Combining 0.5mm and 1.5mm to Obtain Combined Bumphunt Result

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The likelihood…unbinned and binned

$$L(\nu) = \frac{\mu^n}{n!} e^{-\mu} \prod_{i=1}^{n} \left[N_{sig} P_{sig}(m_i|\nu) + N_{bkg} P_{bkg}(m_i|\nu)\right]$$

Binned
and
"-ln"ed

(Nsig and Nbkg are inconsistently included in $\boldsymbol{\nu}$)

$$-\,\mathtt{ln}\lambda(\nu) = \sum_{i=1}^{N_{bins}} \left[\mu_i(m_i|\nu) - n_i + n_i \mathtt{ln} \frac{n_i}{\mu_i(m_i|\nu)}\right]$$

The PDFs are buried in the expected number of events in each bin (μ_i), namely:

$$\mu_i(\nu) = N_{sig}\mathcal{P}_{sig}(m_i|\nu) + N_{bkg}\mathcal{P}_{bkg}(m_i|\nu)$$

…where m_i is the mass at the bin center…or, even better:

$$\mu_i(\nu) = \int_{m_{min}}^{m_{max}} \left[N_{sig}\mathcal{P}_{sig}(m_i|\nu) + N_{bkg}\mathcal{P}_{bkg}(m_i|\nu)\right] dm$$

*** max and min are the bin boundaries

combining datasets → multiply likelihoods

$$L(\nu) = \prod_{i=1}^{n} \mathcal{P}_n(n_i|\nu) \prod_{i=1}^{m} \mathcal{P}_m(m_i|\nu)$$

$\boldsymbol{\nu}$ now includes all parameters for both datasets

$$-\,\mathtt{ln}L(\nu) = -\left(\sum_{i=1}^{n} \mathtt{ln}\left[\mathcal{P}_n(n_i|\nu)\right] + \sum_{i=1}^{m} \mathtt{ln}\left[\mathcal{P}_m(m_i|\nu)\right] \right)$$

Combining 0.5mm and 1.5mm BH datasets

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- Need to link the two likelihoods for combination to make any sense…i.e.

$$-\,\mathtt{ln}L(\theta,\phi) = -\left(\sum_{i=1}^{n}\mathtt{ln}\left[\mathcal{P}_n(n_i|\theta)\right] + \sum_{i=1}^{m}\mathtt{ln}\left[\mathcal{P}_m(m_i|\phi)\right]\right)$$

Independent Likelihoods…no reason to combine

- Current way we do the BH, 0.5mm and 1.5mm LKLs are independent..
  - acceptance (kinematics!), efficiency, radiative fraction, luminosity different between 0.5/1.5
  - For signal: we can scale N_{sig} so that 0.5 & 1.5 mm are on same basis
    - Signal resolution probably should stay independent between two samples
  - For background: potentially correct for acceptance & efficiency for this too and fit common shape & cross-section!, but pretty complicated and likely more effort/ systematic than it's worth

Scaling signal yields between 0.5 and 1.5 mm

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- Basically, the common parameter is ~ epsilon (minus some constants):

$$\epsilon'_{X.Xmm}(m) = \left[\frac{N_{sig}(m)}{\mathcal{L} f_{RAD}(m) a(m)}\right]_{X.Xmm}$$

a(m)==acceptance x efficiency

- So the likelihood (unbinned) becomes:

$$L(\epsilon_{0.5}, \epsilon_{1.5}, \theta, \phi) = \frac{\mu^{n+m}}{n!m!} e^{-\mu} \prod_{i=1}^{n} \left[\epsilon'_{0.5} P_{sig,0.5}(m_i|\theta) + \epsilon'_{bkg,0.5} P_{bkg}(m_i|\theta)\right]$$
$$\prod_{i=1}^{m} \left[\epsilon'_{1.5} P_{sig,1.5}(m_i|\phi) + \epsilon'_{bkg,1.5} P_{bkg}(m_i|\phi)\right]$$

Note that the yields are still the floating parameters! Other parts of ε' are derived from MC or beam.

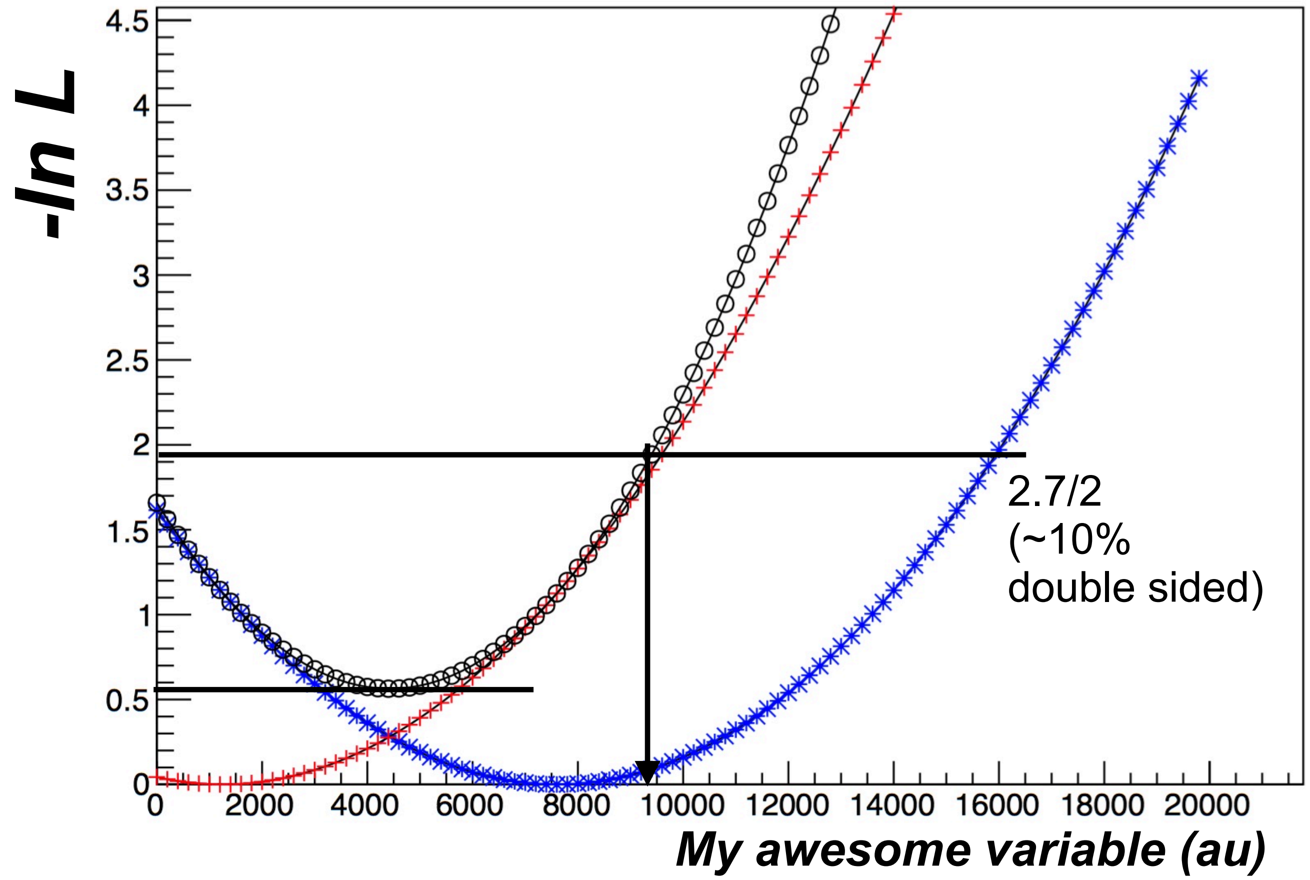
…don't worry about ϵ'_{bkg} for now…matters if actually fitting the samples together

Ok, fine, how do we combine these measurements!

- Two (at least, that I'll talk about) ways to combine these two datasets:

1. Fit the data simultaneously
   - add the constraint that $\epsilon'_{0.5} = \epsilon'_{1.5}$
     - this can be done by multiplying likelihood by a very narrow Gaussian (or whatever)
   - then, procedure is same as what's done in stand-alone search
     - warning, (log) likelihood will probably not be very hyperbolic!
   - this method allows us to easily use pure-toy for limits etc.
2. Combine the likelihoods after stand-alone minimization/scan
   - sum separate likelihood ratio scans (all nuisance parameters floating) vs ε' and convert to probability (-2lnL $\rightarrow \chi^2 \rightarrow$ Prob)
     - must convert N_{sig} to ε'
   - integrate probability scan up to α (e.g. 90%) or get symmetric interval (yeah right)
   - this method is much quicker than above and allows two analyses to be almost entirely independent

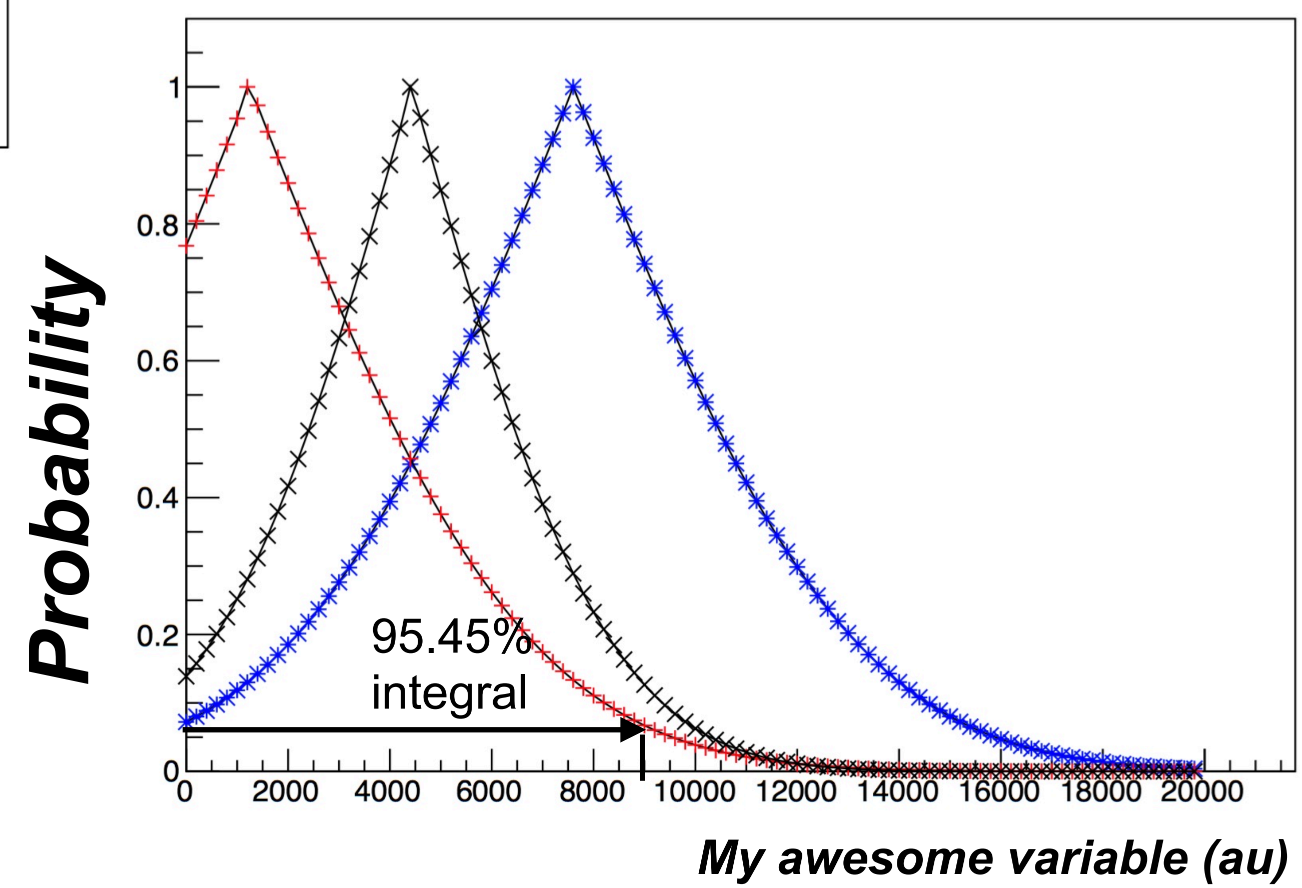
Example: combining likelihoods after the fact



Just an example (never mind the x-axis or what these scans are from)…

…simply add the -nll after subtracting min(nll) (only delta's matter) *after converting your variable to something that should be the same in the two samples*. This combined likelihood should be identical to that obtained if you did a simultaneous fit.

Once you have combined likelihood, can do all the tricks you want.



What's better? Joint fit or combine latter?

- Performing independent fits and combining the likelihood scans is much easier for obtaining statistical-only values
  - systematics are trickier…need to fold into the likelihood by convoluting probabilities.
- Joint fits take some time to set up, longer to run, but give natural way to include systematics …
- Either way works ~equivalently … you pick!