Lattice Calculation of PDF from LaMET at Physical Pion Mass and Large Nucleon Momentum

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Talk based on:

X. Ji, J.-H. Zhang, and Y.Z., Phys. Rev. Lett. 120, 112001 (2018), arXiv: 1706.08962

J.-W. Chen, T. Ishikawa, L. Jin, H.-W. Lin, Y.-B. Yang, J.-H. Zhang, and Y.Z., Phys. Rev. D 97, 014505 (2018), arXiv:1706.01295

I. Stewart and Y.Z., Phys. Rev. D 97, 054512 (2018), arXiv:1709.04933

T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., arXiv:1801.03917

J.-W. Chen, L. Jin, H.-W. Lin, Y.-S. Liu, Y.-B. Yang, J.-H. Zhang, and Y.Z., arXiv: 1803.04393

Outline

- Large momentum effective theory
  - Formalism
  - Procedure of a systematic calculation
- Lattice results

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  - Formalism
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Motivation for a first-principle calculation of PDFs

- Currently our best knowledge of the PDFs comes from the global analysis of high-energy scattering data

1. Extensive experimental analysis motivates a first principle calculation for comparison;

2. First principle calculation might be able to shed light on kinematic regions and flavor structures where experiments cannot constrain so well;

3. The cost of improving calculations seems to be much smaller than building more expensive experiments.

Operator definition of PDF

- Definition of PDFs in QCD factorization theorems:

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P | \bar{\psi}(\xi^-)\gamma^+ U(\xi^-,0)\psi(0) | P \rangle$$

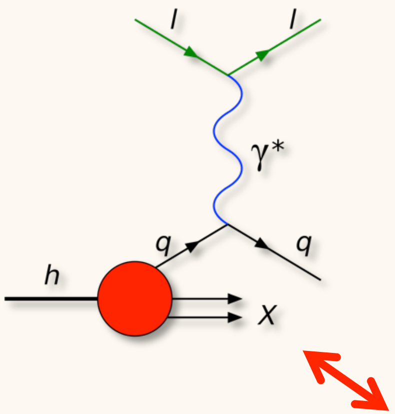
$$\sigma = \sum_{a,b} f_a(x_1) \otimes f_b(x_2) \otimes \sigma_{ab}$$

$$\xi^\pm = (t \pm z)/\sqrt{2} \quad U(\xi^-,0) = P\exp\left[-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right]$$

- Gauge-invariant light-cone correlation;
- Boost invariant distribution, independent of *P*;
- In the light-cone gauge $A^+=0$, has a clear interpretation as parton number density,

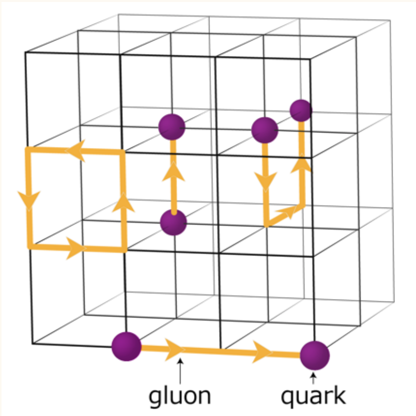
$$q(x) \sim \int dk^+ d^2k_\perp \delta(k^+ - xP^+) \langle P | \hat{n}(k^+,k_\perp) | P \rangle$$

Lattice QCD is the only practical method to solve QCD nonperturbatively so far



Parton model:

- Minkowski space, real time
- Emerges in the IMF, or equivalently, the proton as seen by an observer moving at the speed of light (on the light-cone) $\xi^+ = (t+z)/\sqrt{2} = 0$



PDF not directly accessible from the lattice!

$$e^{iS} \to e^{-S}$$
$$\langle O \rangle = \int D\psi D\bar{\psi} DA\, O(x) e^{-S}$$

Lattice QCD:

- Euclidean space, imaginary time ($t = i\,\tau$)
- Cannot calculate time-dependent quantities generally due to difficulty in analytical continuation in time
- Nucleon static or at finite momentum

Large momentum effective theory

X. Ji, PRL 2013; Sci.China Phys.Mech.Astron. 2014.
Hatta, Ji, Zhao, 2013.

✧ Quasi-PDF:

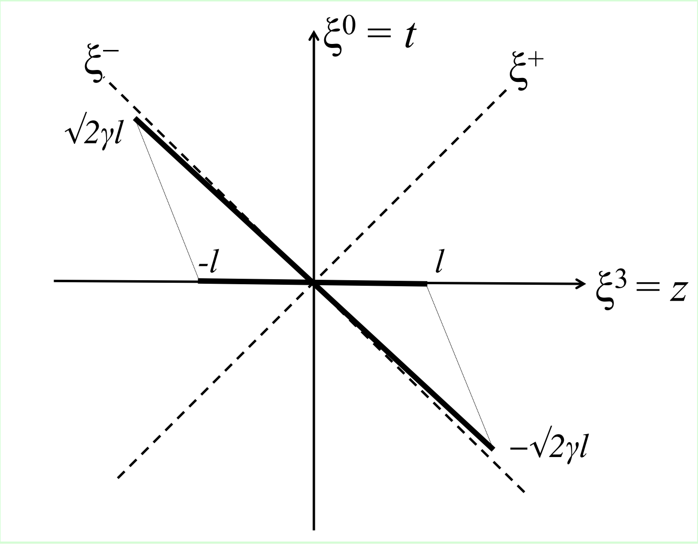
$$\tilde{q}(x,P^z,\Lambda=a^{-1})=\int\frac{dz}{4\pi}e^{ixP^z z}\left\langle P\left|\bar{\psi}(z)\Gamma U(z,0)\psi(0)\right|P\right\rangle$$

$$z^\mu=(0,0,0,z), \qquad \Gamma=\gamma^z \text{ or } \gamma^0$$

$$U(z,0)=P\exp\left[-ig\int_0^z dz' A^z(z')\right]$$

- Equal-time correlation along the z direction, calculable in lattice QCD when $P^z << \Lambda$, and depend on P^z;

- Under an infinite Lorentz boost along the z direction ($P^z >> \Lambda$), the spatial gauge link approaches the light-cone direction, and the quasi-PDF reduces to the (light-cone) PDF.



Large momentum effective theory

- The quasi PDF is related to the PDF through a factorization formula:

$$\tilde{q}(x,P^z,\Lambda)=\int_{-1}^{1}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\Lambda}{\mu},\frac{\mu}{yP^z}\right)q(y,\mu)+O\left(\frac{M^2}{P_z^2},\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$$

Ma and Qiu, 2014;
See also Qiu's talk

- They have the same IR divergences;
- C factor matches their UV difference, and can be calculated in perturbative QCD;
- Power corrections suppressed by large P^z.

Large momentum effective theory

Necessity of a lattice renormalization:

$$\tilde{q}(x,P^z,\Lambda)=\int_{-1}^{1}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\Lambda}{\mu},\frac{\mu}{yP^z}\right)q(y,\mu)+O\left(\frac{M^2}{P_z^2},\frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

- Not necessary as long as one can absorb the UV divergence into the matching coefficient C;

Xiong, Ji, Zhang, Zhao, 2013

- However, linear divergence in the quasi-PDF makes a fixed order calculation of C not sufficient. Therefore, we need a nonperturbative renormalization of the power divergence.

Other proposals

Pseudo-PDF approach: Radyushkin, 2017; Karpie, Orginos, Radyushkin, Zafeiropoulos, 2017

$$\langle P|\bar{\psi}(z)\gamma^0 U(z,0)\psi(0)|P\rangle = 2P^0\tilde{Q}(zP^z, z^2)$$

- Formally, pseudo-PDF is defined from the same equal-time correlation as quasi-PDF, and satisfies an equivalent factorization theorem at small $|z|$;

  Ji, Zhang, Zhao, 2017; T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., 2018

- The nontrivial part is that Radyushkin defined a "reduced Ioffe-time distribution" by forming the ratio of two **nucleon matrix elements**;

$$\frac{\tilde{Q}(zP^z, z^2)}{\tilde{Q}(0, z^2)}$$

A rigorous relation to the PDF can only be established at small $|z|$ as before;
A parametrized form of the PDF to fit all the data points with 1-loop corrections.

See A. Radyushkin and J. Karpie's talks

Higher Moments: Z. Davoudi and M. Savage, PRD 2012;

OPE of current-current correlator: D. Lin and W. Detmold, PRD 2006; A. J. Chambers et al. (QCDSF), PRL 2017;

Hadronic tensor: K.F. Liu (et al.), 1994, 1999, 1998, 2000, 2017.

Lattice cross section: Y.-Q. Ma and J. Qiu, 2014, 2017. See Qiu's talk

Factorization of Euclidean Correlations in Coordinate Space: V. M. Braun and D. Mueller, EPJ C 2008; G. S. Bali, V. M. Braun, A. Schaefer, et al., 2017.

Procedure of Systematic Calculation

1. Simulation of the quasi PDF in lattice QCD

3. Subtraction of higher twist corrections

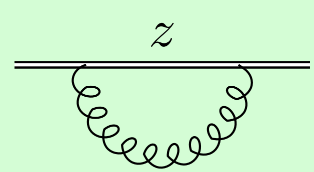
$$\tilde{q}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|} \, C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right) ,$$

2. Renormalization of the lattice quasi PDF, and then taking the continuum limit

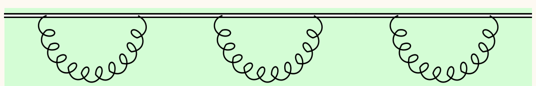
4. Matching to the MSbar PDF.

Renormalization

For an open smooth Wilson line *W(z,0)*, its self energy includes a linear divergence:



$$= \frac{\alpha_S}{2\pi} c_1 \Lambda |z|$$



$$+\ldots\ldots = e^{\delta m^{*}|z|}, \qquad \delta m \sim \Lambda$$

In coordinate space, it can be multiplicatively renormalized as:

$$W^B(z,0) = Z_z e^{\delta m|z|} W^R(z,0)$$

- $e^{\delta m^{*}|z|}$ introduces counterterms that cancel the linear divergences in the Wilson line self energy;
- Z_z depends on the end points and only includes logarithmic divergences.

V. S. Dotsenko and S. N. Vergeles, 1980
N. S. Craigie and H. Dorn, 1981
H. Dorn, 1986

Renormalization

- The gauge-invariant quark Wilson line operator can be renormalized multiplicatively in the coordinate space:

$$\tilde{O}_\Gamma(z) = \bar{\psi}(z)\Gamma W(z,0)\psi(0) = Z_{\psi,z} e^{-\delta m|z|} \left(\bar{\psi}(z)\Gamma W(z,0)\psi(0)\right)^R$$

X. Ji, J.-H. Zhang, and Y.Z., 2017; J. Green et al., 2017
T. Ishikawa, Y.-Q. Ma, J. Qiu, S. Yoshida, 2017.

- No mixing with different flavors (J. Green et al., 2017; T. Ishikawa, Y.-Q. Ma, J. Qiu, S. Yoshida, 2017.);

- Different renormalization schemes can be converted to each other in coordinate space;

Renormalization

On the lattice, there is operator mixing due to broken Lorentz and chiral symmetries:

- For $\Gamma = \gamma^z$, the quark bilinear O_Γ mixes with $\Gamma = 1$ at $O(a^0)$, and mixes with higher dimensional operators starting at $O(a^1)$;
- For $\Gamma = \gamma^0$, the quark bilinear O_Γ does no mix with $\Gamma = 1$ at $O(a^0)$, but mixes with higher dimensional operators starting at $O(a^1)$;

M. Constantinou and H. Panagopoulos, 2017; J. Green et al., 2017; T. Ishikawa et al. (LP3), 2017.

- In either case, there is no power divergent mixing which goes like $1/a^n\,(n>0)$, unlike the moments (Rossi and Testa, 2017).

Regularization-independent momentum subtraction (RI/MOM) scheme

A regularization-independent momentum subtraction scheme (RI/MOM): Martinelli et al., 1994

$$Z_{OM}^{-1}(z,a^{-1},p_z^R,\mu_R)\langle p|\tilde{O}_\Gamma(z)|p\rangle\Big|_{\substack{p^2=\mu_R^2\\ p_z=p_z^R}} = \langle p|\tilde{O}_\Gamma(z)|p\rangle_{\text{tree}}$$

$$Z_{OM}(z,a^{-1},p_z^R,\mu_R) = \frac{\langle p|\tilde{O}_\Gamma(z)|p\rangle\Big|_{\substack{p^2=\mu_R^2\\ p_z=p_z^R}}}{\langle p|\tilde{O}_\Gamma(z)|p\rangle_{\text{tree}}}$$

- Can be implemented nonperturbatively on the lattice.
- Introduce two intermediate scales μ_R and P_z^R, whose dependence shall be cancelled in the matching.

Power corrections

- Consider an operator product expansion as $|z|$->0 for the iso-vector case (in the MSbar scheme):

$$n^{\mu}=(0,0,0,1),\ \mu_0=z$$

$$\tilde{O}_{\gamma^z}(z^2\mu^2)=\sum_{n=0}C_n(\mu^2z^2)\frac{(-iz)^n}{n!}n_{\mu_1}\cdots n_{\mu_n}O^{\mu_0\mu_1\cdots\mu_n}+\text{higher-twist}$$

$$O^{\mu_0\mu_1\cdots\mu_n}=\bar{\psi}(0)\gamma^{(\mu_0}iD^{\mu_1}\cdots iD^{\mu_n)}\psi(0)-\text{trace}$$

- At large P^z, the nucleon contracts in the longitudinal direction. Equal-time correlation dominant within $z\sim 1/P^z$,

$$\langle P|\text{higher-twist}|P\rangle=O(z^2\Lambda^2_{QCD})\sim O(\Lambda^2_{QCD}/P_z^2)$$

$$a_{n+1}(\mu)=\int dy\, y^n q(y,\mu)$$

$$\langle P|O^{\mu_0\mu_1\cdots\mu_n}|P\rangle=2a_{n+1}(\mu)\left[P^{\mu_0}P^{\mu_1}\cdots P^{\mu_n}-trace\right],\quad trace\sim O(M^2/P_z^2)$$

Power corrections

- $O((M/P^z)^2)$ corrections, or target-mass corrections (O Nachtmann, NPB 1973);

  - Expressions derived for all orders of $(M/P^z)^2$ (J.W. Chen et al., NPB, 2016; Also in A. Radyushkin, 2017):

$$\tilde{\tilde{q}}(x) = \sqrt{1+4c}\sum_{n=0}^{\infty}\frac{f_-^n}{f_+^{n+1}}\left[(1+(-1)^n)\tilde{q}(\frac{f_+^{n+1}x}{2f_-^n}) + (1+(-1)^n\tilde{q}(-\frac{f_+^{n+1}x}{2f_-^n})\right],$$

$$c = \frac{M^2}{4P_z^2}, \qquad f_\pm = \sqrt{1+4c} \pm 1.$$

Higher-twist corrections

- $O((\Lambda_{QCD}/P^z)^2)$ corrections

$$
\begin{aligned}
\mathcal{O}_{\rm tr}(z) = &\int_0^z dz_1\, \bar\psi(0)\Big[\gamma^\nu\Gamma\,(0,z_1)\,D_\nu\Gamma\,(z_1,z) \\
&+ \int_0^{z_1} dz_2\,\lambda\cdot\gamma\Gamma\,(0,z_2)\,D^\nu\Gamma\,(z_2,z_1)\,D_\nu\Gamma\,(z_1,z)\Big]\psi(z\lambda).
\end{aligned}
$$

- Need to simulate and renormalize matrix elements of higher-twist operators;
- We can postpone it, and in the end extrapolate to IMF limit after completing all the other corrections.

Matching

- Renormalization on the lattice in a particular scheme "X":

$$Z_X^{-1}(a^{-1},\tilde{\mu})\tilde{q}^B(z,P^z,a^{-1}) = \lim_{a\to 0} Z_X^{-1}(a^{-1},\tilde{\mu})\tilde{q}^B(z,P^z,a^{-1}) + O(a)$$

- Regularization-invariance:

$$\lim_{a\to 0} Z_X^{-1}(a^{-1},\tilde{\mu})\tilde{q}^B(z,P^z,a^{-1}) = Z_X^{-1}(\varepsilon,\tilde{\mu})\tilde{q}^B(z,P^z,\varepsilon), \quad d = 4-2\varepsilon$$

- Matching only needs to be performed in the dimensional regularization scheme in the continuum theory!

Matching Coefficient

- One-loop matching coefficient:

$$\xi = \frac{x}{y}$$

$$p^z = yP^z$$

$$C^{\rm OM}\left(\xi, \frac{\mu_R}{p^z_R}, \frac{\mu}{p^z}, \frac{p^z}{p^z_R}\right) - \delta(1-\xi) \tag{40}$$

$$= \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} - \frac{2(1+\xi^2)-r_R}{(1-\xi)\sqrt{r_R-1}}\arctan\frac{\sqrt{r_R-1}}{2\xi-1} + \frac{r_R}{4\xi(\xi-1)+r_R}\right]_\oplus & \xi>1 \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{4(p^z)^2}{\mu^2} + \frac{1+\xi^2}{1-\xi}\ln\left[\xi(1-\xi)\right] + (2-\xi) - \frac{2\arctan\sqrt{r_R-1}}{\sqrt{r_R-1}}\left\{\frac{1+\xi^2}{1-\xi} - \frac{r_R}{2(1-\xi)}\right\}\right]_+ & 0<\xi<1 \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi} + \frac{2}{\sqrt{r_R-1}}\left[\frac{1+\xi^2}{1-\xi} - \frac{r_R}{2(1-\xi)}\right]\arctan\frac{\sqrt{r_R-1}}{2\xi-1} - \frac{r_R}{4\xi(\xi-1)+r_R}\right]_\ominus & \xi<0 \end{cases}$$

$$+ \frac{\alpha_s C_F}{2\pi}\left\{h(\xi, r_R) - |\eta|\, h\big(1+\eta(\xi-1), r_R\big)\right\},$$

I. Stewart and Y. Z., 2017.

Outline

- Large momentum effective theory
  - Formalism
  - Procedure of a systematic calculation
- Lattice results

Lattice simulation

J.-W. Chen, L. Jin, H.-W. Lin, Y.-S. Liu, Y.-B. Yang, J.-H. Zhang, and Y.Z., arXiv: 1803.04393

Lattice setup:

- Clover valence fermions
- N_f=2+1+1 flavors of HISQ ensemble generated by MILC
- a=0.09fm, L=5.8fm, m_π~135MeV
- N_{cf}=310
- 1-step HYP smeared gauge link, Gaussian momentum smearing for the quark field
- P^z=2.2, 2.6, 3.0 GeV
- Simultaneous fit of correlators on 4 source-sink separations, 0.72, 0.81, 0.90, 1.08 fm.

Matching

- Fourier transform of the spatial correlation:

$$\tilde{q}(x)=\int_{-z_{\max}}^{z_{\max}} dz\,\frac{e^{ixzP^z}}{-ix}\partial_z \tilde{h}_R(z)$$

$|z_{\max}|P^z \sim 15$

Lin et al., 2017

$$=P^z\int_{-z_{\max}}^{z_{\max}} dz\, e^{ixzP^z}\tilde{h}_R(z)-\frac{1}{-ix}[e^{ixz_{\max}P^z}\tilde{h}_R(z_{\max})-e^{-ixz_{\max}P^z}\tilde{h}_R(-z_{\max})]$$

- Remove the oscillatory behavior due to the truncation in the Fourier transform;
- Equivalent to assuming that the spatial correlation beyond $|z_{\max}|$ is flat;
- We can loosen the assumption by considering the Regge behavior of the PDF at small *x*.

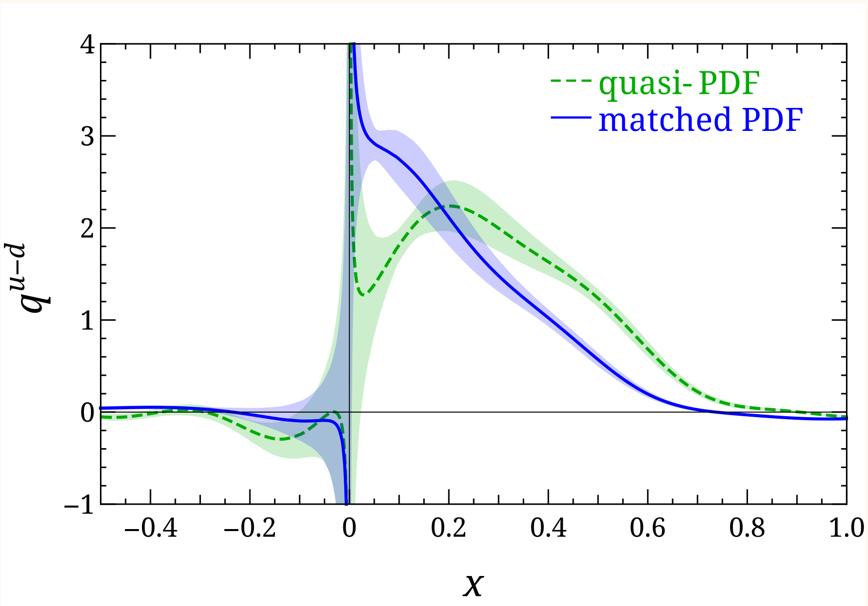
- Inversion of the factorization formula:

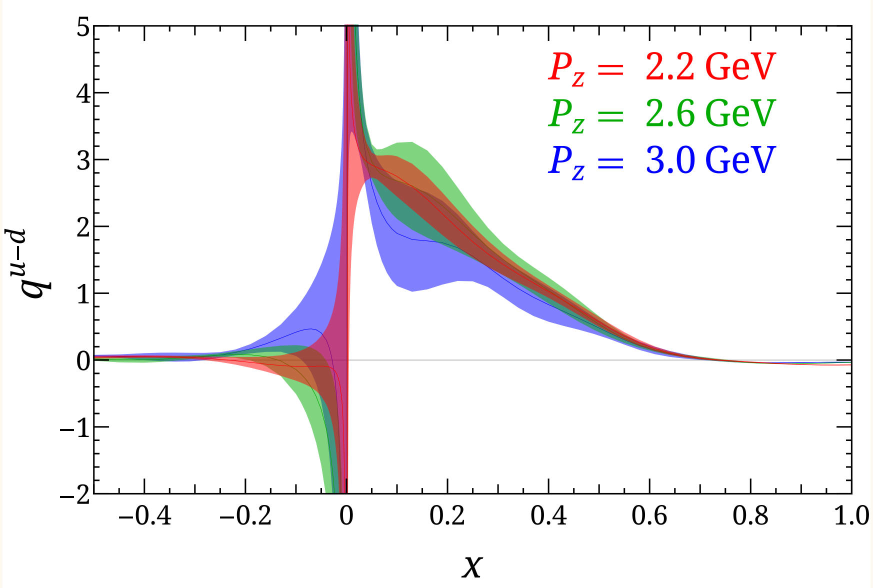
$$\tilde{q}=C\otimes q, \qquad q=C^{-1}\otimes\tilde{q},$$

$$C(\frac{x}{y})=\delta(\frac{x}{y}-1)+\frac{\alpha_s C_F}{2\pi}C^{(1)}(\frac{x}{y}), \qquad C^{-1}(\frac{x}{y})\approx\delta(\frac{x}{y}-1)-\frac{\alpha_s C_F}{2\pi}C^{(1)}(\frac{x}{y}).$$

Final result

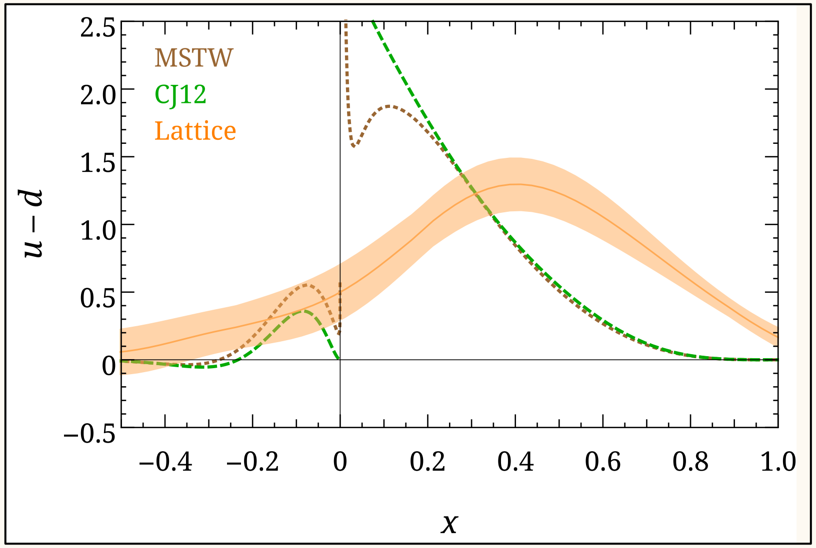
Matched to Msbar PDF at μ =3GeV.

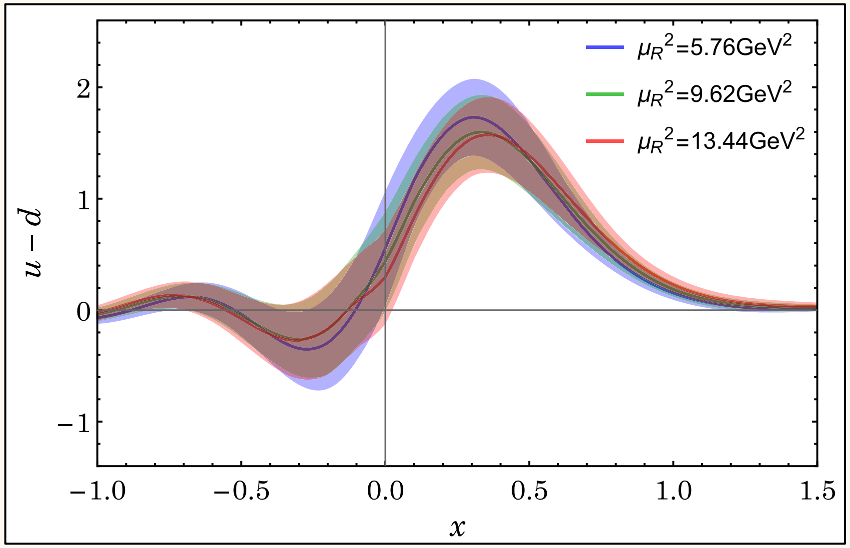




Effect of one-loop matching for the renormalized quasi-PDF at P^z=2.2 GeV and μ_R=3.7 GeV and P_z^R=2.2 GeV.

Final result for different nucleon momentum with μ_R=3.7 GeV and P_z^R=2.2 GeV.

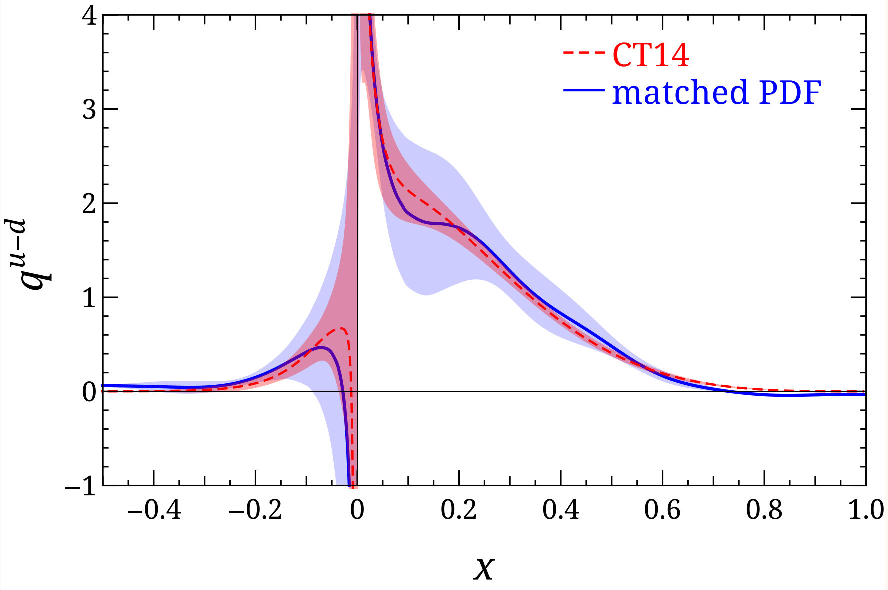




$\Gamma = \gamma^z$. No lattice renormalization. P^z=1.29 GeV, a=0.12fm, m_π~310MeV (Lin et al., 2014)

$\Gamma = \gamma^z$. Lattice renormalization. P^z=1.29 GeV, a=0.12fm, m_π~310MeV (Chen et al. (LP3), 2017)

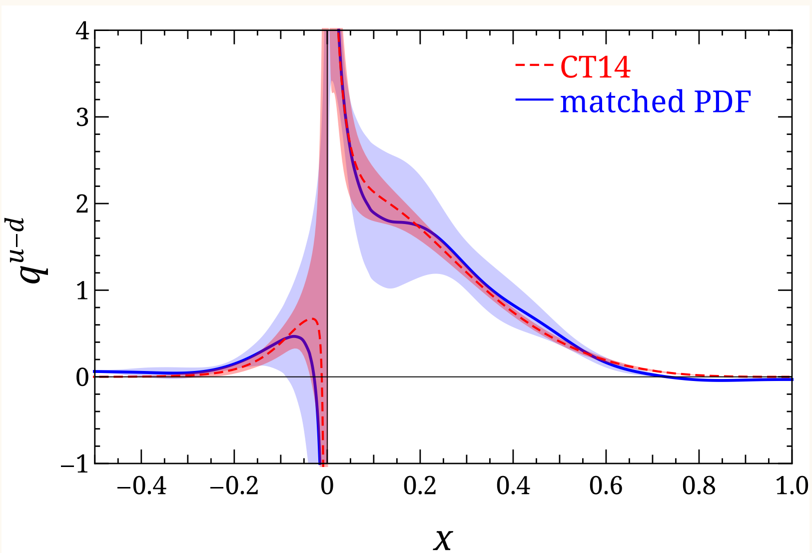
Evolution of calculation on the lattice:
clover valence fermions on N_f=2+1+1 flavors of HISQ generated by MILC, $24^3 \times 64$. (Unpolarized isovector PDF)



$\Gamma = \gamma^t$. Lattice renormalization. a=0.09fm, L=5.8fm, P^z=3.0 GeV, m_π~135MeV (Chen et al. (LP3), 2018)

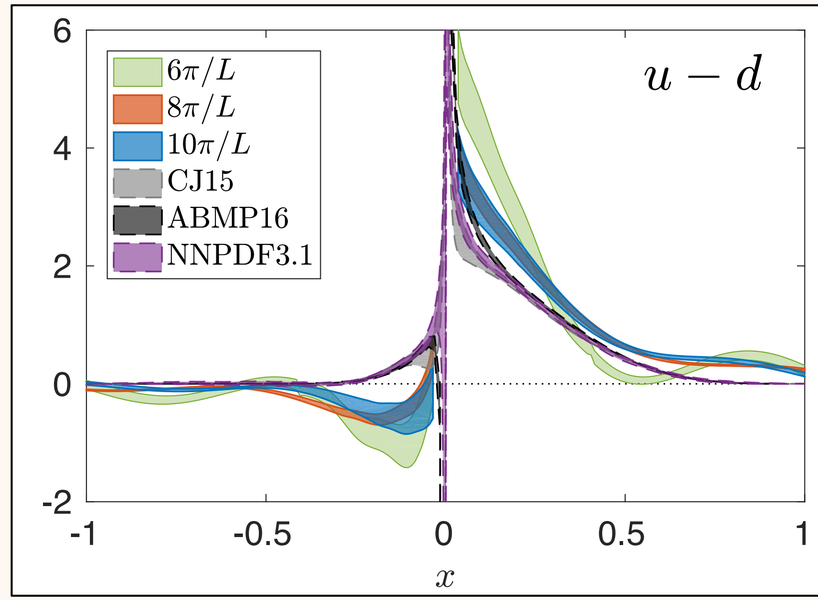
Complete analysis at physical pion mass

J.W. Chen, L. Jin, H.-W. Lin, Y.-S. Liu, Y.-B. Yang, J.-H. Zhang, and Y.Z., (LP3), 2018



$\Gamma = \gamma^t$. Clover valence fermions on N_f=2+1+1 flavors of HISQ generated by MILC, a=0.09fm, L=5.8fm, m_π~135MeV, P^z=3 GeV.
One-step matching from RI/MOM quasi-PDF to MSbar PDF at μ =3 GeV.

C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, and F. Steffens, (ETMC), 2018



$\Gamma = \gamma^t$. Dynamical N_f=2+1+1 twisted mass fermions by ETMC, a=0.09fm, L=4.8fm, m_π~130MeV, P^z=1.4 GeV.
Two steps: matching from RI/MOM quasi-PDF to MSbar quasi-PDF, then to MSbar PDF at μ =2 GeV.

Other distributions

- Pion PDF, LP3 collaboration, arXiv:1804.01483;

See D. Richard's talk on the progress of pion PDF from current-current correlator.

- Distribution amplitudes and GPD;
- Transversity distributions;
  - Pheno results still have fairly large uncertainties;

See M. Radici's talk on the progress from current-current correlator.

  - Can also be calculated with the same procedure in LaMET, and free from operator mixing at $O(a^0)$

M. Constantinou and H. Panagopoulos, 2017; T. Ishikawa et al. (LP3), 2017.

Summary

- ✧ LaMET allows us to calculate the PDF from a Euclidean quasi-PDF on the lattice;
- ✧ A systematic procedure to calculate PDF from the lattice has been set up;
- ✧ A recent calculation at physical pion mass and large nucleon momentum has shown encouraging signs of getting closer to the phenomenological PDF.

Outlook:

- How to resum the large terms in the matching coefficient? 2-loop matching?
- Application to TMDs? Ji, Jin, Yuan, Zhang, and Y.Z., arXiv:1801.05930; M. Ebert, I. Stewart, and Y.Z., in preparation.

Backup Slides

Matching Coefficient

Unrenormalized quasi-PDF at one-loop:

$$\tilde{q}^{(0)}(z,p^z) = 4p^z\zeta\, e^{-izp^z}$$

$$\tilde{q}^{(1)}(z,p^z,0,-p^2) = \frac{\alpha_s C_F}{2\pi}\,(4p^z\zeta)\int_{-\infty}^{\infty}dx\,\left(e^{-ixp^z z} - e^{-ip^z z}\right)h(x,\rho)\,,$$

$$\rho \equiv \frac{(-p^2 - i\varepsilon)}{p_z^2}$$

$$h(x,\rho) \equiv \begin{cases} \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right]\ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} - \frac{\rho}{4x(x-1)+\rho} + 1 & x>1 \\ \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right]\ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} - \frac{2x}{1-x} & 0<x<1 \\ \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right]\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} + \frac{\rho}{4x(x-1)+\rho} - 1 & x<0 \end{cases},$$

- Collinear divergence regulated by $-p^2$, which is not obvious unless one takes the onshell limit;
- Renormalization constant determined in the Euclidean region where $\rho > 1$, needs analytical continuation.

Matching Coefficient

- Renormalized quasi-PDF in the RI/MOM scheme:

$$\tilde{q}^{(1)}_{\rm OM}(x,p^z,p^z_R,\mu_R) = \int\frac{dz}{2\pi}\, e^{ixzp^z}\ \tilde{q}^{(1)}_{\rm OM}(z,p^z,p^z_R,\mu_R)$$
$$= \frac{\alpha_s C_F}{2\pi}(4\zeta)\left\{\int dy\ \left[\delta(y-x)-\delta(1-x)\right]\left[h_0(y,\rho)-h(y,r_R)\right] + h(x,r_R) - |\eta|\, h\big(1+\eta(x-1),r_R\big)\right\},$$

$$\eta \equiv p^z/p^z_R$$

$$h_0(x,\rho)\equiv\begin{cases}\dfrac{1+x^2}{1-x}\ln\dfrac{x}{x-1}+1 & x>1\\ \dfrac{1+x^2}{1-x}\ln\dfrac{4}{\rho}-\dfrac{2x}{1-x} & 0<x<1\\ \dfrac{1+x^2}{1-x}\ln\dfrac{x-1}{x}-1 & x<0\end{cases},$$

$$r_R = (\mu_R/p^z_R)^2$$

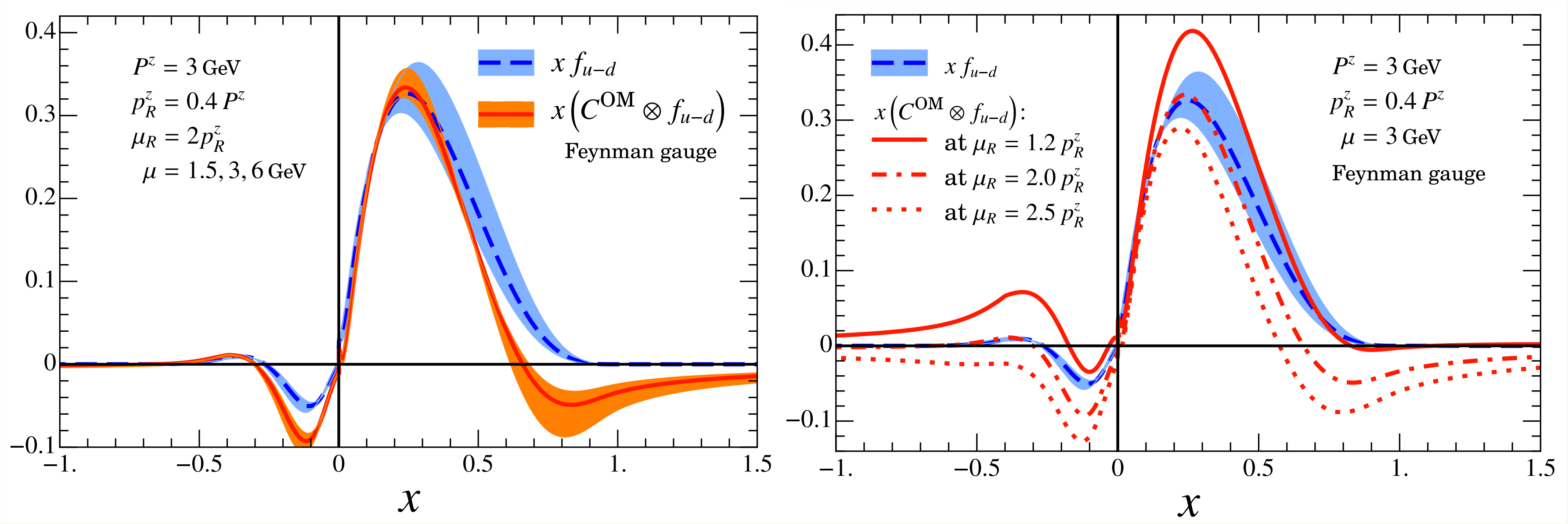
I. Stewart and Y. Z., 2017.

- MSbar PDF:

$$q^{(1)}(x,\mu) = \frac{\alpha_s C_F}{2\pi}(4\zeta)\begin{cases}0 & x>1\\ \left[\dfrac{1+x^2}{1-x}\ln\dfrac{\mu^2}{-p^2}-\dfrac{1+x^2}{1-x}\ln\left[x(1-x)\right]-(2-x)\right]_+ & 0<x<1\\ 0 & x<0\end{cases}$$

Effect of matching on phenomenological PDF

- PDF: MSTW2008nlo



I. Stewart and Y. Z., 2017.

Large momentum effective theory

Large momentum effective theory (LaMET) is a theory that expands in powers of $1/P^z$, where P^z is the proton momentum (Ji, PRL 2013, Sci. China Phys. Mech. Astro., 2014):

1. Construct a Euclidean quasi-observable $\tilde{O}$ which can be calculated in lattice QCD;

2. The IMF limit of $\tilde{O}$ is constructed to be a parton observable O at the operator level;

$$\left|P\neq 0\right\rangle = U(\Lambda(P))\left|P_0=0\right\rangle, \quad U(\Lambda(P=\infty))^{-1}\tilde{O}U(\Lambda(P=\infty))=O$$

$$\left\langle P=\infty\right|\tilde{O}\left|P=\infty\right\rangle = \left\langle P_0=0\right|O\left|P_0=0\right\rangle$$

One cannot calculate it from the lattice!

Large momentum effective theory

3. At finite P^z, the matrix element of $\tilde{O}$ depends on the cut-off Λ of the theory (if not renormalized) and generally P^z, i.e., $\tilde{O}(P^z/\Lambda)$, while that of O depends on the renormalization scale μ (if in the MSbar scheme), i.e., $O(\mu)$;

$$\tilde{O}(P^z/\Lambda) = \left\langle P = P^z \middle| \tilde{O} \middle| P = P^z \right\rangle,$$
$$O(\mu) = \left\langle P = \text{any} \middle| O \middle| P = \text{any} \right\rangle$$

4. Taking the $P^z \rightarrow \infty$ ($P^z >> \Lambda$) limit of $\tilde{O}(P^z/\Lambda)$ is generally ill-defined due to the singularities in quantum field theory,

$$\lim_{P^z \gg \Lambda} \tilde{O}(P^z/\Lambda) = ?$$

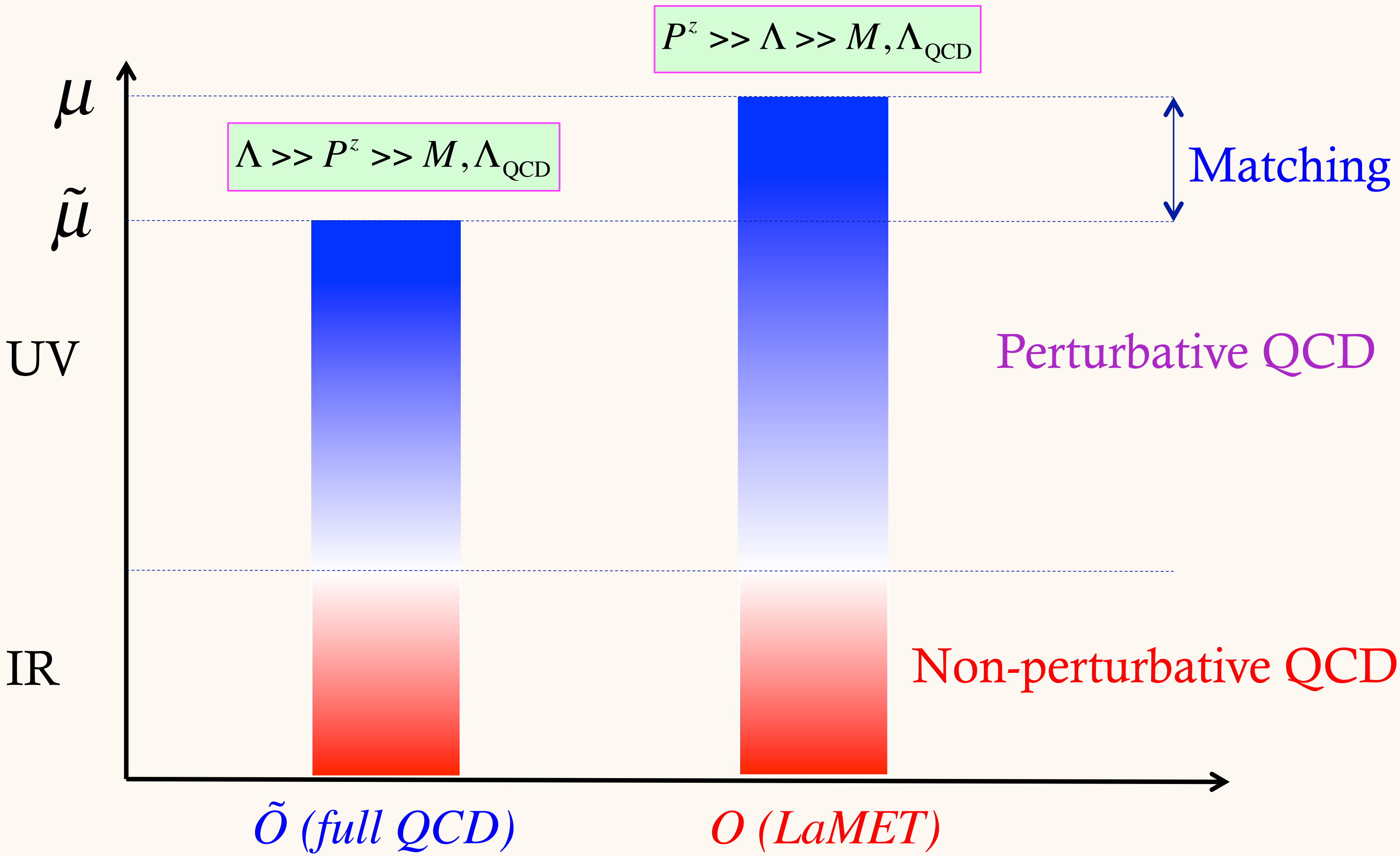
Large momentum effective theory

5. But it can be related to $O(\mu)$ through a factorization formula:

$$\tilde{O}(P^z/\Lambda) = Z(P^z/\Lambda, \mu/\Lambda) \otimes O(\mu) + \frac{c_2}{P_z^2} + \frac{c_4}{P_z^4} + \ldots$$

- P^z is much larger than Λ_{QCD} as well as the proton mass M to suppress the power corrections;
- One can regard $O(\mu)$ as the effective theory observable, and $\tilde{O}(P^z/\Lambda)$ as given by full QCD;
- $O(\mu)$ and $\tilde{O}(P^z/\Lambda)$ have the same infrared (IR) physics, and thus can be perturbatively matched to each other through the leading term.

How matching works



Leading-twist approximation and factorization theorem

- Leading-twist approximation:

$$\frac{1}{2P^z}\langle P|\tilde{O}_{\gamma^z}(z^2\mu^2)|P\rangle = \sum_{n=0} C_n(\mu^2 z^2)\frac{(-izP^z)^n}{n!}a_{n+1}(\mu) + O(\frac{M^2}{P_z^2},\frac{\Lambda_{QCD}^2}{P_z^2})$$
$$= \int dy \sum_{n=0} C_n(\mu^2 z^2)\frac{(-izP^z\cdot y)^n}{n!}q(y,\mu) + O(\frac{M^2}{P_z^2},\frac{\Lambda_{QCD}^2}{P_z^2})$$

- If we define: T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., 2018

$$\tilde{q}(x,P^z,\mu) = \int \frac{dz}{4\pi}\langle P|\tilde{O}_{\gamma^z}(z^2\mu^2)|P\rangle, \quad C(\frac{x}{y},\frac{\mu}{|y|P^z}) = \int \frac{d\zeta}{4\pi} e^{i\frac{x}{y}\zeta}\sum_{n=0} C_n(\frac{\mu^2\zeta^2}{(yP^z)^2})\frac{(-i\zeta)^n}{n!}$$

then we prove

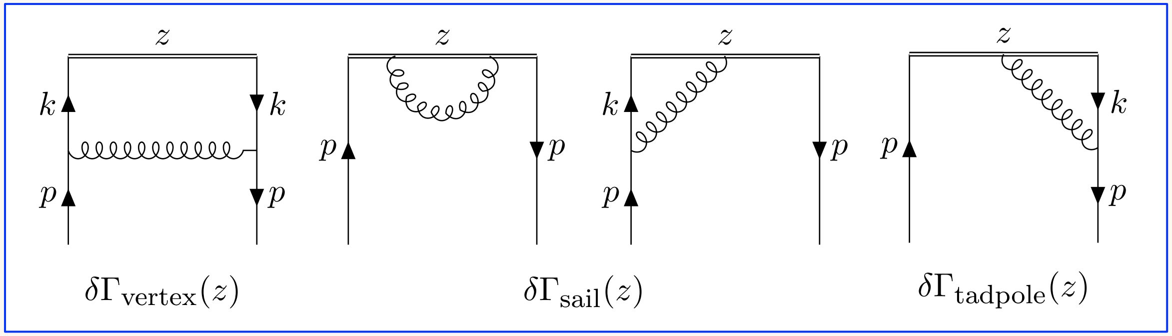
$$\tilde{q}(x,P^z,\mu) = \int \frac{dy}{|y|} C(\frac{x}{y},\frac{\mu}{|y|P^z})q(y,\mu)$$

Matching Coefficient

- The matching coefficient for different renormalization schemes on the lattice have been calculated at one-loop:

  Transverse momentum cut-off scheme: Xiong, Ji, Zhang and Y.Z., 2014; Y. Ma and J. Qiu, 2014;
  MSbar and RI/MOM scheme: I. Stewart and Y. Z., 2017.

- Since the matching does not depend on the IR regulator, one can obtain the matching by calculating the off-shell quark matrix elements of the quasi- and light-cone PDF:



Collaborations

Active collaborations working with the LaMET approach:

- Lattice Parton Physics Project (LP^3) Collaboration (PDF):

  J.W. Chen, T. Ishikawa, L. Jin, H.-W. Lin, Y.-S. Liu, Y.-B. Yang, J.-H. Zhang, R. Zhang, and Y.Z.

- European Twisted Mass Collaboration (ETMC) (PDF):

  C. Alexandrou, M. Constantinou, K.Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese et al.

- χ QCD Collaboration (Gluon polarization):

  A. Alexandru, T. Drapper, M. Glatzmaier, K.F. Liu, R.S. Suffian, Y.-B. Yang, Y.Z., et al.