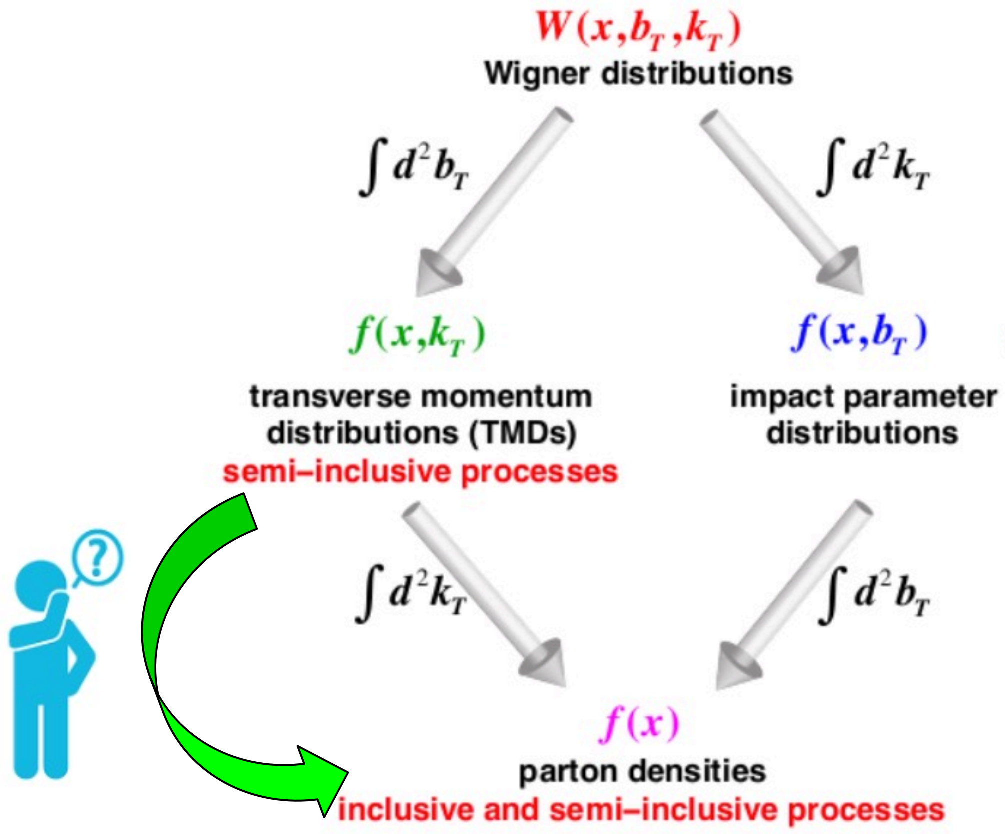
(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q) \equiv f_{1T}^{\perp(1)}(x; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T \to 0; Q^2, \mu_Q)} = \mathbf{0}\,!$$

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

Figure from EIC Whitepaper



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TMDs lose their physical interpretation in the "Original CSS" formalism!

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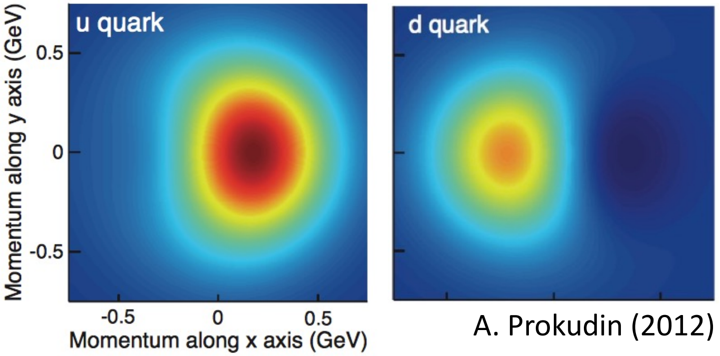
$$\langle k_T^i(x)\rangle_{UT} = \int d^2k_T\, k_T^i \left(-\frac{\vec{k}_T \times \vec{S}_T}{M} f_{1T}^{\perp}(x,k_T) \right)$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target



A. Prokudin (2012)

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$$\int d^2k_T\, f_1(x,k_T;Q^2,\mu_Q) = \tilde{f}_1(x, b_T \to 0; Q^2, \mu_Q) = 0\,!$$

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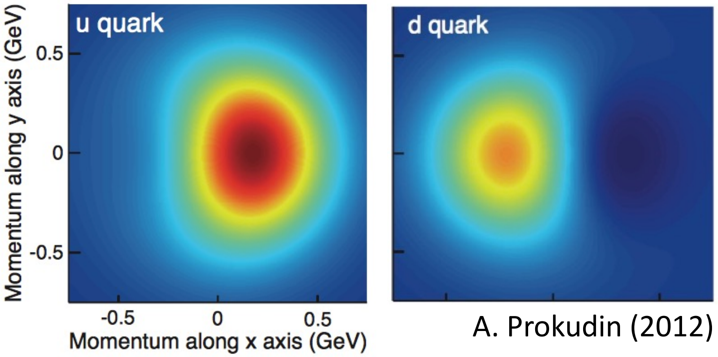

A. Prokudin (2012)

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))*

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$$\Longrightarrow \mu_{b_*} \to \bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))} \text{ so } \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

*Other modifications are discussed in this reference that attempt to improve the agreement of the CSS W+Y formulation with the differential cross section over all transverse momentum regions.

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$$\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

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$$\tilde{\boldsymbol{f}}_{\boldsymbol{1}}(\boldsymbol{x}, \boldsymbol{b_c}(\boldsymbol{b_T}); \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1}(\hat{\boldsymbol{x}}; \bar{\boldsymbol{\mu}})\right)$$
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"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{\boldsymbol{f}}_{\boldsymbol{1}}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) - iM\epsilon^{ij} b_T^i S_T^j \tilde{\boldsymbol{f}}_{\boldsymbol{1T}}^{\boldsymbol{\perp(1)}}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q})$$

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$\Rightarrow$ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes f_1(\hat{x}; \bar{\mu})\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

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b_T -> $b_c(b_T)$

NO b_T -> $b_c(b_T)$ replacement – kinematic factor NOT associated with the scale evolution

b_T -> $b_c(b_T)$

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$$\Rightarrow \quad \mu_{b_*} \to \bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))} \quad \text{so } \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

$$\boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

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$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T, b_c(b_T); Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} - iM\epsilon^{ij} b_T^i S_T^j \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_{1T}^\perp}(\hat{x}_1, \hat{x}_2, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{F_{FT}(\hat{x}_1, \hat{x}_2; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_c(b_T), Q)\right]$$

Analogous modification for fragmentation functions...

$$\tilde{D}_1(z, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{D_1}(z/\hat{z}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes D_1(\hat{z}; \bar{\mu})\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{D_1}(b_c(b_T), Q)\right]$$

$$\tilde{H}_1^{\perp(1)}(z, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{H_1^\perp}(z/\hat{z}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes H_1^{\perp(1)}(\hat{z}; \bar{\mu})\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_c(b_T), Q)\right]$$

We then *define* the momentum-space functions...

$$\boldsymbol{f_1(x, k_T; Q^2, \mu_Q; C_5)} \equiv \int \frac{db_T}{2\pi}\, b_T J_0(k_T b_T)\, \boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\boldsymbol{D_1(z, p_T; Q^2, \mu_Q; C_5)} \equiv \int \frac{db_T}{2\pi}\, b_T J_0(p_T b_T)\, \boldsymbol{\tilde{D}_1(z, b_c(b_T); Q^2, \mu_Q)}$$

$$\frac{\vec{k}_T^2}{2M^2}\, \boldsymbol{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5)} \equiv k_T \int \frac{db_T}{4\pi} b_T^2 J_1(k_T b_T)\, \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\frac{\vec{p}_T^2}{2z^2 M_h^2}\, \boldsymbol{H_1^{\perp}(z, p_T; Q^2, \mu_Q; C_5)} \equiv p_T \int \frac{db_T}{4\pi} b_T^2\, J_1(p_T\, b_T)\, \boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_c(b_T); Q^2, \mu_Q)}$$

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

which leads to...

$$\int d^2\vec{k}_T\, f_1(x,k_T;Q^2,\mu_Q;C_5) = \tilde{f}_1(x,b_c(0);Q^2,\mu_Q) = f_1(x;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, D_1(z,p_T;Q^2,\mu_Q;C_5) = \tilde{D}_1(z,b_c(0);Q^2,\mu_Q) = D_1(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^2}{2M^2}\, f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5) = \tilde{f}_{1T}^{\perp(1)}(x,b_c(0);Q^2,\mu_Q) = \pi\, F_{FT}(x,x;\mu_c) + O(\alpha_s(Q)) + O((m/Q)$$

$$\int d^2\vec{p}_T\,\frac{\vec{p}_T^2}{2z^2M_h^2}H_1^{\perp}(z,p_T;Q^2,\mu_Q;C_5) = \tilde{H}_1^{\perp(1)}(z,b_c(0);Q^2,\mu_Q) = H_1^{\perp(1)}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

At LO in the "Improved CSS" formalism we recover the relations one expects from the "naïve" operator definitions of the functions

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

which leads to...

$$\int d^2\vec{k}_T\, \boldsymbol{f_1(x,k_T;Q^2,\mu_Q;C_5)} = \boldsymbol{\tilde{f}_1(x,b_c(0);Q^2,\mu_Q)} = \boldsymbol{f_1(x;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, \boldsymbol{D_1(z,p_T;Q^2,\mu_Q;C_5)} = \boldsymbol{\tilde{D}_1(z,b_c(0);Q^2,\mu_Q)} = \boldsymbol{D_1(z;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, \boldsymbol{f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5)} = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x,b_c(0);Q^2,\mu_Q)} = \pi\, \boldsymbol{F_{FT}(x,x;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)$$

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^2}{2z^2M_h^2} \boldsymbol{H_1^{\perp}(z,p_T;Q^2,\mu_Q;C_5)} = \boldsymbol{\tilde{H}_1^{\perp(1)}(z,b_c(0);Q^2,\mu_Q)} = \boldsymbol{H_1^{\perp(1)}(z;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

At LO in the "Improved CSS" formalism we recover the relations one expects from the "naïve" operator definitions of the functions

The "Improved CSS" formalism (approximately) restores the physical interpretation of TMDs!

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^2}{2M^2}\,\boldsymbol{f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5)} = \pi\,\boldsymbol{F_{FT}(x,x;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \pi \, F_{FT}(x, x; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T k_T^i \int \frac{db^-}{2\pi} \int \frac{d^2b_T}{(2\pi)^2} e^{ixP^+b^-} e^{-i\vec{k}_T\cdot\vec{b}_T} \langle P, S|\bar{\psi}(0)\gamma^+ \mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$= \frac{1}{2}\int \frac{db^- dy^-}{4\pi} \, e^{ixP^+b^-} \langle P, S|\bar{\psi}(0)\gamma^+ \mathcal{W}(0;y^-) gF^{+i}(y^-)\, \mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$= -\pi \, M\epsilon^{ij} S_T^j \, F_{FT}(x,x;\mu)$$

(Boer, Mulders, Teryaev (1998); Burkardt (2004); Meissner, Metz, Goeke (2007))

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \pi\, F_{FT}(x, x; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

"Naïve" TMD operator – UV renormalization at LO and soft factor at LO, Wilson lines on the lightcone

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T\, k_T^i \boxed{\int \frac{db^-}{2\pi} \int \frac{d^2b_T}{(2\pi)^2} e^{ixP^+b^-} e^{-i\vec{k}_T\cdot\vec{b}_T} \langle P, S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\mathrm{DIS}}(0;b)\psi(b)|P, S\rangle\Big|_{b^+=0}}$$

$$= \frac{1}{2}\int \frac{db^-dy^-}{4\pi}\, e^{ixP^+b^-} \langle P, S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P, S\rangle$$

$$= -\pi\, M\epsilon^{ij} S_T^j\, F_{FT}(x, x; \mu)$$

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \pi\, F_{FT}(x, x; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

This is *NOT* the operator that defines TMDs in CSS

"Naïve" TMD operator – UV renormalization at LO and soft factor at LO, Wilson lines on the lightcone

$\langle k_T^i(x;\mu)\rangle_{UT}$

$$= \frac{1}{2}\int d^2k_T\, k_T^i \boxed{\int \frac{db^-}{2\pi}\int \frac{d^2b_T}{(2\pi)^2}\, e^{ixP^+b^-} e^{-i\vec{k}_T\cdot\vec{b}_T}\, \langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}}$$

$$= \frac{1}{2}\int \frac{db^-dy^-}{4\pi}\, e^{ixP^+b^-}\, \langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$= -\pi\, M\epsilon^{ij}S_T^j\, F_{FT}(x, x; \mu)$$

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^2}{2M^2}\,f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5) = \pi\,F_{FT}(x,x;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T\,k_T^i\int\frac{db^-}{2\pi}\int\frac{d^2b_T}{(2\pi)^2}e^{ixP^+b^-}e^{-i\vec{k}_T\cdot\vec{b}_T}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$= \frac{1}{2}\boxed{\int\frac{db^-dy^-}{4\pi}\,e^{ixP^+b^-}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle}$$

$$= -\pi\,M\epsilon^{ij}S_T^j\,F_{FT}(x,x;\mu)$$

“Naïve” collinear operator – LO term of the UV renormalized correlator, Wilson lines on the lightcone

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^2}{2M^2}\,f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5) = \pi\,F_{FT}(x,x;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

This *IS* the operator for the Qiu-Sterman function that enters the OPE within CSS

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T\,k_T^i \int\frac{db^-}{2\pi}\int\frac{d^2b_T}{(2\pi)^2}\,e^{ixP^+b^-}\,e^{-i\vec{k}_T\cdot\vec{b}_T}\,\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$= \frac{1}{2}\boxed{\int\frac{db^-dy^-}{4\pi}\,e^{ixP^+b^-}\,\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle}$$

$$= -\pi\,M\epsilon^{ij}S_T^j\,F_{FT}(x,x;\mu)$$

“Naïve” collinear operator – LO term of the UV renormalized correlator, Wilson lines on the lightcone

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$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^2}{2M^2}\,f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5)=\pi\,\boxed{F_{FT}(x,x;\mu_c)}+O(\alpha_s(Q))+O((m/Q)^{p'})$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$=\frac{1}{2}\int d^2k_T\,k_T^i\int\frac{db^-}{2\pi}\int\frac{d^2b_T}{(2\pi)^2}e^{ixP^+b^-}e^{-i\vec{k}_T\cdot\vec{b}_T}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$=\frac{1}{2}\int\frac{db^-dy^-}{4\pi}\,e^{ixP^+b^-}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$=-\pi\,M\epsilon^{ij}S_T^j\,F_{FT}(x,x;\mu)$$

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^2}{2M^2}\,f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5) = \pi\,\boxed{F_{FT}(x,x;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T\,k_T^i\int\frac{db^-}{2\pi}\int\frac{d^2b_T}{(2\pi)^2}\,e^{ixP^+b^-}e^{-i\vec{k}_T\cdot\vec{b}_T}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\mathrm{DIS}}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$= \frac{1}{2}\int\frac{db^-dy^-}{4\pi}\,e^{ixP^+b^-}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$= -\pi\,M\epsilon^{ij}S_T^j\,F_{FT}(x,x;\mu)$$

Recall also the Burkardt sum rule $\displaystyle\sum_{a=q,\bar{q},g}\int_0^1 dx\,F_{FT}^a(x,x) = 0$

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \pi\, \boxed{F_{FT}(x, x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T\, k_T^i \int \frac{db^-}{2\pi} \int \frac{d^2b_T}{(2\pi)^2}\, e^{ixP^+b^-} e^{-i\vec{k}_T\cdot\vec{b}_T} \langle P, S|\bar{\psi}(0)\gamma^+ \mathcal{W}_{\mathrm{DIS}}(0;b)\psi(b)|P, S\rangle\Big|_{b^+=0}$$

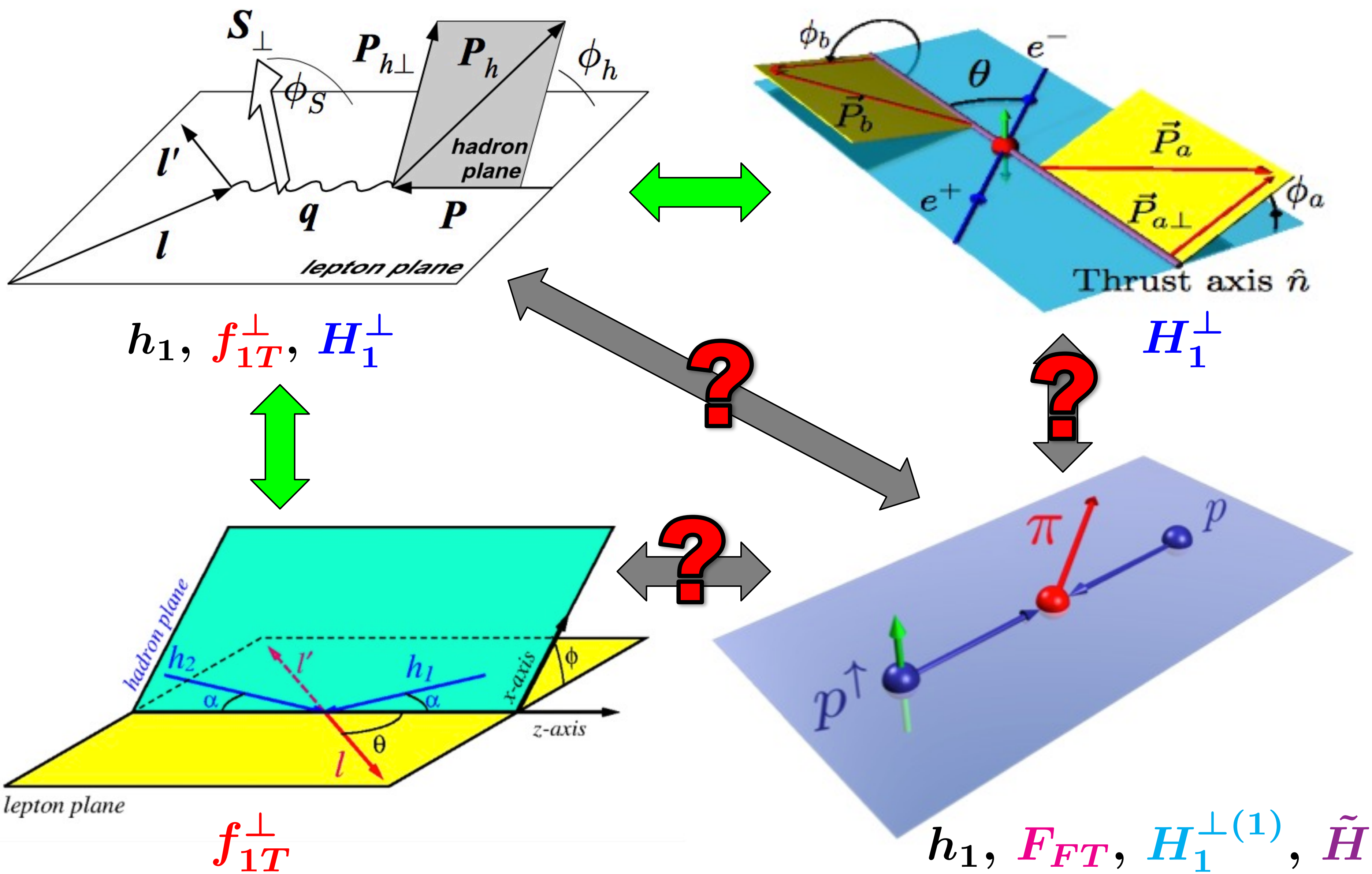
$$= \frac{1}{2}\int \frac{db^- dy^-}{4\pi}\, e^{ixP^+b^-} \langle P, S|\bar{\psi}(0)\gamma^+ \mathcal{W}(0;y^-) gF^{+i}(y^-)\, \mathcal{W}(y^-;b^-)\psi(b^-)|P, S\rangle$$

$$= -\pi\, M\epsilon^{ij} S_T^j\, F_{FT}(x, x;\mu)$$

Recall also the Burkardt sum rule $\sum_{a=q,\bar{q},g} \int_0^1 dx\, F_{FT}^a(x, x) = 0$

The Qiu-Sterman function can fundamentally be understood as an avg. TM, and the first k_T-moment of the Sivers function (using "Improved CSS") retains this interpretation at LO

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))


$S_\perp$
ϕ_S
$P_{h\perp}$
P_h
ϕ_h
hadron plane
l'
q
P
l
lepton plane
h_1, $f_{1T}^\perp$, $H_1^\perp$
ϕ_b
e^-
θ
$\vec{P}_b$
$\vec{P}_a$
e^+
$\vec{P}_{a\perp}$
ϕ_a
Thrust axis $\hat{n}$
$H_1^\perp$
?
?
?
hadron plane
h_2
l'
h_1
x-axis
ϕ
α
α
θ
l
z-axis
lepton plane
$f_{1T}^\perp$
π
p
$p^\uparrow$
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Recall the current phenomenology of TMD observables...

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim \boxed{F_{FT}(x, x; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_T, Q)\right]$$

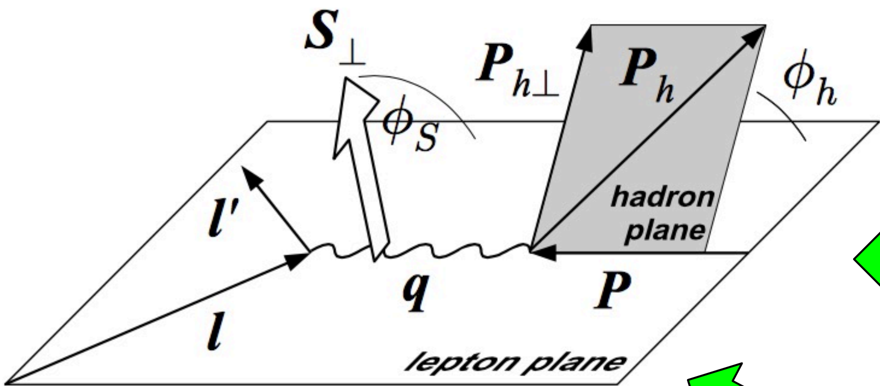
$$S_{NP}^{f_{1T}^\perp}(b_T, Q) \to \boxed{g_{f_{1T}^\perp}(x, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim \boxed{H_1^{\perp(1)}(z; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q)\right]$$

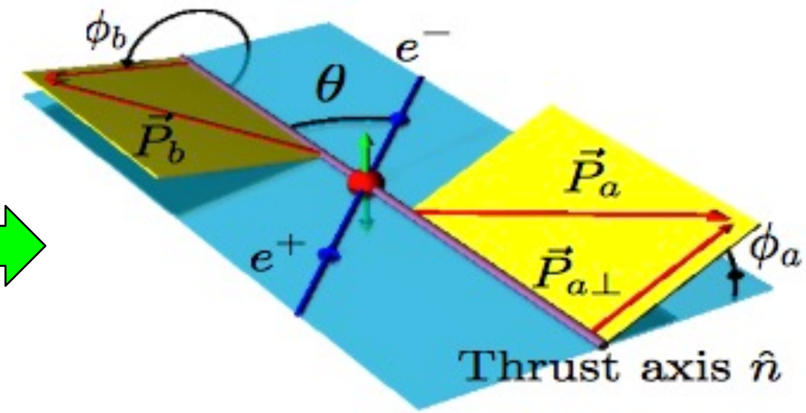
$$S_{NP}^{H_1^\perp}(b_T, Q) \to \boxed{g_{H_1^\perp}(z, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

The **CT3 functions** (along with the NP *g*-functions) are what get extracted in analyses of TSSAs in ***TMD processes*** that use CSS evolution!

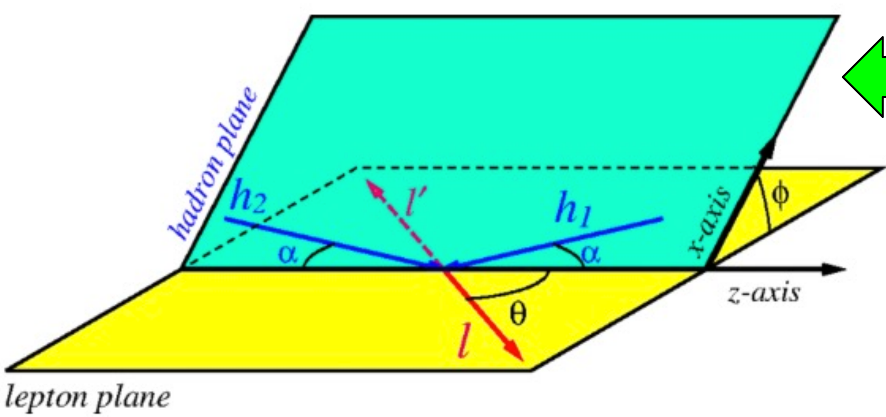
(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))



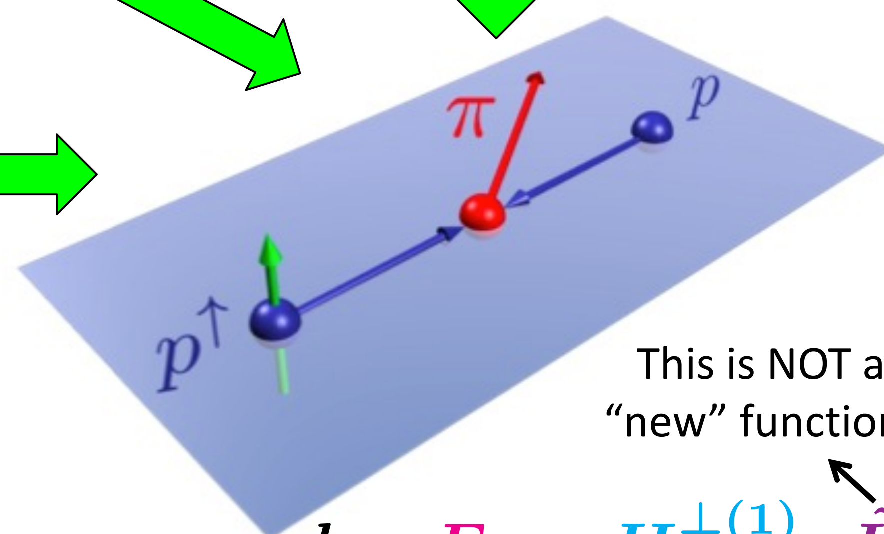
h_1, F_{FT}, $H_1^{\perp(1)}$



$H_1^{\perp(1)}$

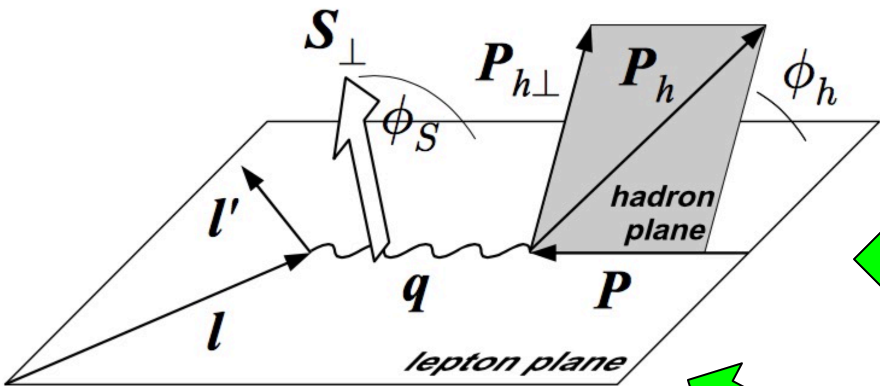


F_{FT}

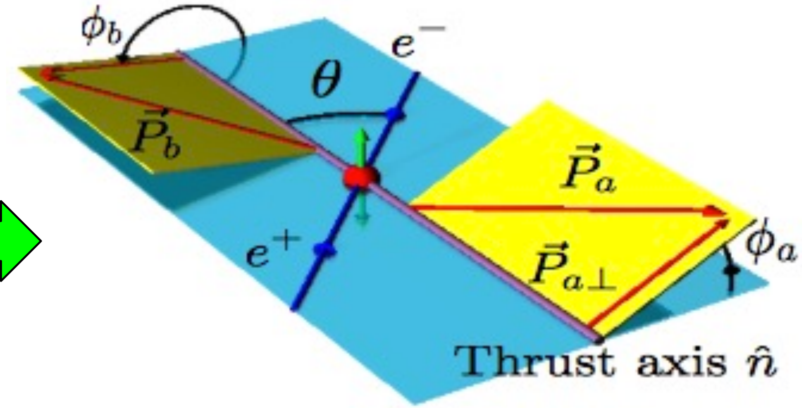


This is NOT a "new" function!

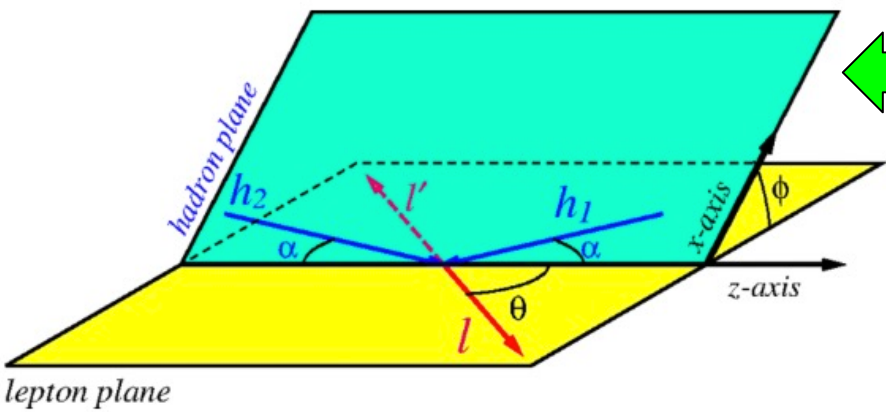
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



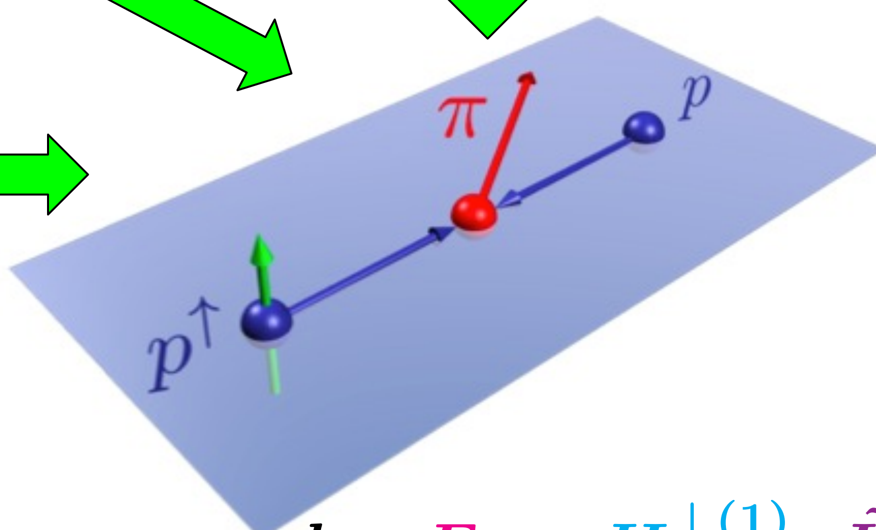
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Summary

- TSSAs have been studied in both TMD processes (SIDIS, e^+e^-, DY) and collinear processes (A_N in *pp* & *lp* collisions).

- The current TMD formalism using "Improved CSS" (iCSS) allows one to rigorously connect these two different types of observables. We have extended the original work on the unpolarized cross section to now include polarization.

- With the iCSS formalism, we are able at LO to restore the physical interpretation of (integrated) TMDs.

- A global analysis of TMD *AND* collinear twist-3 transverse-spin observables is possible.