Twist-2 transverse momentum dependent distributions at NNLO in QCD

QCD Evolution 2018, Santa Fe, NM, US, May 20-24

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Based on:
arXiv: 1702.06558
arXiv: 1805.07243 **New!**

Outline

* Introduction
    * Factorization theorems with TMDs
    * Small-b operator product expansion
* Transversity and Pretzelosity at NLO
* Transversity and Pretzelosity at NNLO
* Conclusions

Factorization theorems with TMDs
Definition of Operators

TMD factorization theorems
Consistent treatment of rapidity divergences in Spin (in)dependent TMDs

→

Self contained definition of TMD operators
↓
Without referring to a scattering process

- Quark and gluon components of the generic TMDs

$$\Phi_{ij}(x,\boldsymbol{b}) = \int \frac{d\lambda}{2\pi} e^{-ixp^+\lambda} \bar{q}_i\left(\lambda n + \boldsymbol{b}\right) \mathcal{W}(\lambda,\boldsymbol{b}) q_j\left(0\right)$$

$$\Phi_{\mu\nu}(x,\boldsymbol{b}) = \frac{1}{xp^+} \int \frac{d\lambda}{2\pi} e^{-ixp^+\lambda} F_{+\mu}\left(\lambda n + \boldsymbol{b}\right) \mathcal{W}(\lambda,\boldsymbol{b}) F_{+\nu}\left(0\right)$$

- The soft function renormalizes the rapidity divergences

$$S(\boldsymbol{b}) = \frac{\mathrm{Tr}_{\mathrm{color}}}{N_c} \langle 0| \left[S_n^{T\dagger} \tilde{S}_{\bar{n}}^T \right](\boldsymbol{b}) \left[\tilde{S}_{\bar{n}}^{T\dagger} S_n^T \right](0) |0\rangle$$

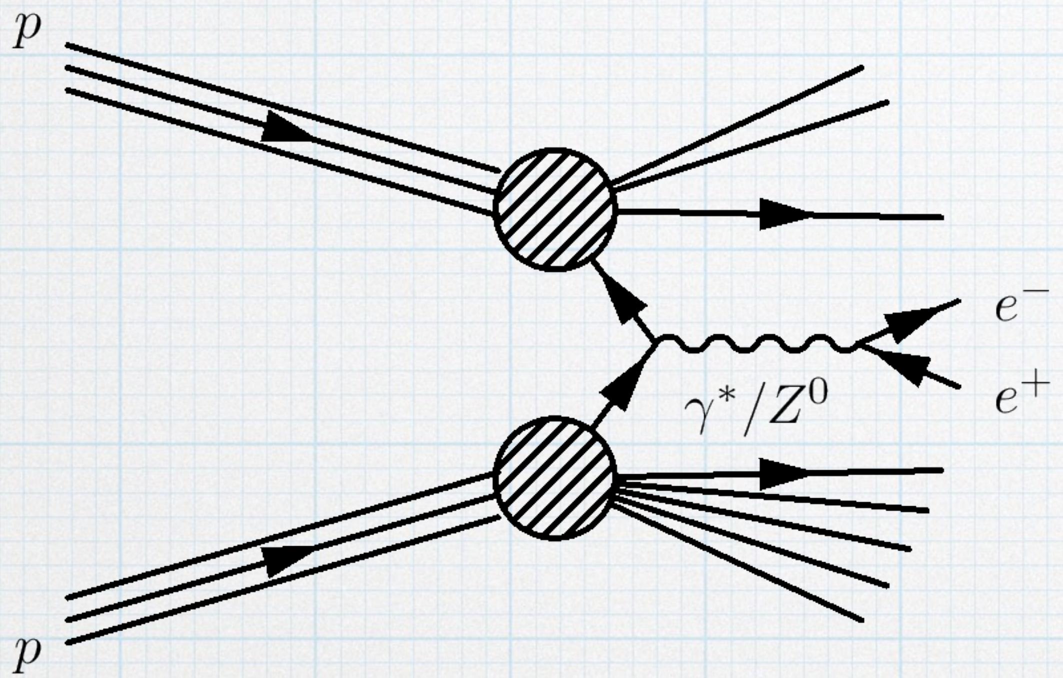
$$S(\boldsymbol{b}) = \exp\left(A(\boldsymbol{b},\epsilon) \ln(\delta^+\delta^-) + B(\boldsymbol{b},\epsilon) \right)$$

R-factor

$$R_{\delta\text{-reg.}} = \frac{1}{\sqrt{S(\boldsymbol{b})}}$$

Its logs are linear in $\ln(\delta^+\delta^-)$
It allows to split r.d. and define individual TMDs!

Factorization theorems with TMDs
Drell-Yan cross section



We write the cross section in terms of a product of TMDPDFs!

DIFFERENT POLARIZATIONS!

Factorization theorems allow us to write cross sections as

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c} \frac{\mathcal{P}}{sQ^2} \sum_{GG'} z_{ll'}^{GG'}(q) \sum_{ff'} z_{FF'}^{GG'} |C_V(q,\mu)|^2$$

$$\int \frac{d^2\boldsymbol{b}}{4\pi} e^{i(\boldsymbol{b}\boldsymbol{q})} F_{f\leftarrow h_1}(x_1, \boldsymbol{b}; \mu, \zeta) F_{f'\leftarrow h_2}(x_2, \boldsymbol{b}; \mu, \zeta) + Y$$

Small-b operator product expansion

Small-b OPE ⇨ Relation between **TMD operators** and **lightcone operators**

$$\Phi_{ij}(x,\boldsymbol{b}) = \Big[(C_{q\leftarrow q}(\boldsymbol{b}))_{ij}^{ab} \otimes \phi_{ab}\Big](x) + \Big[(C_{q\leftarrow g}(\boldsymbol{b}))_{ij}^{\alpha\beta} \otimes \phi_{\alpha\beta}\Big](x) + ...,$$

$$\Phi_{\mu\nu}(x,\boldsymbol{b}) = \Big[(C_{g\leftarrow q}(\boldsymbol{b}))_{\mu\nu}^{ab} \otimes \phi_{ab}\Big](x) + \Big[(C_{g\leftarrow g}(\boldsymbol{b}))_{\mu\nu}^{\alpha\beta} \otimes \phi_{\alpha\beta}\Big](x) + ...$$

Projectors over polarizations

$$\Phi_q^{[\Gamma]} = \frac{\mathrm{Tr}(\Gamma\Phi)}{2} \qquad \Phi_g^{[\Gamma]} = \Gamma^{\mu\nu}\Phi_{\mu\nu}$$

Small-b OPE: Cancellation of rapidity divergences

- **Small-b OPE** for a generic TMD quark operator

$$\Phi_q^{[\Gamma]} = \Gamma^{ab}\phi_{ab} + a_s C_F \boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\Big[\ldots$$

$$+\left(\frac{1}{(1-x)_+} - \ln\left(\frac{\delta}{p^+}\right)\right)\left(\gamma^+\gamma^-\Gamma + \Gamma\gamma^-\gamma^+ + \frac{i\epsilon\gamma^+\not{b}\Gamma}{2\boldsymbol{B}} + \frac{i\epsilon\Gamma\not{b}\gamma^+}{2\boldsymbol{B}}\right)^{ab} + \ldots\Big] \otimes \phi_{ab} + \mathcal{O}(a_s^2)$$

- General **R-factor**

$$R = 1 + 2a_s C_F \boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\left(\mathbf{L}_{\sqrt{\zeta}} + 2\ln\left(\frac{\delta}{p^+}\right) - \psi(-\epsilon) - \gamma_E\right) + \mathcal{O}(a_s^2)$$

Cancellation of rapidity divergences in $R\Phi$ →

$$\gamma^+\Gamma = \Gamma\gamma^+ = 0$$
$$\Gamma^{+\mu} = \Gamma^{-\mu} = \Gamma^{\mu+} = \Gamma^{\mu-} = 0$$

$$\Gamma^q = \{\gamma^+, \gamma^+\gamma^5, \sigma^{+\mu}\}$$

$$\Gamma^g = \{g_T^{\mu\nu}, \epsilon_T^{\mu\nu}, b^\mu b^\nu/\boldsymbol{b}^2\}$$

Lorentz structures of "leading dynamical twist" TMDs

Spin dependent TMD decomposition

Hadron matrix elements of TMD decomposed over all posible Lorentz variants

Polarized TMDPDFs

Naturally defined ← Momentum space / b-space (IPS) →

Goeke, Metz, Schegel 0504130, Bacchetta, Boer, Diehl, Mulders 0803.0227

Boer, Gamberg, Musch, Prokudin 1107.5294 Echevarria, Kasemets, Mulders, Pisano 1502.05354

Decomposition over Lorentz variants

Unpolarized quarks

Helicity quarks

$$\Phi_{q\leftarrow h,ij}(x,\boldsymbol{b}) = \langle h|\Phi_{ij}(x,\boldsymbol{b})|h\rangle = \frac{1}{2}\Big(f_1\gamma^-_{ij} + g_{1L}S_L(\gamma_5\gamma^-)_{ij}$$

$$(S_T^\mu i\gamma_5\sigma^{+\mu})_{ij}h_1 + (i\gamma_5\sigma^{+\mu})_{ij}\left(\frac{g_T^{\mu\nu}}{2}+\frac{b^\mu b^\nu}{\boldsymbol{b}^2}\right)\frac{S_T^\nu}{2}h_{1T}^\perp + \ldots\Big)$$

Transversity

Pretzelosity

$$\Phi_{g\leftarrow h,\mu\nu}(x,\boldsymbol{b}) = \langle h|\Phi_{\mu\nu}(x,\boldsymbol{b})|h\rangle = \frac{1}{2}\Big(-g_T^{\mu\nu}f_1^g - i\epsilon_T^{\mu\nu}S_L g_{1L}^g + 2h_1^{\perp g}\left(\frac{g_T^{\mu\nu}}{2}+\frac{b^\mu b^\nu}{\boldsymbol{b}^2}\right) + \ldots\Big)$$

Unpolarized gluons

Helicity gluons

Linearly polarized gluons

	LO	NLO	NNLO
Unpolarized	✓	✓	✓
Helicity	✓	✓	✗
Transversity	✓	✓	✓
Pretzelosity	✓	✓	✓
Linearly polarized gluons	✓	✓	✗

	LO	NLO	NNLO
Unpolarized	✓	✓	✓
Helicity	✓	✓	✗
Transversity	✓	✓	✓
Pretzelosity	✓	✓	✓
Linearly polarized gluons	✓	✓	✗

Transversity and Pretzelosity at NLO

Lorentz structure and matching

Usual spinor structure

$$\Gamma = i\gamma_5\sigma^{+\mu}$$

Scheme dependent

Not mixture with gluons at leading twist

Common spinor structure

$$\Gamma = \sigma^{+\mu}$$

Scheme independent!

Calculating $R\Phi$ and comparing with the general parameterization

$$R\Phi_q^{[\sigma^{+\mu}]} = g_T^{\mu\nu}\delta C_{q\leftarrow q}\otimes \phi_q^{[\sigma^{+\nu}]} + \left(\frac{b^\mu b^\nu}{\boldsymbol{b}^2} + \frac{g_T^{\mu\nu}}{2(1-\epsilon)}\right)\delta^\perp C_{q\leftarrow q}\otimes\phi_q^{[\sigma^{+\nu}]}$$

Transversity - Transversity matching

Pretzelosity - Transversity matching

Matching coefficients up to NLO

$$\Phi_{1;f\leftarrow q}(x,\boldsymbol{b}) = \sum_{f'=q,\bar{q}} \delta C^{(\perp)}_{f\leftarrow f'} \otimes h_{1;f'\leftarrow q}(x)$$

TMD **PDF**

Let us solve it recursively!

$$\delta^{(\perp)} C^{[0]}_{f\leftarrow f'} = \Phi^{[0]}_{1;f\leftarrow f'}(x,\boldsymbol{b})$$

$$\delta^{(\perp)} C^{[1]}_{f\leftarrow f'} = \Phi^{[1]}_{1;f\leftarrow f'}(x,\boldsymbol{b}) - h^{[1]}_{1;f\leftarrow f'}(x)$$

Matching coefficients up to NLO

$$H_1^q(z,\boldsymbol{b}) = \int_z^1 \frac{dy}{y^{3-2\epsilon}} \sum_{f=q,\bar{q}} \delta\mathbb{C}_{q\to f}\left(\frac{z}{y}, \mathbf{L}_\mu\right) H_1^f(y) + \mathcal{O}(\boldsymbol{b}^2)$$

TMD

FF

Let us solve it recursively!

$$\delta C_{q\to q}^{[0]} = H_1^{[0]}(z,\boldsymbol{b})$$

$$\delta C_{q\to q}^{[1]} = H_1^{[1]}(z,\boldsymbol{b}) - \frac{H_1^{[1]}(z)}{z^{2-2\epsilon}}$$

Renormalized TMDs up to NLO

$$\Phi(x,\boldsymbol{b};\mu,\zeta) = Z(\mu,\zeta|\epsilon) R(\boldsymbol{b},\mu,\zeta|\epsilon,\delta)\Phi^{\text{unsub.}}(x,\boldsymbol{b}|\epsilon,\delta)$$

$$Z = Z_2^{-1} Z_q$$

Expansion up to NLO

Rapidity divergences cancelled here!

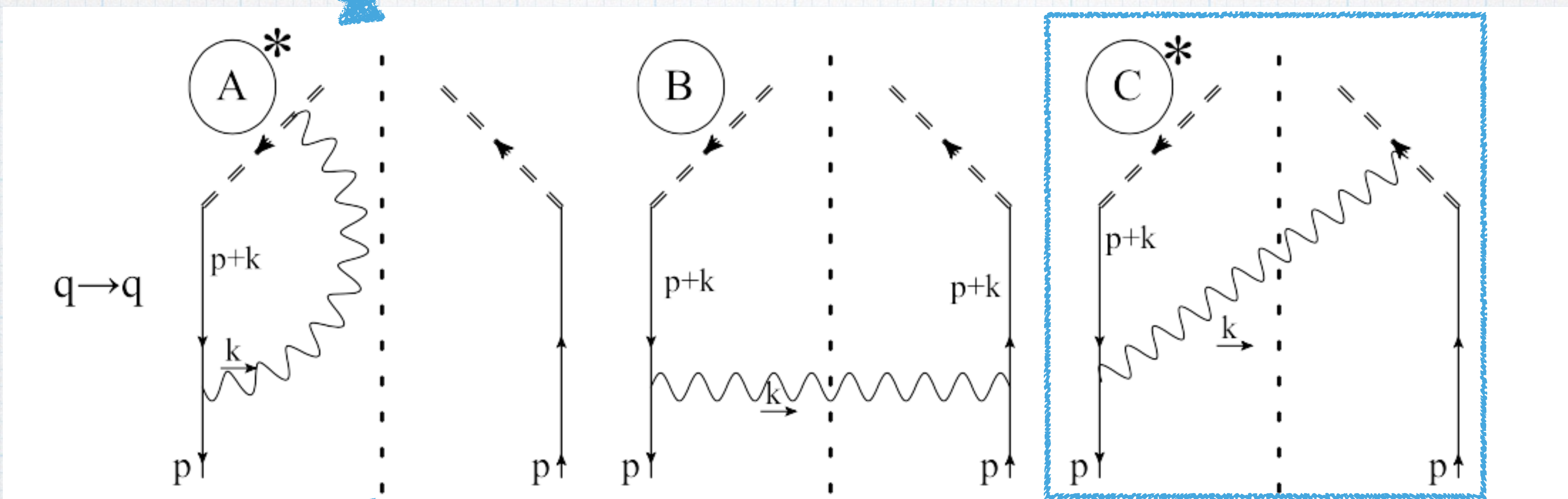
$$\Phi^{[0]}_{f\leftarrow f'} = \Phi^{[0]\text{unsub.}}_{f\leftarrow f'}$$

$$\Phi^{[1]}_{f\leftarrow f'} = \Phi^{[1]\text{unsub.}}_{f\leftarrow f'} - \frac{S^{[1]}\Phi^{[0]\text{unsub.}}_{f\leftarrow f'}}{2} + \left(Z_q^{[1]} - Z_2^{[1]}\right)\Phi^{[0]\text{unsub.}}_{f\leftarrow f'}$$

Diagrams contributing to TMDS at NLO

Transverse projectors

$\sigma^{+\mu}$



$\sigma^{-\nu}$

Project and obtain

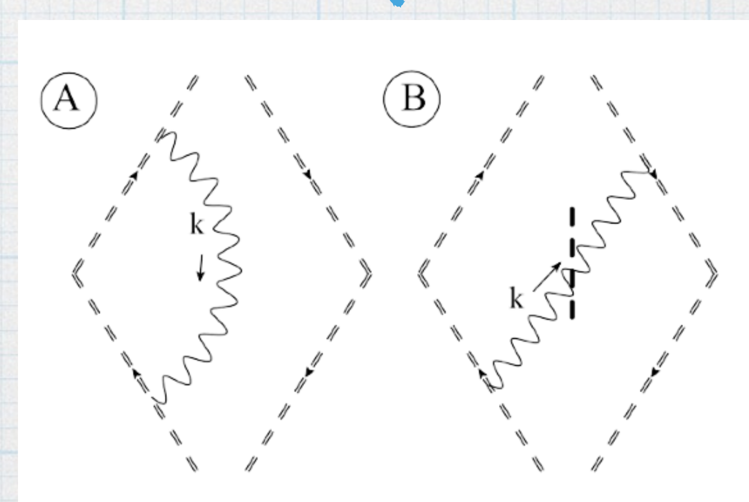
Transversity

$$g_T^{\mu\nu}$$

Pretzelosity

$$\frac{b^\mu b^\nu}{b^2} - \frac{g_T^{\mu\nu}}{2(1-\epsilon)}$$

Rapidity divergences:
Renormalized with SF



The calculation is
striaghtforward
to the unpolarized case
Echevarria, Scimemi, Vladimirov
1604.07869

Matching coefficients up to NLO

Transversity - Transversity small-b expression

$$h_1(x,\boldsymbol{b}) = \Big[\delta C_{q\leftarrow q}(\boldsymbol{b}) \otimes \delta f_q\Big](x) + \mathcal{O}(\boldsymbol{b}^2)$$

Agrees with Bacchetta, Prokudin 1303.2129!

NLO matching coefficient

$$\delta C_{q\leftarrow q} = \delta(\bar{x}) + a_s C_F\Big(-2\mathbf{L}_\mu \delta p_{qq} + \delta(\bar{x})\left(-\mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{l}_\zeta - \zeta_2\right)\Big) + \mathcal{O}(a_s^2)$$

Pretzelosity - Transversity small-b expression

$$h_{1T}^{\perp}(x,\boldsymbol{b}) = \Big[\delta^{\perp} C_{q\leftarrow q}(\boldsymbol{b}) \otimes \delta f_q\Big](x) + \mathcal{O}(\boldsymbol{b}^2) = \Big[\left(0 + \mathcal{O}(a_s^2)\right) \otimes \delta f_q\Big](x) + \mathcal{O}(\boldsymbol{b}^2)$$

NLO matching coefficient

$$\delta^{\perp} C_{q\leftarrow q} = -4 a_s C_F \boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\bar{x}\epsilon^2$$

At NLO the coefficient is $\sim \epsilon$

This observation is supported by the measurement of $\sin(3\phi_h - \phi_S)$ asymmetries by HERMES and COMPASS!

Lefky, Prokudin 1411.0580, Parsamyan PoS(QCDEV2017)042

Matching coefficients up to NLO

Transversity - Transversity Fragmentation small-b expression

$$H_1^q(z,\boldsymbol{b}) = \int_z^1 \frac{dy}{y^{3-2\epsilon}} \sum_{f=q,\bar{q}} \delta\mathbb{C}_{q\to f}\left(\frac{z}{y},\mathbf{L}_\mu\right) H_1^f(y) + \mathcal{O}(\boldsymbol{b}^2)$$

NLO matching coefficient

$$z^2\delta\mathbb{C}_{q\to q} = \delta(\bar{z}) + a_s C_F\left(\left(4\ln z - 2\mathbf{L}_\mu\right)\delta p_{qq} + \delta(\bar{z})\left(-\mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{l}_\zeta - \zeta_2\right)\right)$$

Pretzelosity - Transversity small-b expression

$$h_{1T}^\perp(x,\boldsymbol{b}) = \left[\delta^\perp C_{q\leftarrow q}(\boldsymbol{b})\otimes\delta f_q\right](x) + \mathcal{O}(\boldsymbol{b}^2) = \left[\left(0+\mathcal{O}(a_s^2)\right)\otimes\delta f_q\right](x) + \mathcal{O}(\boldsymbol{b}^2)$$

NLO matching coefficient

$$\delta^\perp C_{q\leftarrow q} = -4a_s C_F \boldsymbol{B}^\epsilon\Gamma(-\epsilon)\bar{x}\epsilon^2$$

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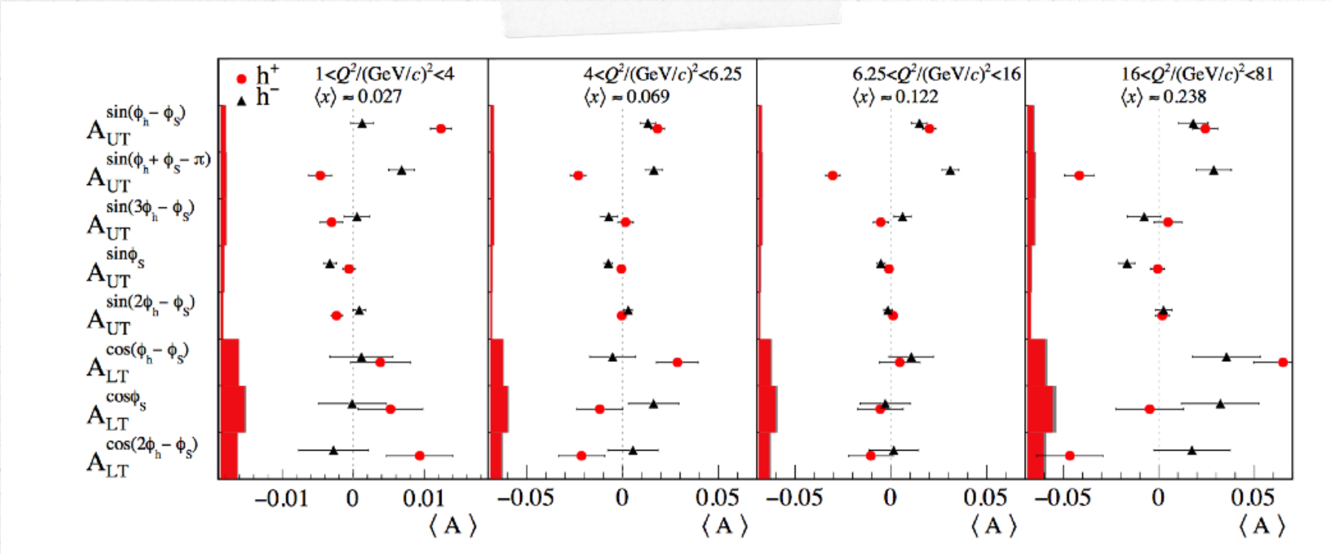
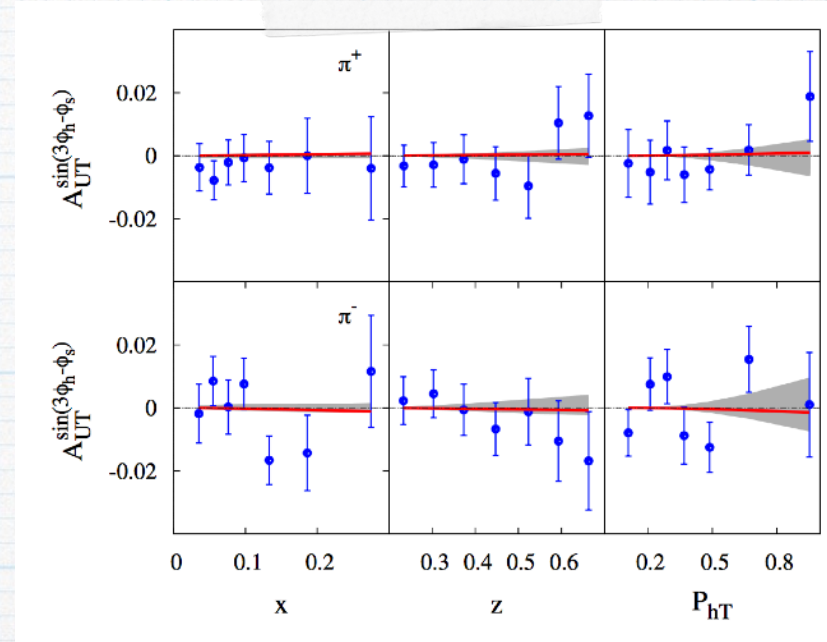
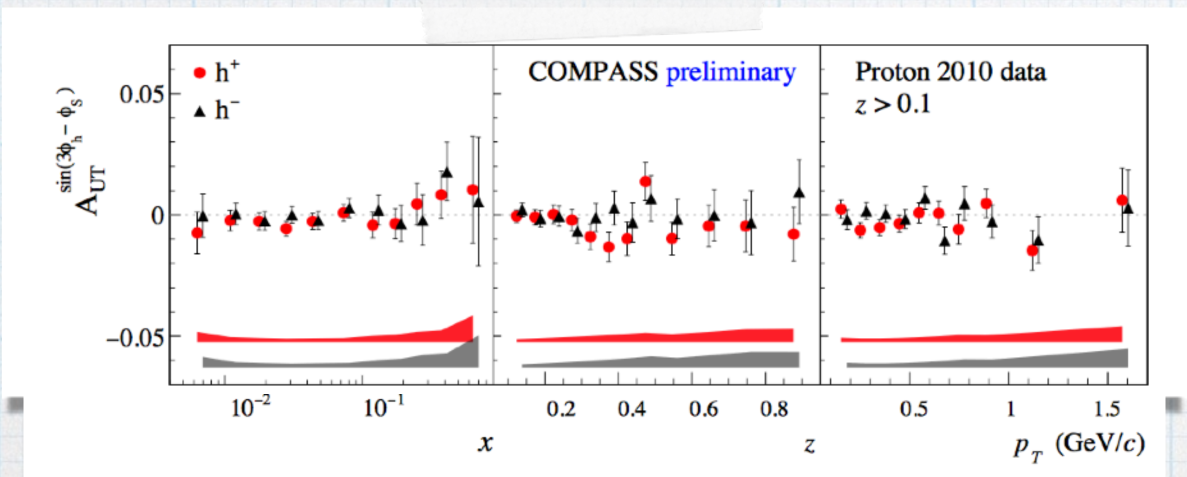
Lefky, Prokudin 1411.0580, Parsamyan PoS(QCDEV2017)042





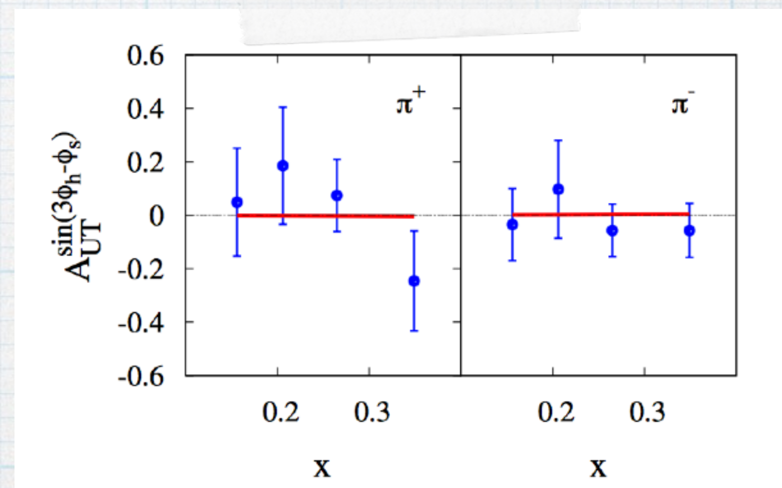
HERMES



COMPASS

Parsamyan PoS(QCDEV2017)042

See Parsamyan's talk



JLAB

Lefky, Prokudin 1411.0580

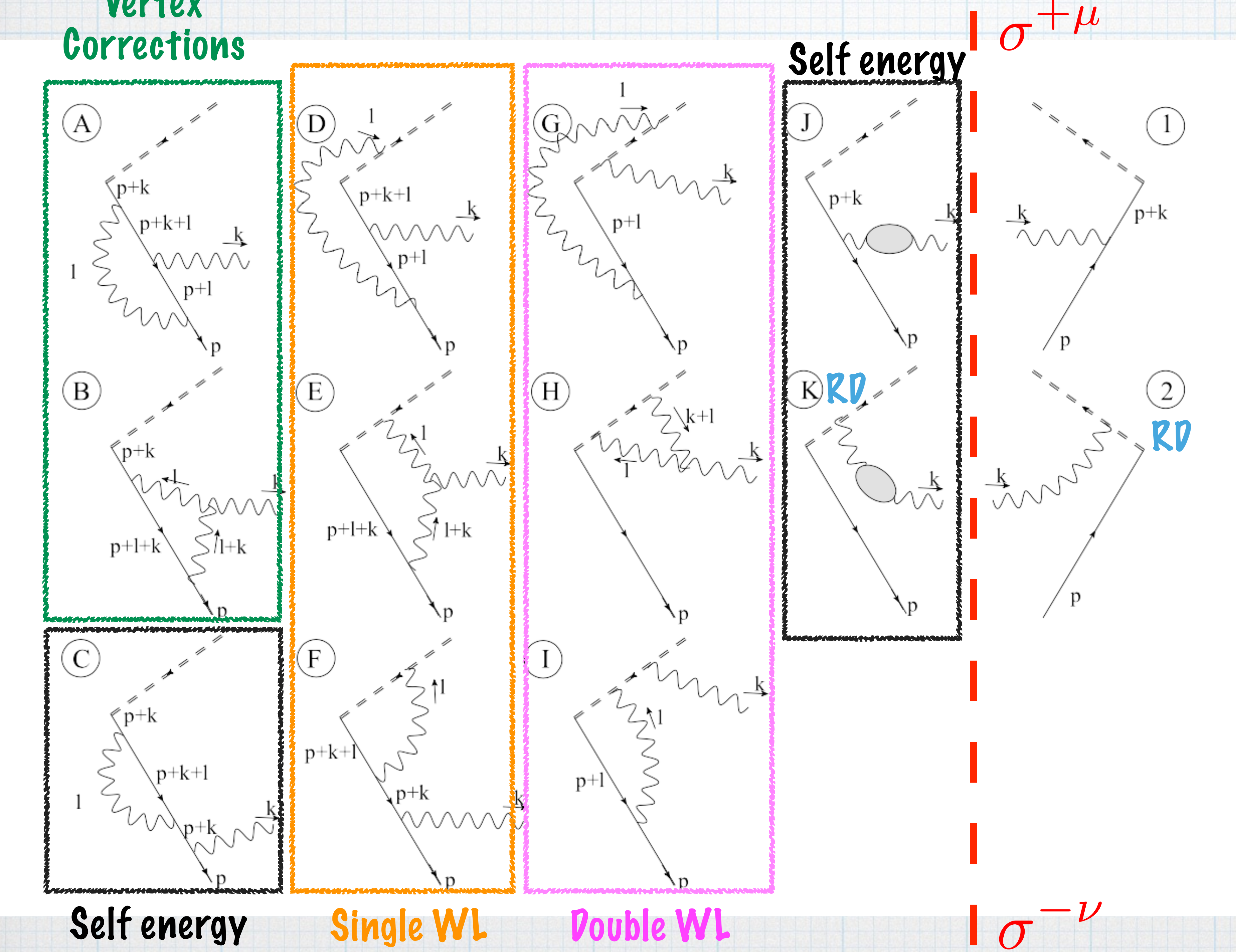
Transversity and Pretzelosity at NNLO

Transversity distribution

Virtual-Real diagrams

Vertex Corrections

Self energy



Self energy

Single WL
RD

Double WL
RD

L.H.S.

R.H.S.

Pole $1/\epsilon^3$

Should be cancelled with vertex correction term in RR diagrams

Pole $1/\epsilon^3$

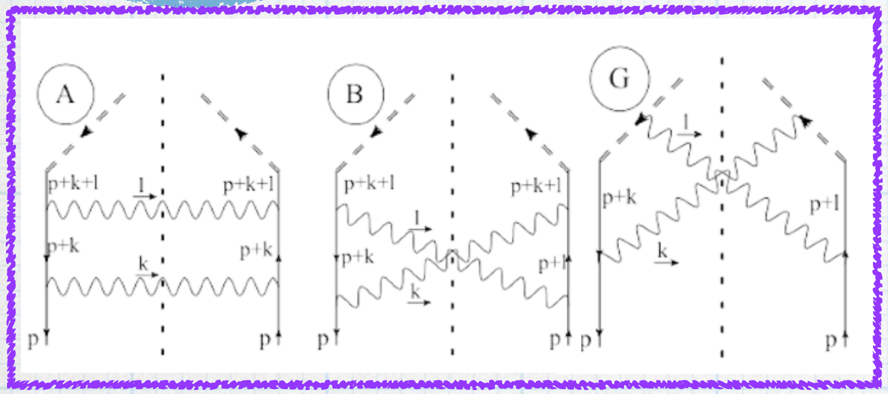
Should be cancelled with single WL term in RR diagrams

These diagrams are exactly zero!

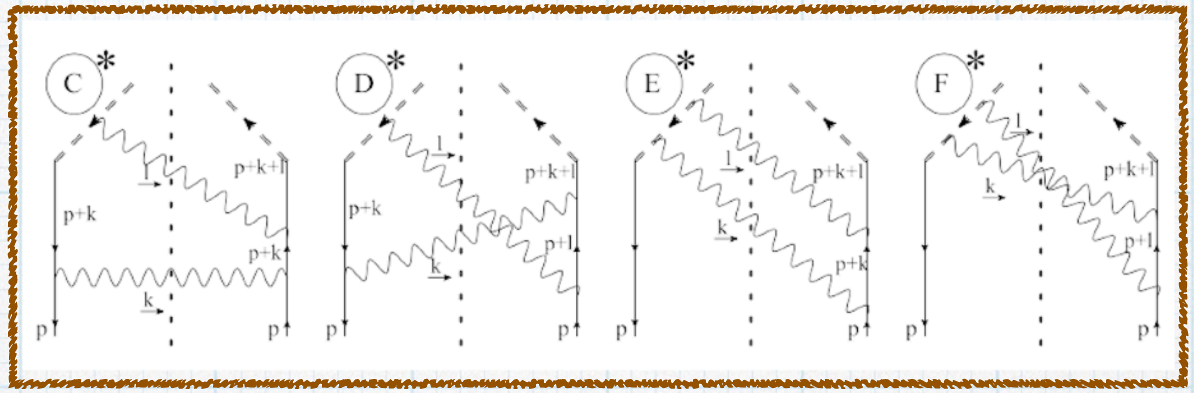
Quark self-energy
+
Gluon self-energy (TrNf)

Real-Real diagrams

q ← q

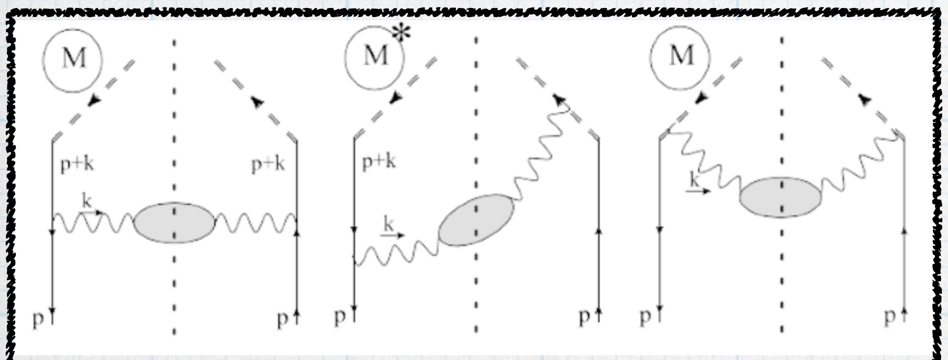


Real ladder

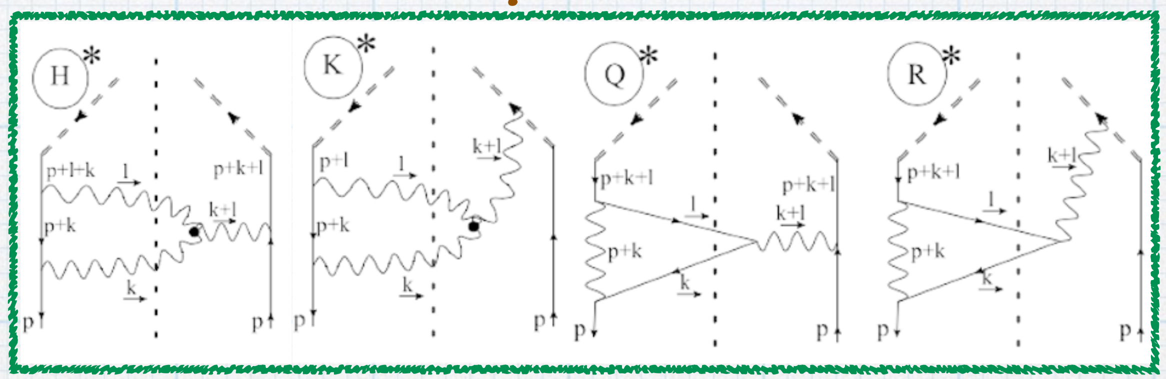


Complex ladder

Pole $1/\epsilon^3$
Cancelled with
vertex correction term
in VR diagrams
As in Unpolarized!



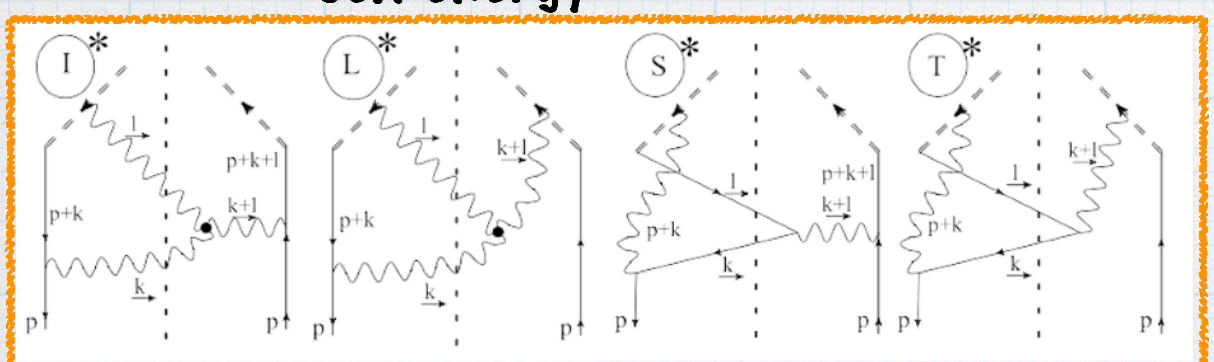
Self energy



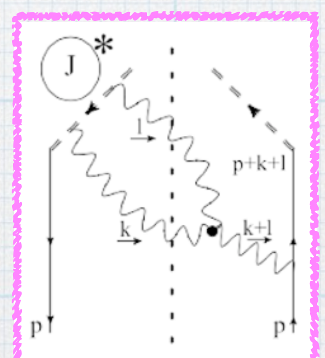
Vertex Corrections

Pole $1/\epsilon^3$
Cancelled with
single WL term
in RR diagrams
As in Unpolarized!

Depend on
TrNf



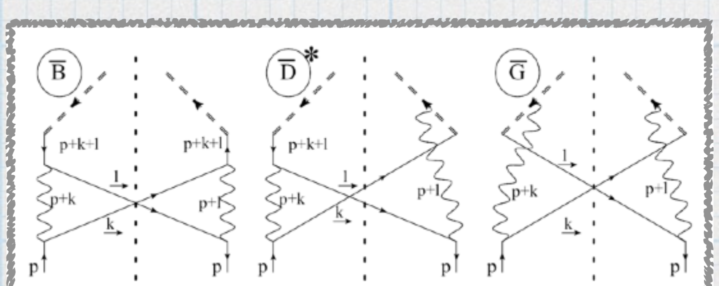
Single WL



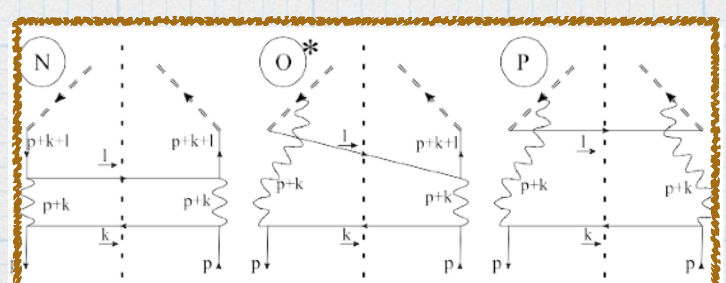
Double WL

Real ladder
Complex ladder
Double WL
→ From $1/\epsilon^2$

q ← $\bar{q}$



No RD
Finite result, without plus-distribted terms and deltas



q ← q'

It is zero!
Odd number of gamma-matrices
In each trace

Renormalization of TMD at NNLO
Cancellation of rapidity divergences

q ← q

RD free!

1-loop Transversity RD free!

$$h_1^{[2]} = \delta\Phi^{[2]} - \frac{S^{[1]}\delta\Phi^{[1]}}{2} - \frac{S^{[2]}\delta\Phi^{[0]}}{2} + \frac{3S^{[1]}S^{[1]}\delta\Phi^{[0]}}{8} + \left(Z_q^{[1]} - Z_2^{[1]}\right)\left(\delta\Phi^{[1]} - \frac{S^{[1]}\delta\Phi^{[0]}}{2}\right)$$

$$+\left(Z_q^{[2]} - Z_2^{[2]} - Z_2^{[1]}Z_q^{[1]} - Z_2^{[1]}Z_2^{[1]}\right)\delta\Phi^{[0]}$$

UV surface term
Pure UV divergence → Z_q Z_2 The same that in unpolarized case!

Sum of all the diagrams

$$\text{diag} = A + B\left(\frac{\delta^+}{p^+}\right)^{-\epsilon} + C\left(\frac{\delta^+}{p^+}\right)^{\epsilon} + D\ln\left(\frac{\delta^+}{p^+}\right) + E\ln^2\left(\frac{\delta^+}{p^+}\right)$$

In the sum of the diagrams the total expression for B and C is zero
IR terms are self-cancelled!

$\bar{q}$ ← q

$$\delta\Phi^{[0]} = 0$$

$$\delta\Phi^{[1]} = 0$$

This channel does not appear up to NNLO →

$$h_1^{[2]} = \delta\Phi^{[2]}$$

No RD here!

Matching coefficients

PDF

Convolution of 1-loop coefficient with 1-loop PDF Cancellation of $\mathbf{L}_\mu/\epsilon$

Renormalized TMD. Free of RDs!

$$\delta C^{[2]}_{q\leftarrow q} = h^{[2]}_{1,q\leftarrow q} - \delta C^{[1]}_{q\leftarrow q} \otimes \delta f^{[1]}_{q\leftarrow q} - \delta f^{[2]}_{q\leftarrow q}$$

Coefficients do not have any divergence!

$$\delta C^{[2]}_{q\leftarrow \bar q} = h^{[2]}_{1,q\leftarrow \bar q} - \delta f^{[2]}_{q\leftarrow \bar q}$$

PDFs at 2-loops

Renormalized TMD. Free of RDs!

No convolution terms
No PDF at 1-loop in this channel

PDFs at 2-loops: Written in terms of 2-loop splitting functions

Vogelsang 9706511
Mikhailov, Vladimirov 0810.1647

$$\delta f^{[2]}_{q\leftarrow q} = \frac{1}{2\epsilon^2}\left(\delta P^{[1]}_{q\leftarrow q}\otimes \delta P^{[1]}_{q\leftarrow q} + \frac{\beta_0}{2}\delta P^{[1]}_{q\leftarrow q}\right) - \frac{1}{2\epsilon}\delta P^{[2]}_{q\leftarrow q}$$

$$\delta f^{[2]}_{q\leftarrow \bar q} = -\frac{1}{2\epsilon}\delta P^{[2]}_{q\leftarrow \bar q}$$

Matching coefficients

FF

Renormalized TMD. Free of RDs!

Convolution of 1-loop coefficient with 1-loop PDF

Cancellation of $\mathbf{L}_\mu/\epsilon$

$$\delta\mathbb{C}^{[2]}_{q\to q} = H^{[2]}_{1,q\to q} - \delta\mathbb{C}^{[1]}_{q\to q} \otimes \frac{\delta d^{[1]}_{q\to q}}{z^{2-2\epsilon}} - \frac{\delta d^{[2]}_{q\to q}}{z^{2-2\epsilon}}$$

Coefficients do not have any divergence!

$$\delta\mathbb{C}^{[2]}_{q\to \bar q} = H^{[2]}_{1,q\to \bar q} - \frac{\delta d^{[2]}_{q\to \bar q}}{z^{2-2\epsilon}}$$

FFs at 2-loops

Renormalized TMD. Free of RDs!

No convolution terms

No PDF at 1-loop in this channel

FFs at 2-loops: Written in terms of 2-loop splitting functions

Vogelsang 9706511

Mikhailov, Vladimirov 0810.1647

$$\delta d^{[2]}_{q\to q} = \frac{1}{2\epsilon^2}\left(\delta\mathbb{P}^{[1]}_{q\to q}\otimes\delta\mathbb{P}^{[1]}_{q\to q} + \frac{\beta_0}{2}\delta\mathbb{P}^{[1]}_{q\to q}\right) - \frac{1}{2\epsilon}\delta\mathbb{P}^{[2]}_{q\to q}$$

$$\delta d^{[2]}_{q\to \bar q} = -\frac{1}{2\epsilon}\delta\mathbb{P}^{[2]}_{q\to \bar q}$$

PDF

Results

LO transversity DGLAP kernel

$$\delta p(x) = \frac{2x}{1-x}$$

The matching coefficients are written as

$$\delta C_{f\leftarrow f'}(x, \mathbf{L}_\mu, \mathbf{l}_\zeta) = \sum_{n=0}^{\infty} a_s^n \sum_{k=0}^{n+1} \sum_{l=0}^{n} \mathbf{L}_\mu^k \, \mathbf{l}_\zeta^l \, \delta C_{f\leftarrow f'}^{(n;k,l)}(x)$$

Abelian part of the lowest order of matching coefficient for quark-to-quark case

$$\delta C_{q\leftarrow q}^{(2;0,0)}(x) = C_F^2 \Big\{ \delta p(x) \Big[4\text{Li}_3(\bar{x}) - 20\text{Li}_3(x) - 4\ln\bar{x}\,\text{Li}_2(\bar{x}) + 12\ln x\,\text{Li}_2(x) + 2\ln^2\bar{x}\,\ln x + 2\ln\bar{x}\,\ln^2 x$$
$$+ \frac{3}{2}\ln^2 x + 8\ln x + 20\zeta_3 \Big] - 2\ln\bar{x} + 4\bar{x} + \delta(\bar{x})\frac{5}{4}\zeta_4 \Big\} + \ldots$$

The part of the coefficient that is multiplied by the LO transversity DGLAP kernel literally coincides with the corresponding part in the unpolarized case

$$C^{(2;0,0)}(x) = P^{[1]} F_1(x) + F_2(x) + \delta(\bar{x}) F_3$$

Unpolarized		Transversity
$P^{[1]} = \frac{1+x^2}{1-x}$		$P^{[1]} = \frac{2x}{1-x}$
F_1	$=$	F_1
F_2	$\neq$	F_2
F_3	$=$	F_3

FF

Results

LO transversity DGLAP kernel

$$\delta p(z) = \frac{2z}{1-z}$$

The matching coefficients are written as

$$\delta\mathbb{C}_{f\to f'}(z,\mathbf{L}_\mu,\mathbf{l}_\zeta) = \sum_{n=0}^{\infty} a_s^n \sum_{k=0}^{n+1}\sum_{l=0}^{n} \mathbf{L}_\mu^k \mathbf{l}_\zeta^l \, \delta\mathbb{C}_{f\to f'}^{(n;k,l)}(z)$$

Abelian part of the lowest order of matching coefficient for quark-to-quark case

$$z^2\delta\mathbb{C}_{q\to q}^{(2;0,0)}(z) = C_F^2\Big\{\delta p(z)\Big[40\text{Li}_3(z) - 4\text{Li}_3(\bar z) + 4\ln\bar z\,\text{Li}_2(\bar z) - 16\ln z\,\text{Li}_2(z) - \frac{40}{3}\ln^3 z + 18\ln^2 z\ln\bar z - 2\ln^2\bar z\ln z$$
$$+\frac{15}{2}\ln^2 z - 8(1+\zeta_2)\ln z - 40\zeta_3\Big] + 4\bar z(1+\ln z) + 2z(\ln\bar z - \ln z) + \delta(\bar z)\frac{5}{4}\zeta_4\Big\} + \ldots$$

The part of the coefficient that are multiplied by the LO transversity DGLAP kernel literally coincides with the corresponding part in the unpolarized case

$$C^{(2;0,0)}(z) = P^{[1]}F_1(z) + F_2(z) + \delta(\bar z)F_3$$

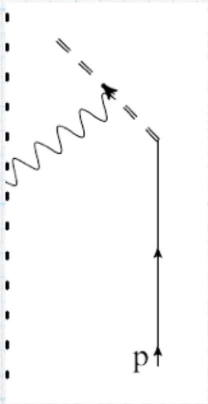
Unpolarized		Transversity
$P^{[1]} = \frac{1+z^2}{1-z}$		$P^{[1]} = \frac{2z}{1-z}$
F_1	$=$	F_1
F_2	$\neq$	F_2
F_3	$=$	F_3

Pretzelosity distribution

Reduction of the number of diagrams

Diagrams with a non-interacting quark are exactly zero

$$\sigma^{+\mu}\left(\frac{\boldsymbol{b}^\mu \boldsymbol{b}^\nu}{\boldsymbol{b}^2} - \frac{g_T^{\mu\nu}}{2(1-\epsilon)}\right)\sigma^{-\nu} = 0$$



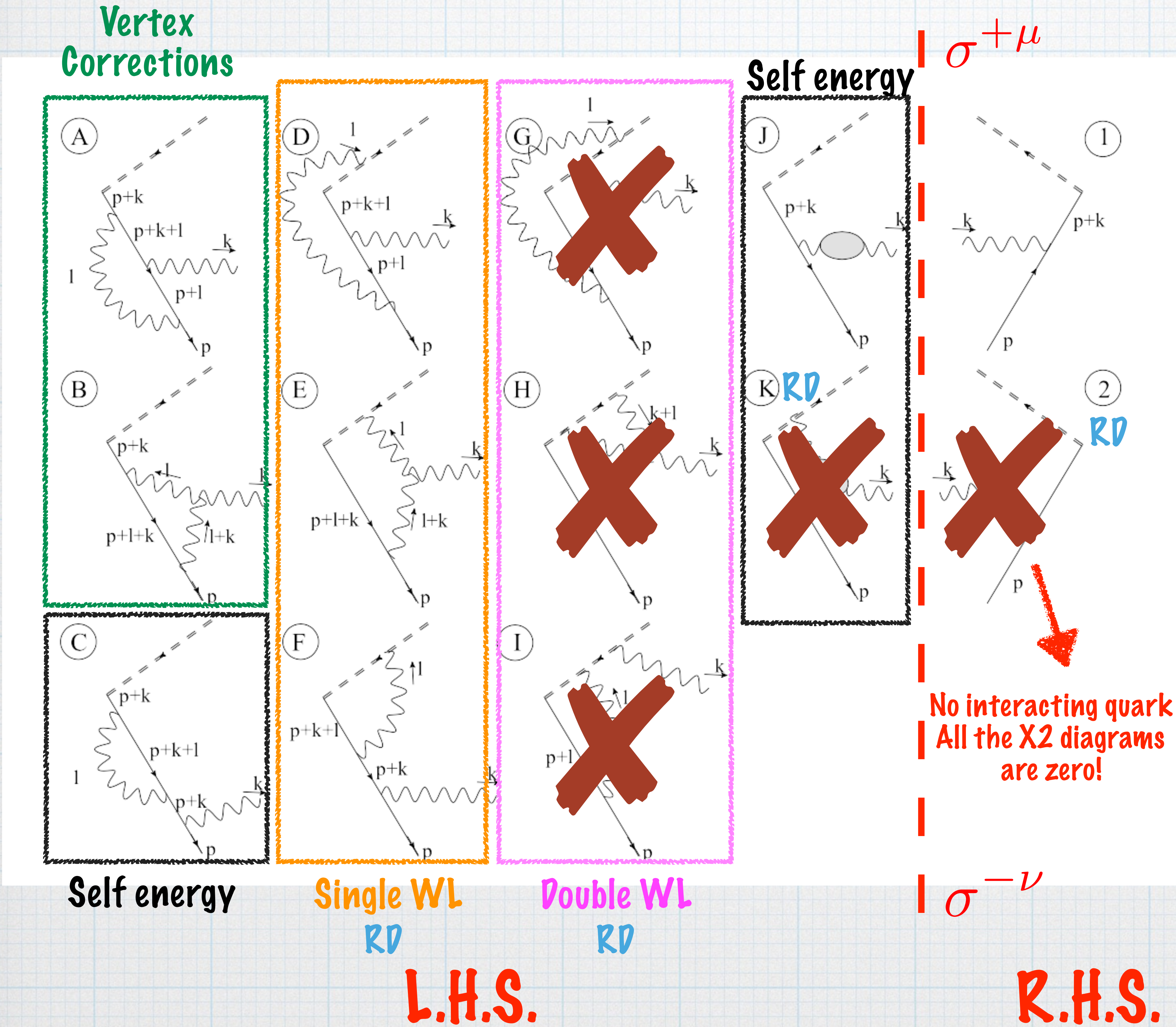
$= 0$

As in the transversity case → Odd number of gamma matrices in each trace in $q \leftarrow q'$ → It is zero!

At NNLO we have the same two cases that in transversity

1-loop result is ϵ -suppressed
Two loop diagrams are less divergent than in another TMDs
All the diagrams have no poles in ϵ

Non-zero Virtual-Real diagrams



No RDs
Finite diagrams
Vertex-correction QCD x 1-loop

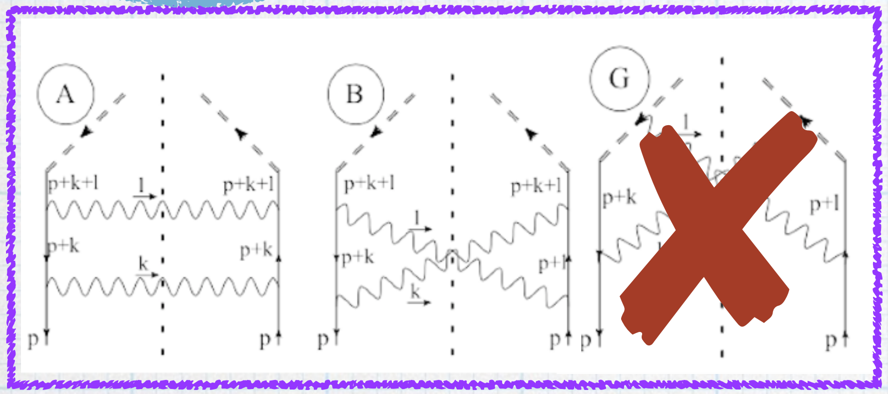
RDs
Finite diagrams
Combined with RR diagrams by color factor RDs should be cancelled

These diagrams are exactly zero!

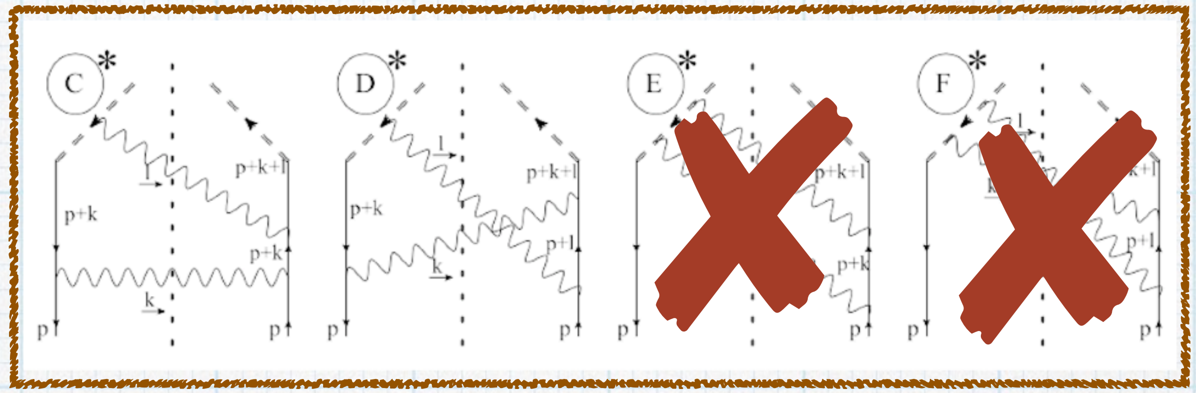
Pretzelosity at NNLO does not depend on TrNf
Sum of these diagrams with RR should be zero

Non-zero Real-Real diagrams

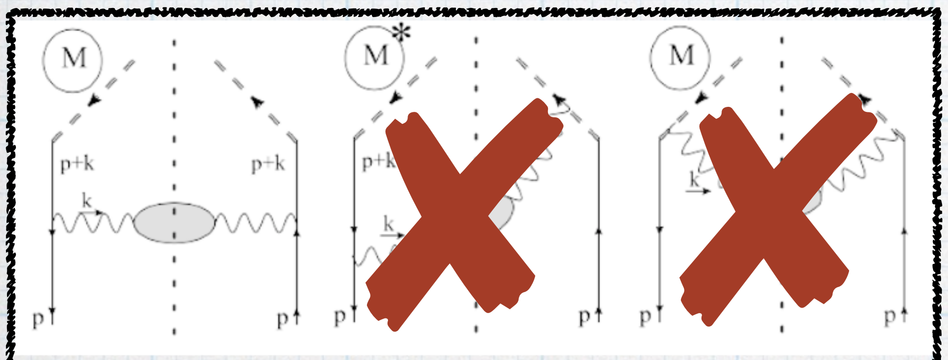
q ← q



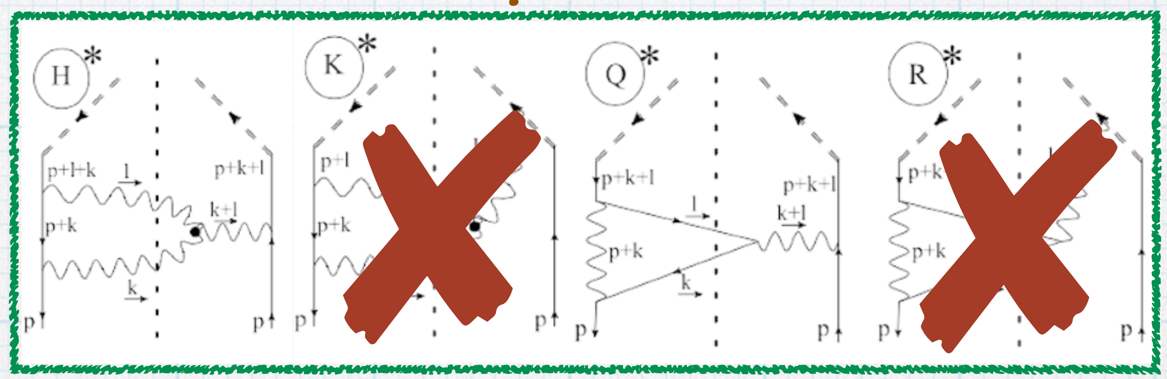
Real ladder



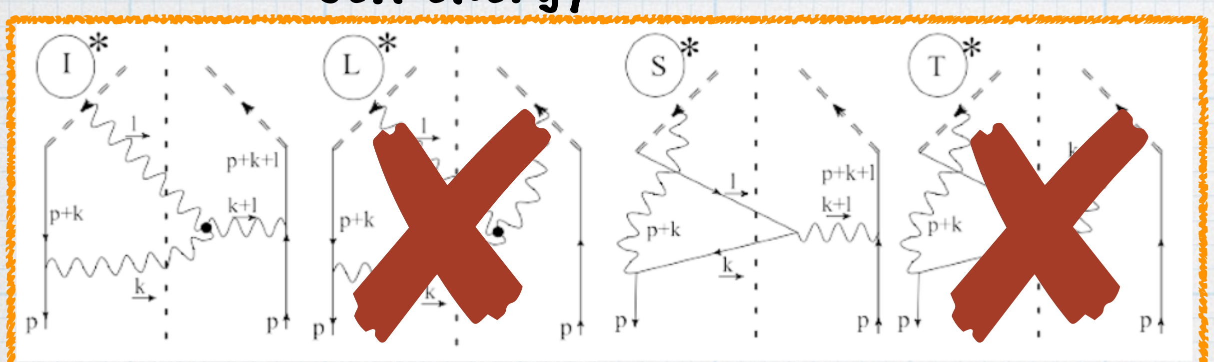
Complex ladder



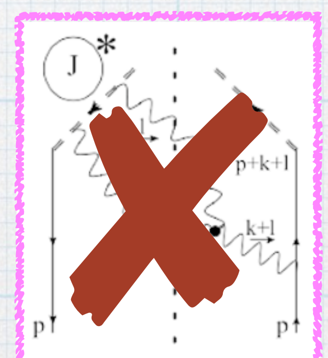
Self energy



Vertex Corrections



Single WL



Double WL

No RDs
Finite diagrams

No RDs
Finite diagrams

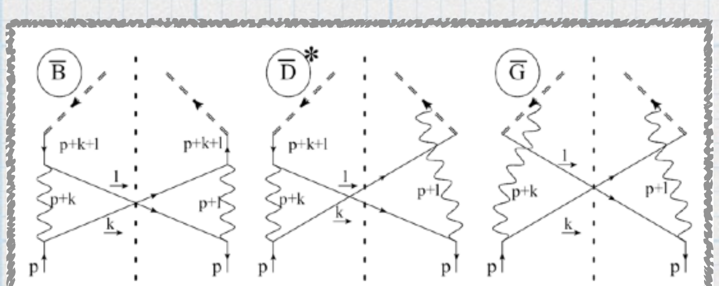
Only RD in diag I
With VR RDs should be cancelled

RDs in both diagrams
With VR should be cancelled

Depend on TrNf
Cancelled with VR

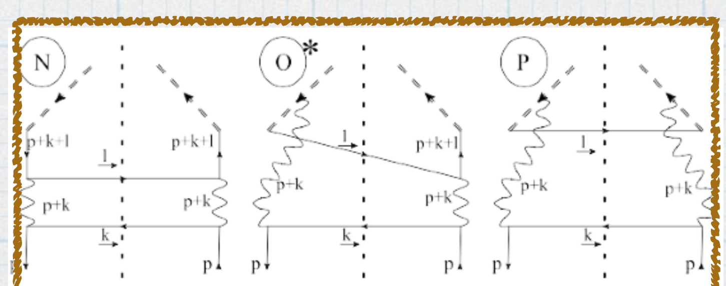
Double WL is zero

q ← q̄



No RD
Finite result, without plus-distribted terms and deltas

q ← q'



It is zero!
Odd number of gamma-matrices
In each trace

Cancellation of Rapidity Divergences

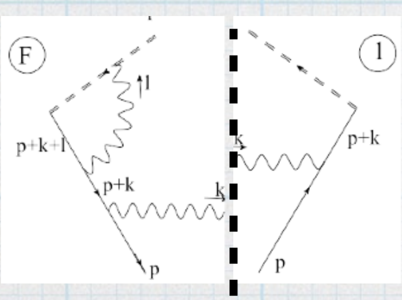
Expression for renormalized TMD

$$h_1^{[2]} = \delta\Phi^{[2]} - \frac{S^{[1]}\delta\Phi^{[1]}}{2} - \frac{S^{[2]}\delta\Phi^{[0]}}{2} + \frac{3S^{[1]}S^{[1]}\delta\Phi^{[0]}}{8} + \left(Z_q^{[1]} - Z_2^{[1]}\right)\left(\delta\Phi^{[1]} - \frac{S^{[1]}\delta\Phi^{[0]}}{2}\right)$$

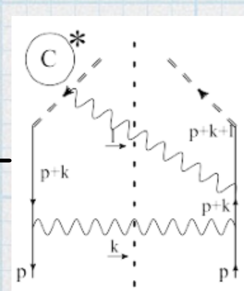
$$+ \left(Z_q^{[2]} - Z_2^{[2]} - Z_2^{[1]}Z_q^{[1]} - Z_2^{[1]}Z_2^{[1]}\right)\delta^{\perp}\Phi^{[0]}$$

We have different combinations of diagrams and SF to cancel RDs depending on their color factors

$$C_F^2$$

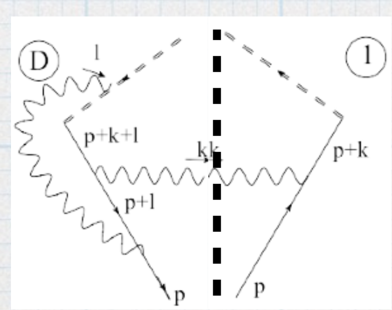


$+$

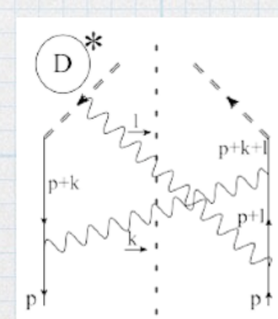


$$-\frac{S^{[1]}\delta^{\perp}\Phi^{[1]}}{2}$$

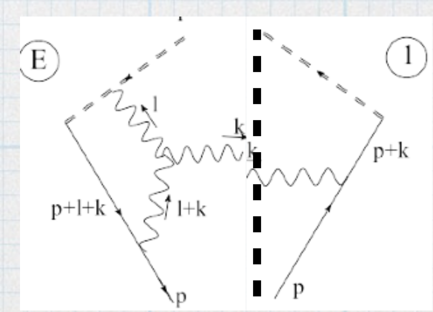
$$C_F^2 - \frac{C_A C_F}{2}$$



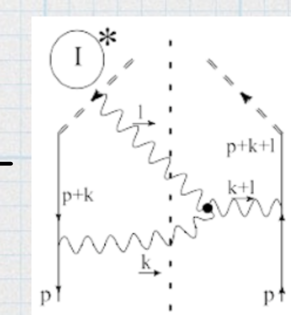
$+$



$$-\frac{C_A C_F}{2}$$



$+$



Results

$q \leftarrow \bar{q}$

$$\delta^{\perp} C^{[2]}_{q\leftarrow\bar{q}} = 0$$

First two diagrams are finite
Third is zero
Sum of the diagrams is $\mathcal{O}(\epsilon)$!

$q \leftarrow q'$

$$\delta^{\perp} C^{[2]}_{q\leftarrow q'} = 0$$

Zero from the beginning
Odd number of gamma matrices

Results

$q \leftarrow q$

$$\delta^{\perp} C^{[2]}_{q\leftarrow q} = 0$$

This cancelation is **highly non-trivial**!

$$\delta^{\perp}\Phi^{[2]}_{f\leftarrow f'} = C_F^2 A_F + C_F\left(C_F - \frac{C_A}{2}\right) A_{FA} + \frac{C_A}{2} A_A + C_F N_f A_N$$

$$A_{FA} = A_A + \mathcal{O}(\epsilon)$$

$$A_N = \mathcal{O}(\epsilon)$$

There is an ϵ-suppression of the $C_A\,C_F$ and N_f parts of the TMD!

$$\delta^{\perp} C^{[2]}_{q\leftarrow q}(x,\boldsymbol{b}) = h^{\perp[2]}_{1T,q\leftarrow q}(x,\boldsymbol{b}) - \left[\delta^{\perp} C^{[1]}_{q\leftarrow q}(\boldsymbol{b}) \otimes \delta f^{[1]}_{q\leftarrow q}\right](x)$$

So, after renormalization

$$h^{\perp[2]}_{1T,q\leftarrow q}(x,\boldsymbol{b}) = -4C_F^2\left(\bar{x}(3+4\ln\bar{x}) + 4x\ln x\right)$$

$$\left[\delta^{\perp} C^{[1]}_{q\leftarrow q}(\boldsymbol{b}) \otimes \delta f^{[1]}_{q\leftarrow q}\right](x) = -4C_F^2\left(\bar{x}(3+4\ln\bar{x}) + 4x\ln x\right)$$

Actually the result is zero!
$\mathcal{O}(\epsilon)$

LO at twist-4?

Results

Conjecture:

$$\delta^{\perp}C_{q\leftarrow f}(x,\boldsymbol{b})=0$$

At all orders in P.T.!

LO of large-Nf matching is zero
Supports the conjeture!

q ← q

$$\delta^{\perp}C^{[2]}_{q\leftarrow q}=0$$

This cancelation is **highly non-trivial!**

$$\delta^{\perp}\Phi^{[2]}_{f\leftarrow f'}=C_F^2A_F+C_F\left(C_F-\frac{C_A}{2}\right)A_{FA}+\frac{C_A}{2}A_A+C_FN_fA_N$$

$$A_{FA}=A_A+\mathcal{O}(\epsilon)$$

$$A_N=\mathcal{O}(\epsilon)$$

There is an ϵ-suppression of the C_A C_F and N_f parts of the TMD!

$$\delta^{\perp}C^{[2]}_{q\leftarrow q}(x,\boldsymbol{b})=h^{\perp[2]}_{1T,q\leftarrow q}(x,\boldsymbol{b})-\left[\delta^{\perp}C^{[1]}_{q\leftarrow q}(\boldsymbol{b})\otimes\delta f^{[1]}_{q\leftarrow q}\right](x)$$

So, after renormalization

$$h^{\perp[2]}_{1T,q\leftarrow q}(x,\boldsymbol{b})=-4C_F^2\left(\bar{x}(3+4\ln\bar{x})+4x\ln x\right)$$

$$\left[\delta^{\perp}C^{[1]}_{q\leftarrow q}(\boldsymbol{b})\otimes\delta f^{[1]}_{q\leftarrow q}\right](x)=-4C_F^2\left(\bar{x}(3+4\ln\bar{x})+4x\ln x\right)$$

Actually the result is zero!
$\mathcal{O}(\epsilon)$

LO at twist-4?

Conclusions

* We have a polarized TMD (transversity) calculated the at same order that the unpolarized one. This feature allows tests of independence of polarization of the TMD Evolution
* For the transversity TMD we have information both for PDFs and FFs, which allows further tests of TMD evolution
* It is welcome to know and to have grids of collinear transversity extracted at NNLO. See Radici's talk
* Resume of our calculation:
  * Transversity has a matching coefficient calculated in an analogous way of the unpolarized function.
    * Rapidity divergences cancelled (Polarized Factorization theorems at NNLO)
    * Z's do not depend on the polarization.
  * Pretzelosity has a matching coefficient that
    * Is ϵ-suppressed at NLO, explaining phenomenological analysis
    * Zero (ϵ-suppressed) at NNLO for all the different channels. Conjecture: zero at all order in P.T.
    * LO is twist-4 matching?

Thanks!!!

Back up

δ-regularization

$$W_n = P\exp\left(-ig\int_0^\infty d\sigma(n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma(n\cdot A)(n\sigma)e^{-\delta\sigma x}\right)$$

$$S_n = P\exp\left(-ig\int_0^\infty d\sigma(n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma(n\cdot A)(n\sigma)e^{-\delta\sigma}\right)$$

At diagram level → **Eikonal propagators**

$$\frac{1}{(k_1^+ + i0)(k_1^+ + k_2^+ + i0)...(k_1^+ + ... + k_n^+ + i0)} \to \frac{1}{(k_1^+ + i\delta)(k_1^+ + k_2^+ + 2i\delta)...(k_1^+ + ... + k_n^+ + ni\delta)}$$

This regularization makes zero-bin equal to soft factor

R-factor is scheme dependent!

$$R = \frac{\sqrt{S(\boldsymbol{b})}}{\text{zero-bin}} \xrightarrow{\delta-\text{reg.}} R_{\delta-\text{reg.}} = \frac{1}{\sqrt{S(\boldsymbol{b})}}$$

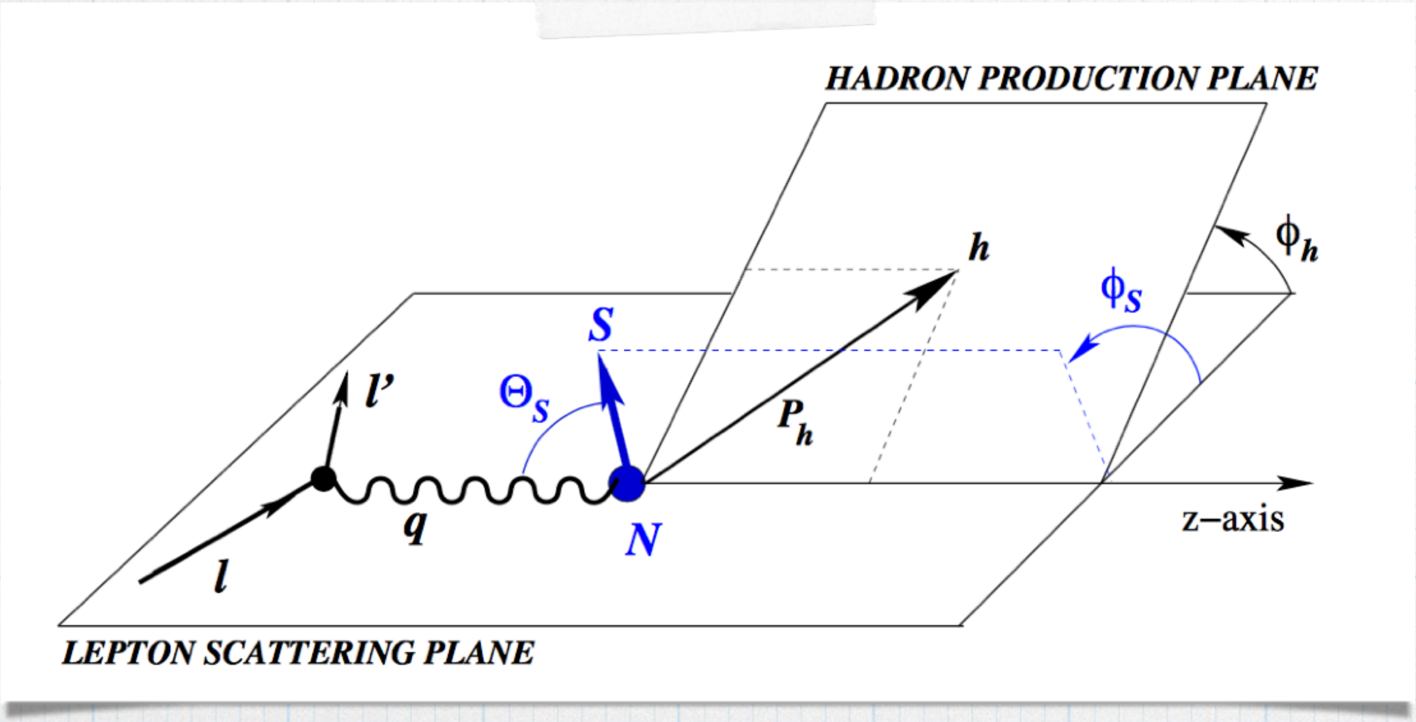
Non-abelian exponentiation satisfied at all orders!

δ-regularization violates gauge properties of WL by power suppressed in δ terms
Only calculation at $\delta \to 0$ is legitimate!

Pretzelosity distribution

Cuadrupole modulation of parton density in the distribution of transversely polarized quarks in a transversely polarized nucleon

A polarized proton might not be spherically symmetric



H.Avakian et al. 0805.3355

Pretzelosity distribution in **convolution** with the Collins FF generates $\sin(3\phi_h - \phi_S)$ asymmetry in **SIDIS (HERMES & COMPASS)** and future facilities (**EIC, LHC-b**)

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[w_{\rm kin} h_{1T}^{\perp} H_1^{\perp}\right]$$

Experimentally measured: SSA

$$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dP_{hT}} = \frac{\alpha^2 2P_{hT}}{xyQ^2}\left\{\left(1-y+\frac{1}{2}y^2\right)(F_{UU,T}+\varepsilon F_{UU,L}) + S_T(1-y)\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)} + \ldots\right\}$$

Linearly polarized gluons matching coefficients

Small-b expression for the linearly polarized gluon TMDPDF

$$h_1^{\perp g}(x,\boldsymbol{b}) = [\delta^L C_{g\leftarrow q}(\boldsymbol{b}) \otimes f_q](x) + [\delta^L C_{g\leftarrow g}(\boldsymbol{b}) \otimes f_g](x) + \mathcal{O}(\boldsymbol{b}^2)$$

NLO **matching coefficients**

$$\delta^L C_{g\leftarrow g} = -4a_s C_A \frac{\bar{x}}{x} + \mathcal{O}(a_s^2) \qquad \delta^L C_{g\leftarrow q} = -4a_s C_F \frac{\bar{x}}{x} + \mathcal{O}(a_s^2)$$

These results agree with the obtained in
T. Becher et al. 1212.2621!!

Helicity distribution

Schemes for γ^5 in DR. Small-b OPE

Lorentz structures

$\Gamma = \gamma^+\gamma^5 \quad \Gamma^{\mu\nu} = i\epsilon_T^{\mu\nu}$

→ γ^5 needs a definition in DR! →

$$\gamma^+\gamma^5 = \frac{i}{3!}\epsilon^{+\nu\alpha\beta}\gamma_\nu\gamma_\alpha\gamma_\beta$$

↗ HVBM 4-dimensional

↘ Larin d-dimensional

Larin scheme is more convenient than HVBM because it does not violate Lorentz invariance, but it violates the definition of the leading dynamical twist

$$\gamma^+\Gamma = \gamma^+\left(\gamma^+\gamma^5\right)_{\text{Larin}} = \frac{i}{3!}\epsilon^{+\nu\alpha\beta}\gamma^+\gamma_\nu\gamma_\alpha\gamma_\beta \neq 0$$

Light modification of Larin scheme ⇒ Larin+

$$(\gamma^+\gamma^5)_{\text{Larin}^+} = \frac{i\epsilon^{+-\alpha\beta}}{2!}\gamma^+\gamma_\alpha\gamma_\beta = \frac{i\epsilon_T^{\alpha\beta}}{2!}\gamma^+\gamma_\alpha\gamma_\beta$$

Helicity TMD distribution in the regime of small-b

$$g_{1L}(x,\boldsymbol{b}) = [\Delta C_{q\leftarrow q}(\boldsymbol{b})\otimes\Delta f_q](x) + [\Delta C_{q\leftarrow g}(\boldsymbol{b})\otimes\Delta f_g](x) + \mathcal{O}(\boldsymbol{b}^2)$$

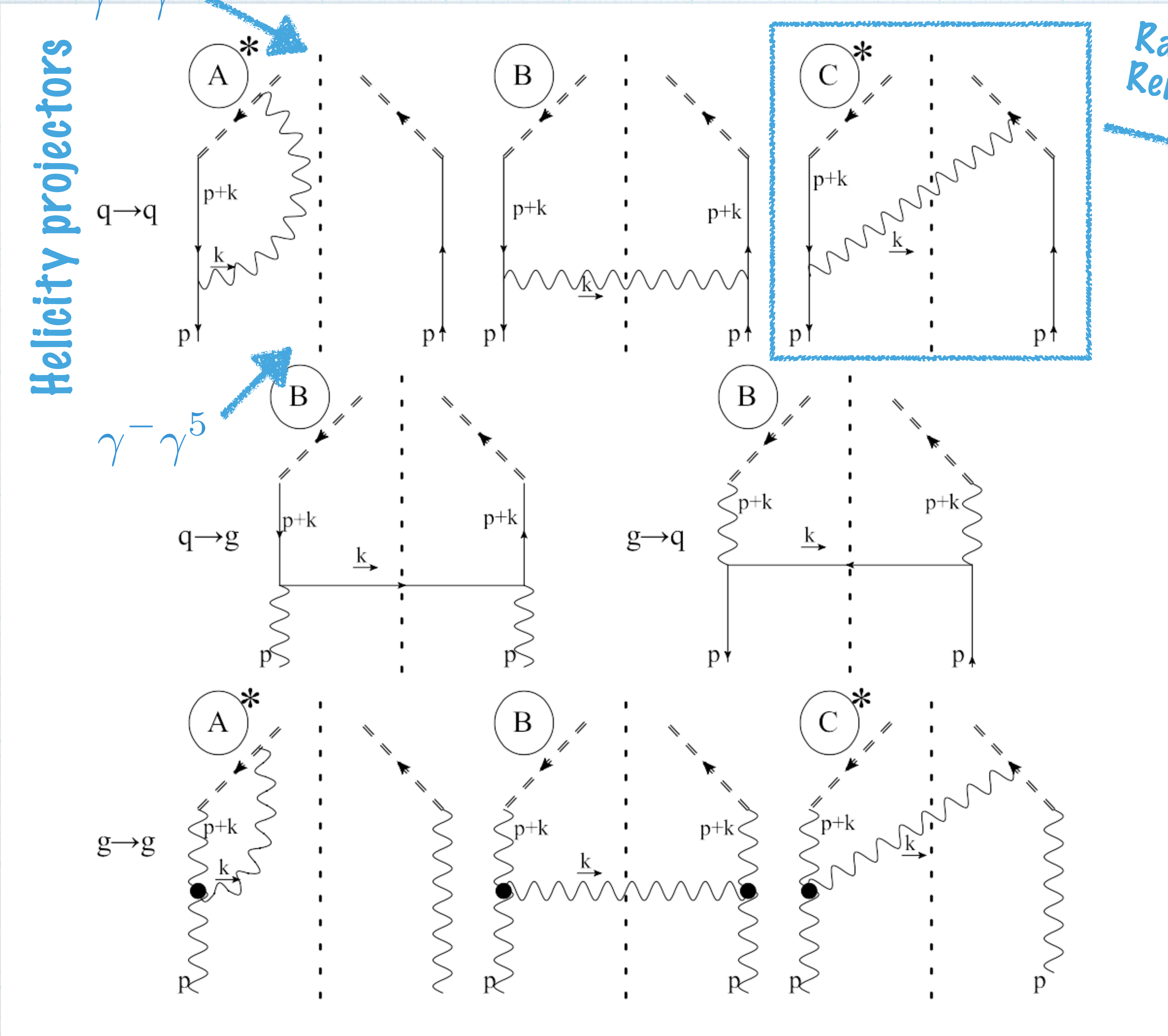
$$g_{1L}^g(x,\boldsymbol{b}) = [\Delta C_{g\leftarrow q}(\boldsymbol{b})\otimes\Delta f_q](x) + [\Delta C_{g\leftarrow g}(\boldsymbol{b})\otimes\Delta f_g](x) + \mathcal{O}(\boldsymbol{b}^2)$$

Diagrams contributing to TMDS at NLO

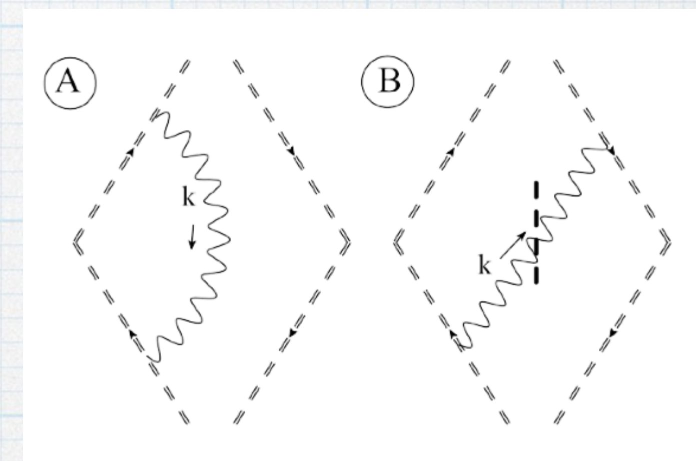
Helicity projectors

$\gamma^+\gamma^5$

$\gamma^-\gamma^5$



Rapidity divergences:
Renormalized with SF



The calculation is
striaghtforward
to the unpolarized case

M.G.Echevarria et al.: 1604.07869

Matching coefficients: scheme dependence

$$\Delta C_{q\leftarrow q} = \delta(\bar{x}) + a_s C_F \left\{ 2\boldsymbol{B}^\epsilon \Gamma(-\epsilon) \left[\frac{2}{(1-x)_+} - 2 + \bar{x}(1+\epsilon)\mathcal{H}_{\text{sch.}} + \delta(\bar{x}) \left(\mathbf{L}_{\sqrt{\zeta}} - \psi(-\epsilon) - \gamma_E \right) \right] \right\}_{\epsilon\text{-finite}}$$

$$\Delta C_{q\leftarrow g} = a_s C_F \left\{ 2\boldsymbol{B}^\epsilon \Gamma(-\epsilon) \left[x - \bar{x}\mathcal{H}_{\text{sch.}} \right] \right\}_{\epsilon\text{-finite}}$$

$$\Delta C_{g\leftarrow q} = a_s C_F \left\{ 2\boldsymbol{B}^\epsilon \Gamma(-\epsilon) \left[1 + \bar{x}\mathcal{H}_{\text{sch.}} \right] \right\}_{\epsilon\text{-finite}}$$

$$\Delta C_{g\leftarrow g} = \delta(\bar{x}) + a_s C_A \left\{ 2\boldsymbol{B}^\epsilon \Gamma(-\epsilon) \frac{1}{x} \left[\frac{2}{(1-x)_+} - 2 - 2x^2 + 2x\bar{x}\mathcal{H}_{\text{sch.}} + \delta(\bar{x}) \left(\mathbf{L}_{\sqrt{\zeta}} - \psi(-\epsilon) - \gamma_E \right) \right] \right\}_{\epsilon\text{-finite}}$$

$$\mathcal{H}_{\text{sch.}} = \begin{cases} 1 + 2\epsilon & \text{HVBM} \\ \dfrac{1+\epsilon}{1-\epsilon} & Larin^+ \end{cases}$$

→ **At NLO there is not scheme dependence!**

Helicity matching coefficients: NLO results

At $\epsilon \to 0$ we have the NLO coefficients

$$\Delta C_{q\leftarrow q} \equiv C_{q\leftarrow q} = \delta(\bar{x}) + a_s C_F \Big(-2\mathbf{L}_\mu \Delta p_{qq} + 2\bar{x} + \delta(\bar{x}) \left(-\mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{l}_\zeta - \zeta_2\right) \Big) + \mathcal{O}(a_s^2)$$

$$\Delta C_{q\leftarrow g} = a_s T_F \left(-2\mathbf{L}_\mu \Delta p_{qg} + 4\bar{x}\right) + \mathcal{O}(a_s^2)$$

$$\Delta C_{g\leftarrow q} = a_s C_F \left(-2\mathbf{L}_\mu \Delta p_{gq} - 4\bar{x}\right) + \mathcal{O}(a_s^2)$$

$$\Delta C_{g\leftarrow g} = \delta(\bar{x}) + a_s C_A \Big(-2\mathbf{L}_\mu \Delta p_{gg} - 8\bar{x} + \delta(\bar{x}) \left(-\mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{l}_\zeta - \zeta_2\right) \Big) + \mathcal{O}(a_s^2)$$

These results agree with the obtained in
M.G.Echevarría et al. 1502.05354
A.Bacchetta A.Prokudin 1303.2129!!

Drawback of schemes. Z^5_{qq} renormalization constant

Drawback of both schemes ⇒ Violation of **Adler-Bardeen** theorem ⇒ Non renormalization of the axial anomaly

Fixed by an extra renormalization constant, Z^5_{qq} ⇒ Derived from a external condition

S.A. Larin 9302240, V.Matiouine et al 076002, V.Ravindran et al. 0311304

Only affect to the quark-to-quark part

- At large q_T TMD factorization reproduces collinear factorization ⇒ It is natural to **normalize Helicity distribution** ⇒ It reproduces polarized DY which is normalized to unpolarized DY
- Equivalent in TMDs ⇒ Equality in **polarized** and **unpolarized** coefficients

$$\Big[Z^5_{qq}(\boldsymbol{b}) \otimes \Delta C_{q\leftarrow q}(\boldsymbol{b})\Big](x) = C_{q\leftarrow q}(x, \boldsymbol{b})$$

↓

$$Z^5_{qq} = \delta(\bar{x}) + 2a_s C_F \boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\left(1 - \epsilon - (1+\epsilon)\mathcal{H}_{\text{sch.}}\right)\bar{x}$$