Contributions of up and down quarks to TMD observables from Lattice QCD

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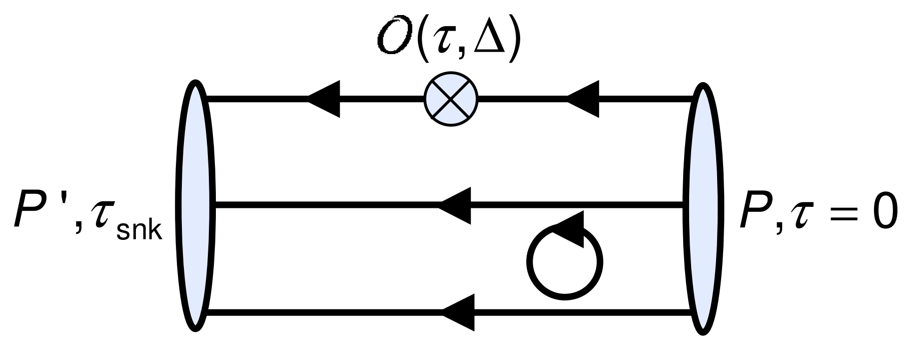
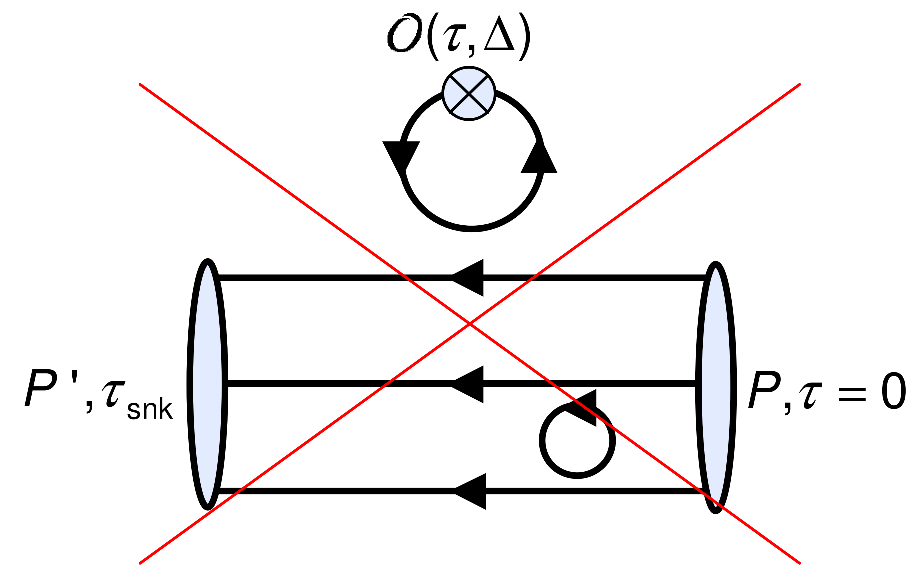
Lattice QCD: Isovector $u - d$ observables are simpler to evaluate





- Disconnected diagrams cancel for isovector insertions, not for u and d quarks separately
- Renormalization: Isoscalar operators mix with gluonic operators

Nonetheless, flavor-separated quantities are phenomenologically interesting . . .

- At moderate pion masses (here: ~ 300 MeV), disconnected contributions are usually small for light-flavor observables
- Will present results for flavor-separated TMD observables, **connected contributions only!**
- Will include results for twist-three unpolarized chiral-odd TMD e

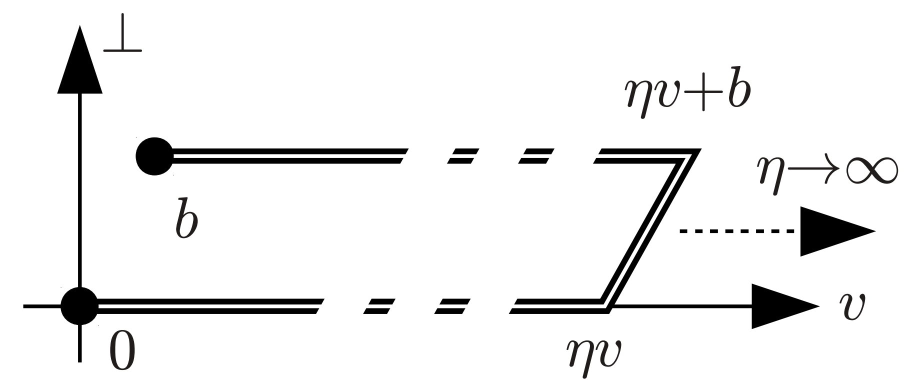
Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots) \equiv \frac{1}{2}\langle P,S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0,\ldots,b]\ q(b)\ |P,S\rangle$$

$$\Phi^{[\Gamma]}(x,k_T,P,S,\ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b\cdot P)}{(2\pi)P^+} \exp\left(ix(b\cdot P) - ib_T\cdot k_T\right) \left.\frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)}{\widetilde{\mathcal{S}}(b^2,\ldots)}\right|_{b^+=0}$$

Ultimately, will only consider ratios in which soft factors $\widetilde{\mathcal{S}}$ cancel

Gauge link structure motivated by SIDIS



Regulate rapidity divergences by using spacelike staple direction v. Parametrize in terms of Collins-Soper parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for $\hat{\zeta} \to \infty$. Perturbative evolution equations for large $\hat{\zeta}$.

Decomposition of Φ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[\frac{\epsilon_{ij}k_i S_j}{m_N} f_{1T}^{\perp}\right] \text{odd}$$

$$\Phi^{[\gamma^+\gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_N} g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_N^2} h_{1T}^{\perp} + \frac{\Lambda k_i}{m_N} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij}k_j}{m_N} h_1^{\perp}\right] \text{odd}$$

Decomposition of $\widetilde{\Phi}$ into amplitudes

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\begin{aligned}
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}} &= \widetilde{A}_{2B} + i m_N \epsilon_{ij} b_i S_j \widetilde{A}_{12B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+\gamma^5]}_{\text{unsubtr.}} &= -\Lambda \widetilde{A}_{6B} + i[(b\cdot P)\Lambda - m_N(b_T\cdot S_T)]\widetilde{A}_{7B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[i\sigma^{i+}\gamma^5]}_{\text{unsubtr.}} &= i m_N \epsilon_{ij} b_j \widetilde{A}_{4B} - S_i \widetilde{A}_{9B} \\
&\quad - i m_N \Lambda b_i \widetilde{A}_{10B} + m_N[(b\cdot P)\Lambda - m_N(b_T\cdot S_T)] b_i \widetilde{A}_{11B}
\end{aligned}$$

Twist three: $$\frac{1}{2}\widetilde{\Phi}^{[1]}_{\text{unsubtr.}} = m_N \widetilde{A}_1 - \frac{i m_N^2}{v\cdot P}\epsilon^{\mu\nu\rho\sigma} P_\mu b_\nu v_\rho S_\sigma \widetilde{B}_5$$

(Decompositions analogous to work by Metz et al. in momentum space)

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2 k_T \, \exp(i b_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left(-\frac{2}{m_N^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

In limit $|b_T| \to 0$, recover k_T-moments:

$$\tilde{f}^{(n)}(x, 0, \ldots) \equiv \int d^2 k_T \left(\frac{k_T^2}{2 m_N^2} \right)^n f(x, k_T^2, \ldots) \equiv f^{(n)}(x)$$

ill-defined for large k_T, so will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

In this study, only consider first x-moments (accessible at $b \cdot P = 0$), rather than scanning range of $b \cdot P$:

$$f^{[1]}(k_T^2, \ldots) \equiv \int_{-1}^{1} dx \, f(x, k_T^2, \ldots)$$

$\longrightarrow$ Bessel-weighted asymmetries (Boer, Gamberg, Musch, Prokudin, JHEP 1110 (2011) 021)

Relation between Fourier-transformed TMDs and invariant amplitudes $\widetilde{A}_i$

Invariant amplitudes directly give selected x-integrated TMDs in Fourier (b_T) space (showing just the ones relevant for Sivers, Boer-Mulders shifts), up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

Generalized shifts

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

Boer-Mulders shift:

$$\langle k_y \rangle_{UT} \equiv m_N \frac{\tilde{h}_1^{\perp [1](1)}}{\tilde{f}_1^{[1](0)}} = \left. \frac{\int dx \int d^2 k_T \, k_y \Phi^{[\gamma^+ + s^j i\sigma^{j+}\gamma^5]}(x, k_T, P, \ldots)}{\int dx \int d^2 k_T \, \Phi^{[\gamma^+ + s^j i\sigma^{j+}\gamma^5]}(x, k_T, P, \ldots)} \right|_{s_T=(1,0)}$$

Average transverse momentum of quarks polarized in the orthogonal transverse ("T") direction in an unpolarized ("U") hadron; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Issue: k_T-moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* b_T^2,

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) \equiv m_N \frac{\tilde{h}_1^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

(remember singular $b_T \to 0$ limit corresponds to taking k_T-moment). "Generalized shift".

Generalized shifts from amplitudes

Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) \equiv m_N \frac{\tilde{h}_1^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = m_N \frac{\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Sivers shift (in a polarized hadron):

$$\langle k_y \rangle_{TU}(b_T^2, \ldots) = -m_N \frac{\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Worm-gear (g_{1T}) shift:

$$\langle k_x \rangle_{TL}(b_T^2, \ldots) = -m_N \frac{\widetilde{A}_{7B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

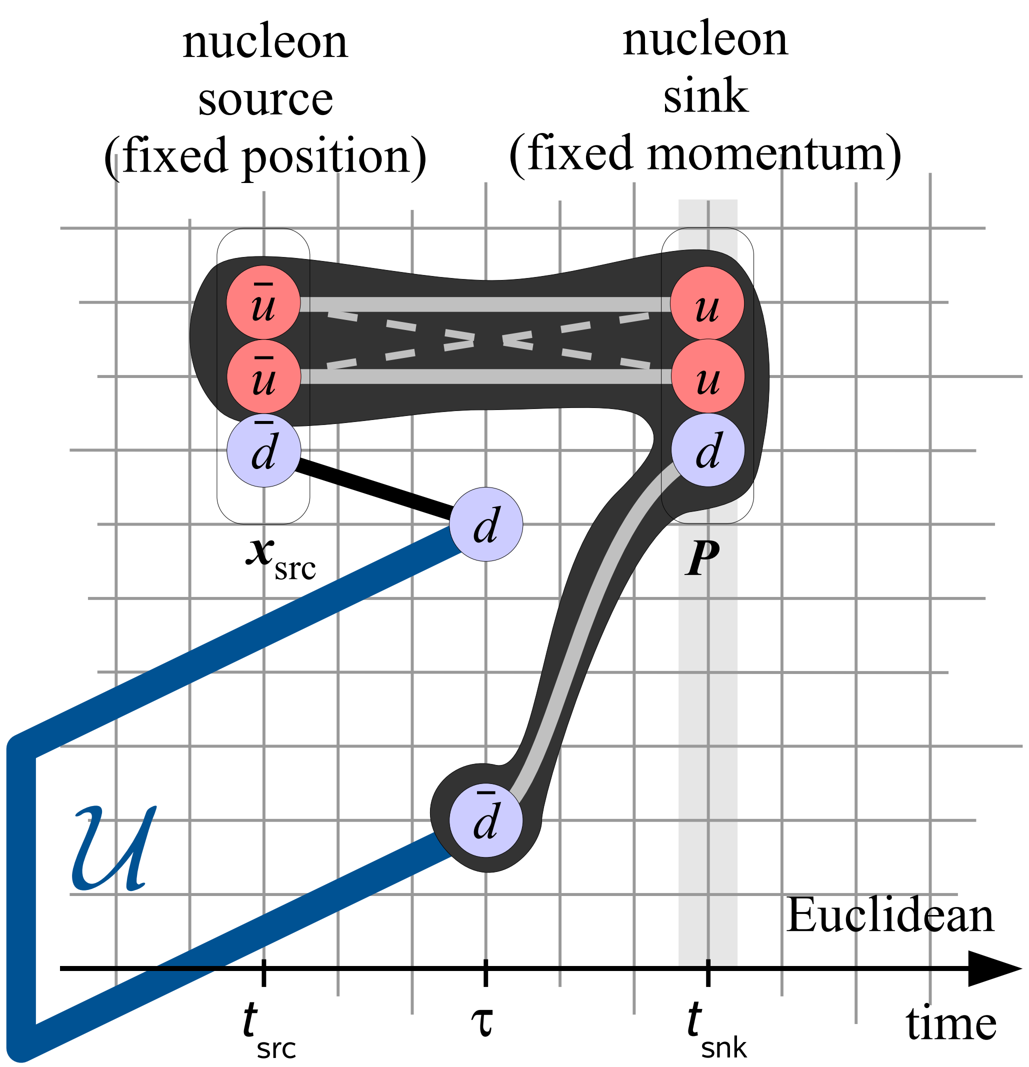
Generalized shifts from amplitudes

Generalized tensor charge (no k-weighting) :

$$\frac{\tilde{h}_1^{[1](0)}}{\tilde{f}_1^{[1](0)}}(b_T^2, \ldots) = -\frac{\widetilde{A}_{9B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P) - (m_N^2 b^2/2)\widetilde{A}_{11B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Twist-three unpolarized chiral-odd TMD e charge (no k-weighting) :

$$\frac{\tilde{e}^{[1](0)}}{\tilde{f}_1^{[1](0)}}(b_T^2, \ldots) = \frac{\widetilde{A}_1(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$



Lattice setup

- Evaluate directly $\widetilde{\Phi}^{[\Gamma]}_{\rm unsubtr.}(b, P, S, \hat{\zeta}, \mu)$
  $$\equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$
- Euclidean time: Place entire operator at one time slice, i.e., b, ηv purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\widetilde{A}_i$ invariants permits direct translation of results back to original frame; form desired $\widetilde{A}_i$ ratios.
- Use variety of P, b, ηv; here $b \perp P$, $b \perp v$ (lowest x-moment, kinematical choices/constraints)
- Extrapolate $\eta \to \infty$, $\hat{\zeta} \to \infty$ numerically.

Ensembles

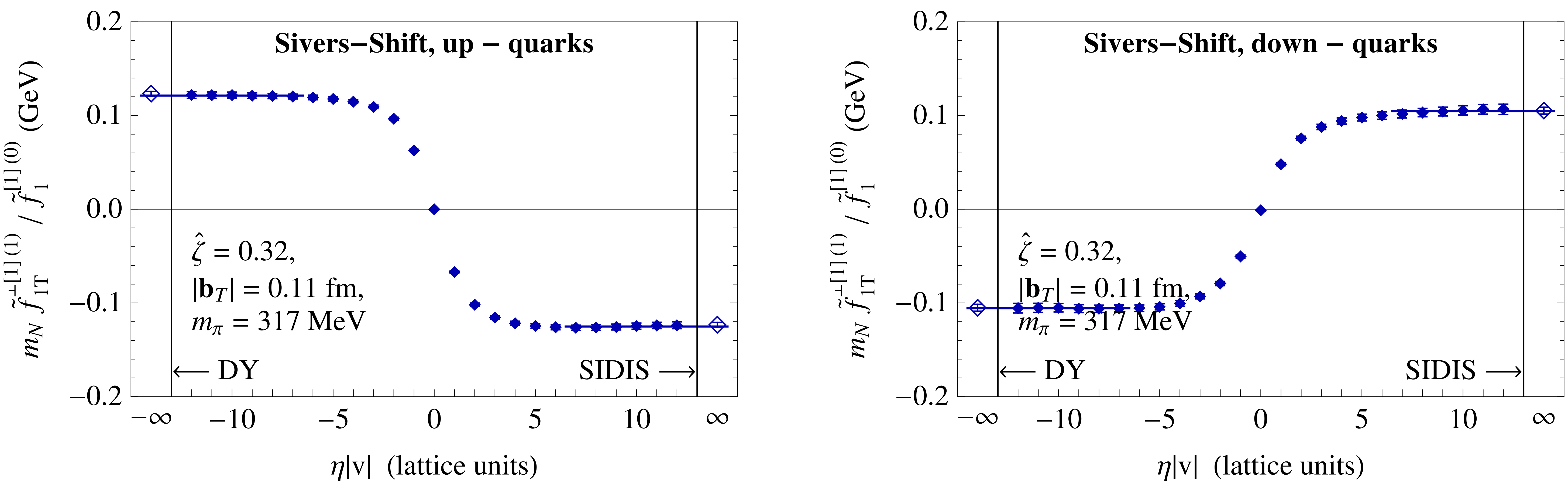
Primary data: Clover fermions, $m_\pi = 317\,\mathrm{MeV}$, $a = 0.114\,\mathrm{fm}$
produced by K. Orginos and JLab collaborators

Auxiliary data: Domain wall fermions, $m_\pi = 297\,\mathrm{MeV}$, $a = 0.084\,\mathrm{fm}$
produced by RBC/UKQCD

Restricted range of $\hat{\zeta}$ up to $\hat{\zeta} \sim 0.3 \ldots 0.4$

Results: Sivers shift

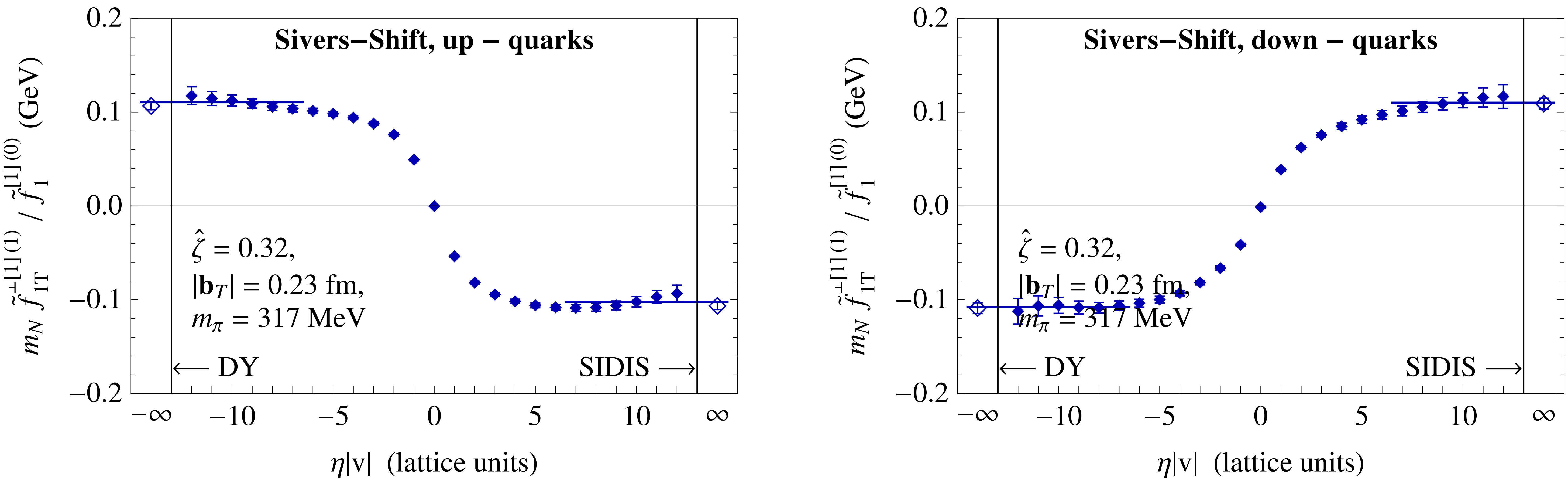
Dependence on staple extent; sequence of panels at different $|b_T|$



NOTE: Up-quark data are normalized to give the contribution from 1 up quark

Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



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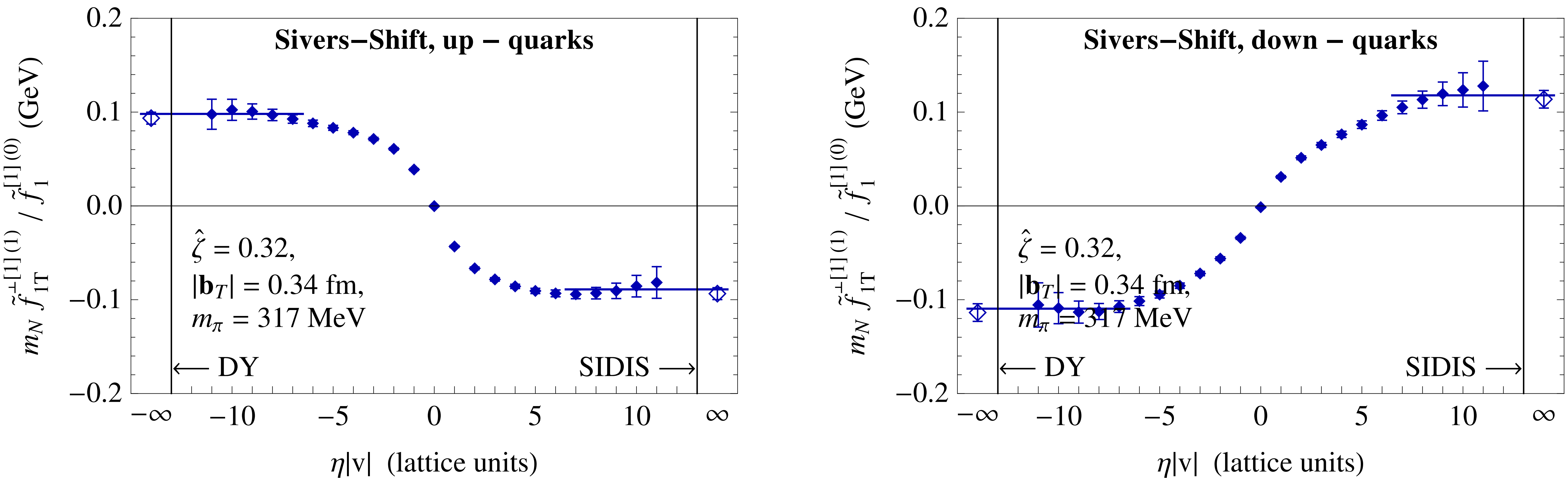

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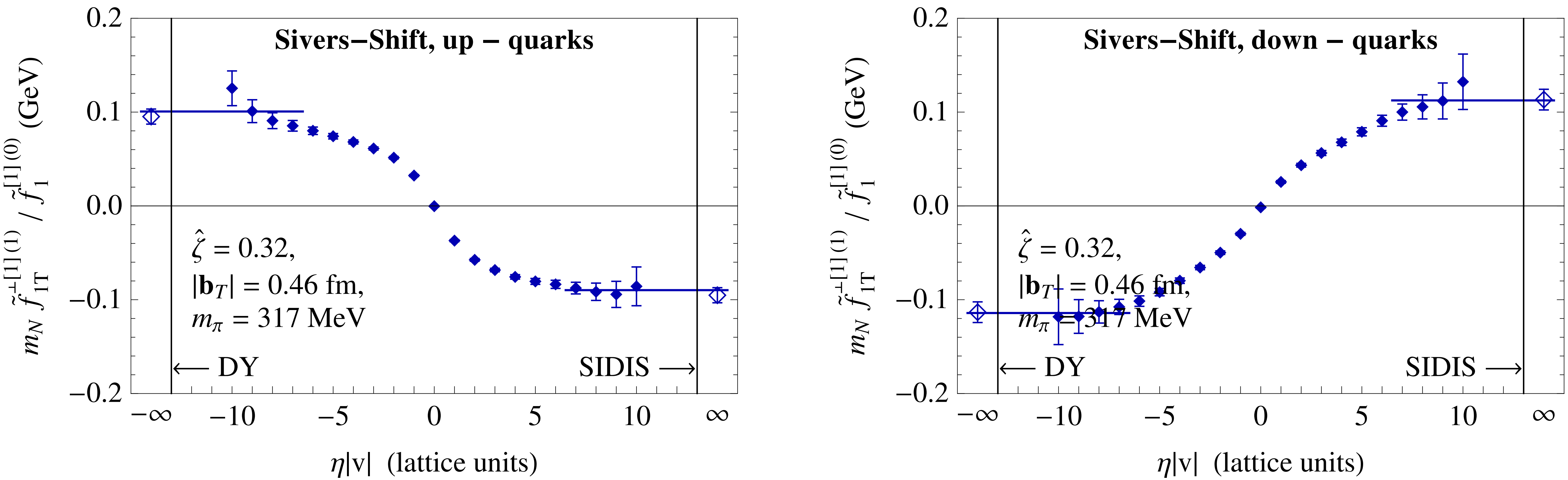
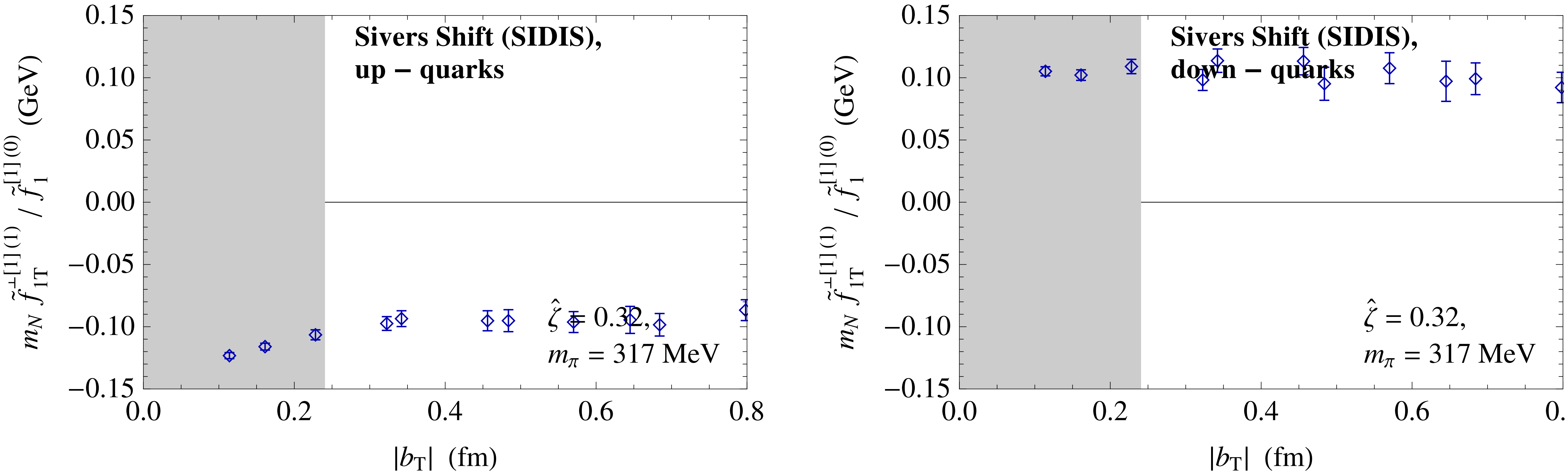


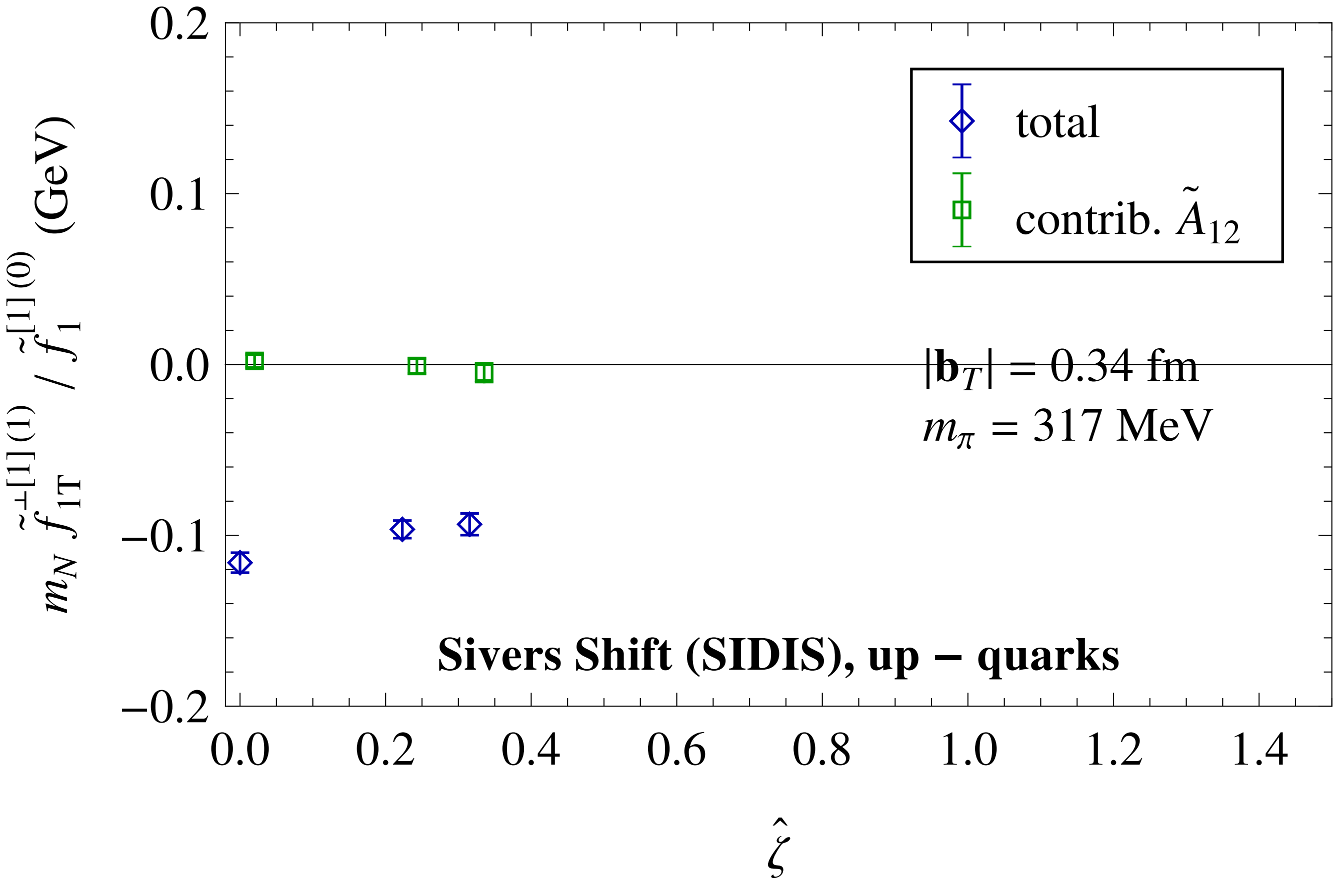
Results: Sivers shift

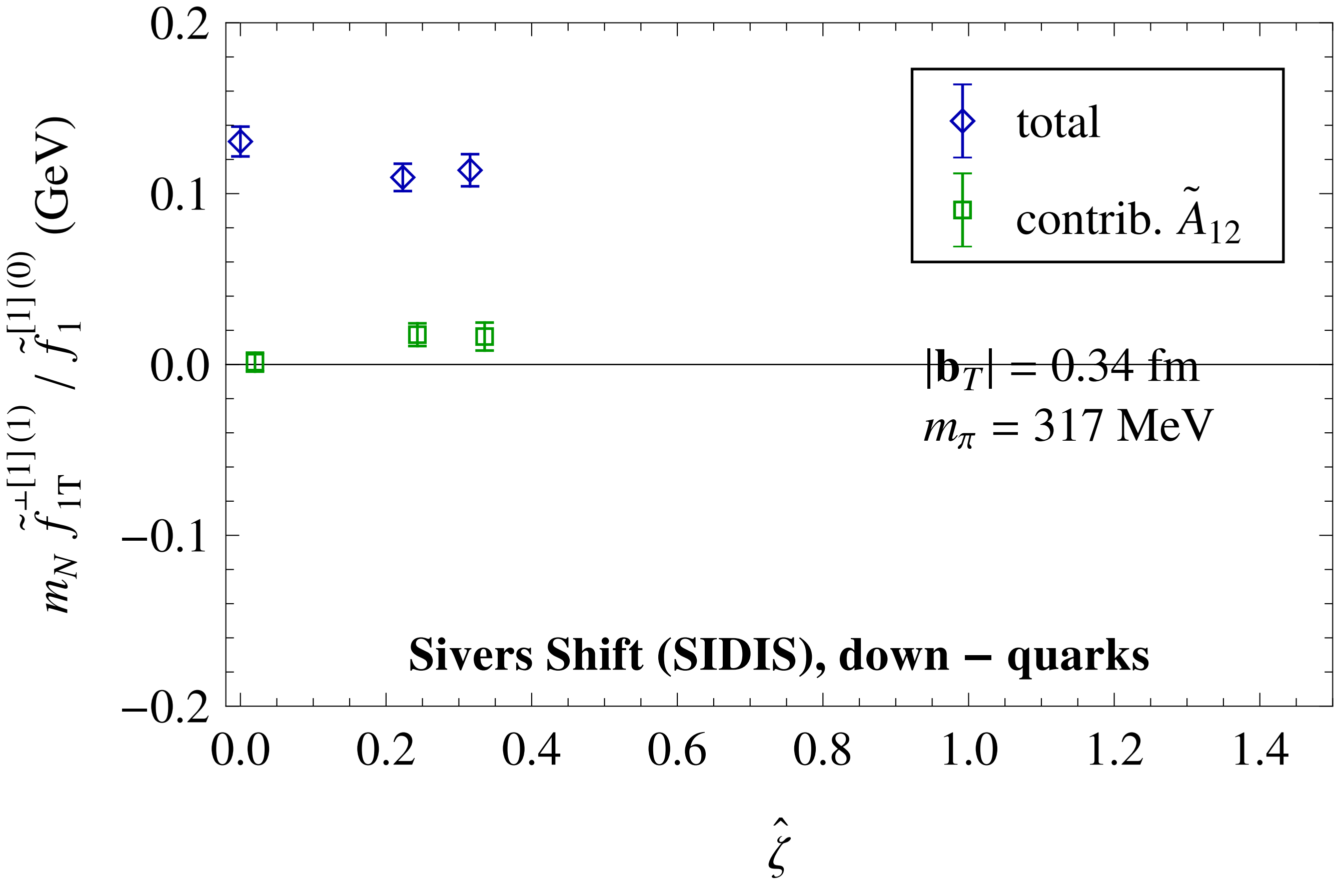
Dependence of SIDIS limit on $|b_T|$



Results: Sivers shift

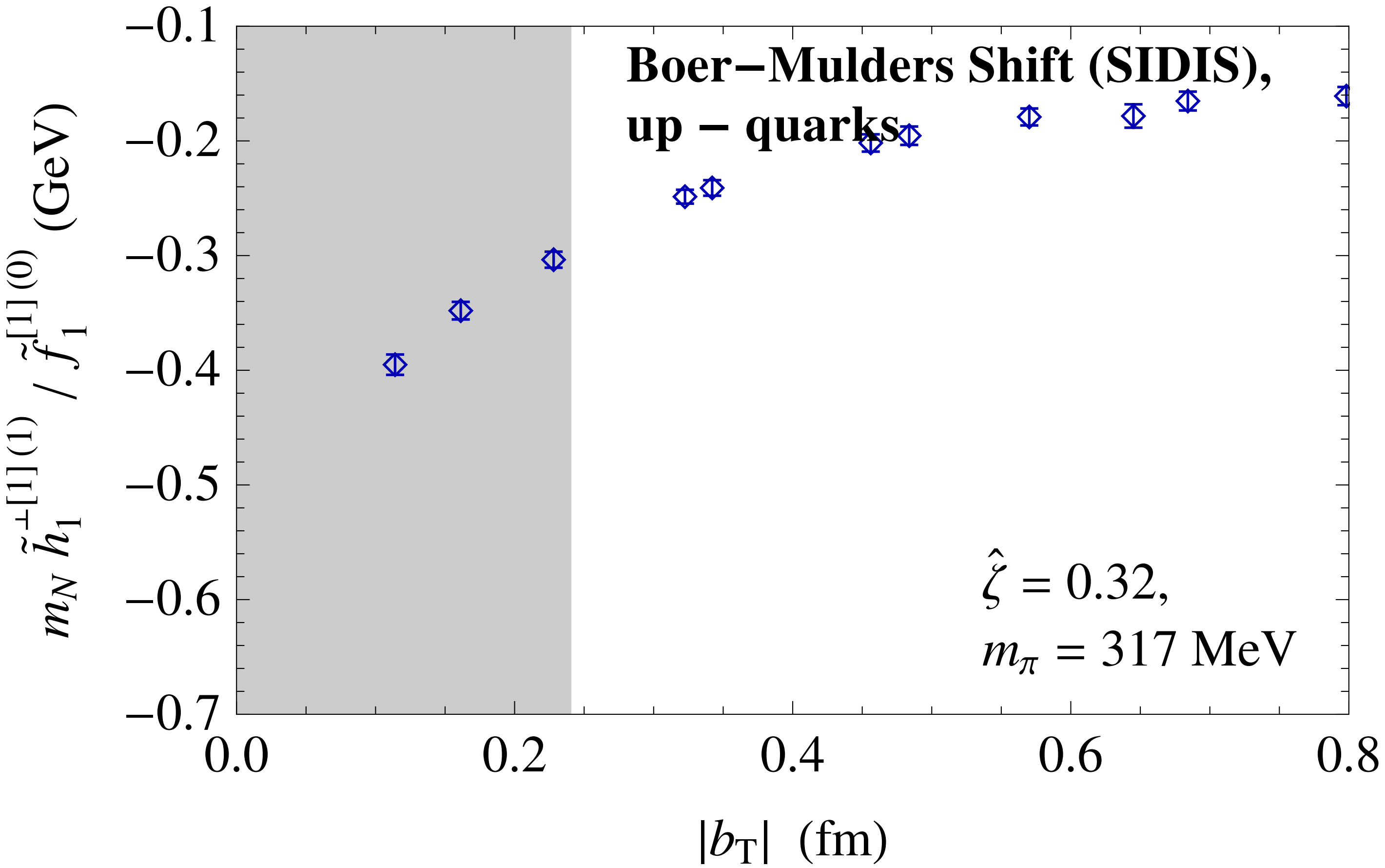
Dependence of SIDIS limit on $\hat{\zeta}$

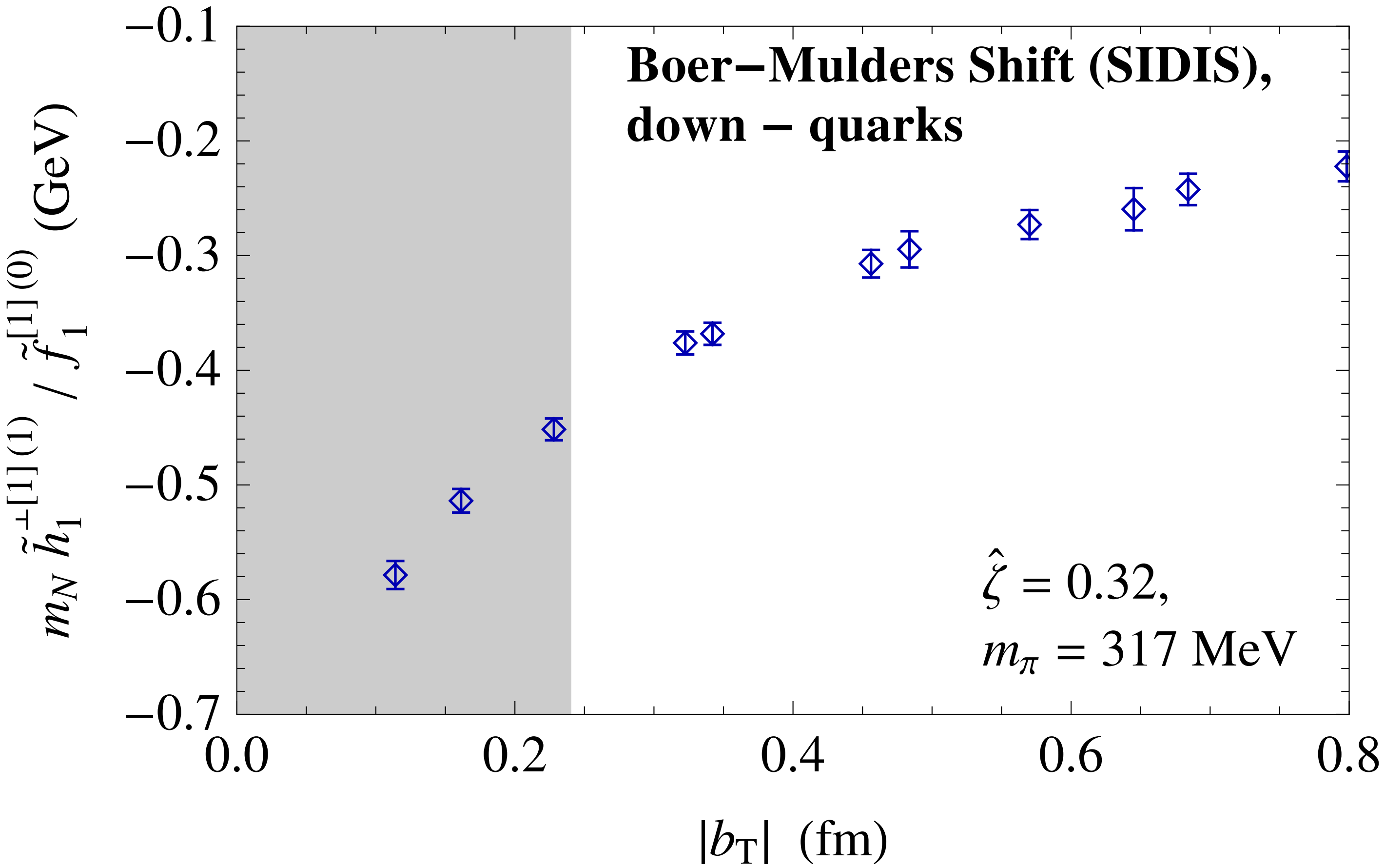




Results: Boer-Mulders shift

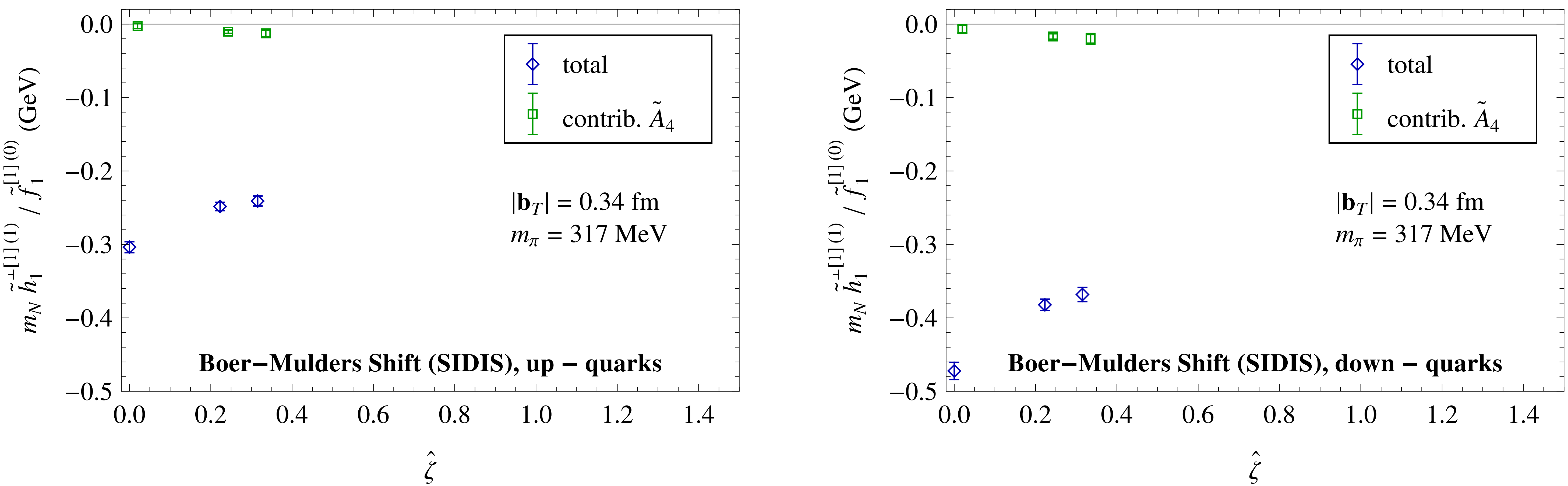
Dependence of SIDIS limit on $|b_T|$





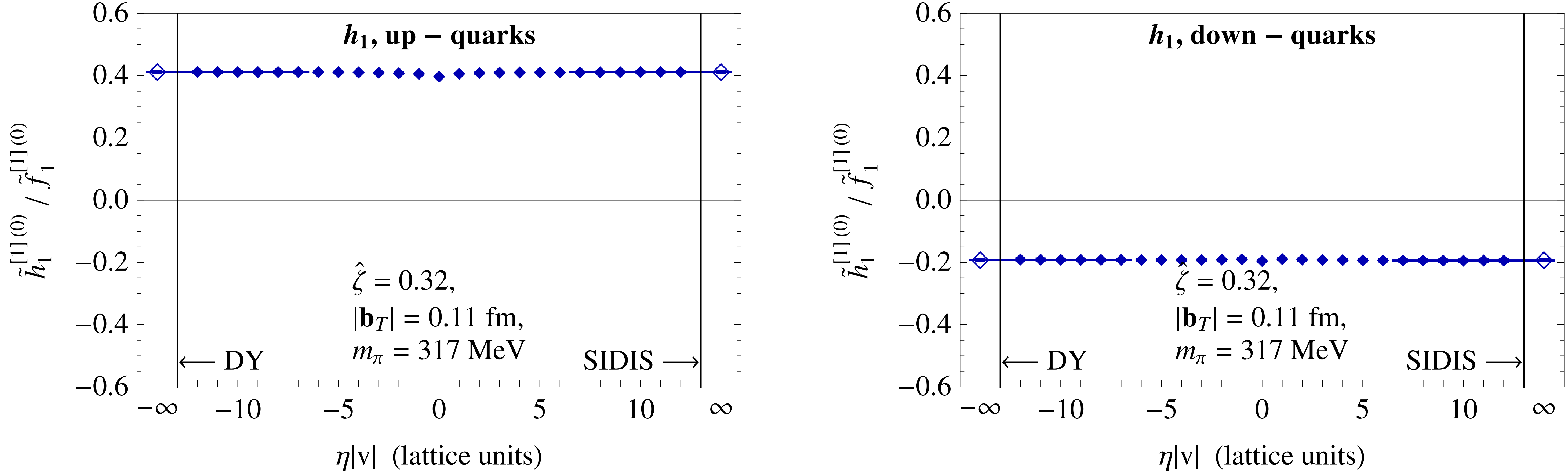
Results: Boer-Mulders shift

Dependence of SIDIS limit on $\hat{\zeta}$



Results: Transversity

Dependence on staple extent; sequence of panels at different $|b_T|$



Results: Transversity

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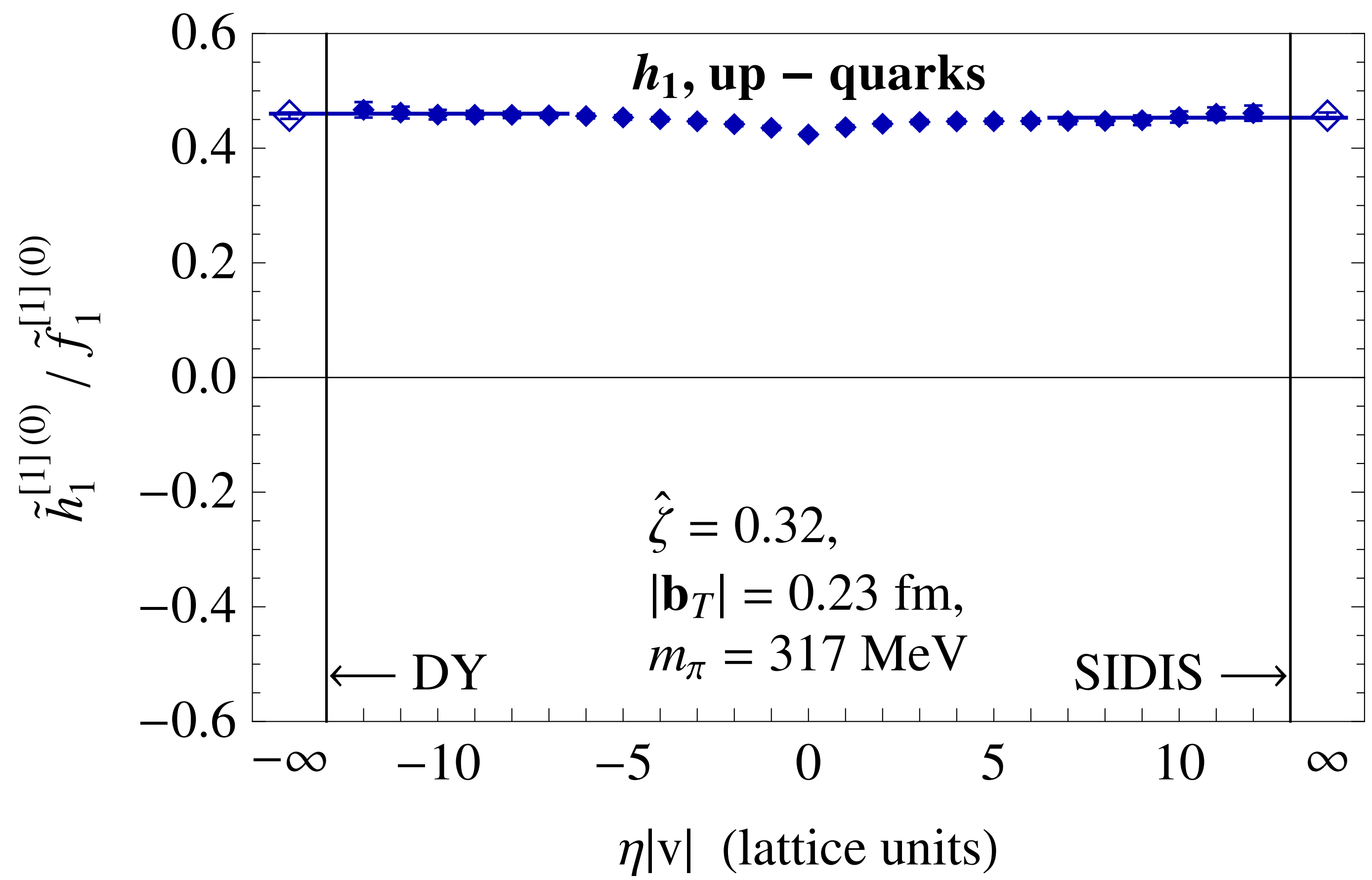

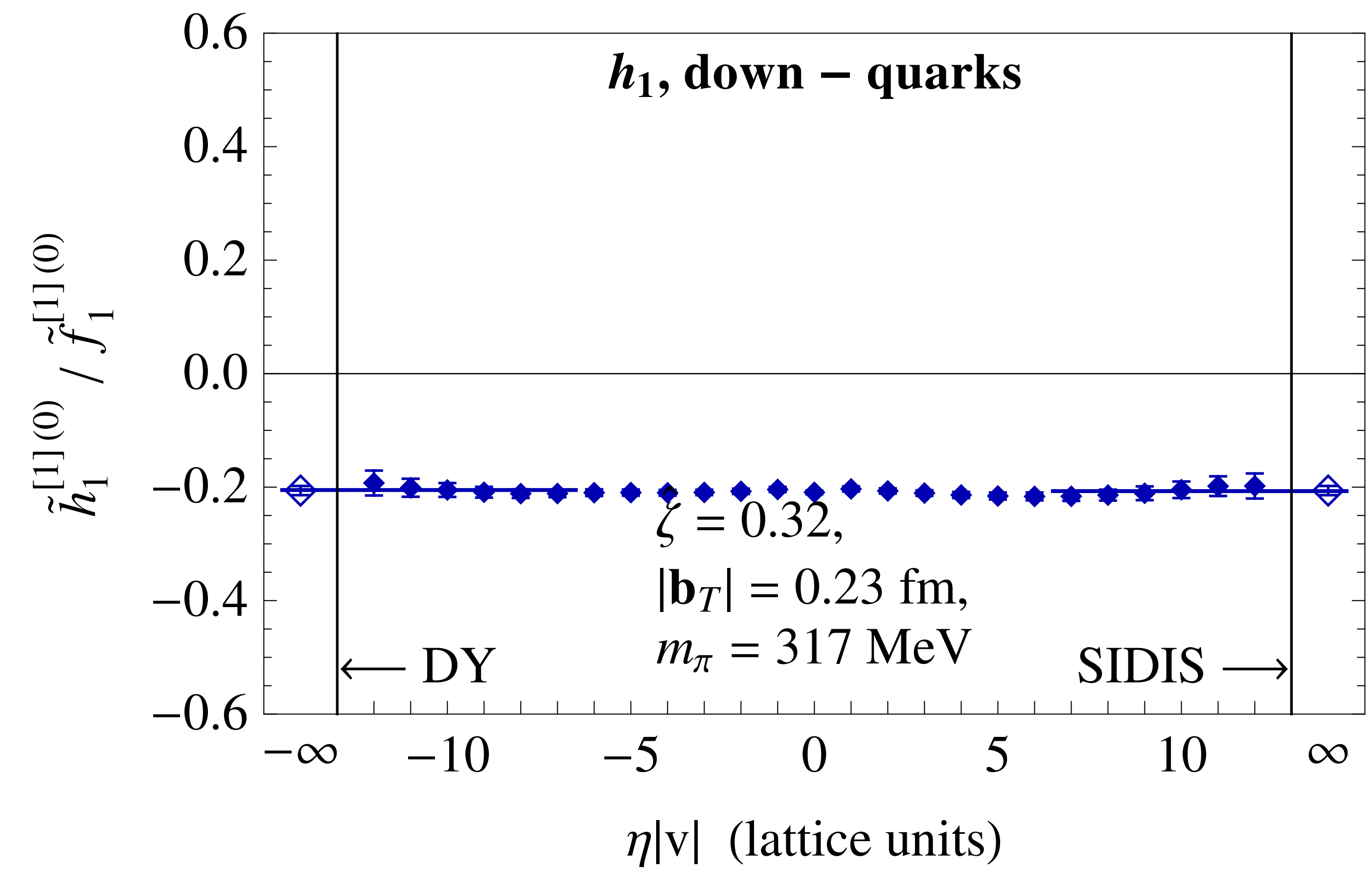

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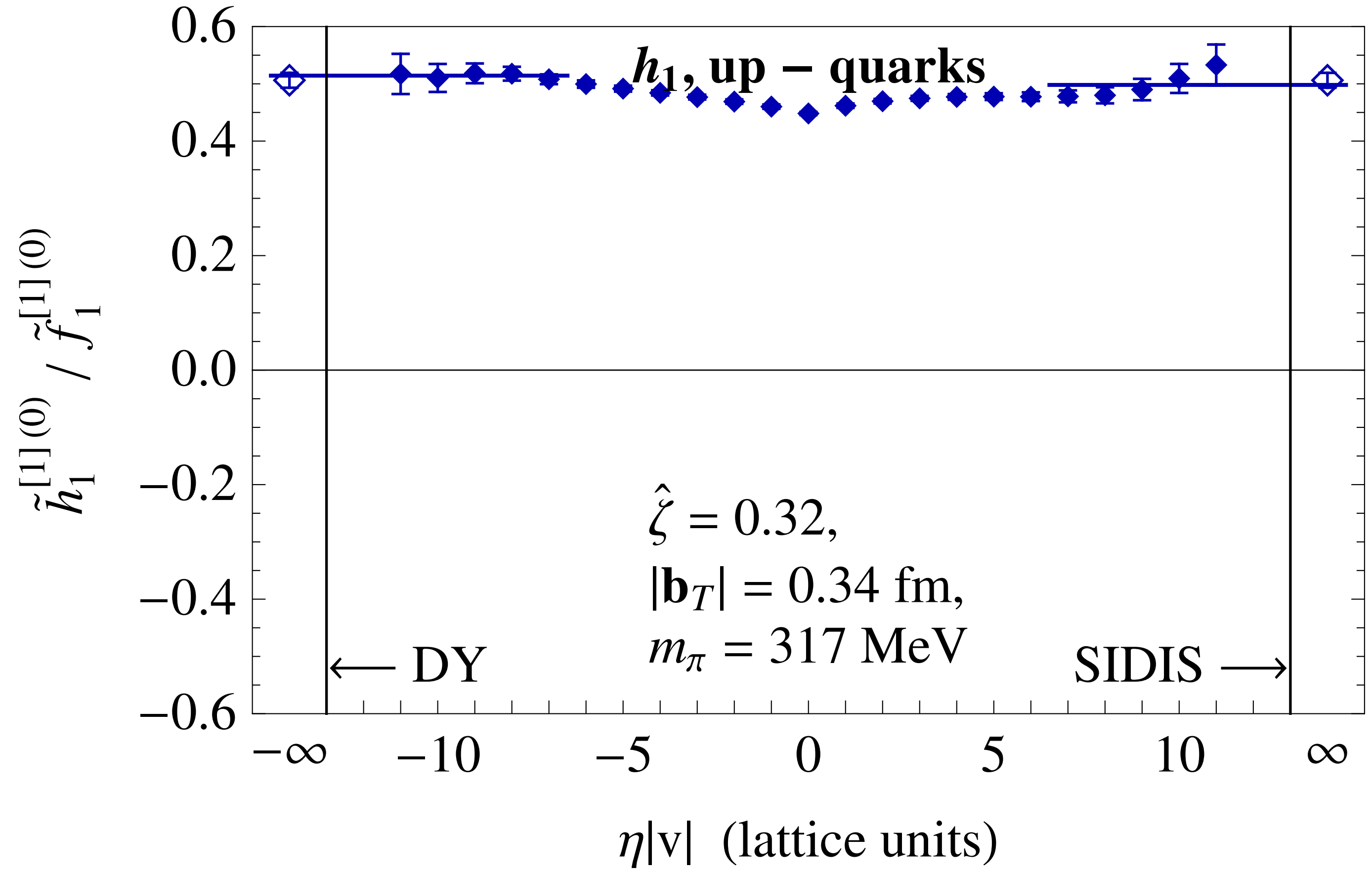

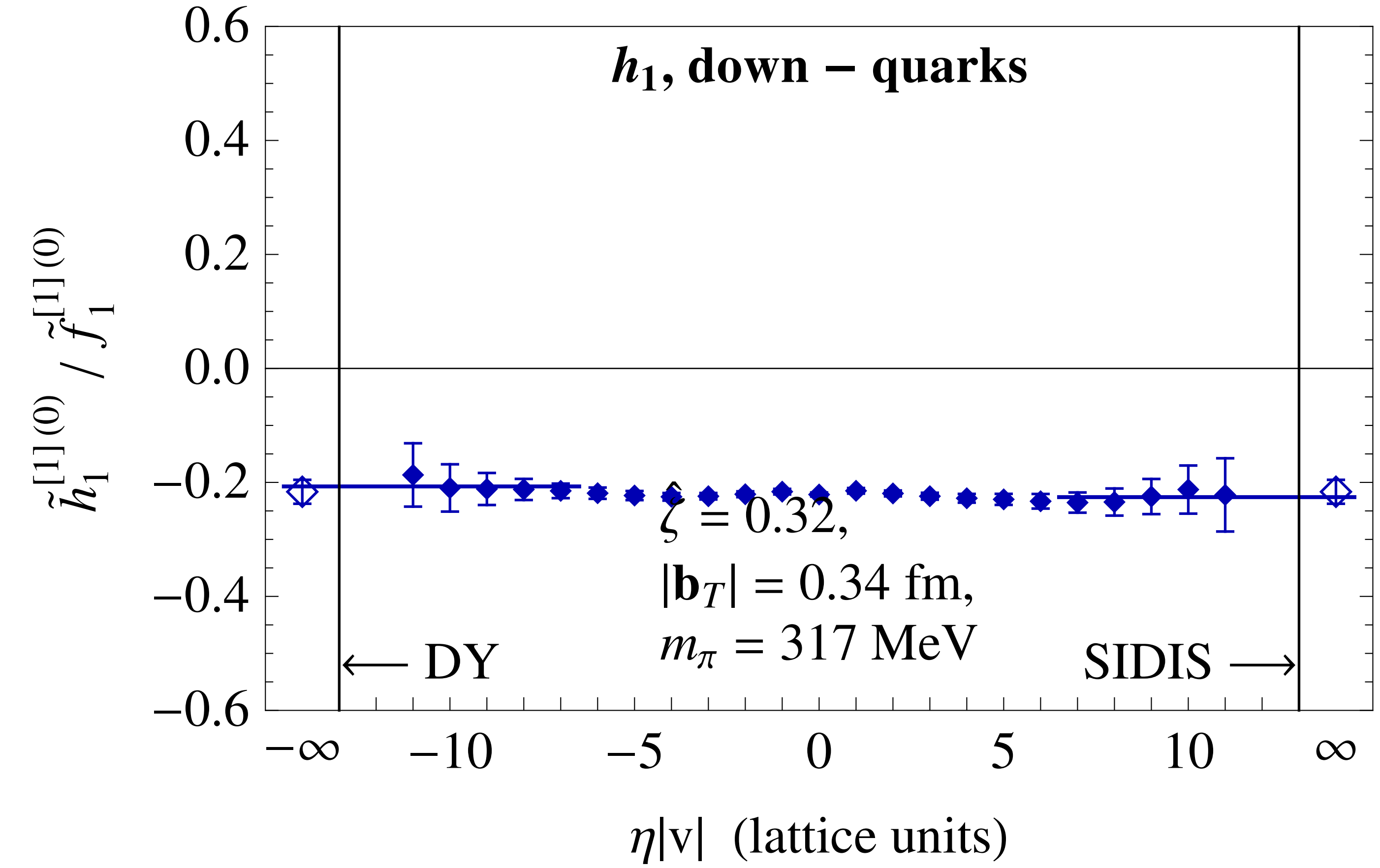

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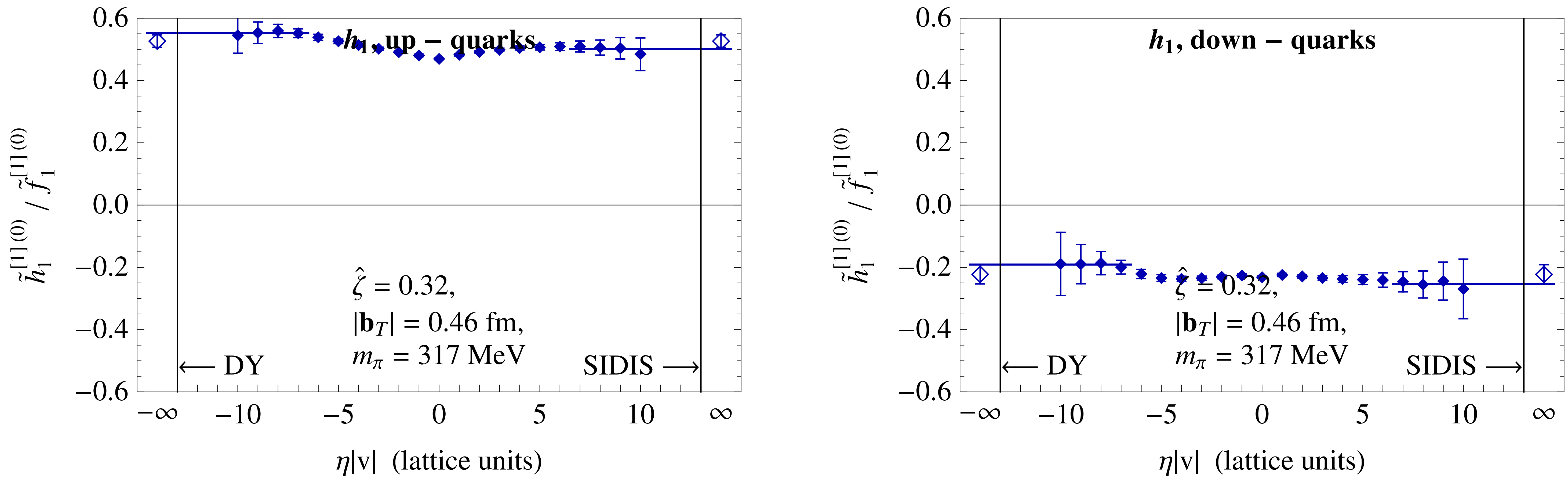
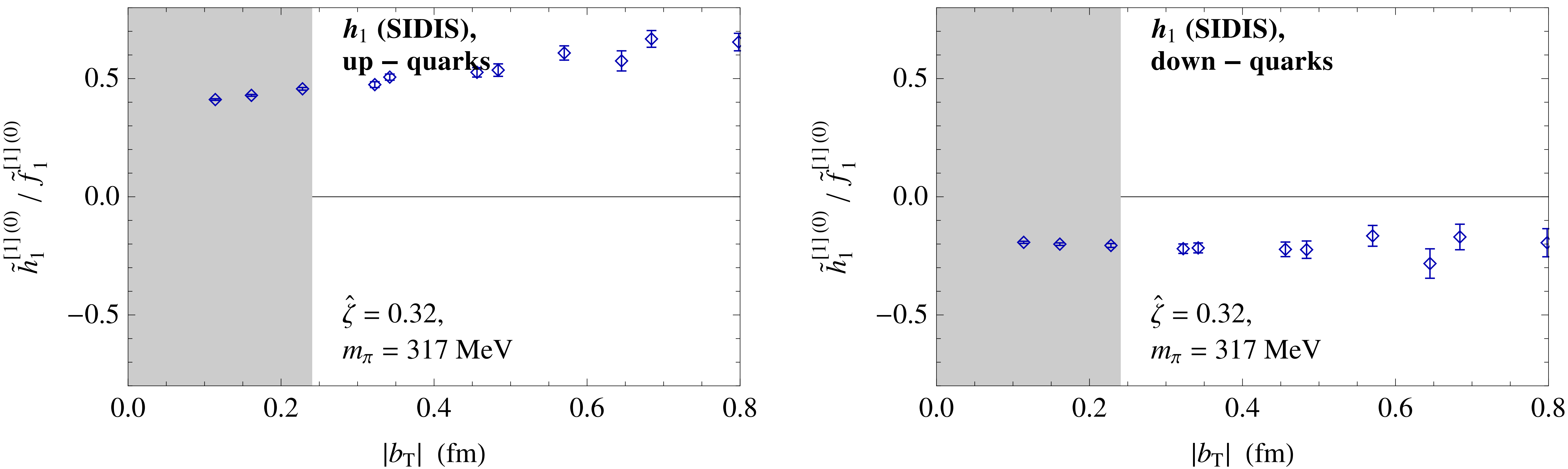


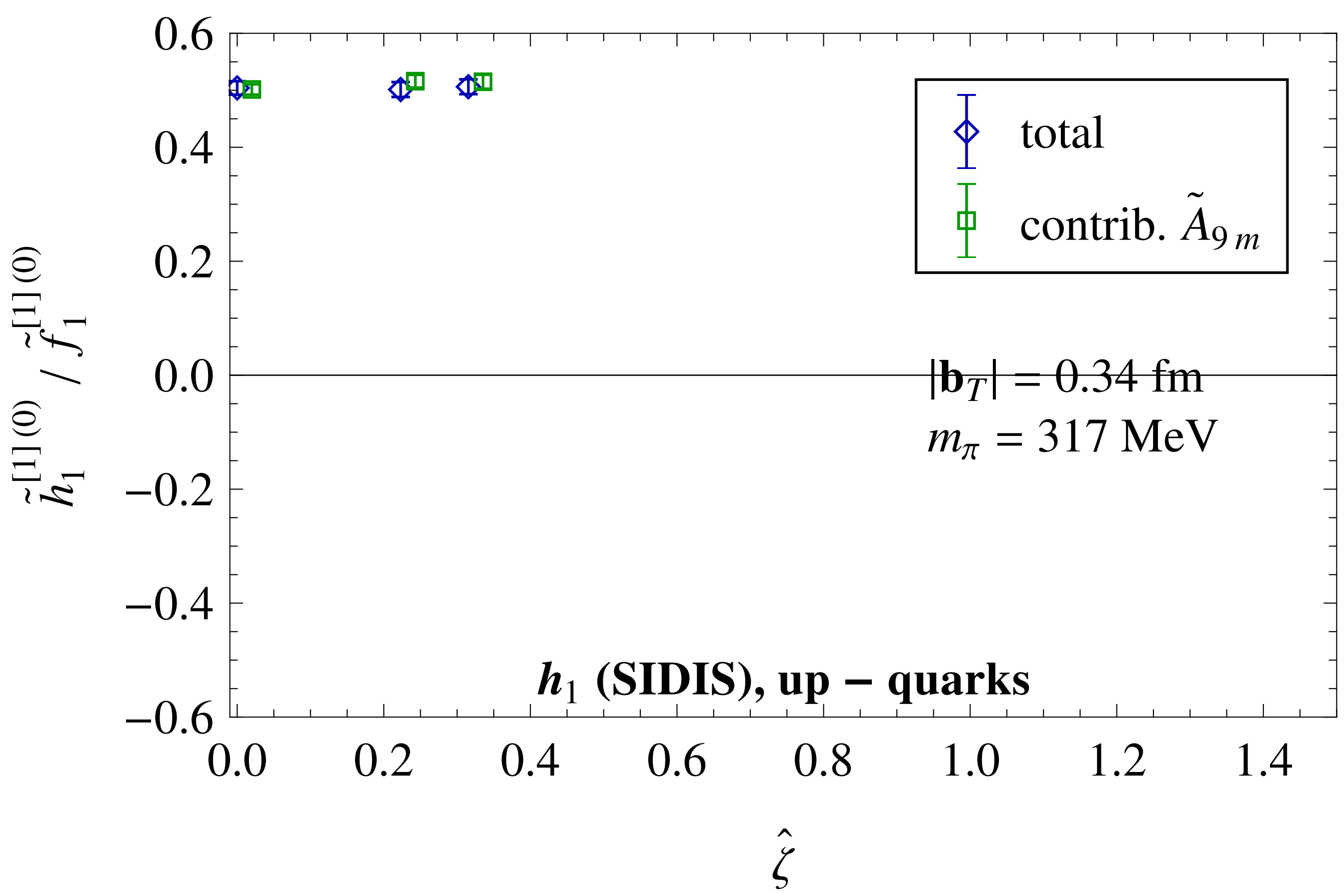
Results: Transversity

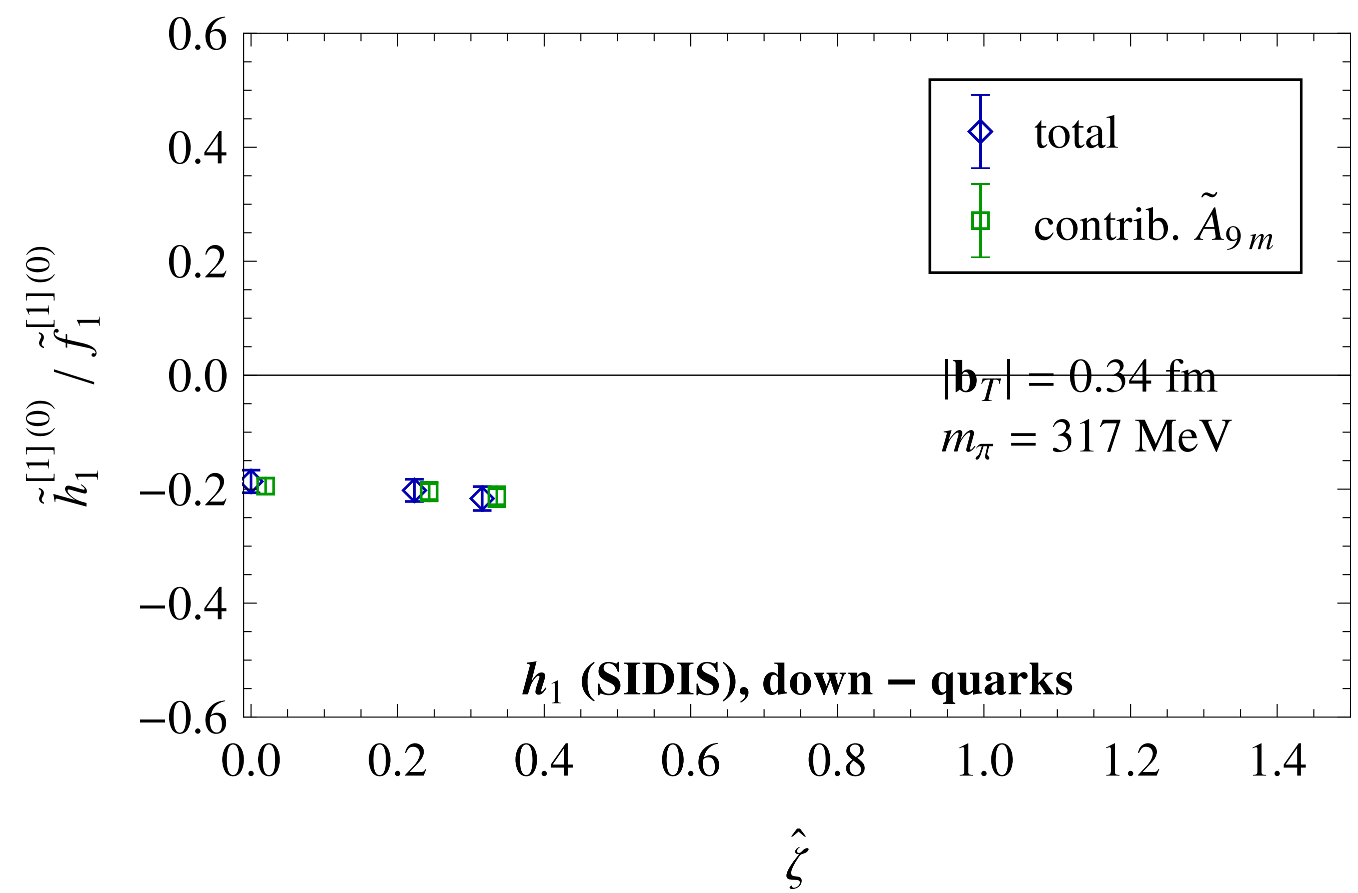
Dependence of SIDIS/DY limit on $|b_T|$



Results: Transversity

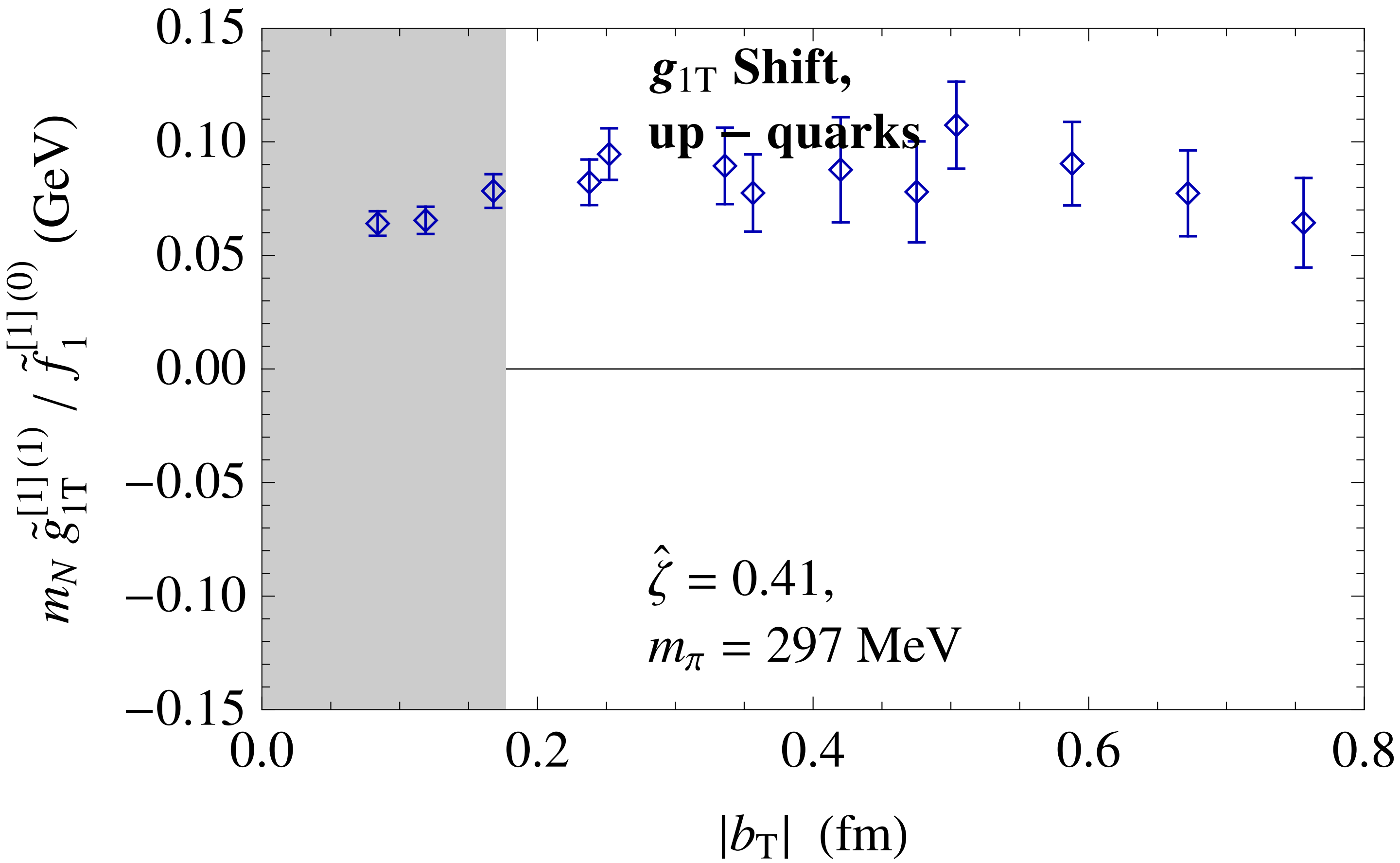
Dependence of SIDIS/DY limit on $\hat{\zeta}$

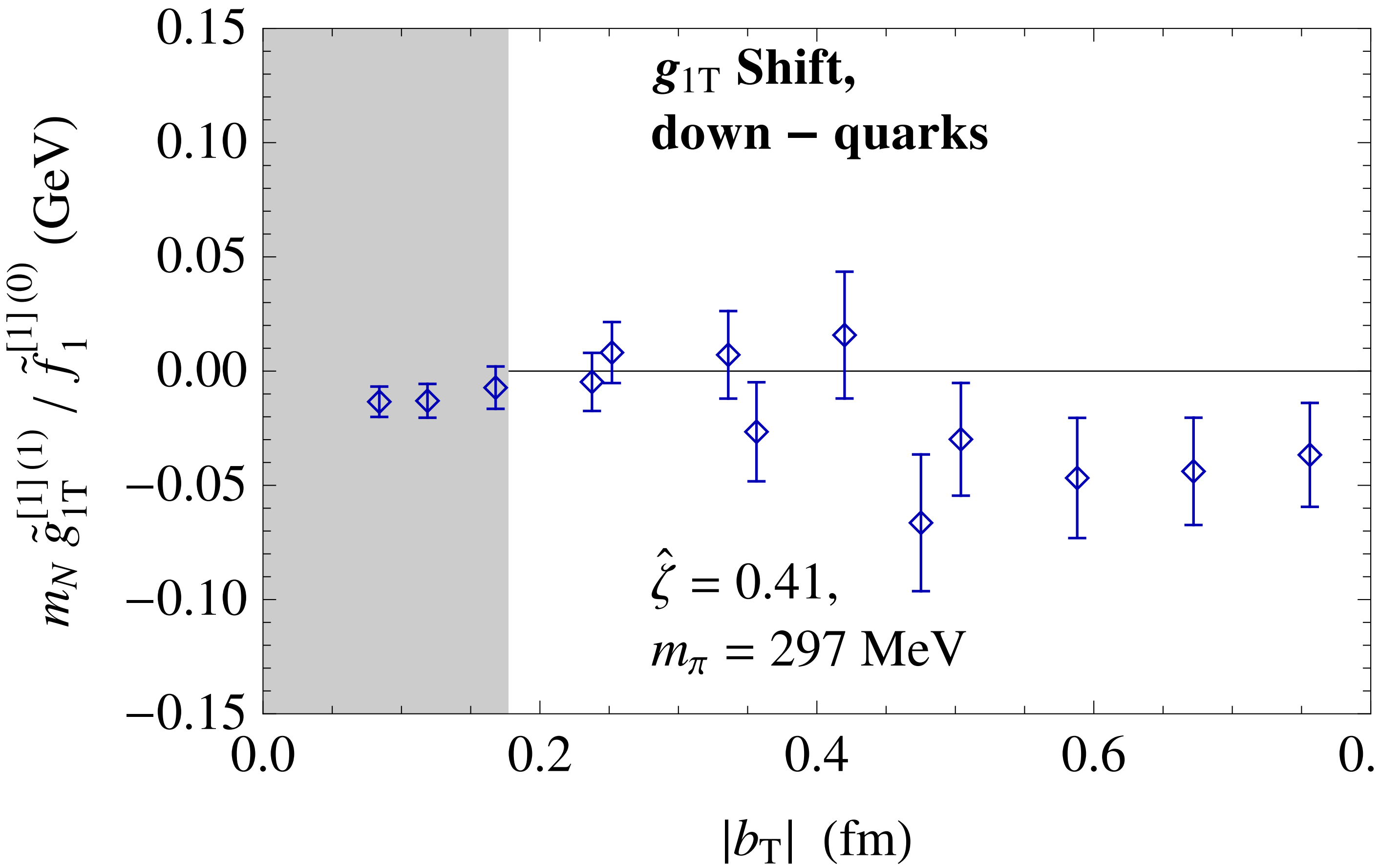




Results: g_{1T} worm gear shift

Dependence of SIDIS/DY limit on $|b_T|$





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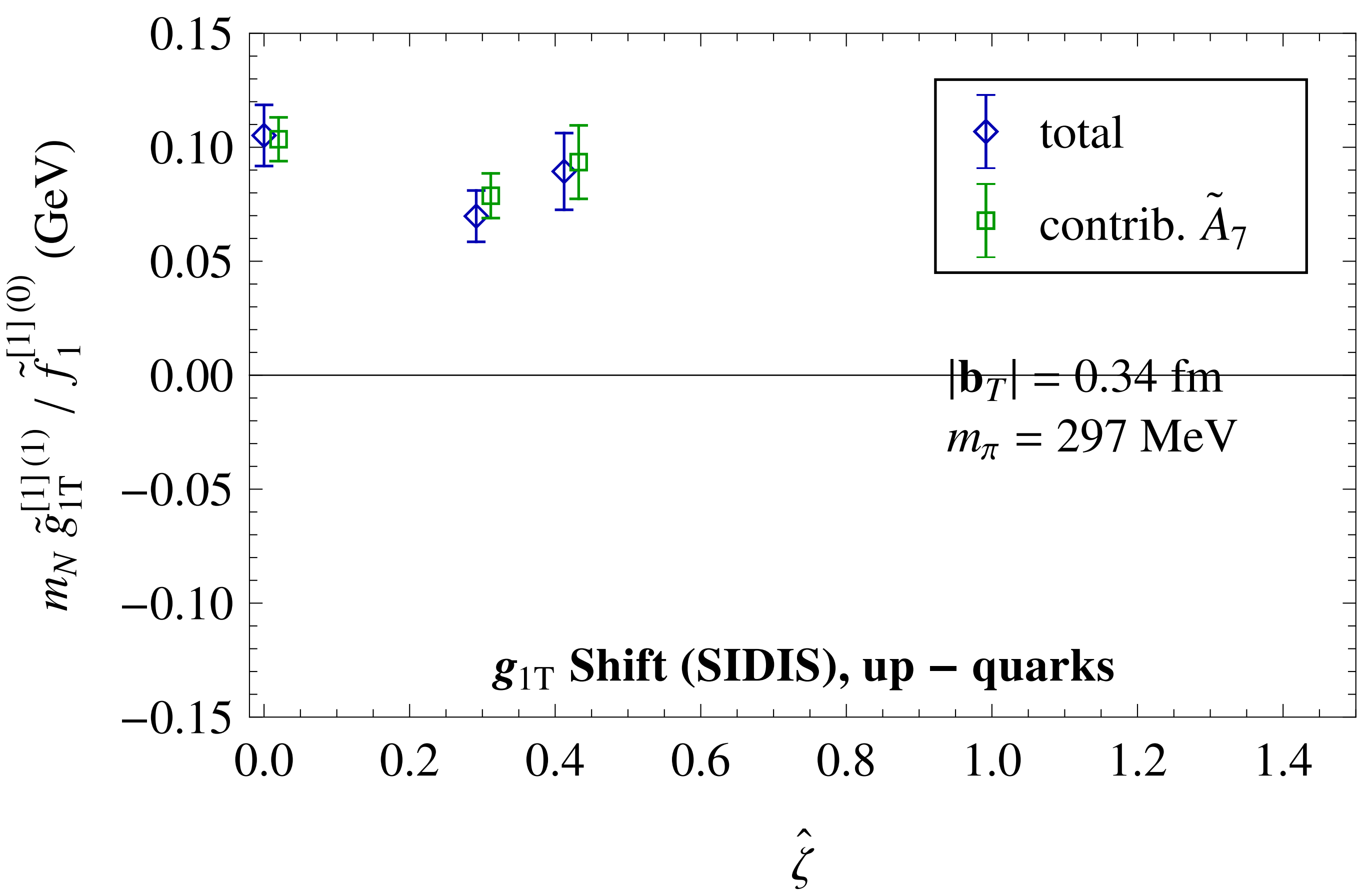
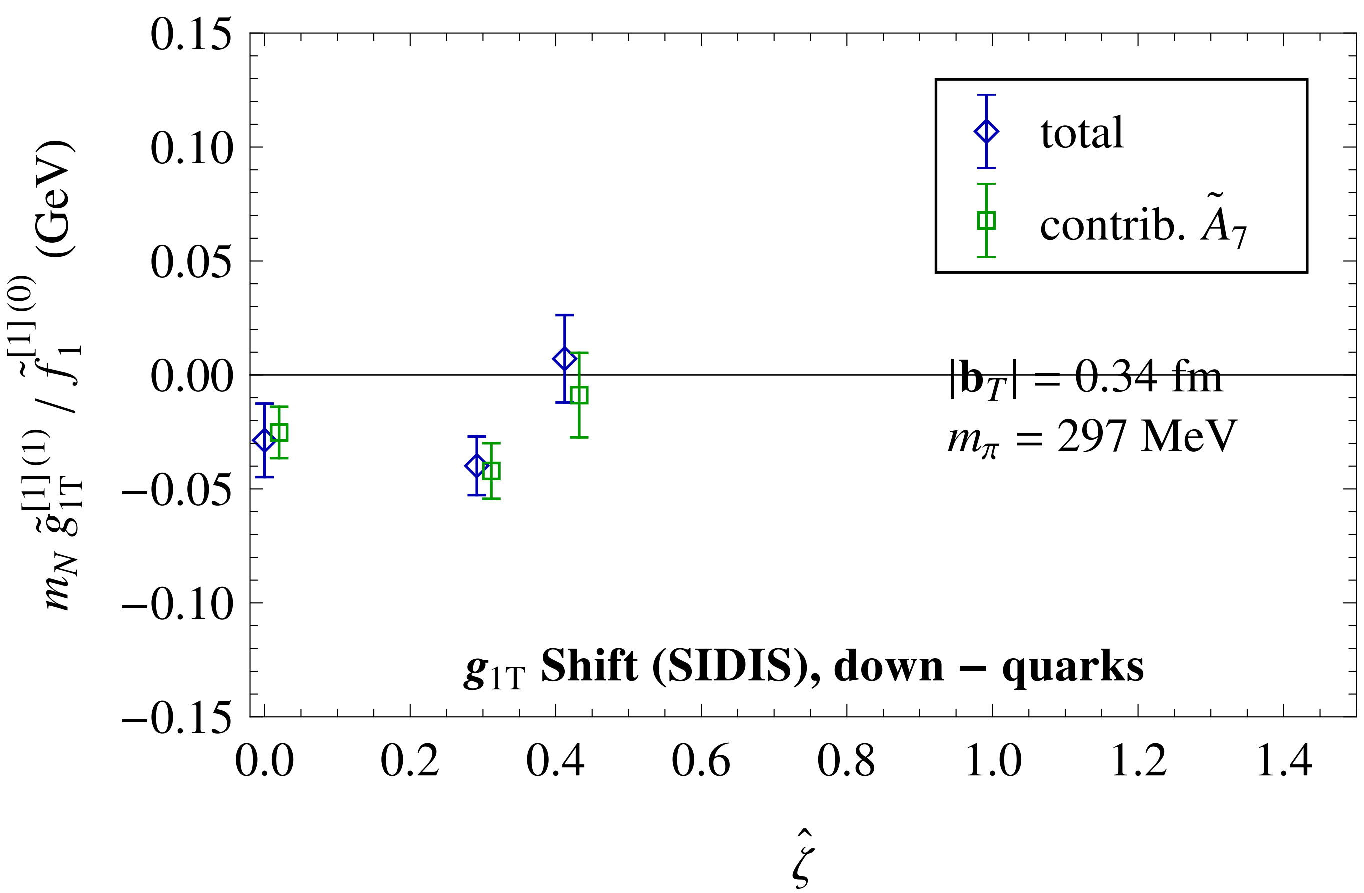
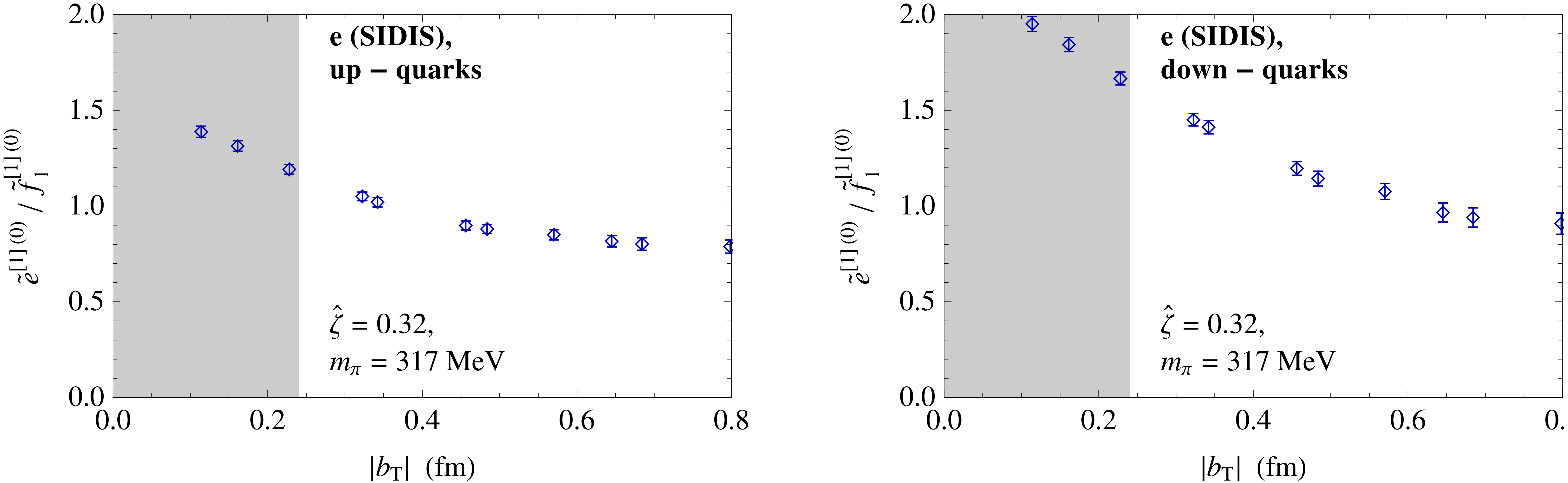




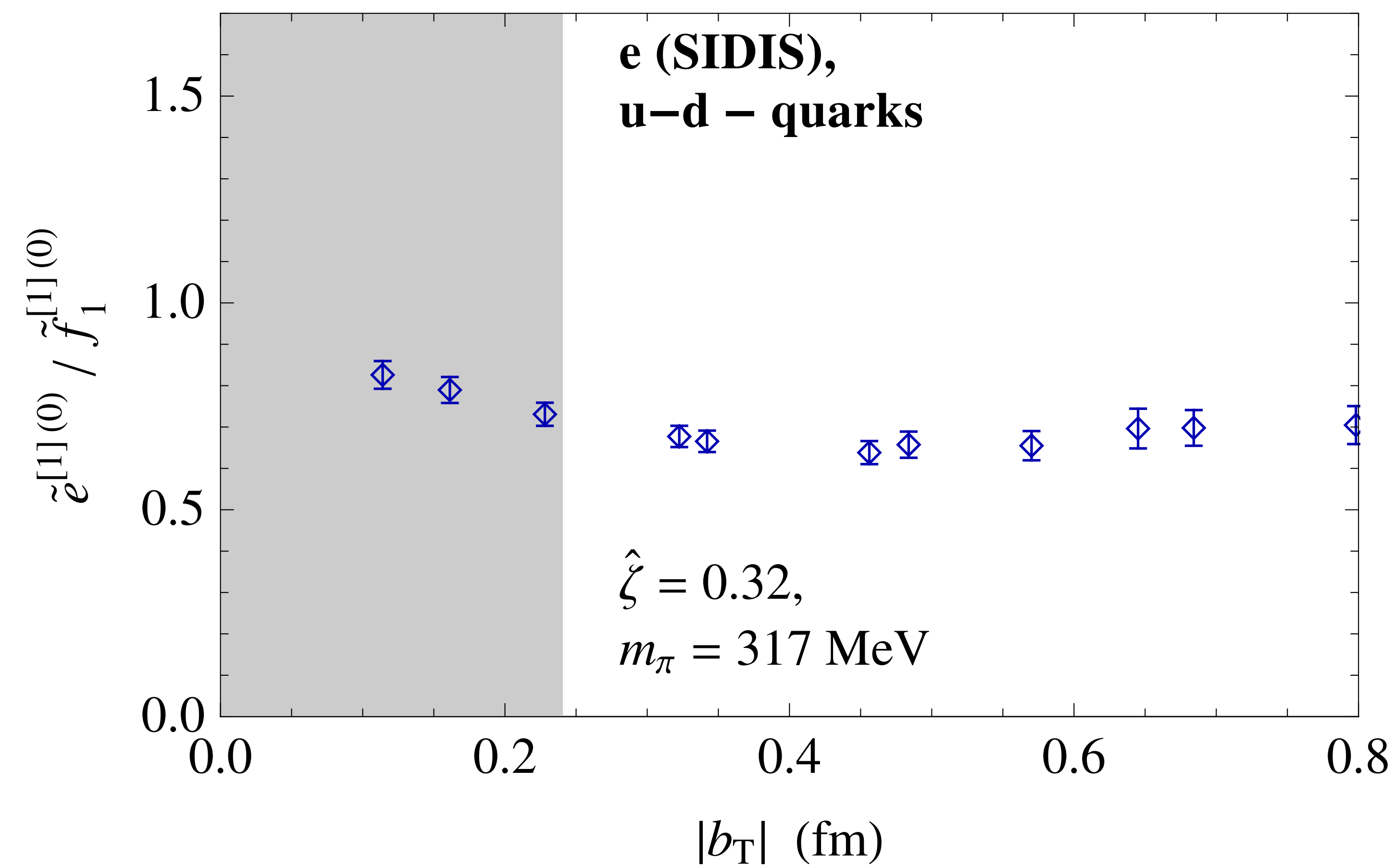
Results: Twist-three unpolarized chiral-odd TMD e

Dependence of SIDIS/DY limit on $|b_T|$



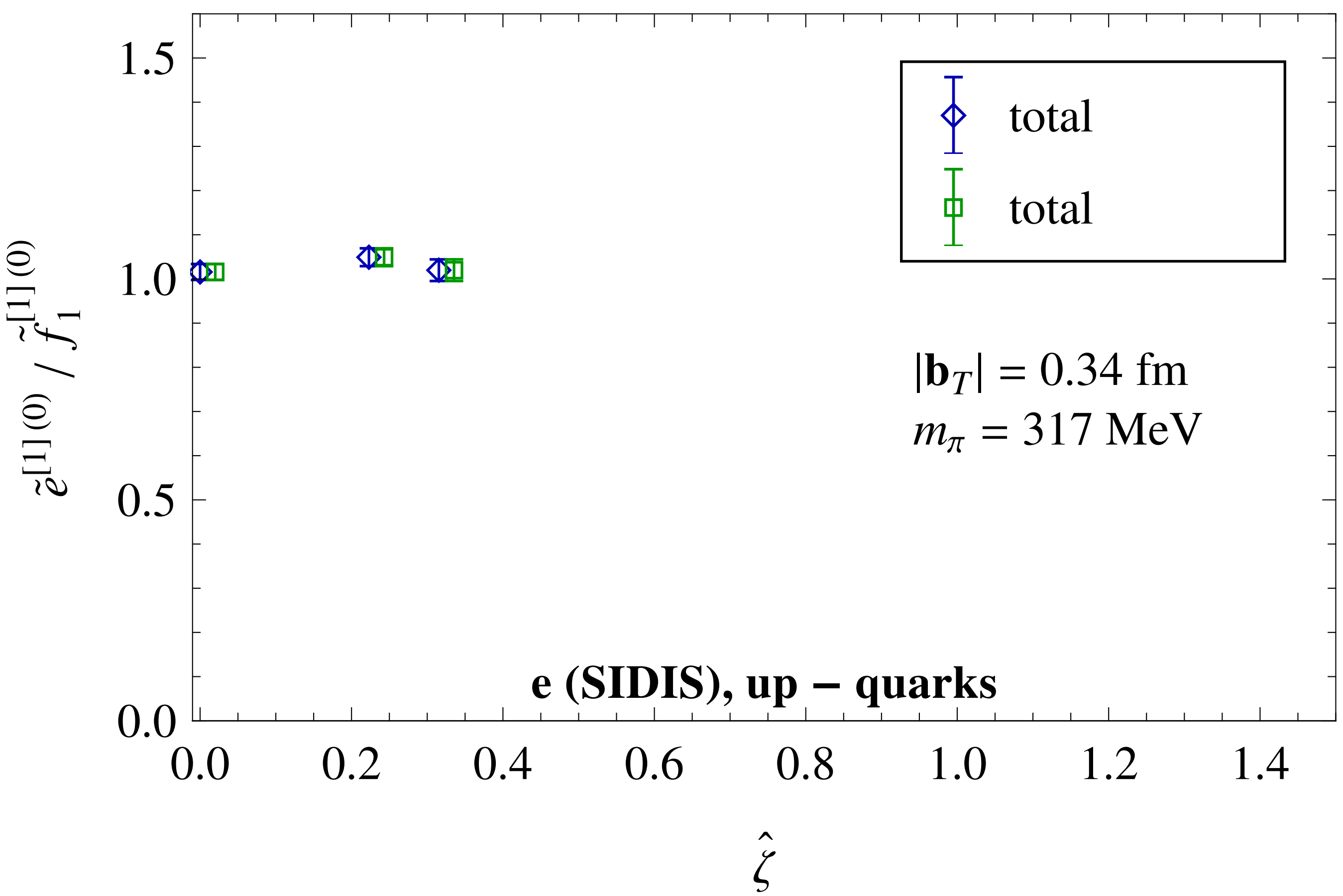
Results: Twist-three unpolarized chiral-odd TMD e − isovector combination

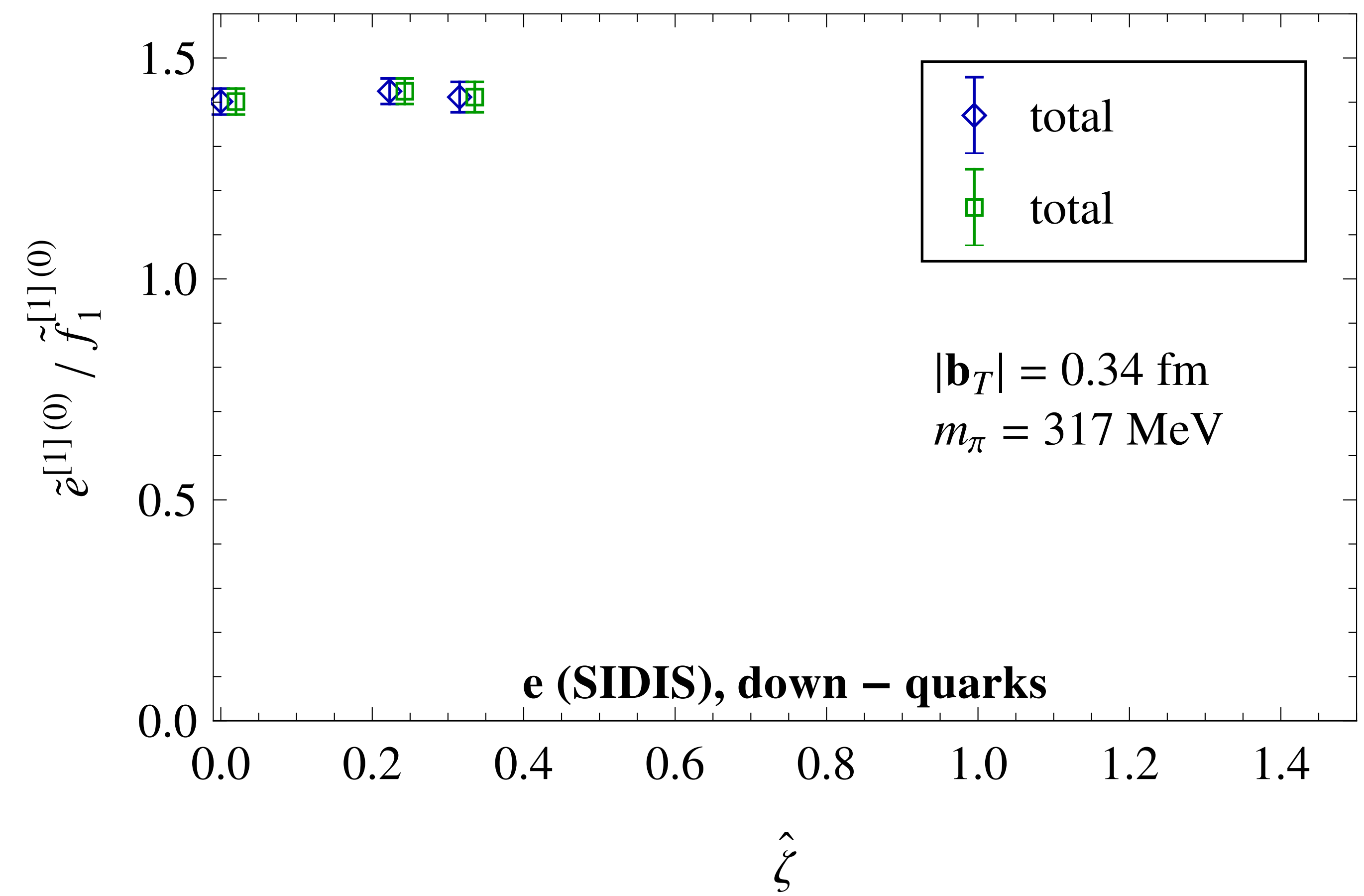
Dependence of SIDIS/DY limit on $|b_T|$



Results: Twist-three unpolarized chiral-odd TMD e

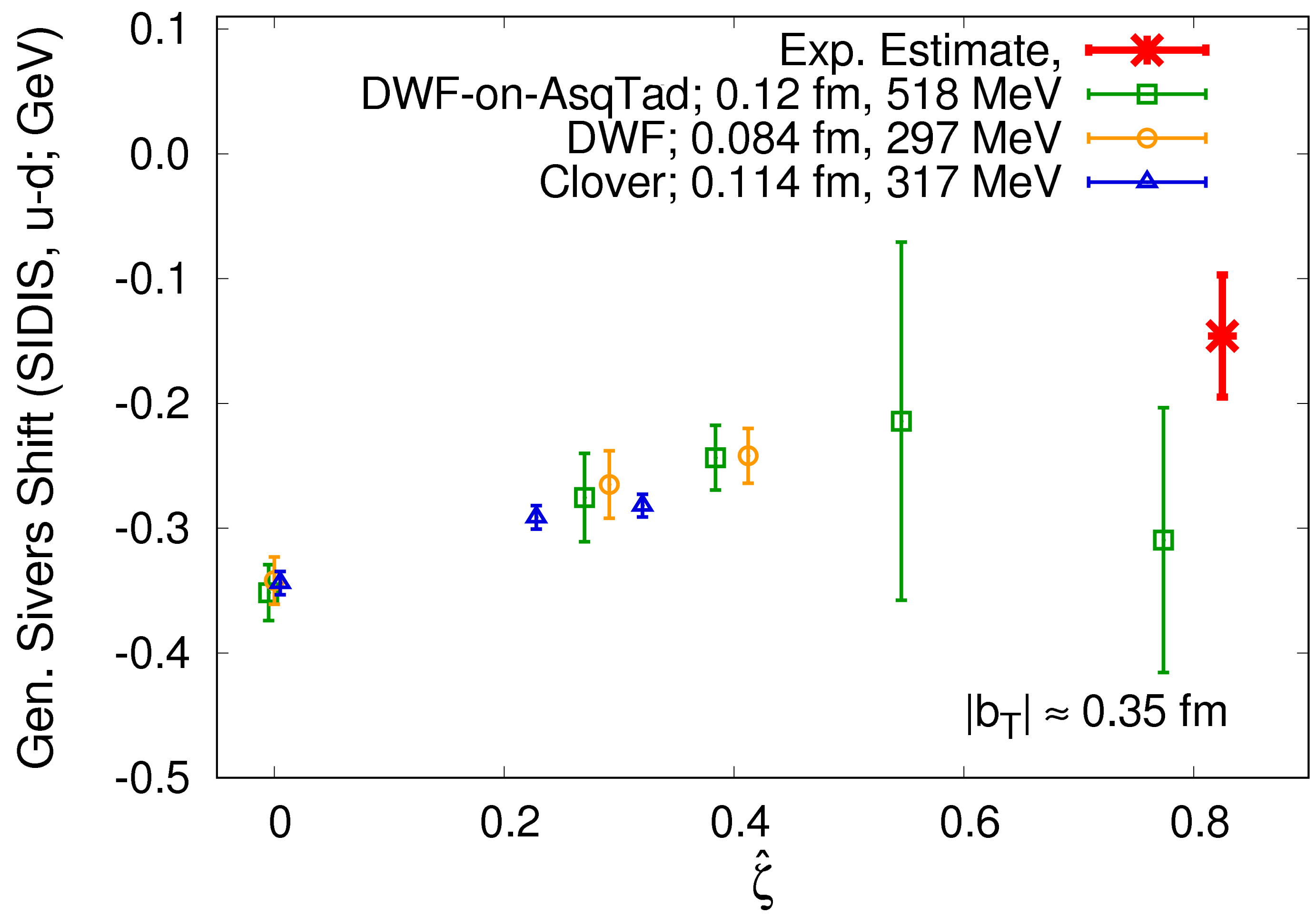
Dependence of SIDIS/DY limit on $\hat{\zeta}$





Results: Sivers shift summary

Dependence of SIDIS limit on $\hat{\zeta}$



Experimental value from global fit to HERMES, COMPASS and JLab data,
M. Echevarria, A. Idilbi, Z.-B. Kang and I. Vitev, Phys. Rev. D 89 (2014) 074013

Summary and Outlook

- Continued exploration of TMD and GTMD observables in Lattice QCD using bilocal quark operators with staple-shaped gauge link structures.
- Connected contributions to flavor-separated TMD observables at 300 MeV pion mass presented, including transverse polarization leading twist and twist-three TMD e.
- On deck: Dependence of the Sivers shift on momentum fraction x; improved evaluation of quark orbital angular momentum.
- Current data production at physical pion mass. Will also include longitudinal polarization TMD and GTMD observables, large hadron momenta (boosted sources).