Effective Field Theory Approach for Quarkonium at Low pT

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Outline

Part 1

Quarkonium production at moderate pT (standard NRQCD): $p_T^{\mathcal{Q}} \sim m_{\mathcal{Q}}$

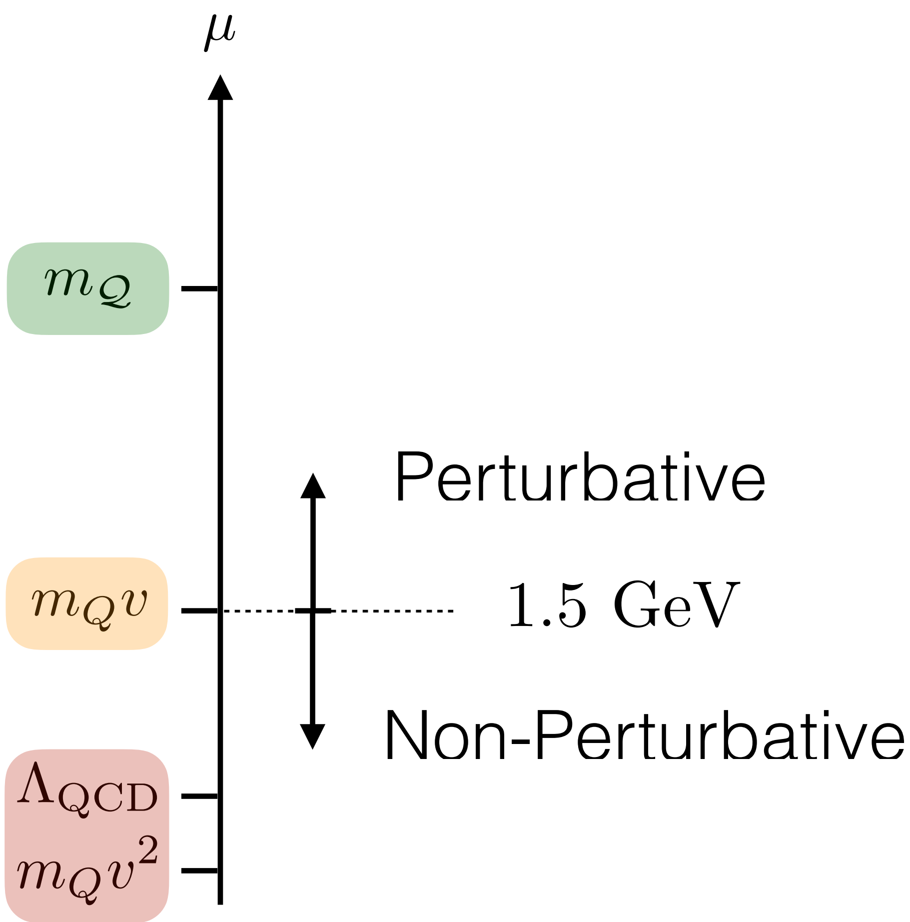
Quarkonium production at high pT (parton fragmentation): $p_T^{\mathcal{Q}} \gg m_{\mathcal{Q}}$

Part 2

Quarkonium production at low pT (TMD region): $p_T^{\mathcal{Q}} \ll m_{\mathcal{Q}}$

NRQCD scales

NRQCD = Non-Relativistic QCD



$b\bar{b}: \quad v^2 \sim 0.1$ bottomonium

$c\bar{c}: \quad v^2 \sim 0.3$ charmonium

→ Relative velocity of the heavy quark and antiquark in the quarkonium

typical momentum of heavy quark: $|\mathbf{p}_Q| \sim m_Q v$ (soft)

typical kinetic energy of heavy quark: $K_Q \sim m_Q v^2$ (ultra-soft)

NRQCD Lagrangian

NRQCD = Non-Relativistic QCD

soft: $p_s^\mu \sim m_Q v(1,1,1,1)$

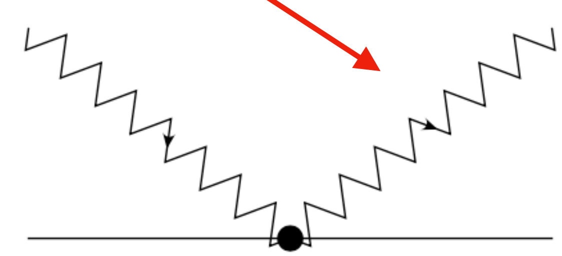
ultra-soft: $p_{us}^\mu \sim m_Q v^2(1,1,1,1)$

ultra-soft

subheading

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_p \left|p^\mu A_p^\nu - p^\nu A_p^\mu\right|^2 + \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger \left\{ iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} \right\}\psi_{\mathbf{p}} \\
& -4\pi\alpha_s \sum_{q,q'\mathbf{p},\mathbf{p}'} \left\{ \frac{1}{q^0}\psi^\dagger_{\mathbf{p}'}\left[A^0_{q'},A^0_q\right]\psi_{\mathbf{p}} \right. \\
& \left. + \frac{g^{\nu 0}(q'-p+p')^\mu - g^{\mu 0}(q-p+p')^\nu + g^{\mu\nu}(q-q')^0}{(\mathbf{p}'-\mathbf{p})^{\mathbf{2}}}\psi^\dagger_{\mathbf{p}'}\left[A^\nu_{q'},A^\mu_q\right]\psi_{\mathbf{p}}\right\} \\
& +\psi\leftrightarrow\chi,\ T\leftrightarrow\bar{T} \\
& +\sum_{\mathbf{p},\mathbf{q}} \frac{4\pi\alpha_s}{(\mathbf{p}-\mathbf{q})^{\mathbf{2}}}\psi^\dagger_{\mathbf{q}}T^A\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{q}}\bar{T}^A\chi_{-\mathbf{p}} + \dots
\end{aligned}$$

soft



arXiv:hep-ph/9910209 M. E. Luke, A. V. Manohar, I. Z. Rothstein

NRQCD Factorization

LDME: Long Distance Matrix Elements

$$d\sigma(a+b\to\mathcal{Q}+X)=\sum_n d\sigma(a+b\to Q\overline{Q}(n)+X)\langle\mathcal{O}_n^{\mathcal{Q}}\rangle$$

Perturbative expansion in the strong coupling.

NRQCD Scaling Rules

$$d\sigma_0(1+\alpha_s C_1+\alpha_s^2 C_2+\ldots)$$

$$\langle\mathcal{O}(^{2S+1}L_J^{[1,8]})\rangle\sim v^{3+2L+2E+4M}$$

$$Q\bar{Q}(n)\xrightarrow{\langle\mathcal{O}_n^{\mathcal{Q}}\rangle}\mathcal{Q}$$

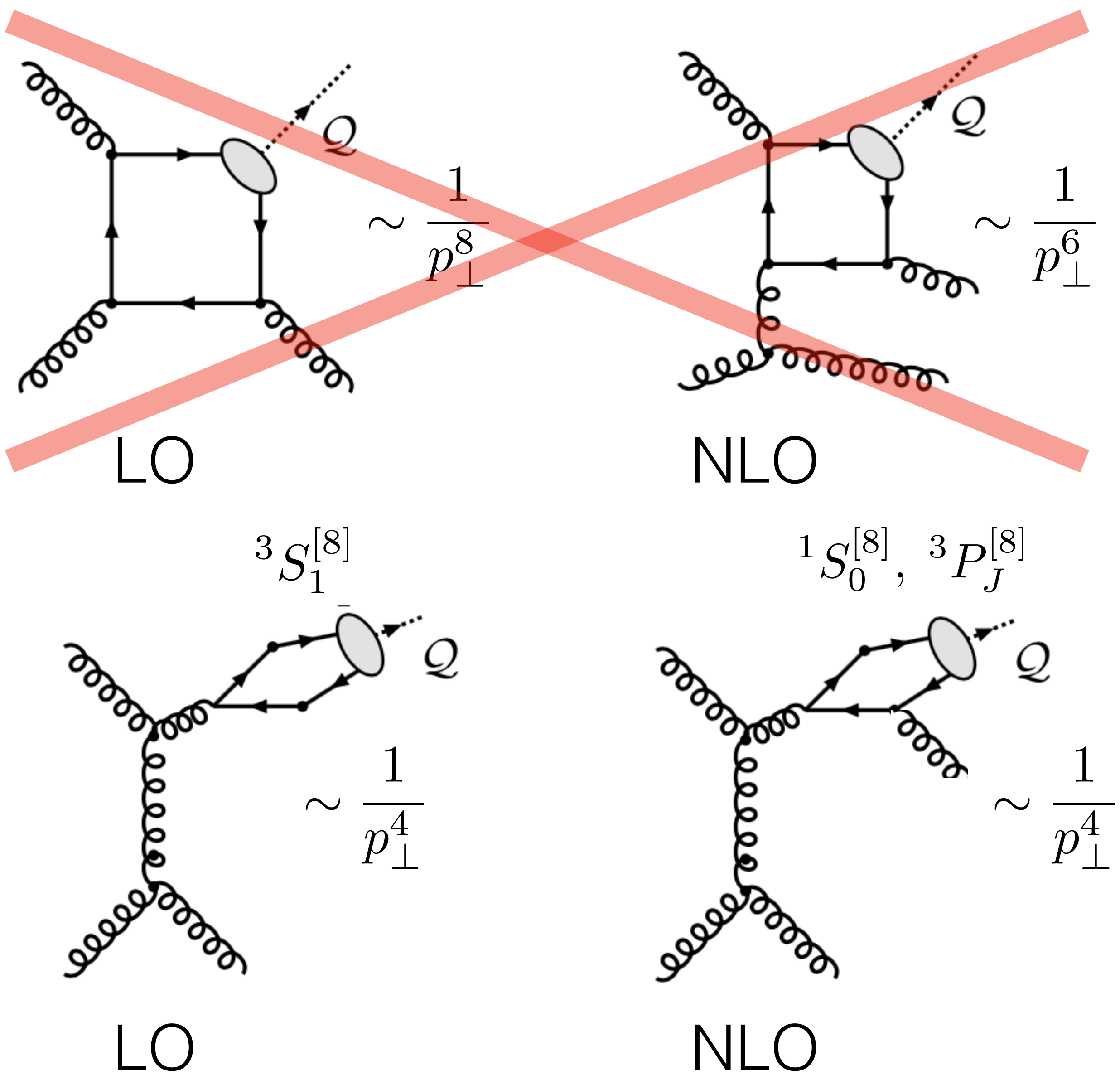
$$\mathcal{O}_n^{\mathcal{Q}}=\mathcal{O}_2^{n\,\dagger}\left(\sum_X|X+\mathcal{Q}\rangle\langle X+\mathcal{Q}|\right)\mathcal{O}_2^n$$

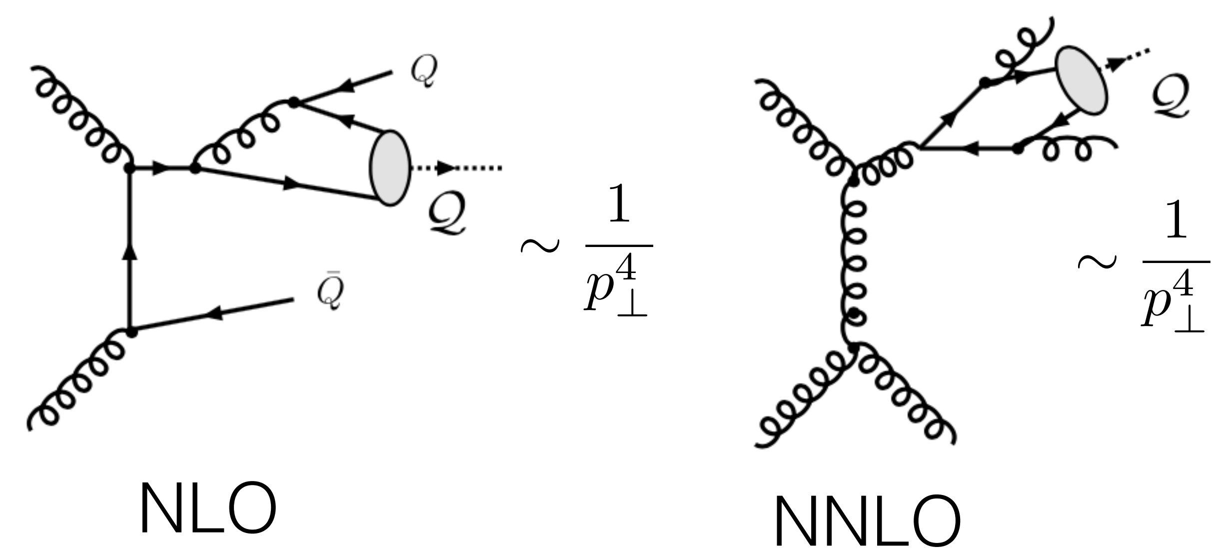
ultra-soft + soft

$$\mathcal{O}_2^n=\psi^\dagger\mathcal{K}^n\chi$$

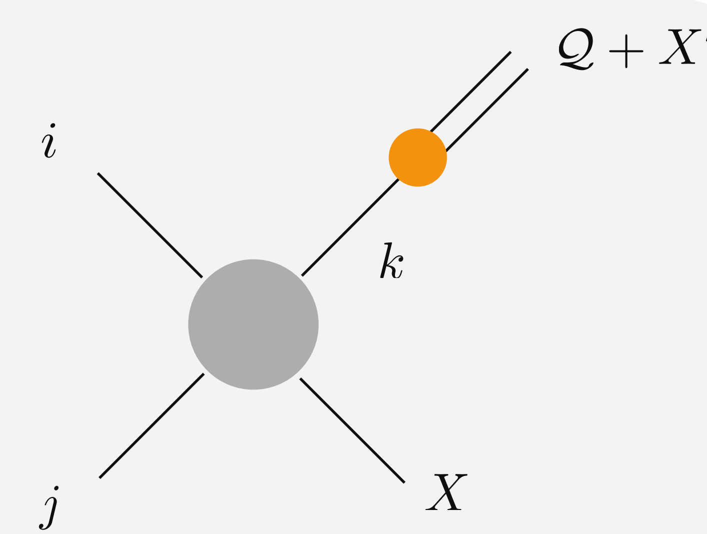
$$n={}^{2S+1}L_J^{[c]}$$

NRQCD at large pT





(single) Parton fragmentation process



NRQCD at large pT

Leading Power (LP) Factorization

$$\frac{d\sigma_h}{dp_\perp}(p_\perp) = \sum_i \int_z^1 \frac{dx}{x}\, \frac{d\sigma_i}{dp_\perp}\left(\frac{p_\perp}{x}, \mu\right) D_{i/h}(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{p_\perp^2}\right)$$

Expansion in: $\frac{m_\mathcal{Q}}{p_\perp}$

At sufficiently large pT the fragmentation processes will dominate the cross section:

Only few diagrams for each mechanism

Large Logarithms

$\ln(p_T/m_\mathcal{Q})$

NRQCD at large pT

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$$\ln\left(\frac{\mu}{p_T}\right) - \ln\left(\frac{\mu}{2m_Q}\right)$$

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Expansion in: $\frac{m_Q}{p_\perp}$

$$\ln\left(\frac{\mu}{p_T}\right) - \ln\left(\frac{\mu}{2m_Q}\right)$$

DGLAP Evolution

$$\mu\frac{d}{d\mu}D_{i/h}(z,\mu) = \sum_i \int_z^1 \frac{dx}{x} P_{ij}(x) D_{i/h}\left(\frac{z}{x}, \mu\right)$$

Resummation: $\ln(p_T/m_h)$

NRQCD at large pT

Leading Power (LP) Factorization

$$\frac{d\sigma_h}{dp_\perp}(p_\perp) = \sum_i \int_z^1 \frac{dx}{x} \frac{d\sigma_i}{dp_\perp}\left(\frac{p_\perp}{x}, \mu\right) D_{i/h}(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{p_\perp^2}\right)$$

Expansion in: $\frac{m_Q}{p_\perp}$

$$\ln\left(\frac{\mu}{p_T}\right) - \ln\left(\frac{\mu}{2m_Q}\right) \qquad d_{i/n}(x,\mu)\langle\mathcal{O}_n^h\rangle$$

DGLAP Evolution

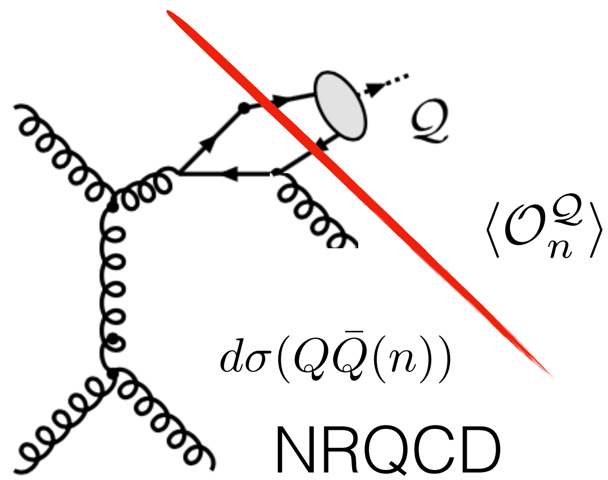
$$\mu\frac{d}{d\mu}D_{i/h}(z,\mu) = \sum_i \int_z^1 \frac{dx}{x} P_{ij}(x) D_{i/h}\left(\frac{z}{x}, \mu\right)$$

Resummation: $\ln(p_T/m_h)$

Part 1: Summary

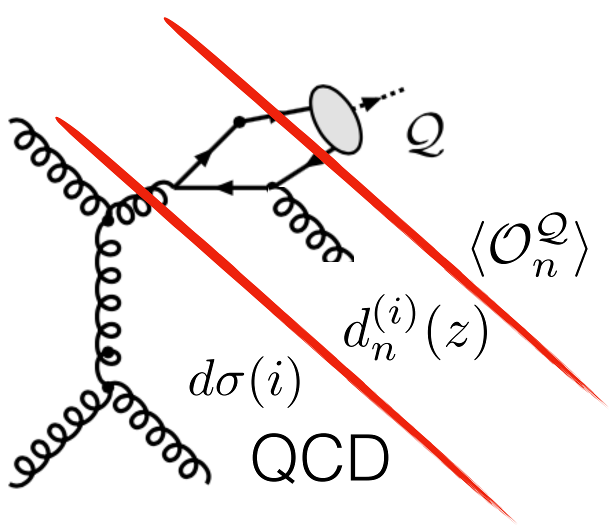
Fixed order NRQCD

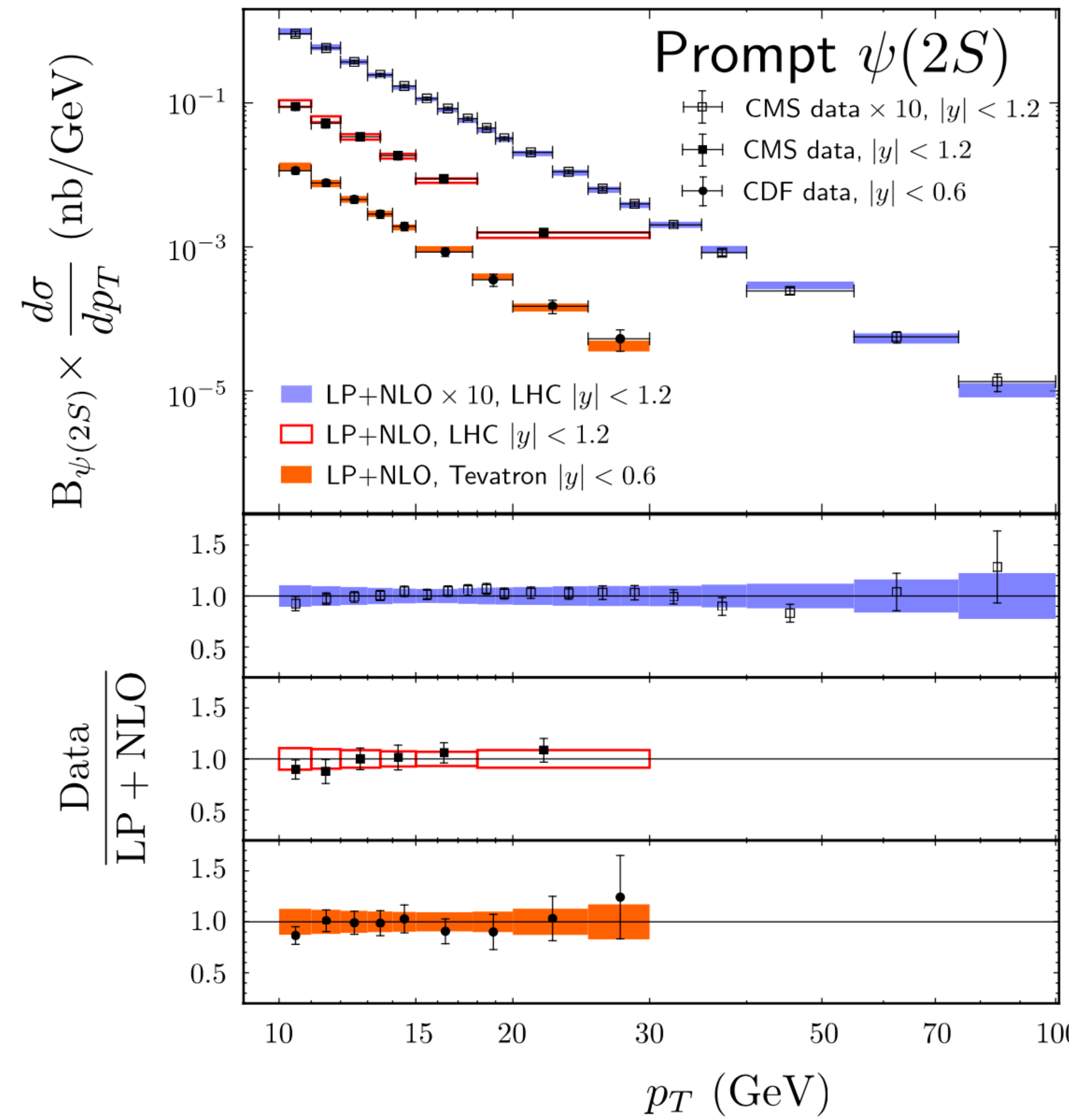
Preferred at
$p_T \sim m_{\mathcal{Q}}$
corrections of
$p_T/m_{\mathcal{Q}}$



Leading Power NRQCD

Preferred at
$p_T \gg m_{\mathcal{Q}}$
Resummation of
$\ln(p_T/m_{\mathcal{Q}})$





G. T. Bodwin, K-T. Chao, H. S. Chung, U-R. Kim, J. Lee, and Y-Q. Ma (PRD) 2016

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Part 2

Quarkonium production at low pT (TMD region): $p_T^{\mathcal{Q}} \ll m_{\mathcal{Q}}$

bottomonium: $m_{\Upsilon(1S)} = 9.5\mathrm{GeV}$

Quarkonium at low pT (previous attempts)

Many attempts that approach the problem in CEM and CSM

Cannot be improved the same way EFTs are.

CEM and CSM fail in other regions

NRQCD attempts (including resumption)

arXiv:1210.3432, Peng Sun, C.-P. Yuan, and Feng Yuan

→ No factorization. NRQCD factorization holds down to low pT?

Read logs from the small pT limit of NLO NRQCD

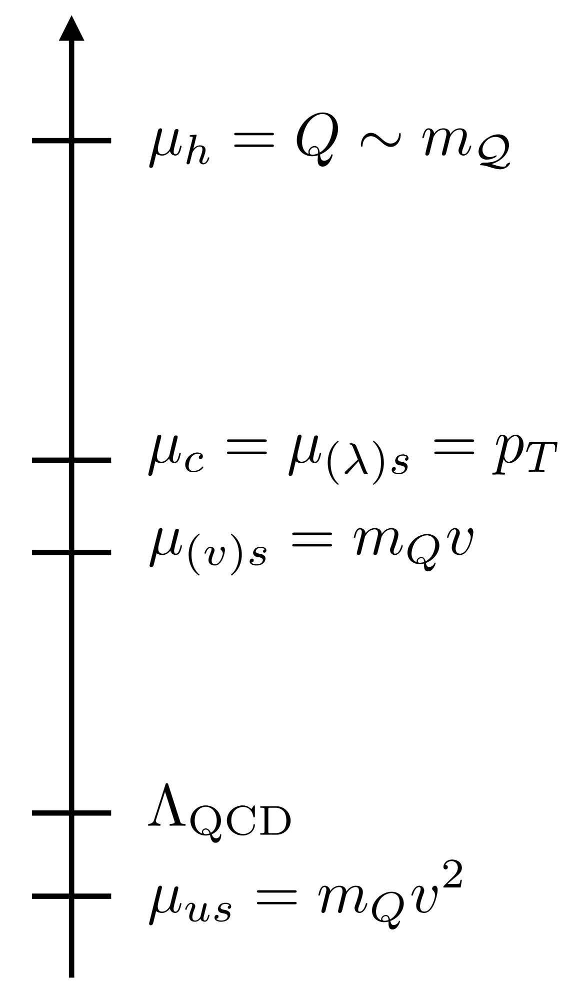
NRQCD+SCET scales + Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SCET} + \mathcal{L}_{\rm NRQCD}$$

- $p_c^\mu = (p_c^+, p_c^-, \vec{p}_c^{\,\perp}) \sim Q(\lambda^2, 1, \lambda)$ $\qquad \lambda = \dfrac{p_T}{Q}$

- $p_{(\lambda)s}^\mu = (p_{(\lambda)s}^+, p_{(\lambda)s}^-, \vec{p}_{(\lambda)s}^{\,\perp}) \sim Q(\lambda, \lambda, \lambda)$

- $p_{(v)s}^\mu = (p_{(v)s}^+, p_{(v)s}^-, \vec{p}_{(v)s}^{\,\perp}) \sim Q(v, v, v)$



NRQCD at low pT - Factorization

$$\mathcal{O}^{\rm QCD}(2+2) \to \sum_n C^n_{(2+2)} \Big(\mathcal{O}_2^{\rm SCET} \times \mathcal{O}_2^n\Big)$$

$$\mathcal{O}^{\rm QCD}_{2+2} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}_2^n = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}_2^{\rm SCET} = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

Factorize the cross section using BPS field redefinition for decoupling collinear and heavy from ultra-soft modes.

$$\frac{d\sigma}{dy\, d^2\mathbf{p}_T} = \sum_n \left(\sigma_0(n)\frac{m^2}{s}\right) \times H^n_{ij} \times \mathcal{B}_i(x_1) \otimes \mathcal{B}_j(x_2) \otimes S^{\mathcal{Q}}_n \times \Big(1 + \mathcal{O}(\lambda)\Big)$$

NRQCD at low pT - Hard function

$$\mathcal{O}^{\rm QCD}(2+2) \to \sum_n C^n_{(2+2)} \Big(\mathcal{O}_2^{\rm SCET} \times \mathcal{O}_2^n \Big)$$

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Hard function: $H^n_{ij} \sim C^n_{(2+2)} (C^n_{(2+2)})^*$

NRQCD at low pT - Beam function

$$\mathcal{O}^{\rm QCD}(2+2) \to \sum_n C^n_{(2+2)}\Big(\mathcal{O}^{\rm SCET}_2 \times \mathcal{O}^n_2\Big)$$

$$\mathcal{O}^{\rm QCD}_{2+2} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}^n_2 = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}^{\rm SCET}_2 = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

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Beam Function: $$\mathcal{B}_{i/h}(x_i, p_T) = \int_{x_i}^1 \frac{dx}{x} C_{i/j}(x, p_T) f_{j/h}(x_i/x)$$

short distance matching coefficients ($C_{i/j}$)

collinear PDFs ($f_{j/h}$)

NRQCD at low pT - Shape function

$$\mathcal{O}^{\text{QCD}}_{2+2} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}^n_2 = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}^{\text{SCET}}_2 = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

$$\mathcal{O}^{\text{QCD}}(2+2) \to \sum_n C^n_{(2+2)} \Big(\mathcal{O}^{\text{SCET}}_2 \times \mathcal{O}^n_2 \Big)$$

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Quarkonium TMD shape function:

$$S^{\mathcal{Q}}_n = \frac{1}{\mathcal{N}} \text{Tr}\Big[\sum_{X_s} \Big\langle 0 \Big| (\chi^\dagger \mathcal{K}^\dagger_n \psi)\, \mathcal{Y}^\dagger_{(\lambda)s,n_{\bar{B}}} \mathcal{Y}^\dagger_{(\lambda)s,n_B} \delta^{(2)}(\mathbf{p}_T - \vec{\mathcal{P}}_\perp) \Big| X_s + \mathcal{Q} \Big\rangle$$
$$\times \Big\langle X_s + \mathcal{Q} \Big| \mathcal{Y}_{(\lambda)s,n_B} \mathcal{Y}_{(\lambda)s,n_{\bar{B}}}\, (\psi^\dagger \mathcal{K}_n \chi) \Big| 0 \Big\rangle \Big]$$

NRQCD at low pT - Shape function

$$\mathcal{O}^{\rm QCD}_{2+2} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}^{n}_{2} = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}^{\rm SCET}_{2} = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

$$\mathcal{O}^{\rm QCD}(2+2) \to \sum_n C^n_{(2+2)} \Big(\mathcal{O}^{\rm SCET}_2 \times \mathcal{O}^n_2\Big)$$

Factorize the cross section using BPS field redefinition for decoupling collinear and heavy from ultra-soft modes.

$$\frac{d\sigma}{dy\, d^2\mathbf{p}_T} = \sum_n \left(\sigma_0(n)\frac{m^2}{s}\right) \times H^n_{ij} \times \mathcal{B}_i(x_1) \otimes \mathcal{B}_j(x_2) \otimes S^{\mathcal{Q}}_n \times \Big(1+\mathcal{O}(\lambda)\Big)$$

Quarkonium TMD shape function: octet case, not IR finite cross section !

$$S^{\mathcal{Q}}_n = \frac{1}{\mathcal{N}} \mathrm{Tr}\Big[\sum_{X_s} \Big\langle 0 \Big| (\chi^\dagger \mathcal{K}^\dagger_n \psi)\, \mathcal{Y}^\dagger_{(\lambda)s,n_{\bar{B}}} \mathcal{Y}^\dagger_{(\lambda)s,n_B} \delta^{(2)}(\mathbf{p}_T - \vec{\mathcal{P}}_\perp) \Big| X_s + \mathcal{Q} \Big\rangle \times \Big\langle X_s + \mathcal{Q} \Big| \mathcal{Y}_{(\lambda)s,n_B} \mathcal{Y}_{(\lambda)s,n_{\bar{B}}}\, (\psi^\dagger \mathcal{K}_n \chi) \Big| 0 \Big\rangle\Big]$$

NRQCD at low pT - Shape function

The correct operator definition of the shape function is the same as before but with : $\mathcal{O}_2^n \to (\psi Y_u)^\dagger \mathcal{K}_n (Y_u \chi)$

$$\psi \to Y_u \psi$$

$$Y_u(x) = \mathrm{T}\Big[\exp\Big(ig\int_{-\infty}^{0} dt'\; u\cdot A_s^a(\vec{x}, t+t')T^a\Big)\Big]$$

$$u^\mu = (1,0,0,0)$$

Singlet	Octet
$\mathcal{K}_1 = \delta_{ab}\,\Gamma({}^{2S+1}L_J)$	$\mathcal{K}_8 = T^A_{ab}\,\Gamma({}^{2S+1}L_J)$
$Y_u^\dagger Y_u = 1$	$Y_u^\dagger T^A Y_u = \mathcal{Y}_u^{AB} T^B$
$\mathcal{O}_2^{[1]} \to \mathcal{O}_2^{[1]}$	$\mathcal{O}_2^{[8]} \to \mathcal{O}_2^{[8]} \mathcal{Y}_u$

NRQCD at low pT - Soft Wilson lines

arXiv:hep-ph/0501235, hep-ph/0509021: G. C. Nayak, J.-W. Qiu, G. F. Sterman

→ light-like wilson-lines for color octet LDMEs for gauge invariance and factorization for fragmentation processes

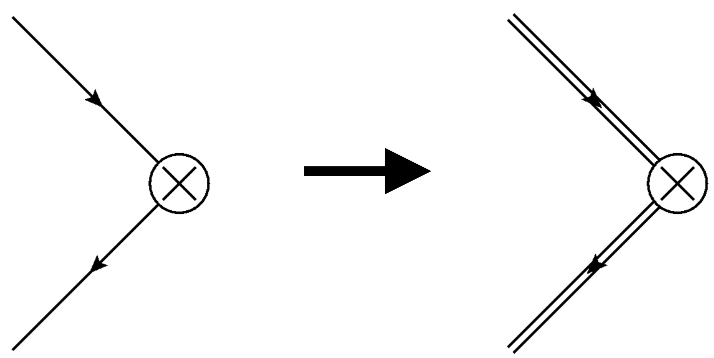
$$Y_u(x) = \mathrm{P}\Big[\exp\Big(ig\int_{-\infty}^{0} dt' u \cdot A_s^a(x^\mu + u^\mu t')\Big)\Big] \qquad u^\mu = (1, \vec{u})$$

SCET 2017 - Talk by Prashant Shrivastava in collaboration with Iain W. Stewart on soft-gauge invariant v-NRQCD

→ soft Wilson-lines for gauge invariance of the Lagrangian

NRQCD at low pT - Soft Wilson lines

QCD Non-relativistic limit



$$= \chi^\dagger \, \mathcal{K}_n \, \psi$$

NRQCD at low pT - Soft Wilson lines

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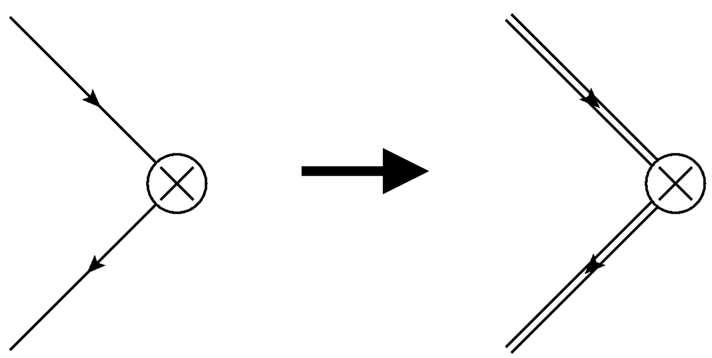

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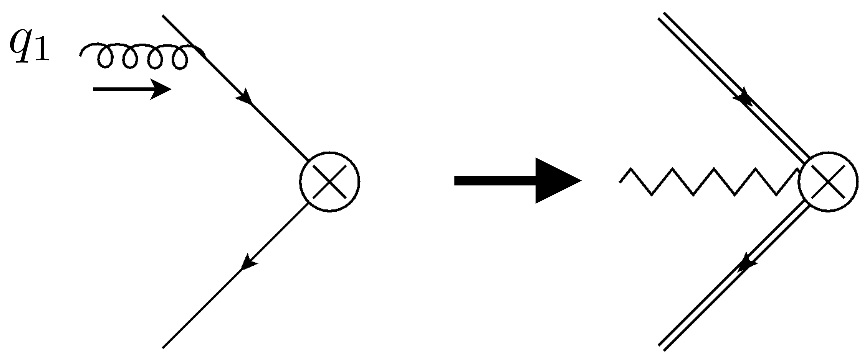
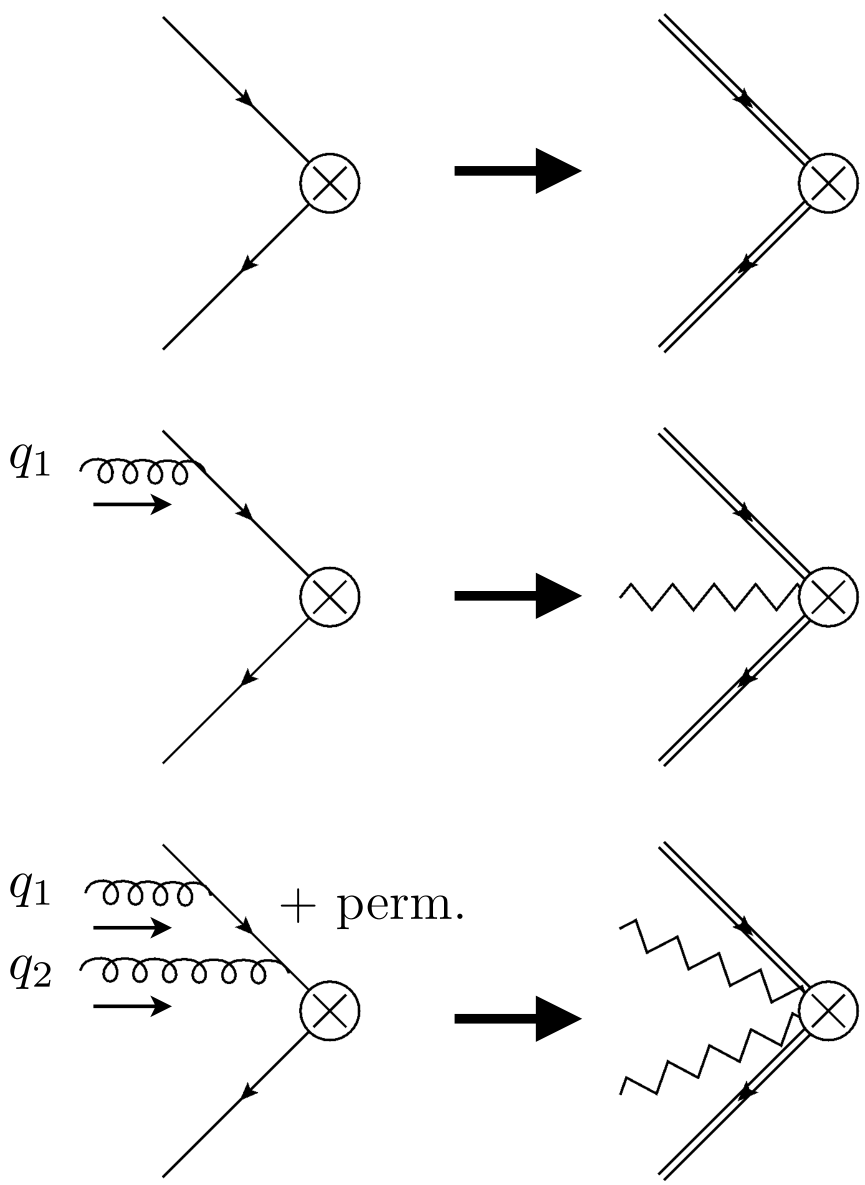


$$= \chi^\dagger \, \mathcal{K}_n \left(\frac{-g \; u \cdot A_s(q_1)}{u \cdot q_1} \right) \psi$$

$$u^\mu = (1, 0, 0, 0)$$

NRQCD at low pT - Soft Wilson lines

QCD Non-relativistic limit



$$= \chi^\dagger \, \mathcal{K}_n \, \psi$$

$$= \chi^\dagger \, \mathcal{K}_n \Big(\frac{-g \; u \cdot A_s(q_1)}{u \cdot q_1} \Big) \psi \qquad u^\mu = (1,0,0,0)$$

$$= \chi^\dagger \, \mathcal{K}_n \Big(\frac{g^2 \; [u \cdot A_s(q_1)][u \cdot A_s(q_2)]}{2 \; [u \cdot q_1][u \cdot (q_1 + q_2)]} \Big) \psi \quad \text{+ perm.}$$

$$\rightarrow \quad Y_u = \sum_k \sum_{\text{perm}} \frac{(-g)^k [u \cdot A_s(q_1)] \cdots [u \cdot A_s(q_k)]}{(k!)[u \cdot q_1] \cdots [u \cdot (q_1 + q_2 + \cdots q_k)]}$$

NRQCD at low pT - Soft Wilson lines

(1) This soft Wilson line is it a “v-soft” or a “λ-soft”?

(2) What are the different hierarchies between the two soft scales?

(3) Can we do a perturbative calculation of the shape function to check consistency of out factorization?

(4) How this factorization is related with the standard NRQCD factorization if the small pT limit is taken?

NRQCD at low pT - Soft Wilson lines

(1) This soft Wilson line is it a "v-soft" or a "λ-soft"?

There is **no** hierarchy → No two modes to choose from.

There is hierarchy → Question 2

NRQCD at low pT - Soft Wilson lines

(2) What are the different hierarchies between the two soft scales?

Region (1)

$p_T \gg m_Q v \gtrsim \Lambda_{\rm QCD}$

Two separated soft modes

$$A_s \to A_{(\lambda)s} + A_{(v)s}$$

$$Y_{s,u} \to Y_{(\lambda)s,u} \times Y_{(v)s,u}$$

$$|X_s\rangle \to |X_{(\lambda)s}\rangle \times |X_{(v)s}\rangle$$

$$S_n^{\mathcal{Q}}(p_T) \to S_c(p_T) \times \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

Region (2)

$p_T \sim m_Q v \sim \Lambda_{\rm QCD}$

No hierarchy of scales

Shape function introduces **new** non-perturbative effects

No further re-factorization

Region (3)

$p_T \sim m_Q v \gg \Lambda_{\rm QCD}$

Hadronization effects happen at much lower scale but no re-factorization is possible.

Can be matched onto a **new** set of LDMEs

$$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$$

NRQCD at low pT - Soft Wilson lines

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$p_T \gg m_Q v \gtrsim \Lambda_{\rm QCD}$

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Region (2) ✓

$p_T \sim m_Q v \sim \Lambda_{\rm QCD}$

$$S_n^{[Q\bar{Q}]}\Big|_{\rm NLO} = S_{[c]}\Big|_{\rm NLO} + \langle \mathcal{O}_n^{Q\bar{Q}} \rangle\Big|_{\rm NLO}$$

$$\gamma_\mu^H + \gamma_\mu^S + 2\gamma_\mu^{\mathcal{B}} = 0$$

$$\gamma_\nu^S + 2\gamma_\nu^{\mathcal{B}} = 0$$

Region (3)

$p_T \sim m_Q v \gg \Lambda_{\rm QCD}$

Hadronization effects happen at much lower scale but no re-factorization is possible.

Can be matched onto a **new** set of LDMEs

$$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$$

Rapidity regulator and rapidity-RG:
arXiv:1202.0814, J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein

NRQCD at low pT - Soft Wilson lines

(4) How this factorization is related with the standard NRQCD factorization if the small pT limit is taken?

Region (1) ✓

$p_T \gg m_Q v \gtrsim \Lambda_{\rm QCD}$

Two separated soft modes

$$A_s \to A_{(\lambda)s} + A_{(v)s}$$
$$Y_{s,u} \to Y_{(\lambda)s,u} \times Y_{(v)s,u}$$
$$|X_s\rangle \to |X_{(\lambda)s}\rangle \times |X_{(v)s}\rangle$$

$$S_n^{\mathcal{Q}}(p_T) \to S_c(p_T) \times \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

Region (2) ✓

$p_T \sim m_Q v \sim \Lambda_{\rm QCD}$

$$S_n^{[Q\bar{Q}]}\Big|_{\rm NLO} = S_{[c]}\Big|_{\rm NLO} + \langle \mathcal{O}_n^{Q\bar{Q}} \rangle\Big|_{\rm NLO}$$

$$\gamma_\mu^H + \gamma_\mu^S + 2\gamma_\mu^{\mathcal{B}} = 0$$

$$\gamma_\nu^S + 2\gamma_\nu^{\mathcal{B}} = 0$$

Region (3)

$p_T \sim m_Q v \gg \Lambda_{\rm QCD}$

Hadronization effects happen at much lower scale but no re-factorization is possible.

Can be matched onto a **new** set of LDMEs

$$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$$

Rapidity regulator and rapidity-RG:
arXiv:1202.0814, J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein

NRQCD at low pT - Soft Wilson lines

(?)

Region (1)

$p_T \gg m_Q v \gtrsim \Lambda_{\rm QCD}$

Two separated soft modes

$$A_s \to A_{(\lambda)s} + A_{(v)s}$$

$$Y_{s,u} \to Y_{(\lambda)s,u} \times Y_{(v)s,u}$$

$$|X_s\rangle \to |X_{(\lambda)s}\rangle \times |X_{(v)s}\rangle$$

$$S_n^{\mathcal{Q}}(p_T) \to S_c(p_T) \times \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

Region (2)

$p_T \sim m_Q v \sim \Lambda_{\rm QCD}$

No hierarchy of scales

Shape function introduces **new** non-perturbative effects

No further re-factorization

Region (3)

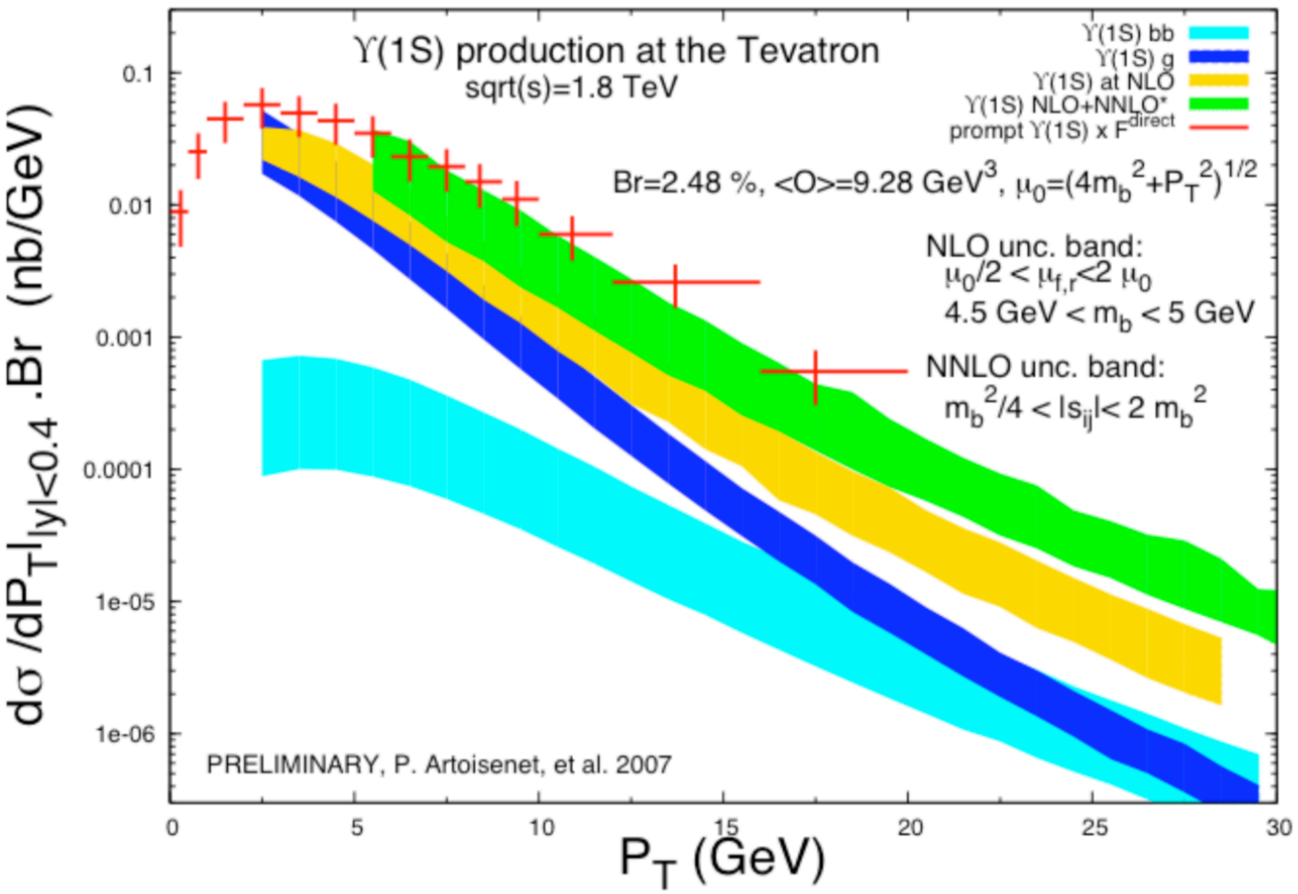
$p_T \sim m_Q v \gg \Lambda_{\rm QCD}$

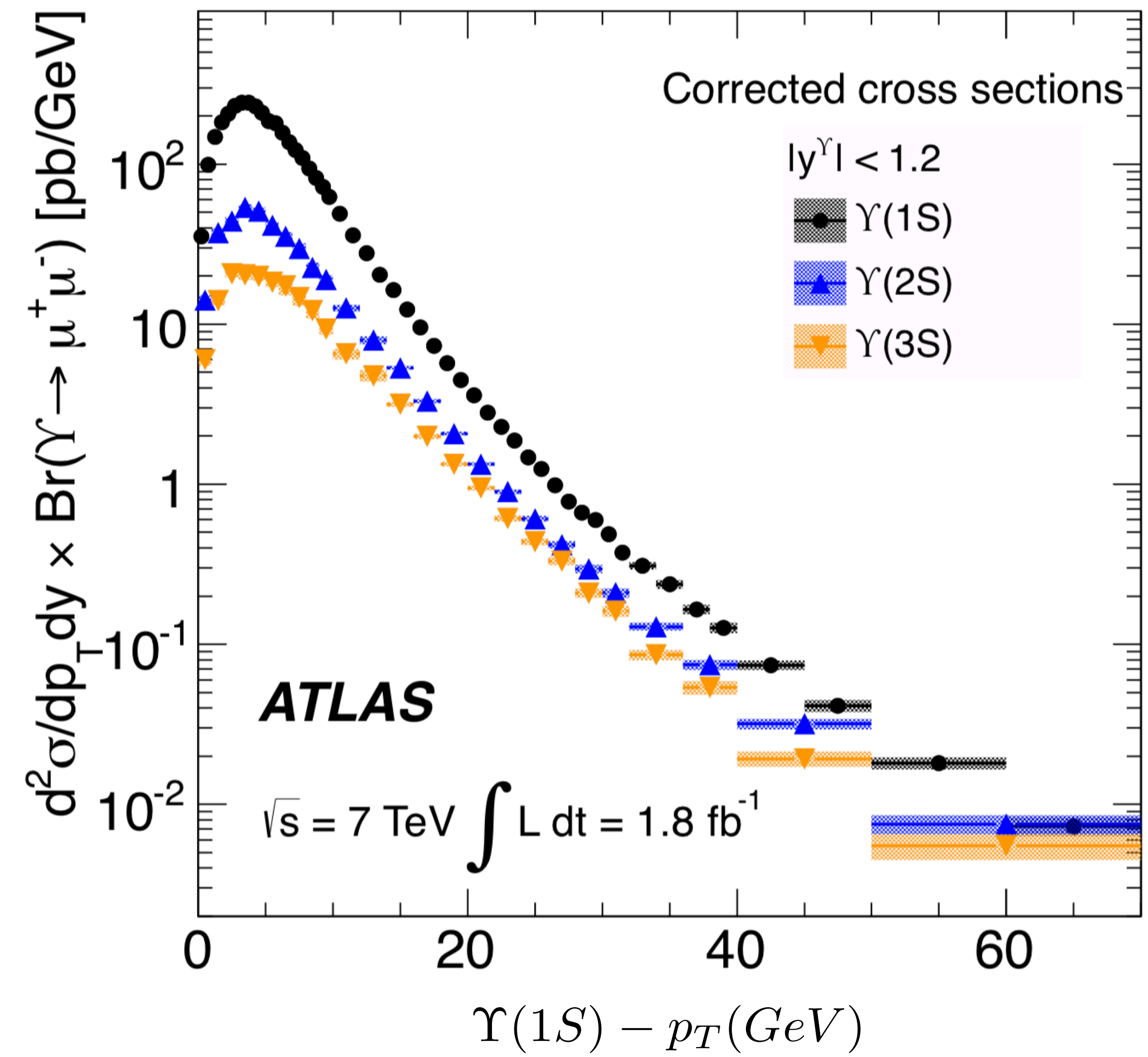
Hadronization effects happen at much lower scale but no re-factorization is possible.

Can be matched onto a **new** set of LDMEs

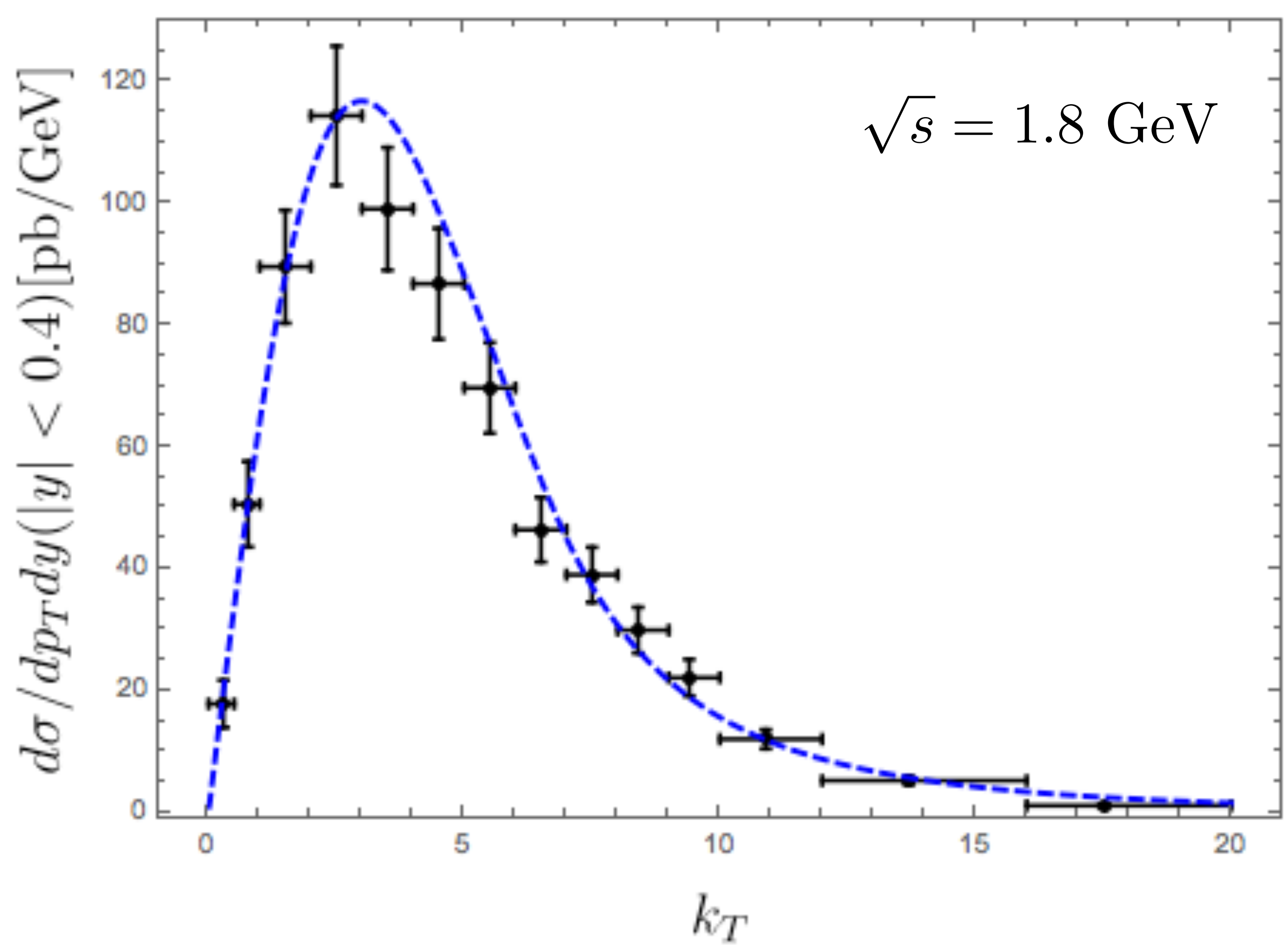
$$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$$

Quarkonium at low pT (Experimental data)





NRQCD at low pT - Numerics (preliminary)



- Region (1) + (2) factorization

- All leading and subheading channels.

- Octet LDMEs are fitted.

Summary

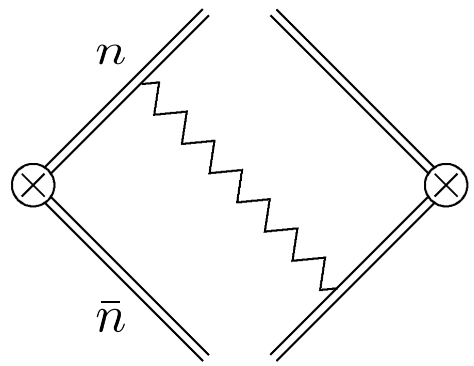
- Time-like Wilson line is necessary for consistency in the case of color octet mechanisms
- Quarkonium production in low pT standard LDME are “promoted” to shape functions
- Small pT limit of NRQCD is recovered only in the limit: $p_T \gg m_Q v$
- Shape function introduces new non-perturbative effects.
- Extension to:
    - polarized cross sections
    - TMD-fragmentation
    - In-jet Quarkonia

Thank you

Back-up

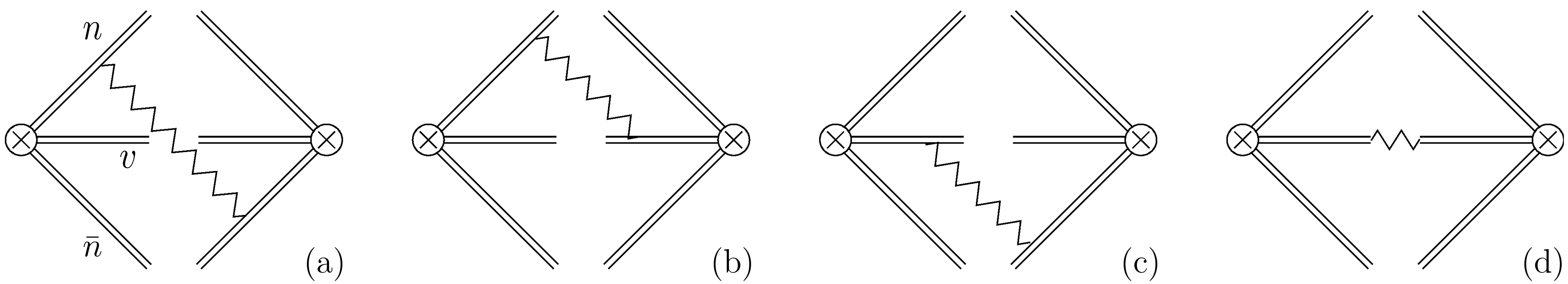
Quarkonium at low pT (shape function perturbative)

soft contribution to singlet:



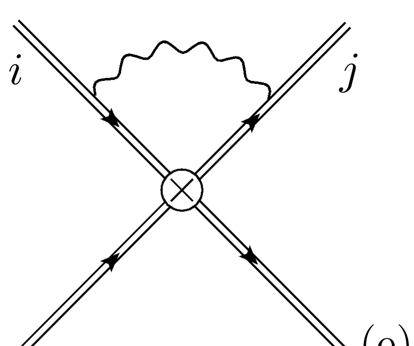
+ mirror diagram

soft contribution to octet:

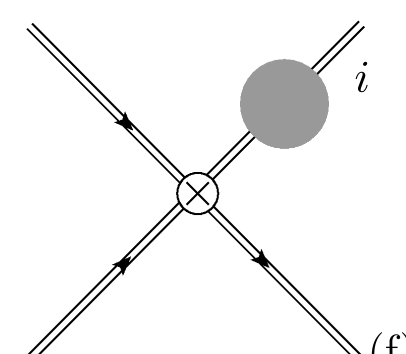


+ mirror diagram + mirror diagram + mirror diagram

ultra-soft gluon exchanges: $\frac{1}{2}\sum_{i\neq j}$



self-energy diagrams: $\sum_{i}$



Quarkonium at low pT (shape function perturbative)

$$\boxed{S_{ij}^{[8]} = S_{ij}^{[1]} + \frac{\alpha_s C_A}{2\pi}\Big\{\frac{1}{\epsilon}\delta^{(2)}(\mathbf{p}_T) - 2\mathcal{L}_0(\mathbf{p}_T^2,\mu^2)\Big\}}$$

$$S_{ij}^{[1]}(\mathbf{p}_T) = \delta^{(2)}(\mathbf{p}_T) + \frac{\alpha_s C_{ij}}{2\pi}\Big\{\frac{4}{\eta}\Big[2\mathcal{L}_0(\mathbf{p}_T^2,\mu^2) - \frac{1}{\epsilon}\delta^{(2)}(\mathbf{p}_T)\Big] + \frac{2}{\epsilon}\Big[\frac{1}{\epsilon} - \ln\Big(\frac{\nu^2}{\mu^2}\Big)\Big]\delta^{(2)}(\mathbf{p}_T) - \frac{\pi^2}{6}\delta(\mathbf{p}_T)$$

$$- 4\mathcal{L}_1(\mathbf{p}_T^2,\mu^2) + 4\mathcal{L}_0(\mathbf{p}_T^2,\mu^2)\ln\Big(\frac{\nu^2}{\mu^2}\Big)\Big\}$$

<u>NLO hard function:</u>

$$H_{ij}^{[n],\mathrm{b}} = 1 + \frac{\alpha_s C_{ij}}{2\pi}\Big\{\frac{2}{\epsilon}\Big[\ln\Big(\frac{M^2}{\mu^2}\Big) - \bar{\gamma}_i - \frac{1}{\epsilon}\Big] + 2B(n = {}^{2S+1}L_J^{[c]}) - \ln^2\Big(\frac{M^2}{\mu^2}\Big) - \frac{\pi^2}{6} + 2\bar{\gamma}_i\ln\Big(\frac{M^2}{\mu^2}\Big)\Big\}$$

$$+ \delta_{c8}\frac{\alpha_s C_A}{2\pi}\Big\{ - \frac{1}{\epsilon} + \ln\Big(\frac{M^2}{\mu^2}\Big)\Big\}$$

NRQCD at low pT - Soft Wilson lines

(4) How this factorization is related with the standard NRQCD factorization if the small pT limit is taken?

Region (1)	Region (2)	Region (3)
$p_T \gg m_Q v \gtrsim \Lambda_{\rm QCD}$	$p_T \sim m_Q v \sim \Lambda_{\rm QCD}$	$p_T \sim m_Q v \gg \Lambda_{\rm QCD}$
Two separated soft modes	No hierarchy of scales	Hadronization effects happen at much lower scale but no re-factorization is possible.
$A_s \to A_{(\lambda)s} + A_{(v)s}$ $Y_{s,u} \to Y_{(\lambda)s,u} \times Y_{(v)s,u}$ $\lvert X_s\rangle \to \lvert X_{(\lambda)s}\rangle \times \lvert X_{(v)s}\rangle$	Shape function introduces **new** non-perturbative effects	Can be matched onto a **new** set of LDMEs
$S_n^{\mathcal{Q}}(p_T) \to S_c(p_T) \times \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$	No further re-factorization	$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$

$$\overline{\mathcal{O}}_n^{\mathcal{Q}}(c=1) = \mathcal{O}_n^{(2)\dagger} \mathcal{Y}_{n,s}^{a\dagger} \mathcal{Y}_{\bar{n},s}^{b\dagger} \left(a_{\mathcal{Q}}^\dagger a_{\mathcal{Q}} \right) \mathcal{Y}_{\bar{n},s}^{b} \mathcal{Y}_{n,s}^{a} \mathcal{O}_n^{(2)}$$

$$\overline{\mathcal{O}}_n^{\mathcal{Q}}(c=8) = \mathcal{O}_n^{(2)\dagger} \mathcal{Y}_{v,s}^{c\dagger} \mathcal{Y}_{n,s}^{a\dagger} \mathcal{Y}_{\bar{n},s}^{b\dagger} \left(a_{\mathcal{Q}}^\dagger a_{\mathcal{Q}} \right) \mathcal{Y}_{\bar{n},s}^{b} \mathcal{Y}_{n,s}^{a} \mathcal{Y}_{v,s}^{c} \mathcal{O}_n^{(2)}$$