

# True Muonium Acceptance at 6.6 GeV

a true underdog story

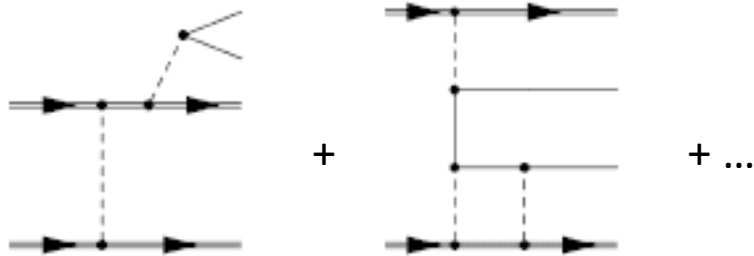
Bradley Yale

Spring 2018 Collaboration Meeting

05/24/2018

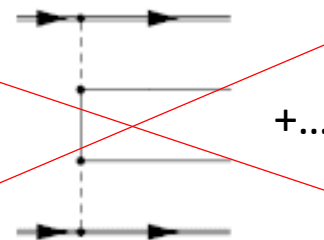
# True Muonium Production

- Muon-antimuon bound state
  - Two spin  $\frac{1}{2}$  fermions, total spin can add to either 0 or 1
- Two mechanisms for production:
  - Odd # of photons, spin-1 triplet states, “ortho” ( $TM \rightarrow e^+ e^-$ )



The 1-photon mechanism is identical to brems-like  $A'$  production, and peaks in the radiative energy region. The 3-photon mechanism peaks in the BH region, so difficult to search for.

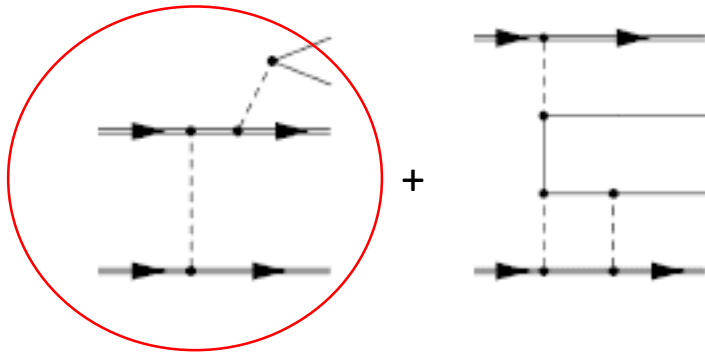
- Even # of photons, spin-0 singlet states, “para” ( $TM \rightarrow \gamma\gamma$ )



Since para-TM decays into photons, HPS is currently unequipped to search for them.

# True Muonium Production

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  - Two spin  $\frac{1}{2}$  fermions, total spin can add to either 0 or 1
- Two mechanisms for production:
  - Odd # of photons, spin-1 triplet states, “ortho” ( $TM \rightarrow e^+ e^-$ )

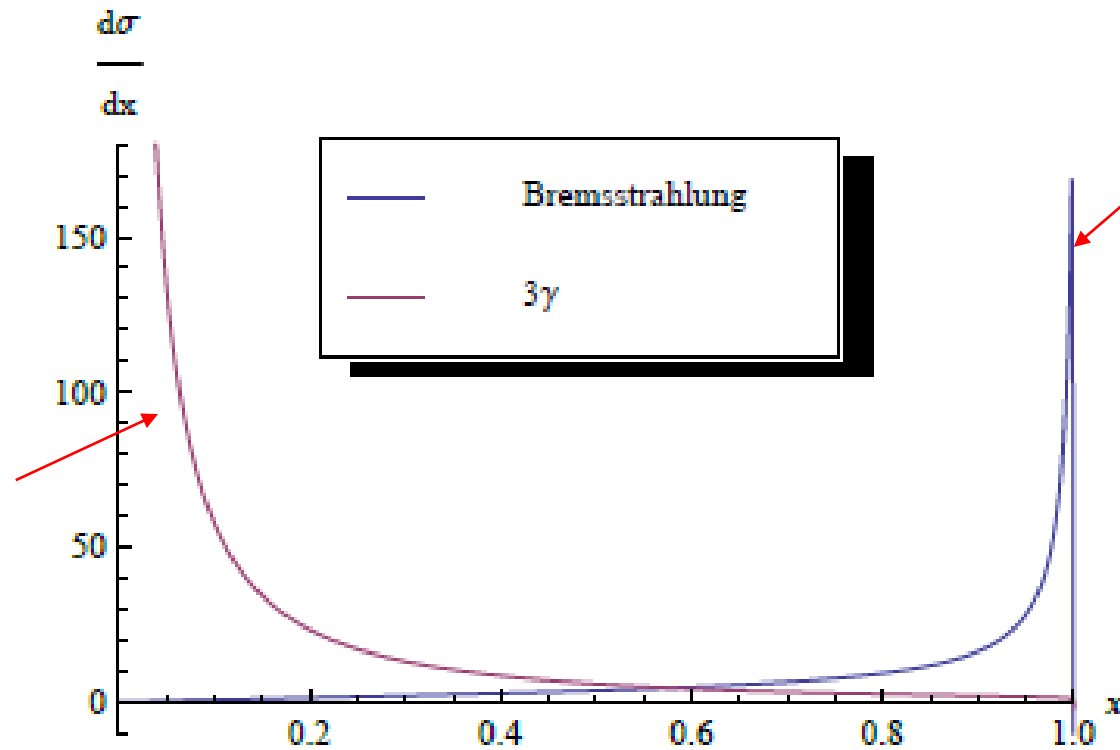


- The 1-photon mechanism is what HPS is most capable of searching for
- Effectively an  $A'$  with known mass and decay length

# TM Production Cross sections (1 vs. 3 photon)

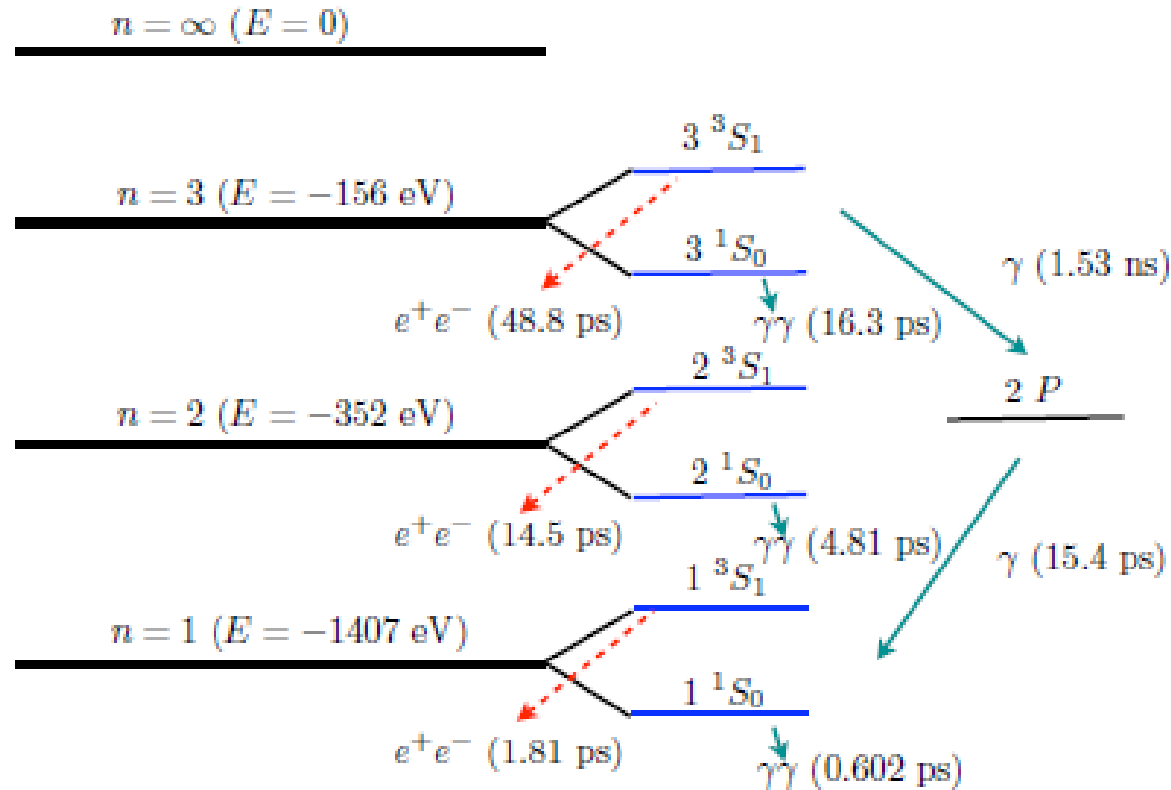
- $x = E/E_{beam}$

3-photon mechanism peaks below likely pair1 trigger thresholds

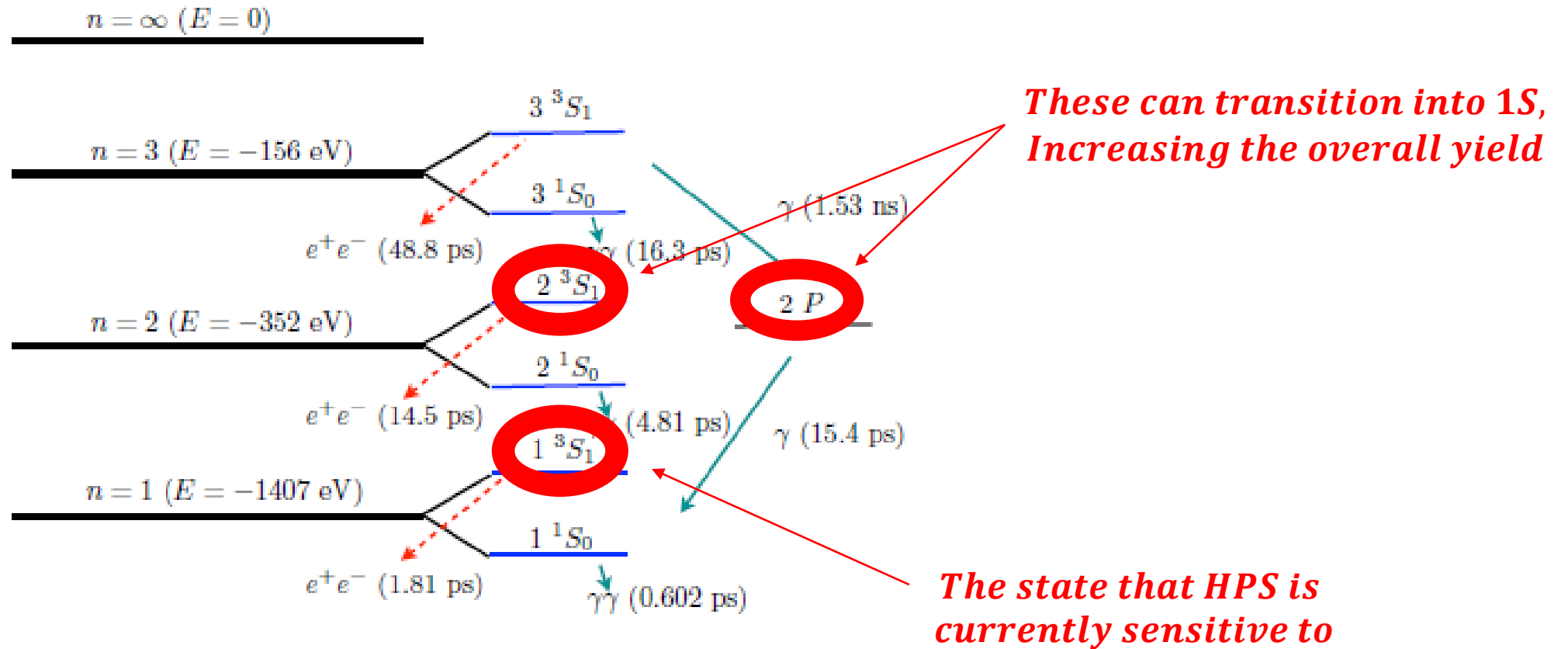


Radiative mechanism peaks at beam energy ("A'-like")

# True Muonium Energy Levels and lifetime



# True Muonium Energy Levels and lifetime



# Part I: True Muonium Production Rate

- To estimate how many detectable True Muonium events will be produced, both the production and dissociation cross sections must be calculated
- Total 1S produced = produced 1S +  $[(2S, 2P) \rightarrow 1S]$  – dissociations
- The small production rate and extremely high dissociation in the target is what makes these events so difficult to detect

# Primary Production Cross Section (radiative)

- Banburski Schuster - arXiv:1206.3961v1 [hep-ph] 18 Jun 2012

$$\left(\frac{d\sigma}{dx}\right)_{rad\ 1S} = \frac{1}{4} \frac{\alpha^7 74^2}{(m_\mu)^2} \frac{x(1-x)}{\left(1-x + (m_e/(2m_\mu))^2\right)^2} \left(1-x + \frac{1}{3}x^2\right) \\ * \left(2\text{Log}[E_{Beam}/m_\mu] + 2\text{Log}[1-x] - \text{Log}[1-x + (m_e/(2m_\mu))^2]\right) - 1)$$

- Integrate (up to  $x = 1 - m_e/E_{beam}$  where  $\frac{d\sigma}{dx} = 0$ )

$$\sigma(4.4\ \text{GeV})_{rad\ 1S} = 0.113724\ \text{pb}$$

$$\sigma(6.6\ \text{GeV})_{rad\ 1S} = 0.26614\ \text{pb}$$

1S production cross section for radiative process only



# Secondary Production/Dissociation (target interaction)

- Transition cross section (to all other bound states + dissociation)

$$d\sigma_{\text{tot}}^{nl} = Z^2 \frac{\alpha^2}{\pi} \left( 1 - F^{nl0, n'l'0}(q) \right) \\ \times \frac{1}{a^2} |\Delta(q, Z)|^2 q dq.$$

**= 5.67937 × 10<sup>16</sup> pb**

1S state → all states + dissociation ("X")

Transfer matrix elements

$$F^{nlm, n'l'm'}(q) = \int_0^\infty \int_0^\pi \int_0^{2\pi} x^2 \text{Sin}(\theta) e^{iqx \text{Cos}(\theta)} \times \\ \psi^{n'l'm'}(x, \theta, \phi)^* \psi^{nlm}(x, \theta, \phi) d\phi d\theta dx$$

Thomas-Fermi-Moliere FF

$$\Delta(q, Z) = 4\pi \sum_{i=1}^3 \frac{\alpha_i}{q^2 + \beta_i^2},$$

where  $\beta_i = \frac{m_e b_i}{121} Z^{1/3}$ , with

$$b_1 = 6.0, b_2 = 1.2, b_3 = 0.3, \\ \alpha_1 = 0.10, \alpha_2 = 0.55, \alpha_3 = 0.35$$

# Dissociation Cross Section (1S)

- Transition cross section (bound state to another bound state)

$$d\sigma^{nl,n'l'} = \left(1 - (-1)^{l-l'}\right) Z^2 \frac{\alpha^2}{\pi} \frac{1}{a^2} q |\Delta(q, Z)|^2 \times \left|F^{nl0,n'l'0}\left(\frac{q}{2}\right)\right|^2 dq$$

- To get dissociation,  $\sigma_{trans+diss.} - \sigma_{transition} = 2.21044 \times 10^{16} \text{ pb}$

Published value for Pb (Schuster):  
 $2.65 \times 10^{16} \text{ pb}$

- Note that  $\sigma_{diss}$  is **17** orders of magnitude greater than  $\sigma_{prod}$ !

# Expected True Muonium Rate (1S)

$$\frac{dN_{1S}}{dz} = N_e(z) \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} - N_{1S}$$

- Solving

$$N_{1S}(l) = N_e \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} \left(1 - e^{-\frac{l}{l_{1S \rightarrow X}}}\right)$$

1-photon mechanism

Proposed 6.6 GeV target

$$= \left(\frac{5625 \text{ electrons}}{2 \text{ ns}}\right) \left(\frac{0.26614 \text{ pb}}{5.67937 * 10^{16} \text{ pb}}\right) \left(1 - e^{-\frac{8.750 \mu\text{m}}{2.792 \mu\text{m}}}\right)$$

= **33.13 events/month**  
(6.6 GeV, 1S state only)

The number of radiative 1S TM events before HPS acceptance

# Expected True Muonium Rate (1S)

$$\frac{dN_{1S}}{dz} = N_e(z) \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} - N_{1S}$$

- Solving

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1-photon + 3-photon

Proposed 6.6 GeV target

$$= \left(\frac{5625 \text{ electrons}}{2 \text{ ns}}\right) \left(\frac{1.27455 \text{ pb}}{5.67937 * 10^{16} \text{ pb}}\right) \left(1 - e^{-\frac{8.750 \mu\text{m}}{2.792 \mu\text{m}}}\right)$$

**= 168.36 events/month**  
**(6.6 GeV, 1S state only)**

Add to this the transitions  
 from 2S,2P to 1S...

# Expected True Muonium Rate (1S,2S,2P)

- 1S yield: 
$$\frac{dN_{1S}}{dz} = N_e \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} - N_{1S} + N_{2P} \frac{\sigma(1S \rightarrow 2P)}{\sigma(1S \rightarrow X)}$$

- 2S yield:

$$\frac{dN_{2S}}{dz} = \frac{1}{\sigma(1S \rightarrow X)} [N_e * \sigma(e^- \rightarrow 2S) - N_{2S} * \sigma(2S \rightarrow X) + N_{2P} * \sigma(2P \rightarrow 2S)]$$

- 2P yield:

$$\begin{aligned} \frac{dN_{2P}}{dz} &= \frac{1}{\sigma(1S \rightarrow X)} [N_e * \sigma(e^- \rightarrow 2P) + N_{1S} * \sigma(1S \rightarrow 2P) + N_{2S} * \sigma(2S \rightarrow 2P) \\ &\quad - N_{2P} * \sigma(2P \rightarrow X)] \end{aligned}$$

# Expected True Muonium Rate (1S,2S,2P)

• 1S yield:  $\frac{dN_{1S}}{dz} = N_e \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} - N_{1S} + N_{2P} \frac{\sigma(1S \rightarrow 2P)}{\sigma(1S \rightarrow X)}$

1.27455 pb (1 + 3 photon mechanisms)

• 2S yield:

5.67937 \* 10<sup>16</sup> pb

$$\frac{dN_{2S}}{dz} = \frac{1}{\sigma(1S \rightarrow X)} [N_e * \sigma(e^- \rightarrow 2S) - N_{2S} * \sigma(2S \rightarrow X) + N_{2P} * \sigma(2P \rightarrow 2S)]$$

4.90741 \* 10<sup>17</sup> pb

• 2P yield:

0.0332675 pb  
=  $\sigma(e^- \rightarrow 1S)/8$

3.1625 \* 10<sup>17</sup> pb

$$\frac{dN_{2P}}{dz} = \frac{1}{\sigma(1S \rightarrow X)} [N_e * \sigma(e^- \rightarrow 2P) + N_{1S} * \sigma(1S \rightarrow 2P) + N_{2S} * \sigma(2S \rightarrow 2P) - N_{2P} * \sigma(2P \rightarrow X)]$$

2.6437 \* 10<sup>16</sup>

6.30545 \* 10<sup>17</sup> pb

# Solving the coupled DE's...

- In one month of running at 6.6 GeV:
  - $N_{1S} = 168.36$  ← from both production diagrams
  - $N_{2S} = 9.46$
  - $N_{2P} = 15.37$

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- If only considering radiative events:
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  - $N_{2P} = 15.37$
- If only considering radiative events:
  - $N_{1S} = 33.13$
- What about acceptance?

# Part II: Simulating True Muonium events

- Lifetime depends on energy level  $n$

$$\tau(n^3S_1 \rightarrow e^+e^-) = \frac{6\hbar n^3}{\alpha^5 mc^2} = 1.81 \text{ ps for } 1S$$

$$\Gamma = \frac{6.582 * 10^{-25} \text{ GeVs}}{1.81 * 10^{-12} \text{ s}} = 3.627 * 10^{-13} \text{ GeV}$$

← Only used to calculate cross section, so we don't really need this...

- These parameters can be used by MadGraph/stdhep to generate the  $1^3S$  state.
- The idea is to find the acceptance of a known # of events, to scale the theoretical yield

# True Muonium Acceptance @ 6.6 GeV

- Initial 6.6 GeV trigger cuts (scaled up from 2.3 GeV)

<seedEnergyLow> 300 MeV

<minHitCount> 2

<pairCoincidence> 3ns

<clusterEnergyLow> 150 MeV

<clusterEnergyHigh> 4.2 GeV

<energySumLow> 1.800 GeV

<energySumHigh> 6 GeV

<energyDifferenceHigh> 3.3 GeV

<coplanarityHigh> 35 deg

<energySlopeParamF> 0.0055 GeV/mm

<energySlopeLow> 700 MeV

6.6 GeV Layer-0 detector used:

HPS-Proposal2017-Nominal-v2-6pt6-fieldmap

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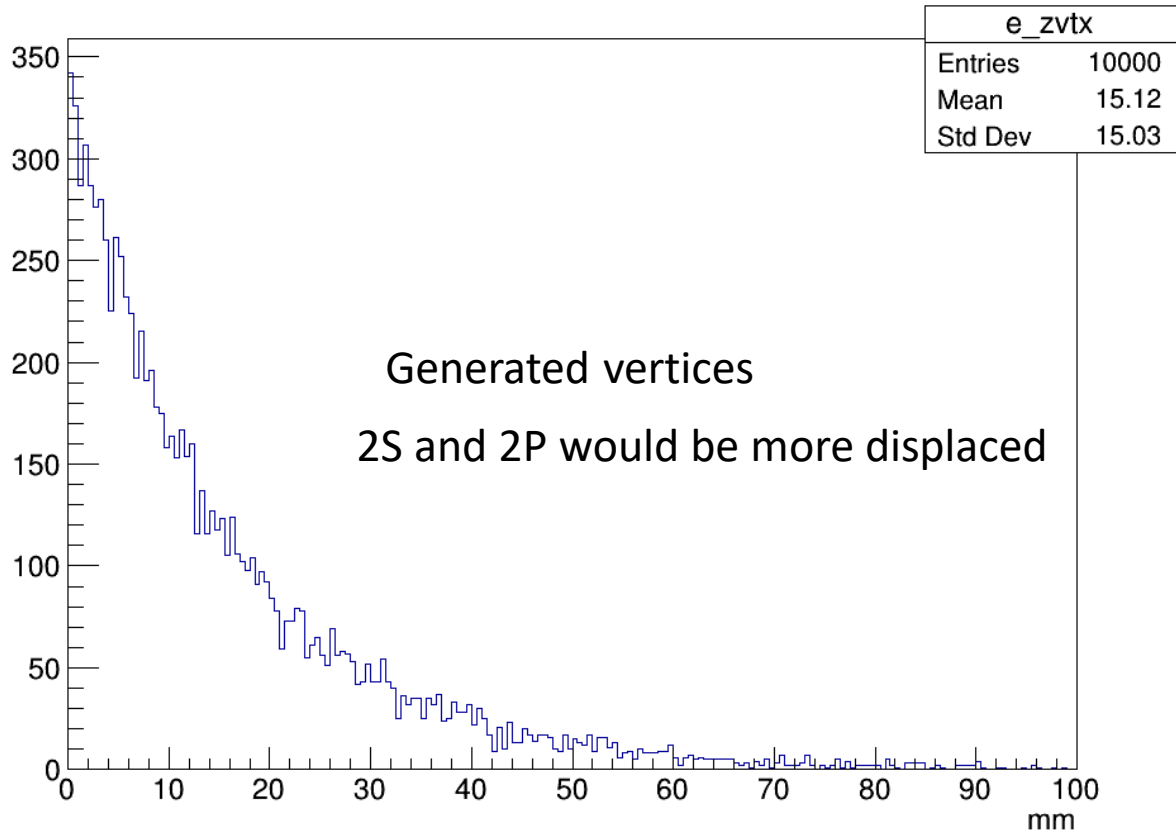
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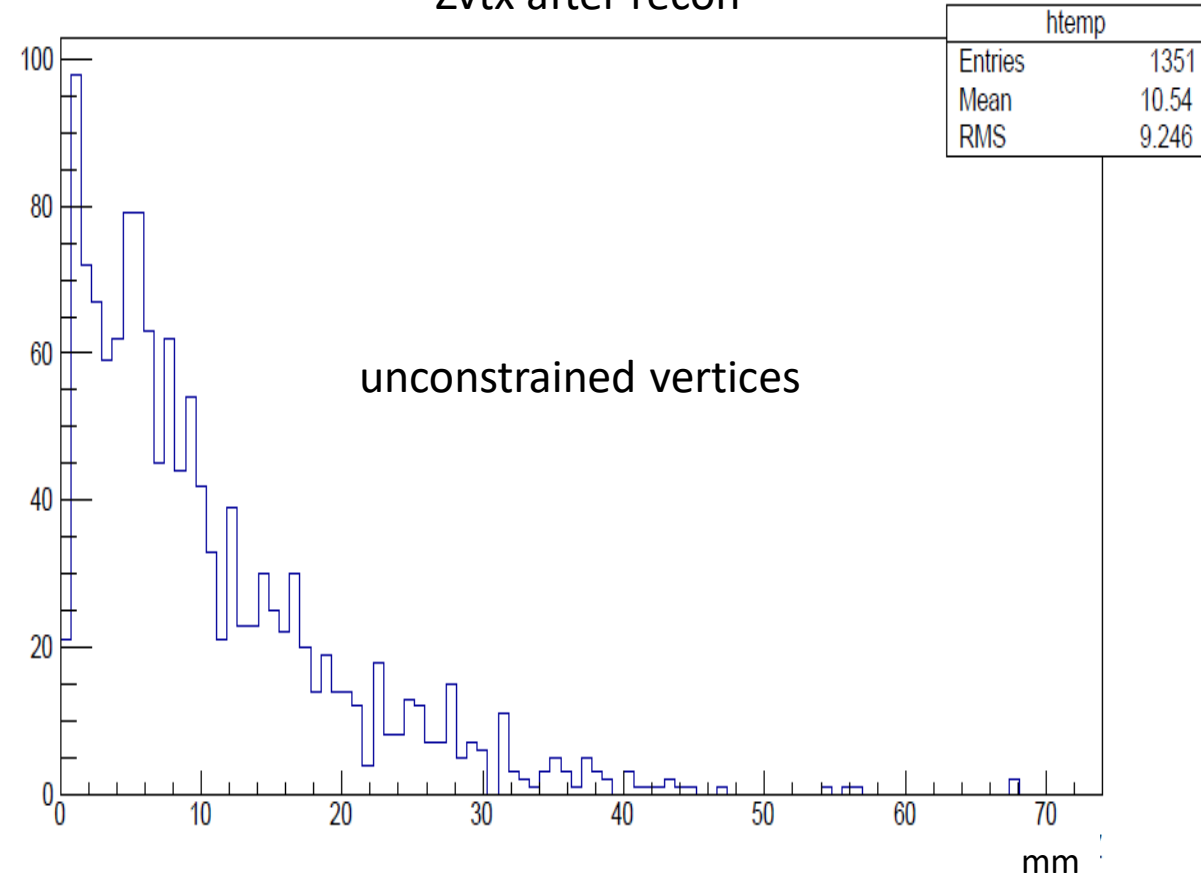
Quick check of the events...

# TM Vertex Acceptance (1S)

e- zvtx from 1S rad

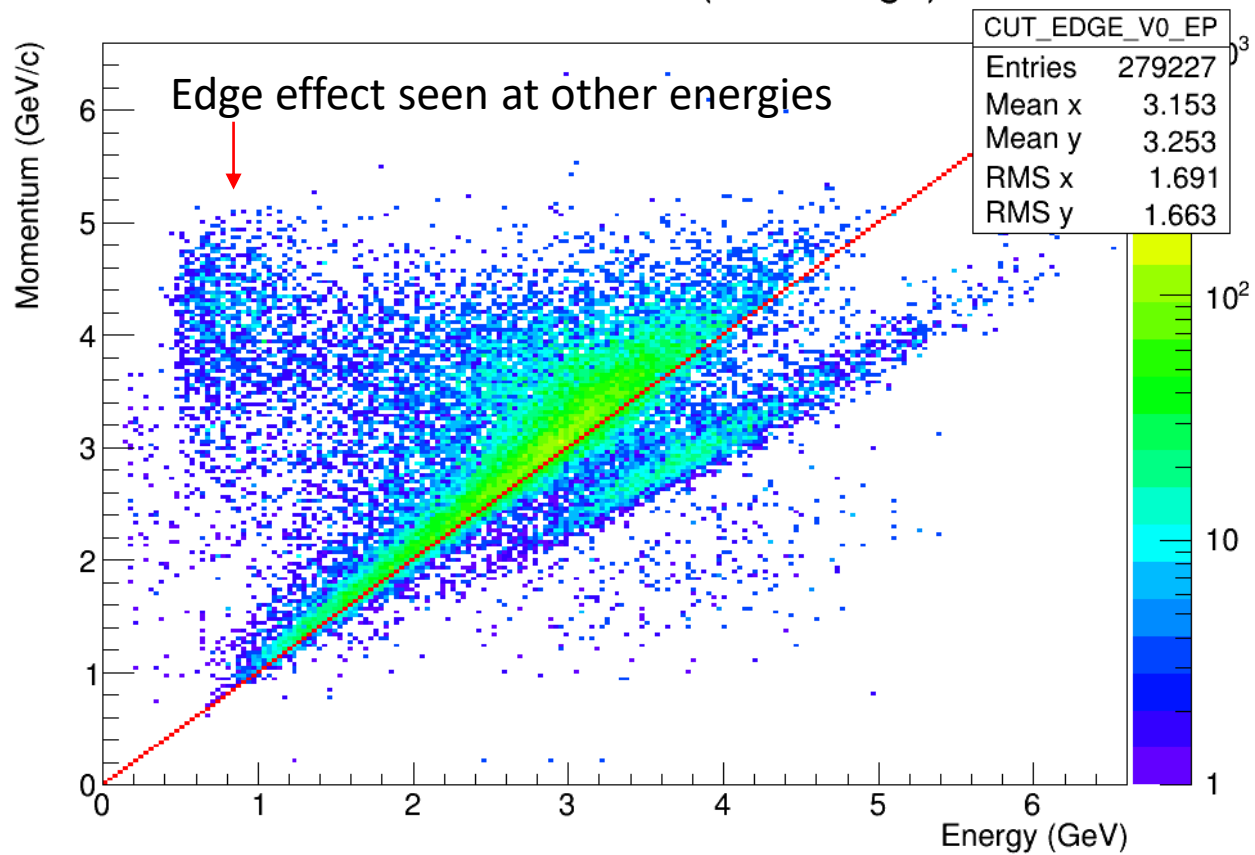


Zvtx after recon

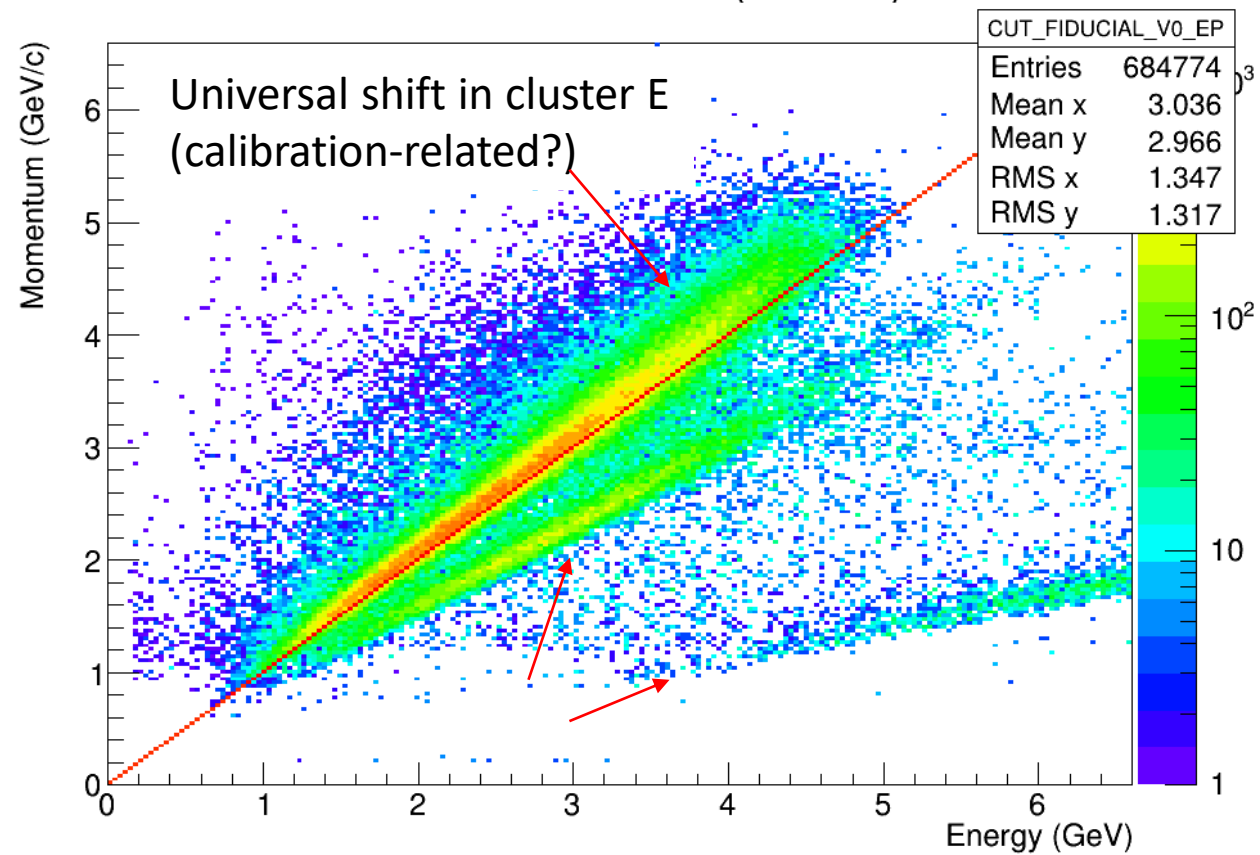


# Momentum vs. Cluster Energy

## V0 Momentum vs. E (Inner Edge)

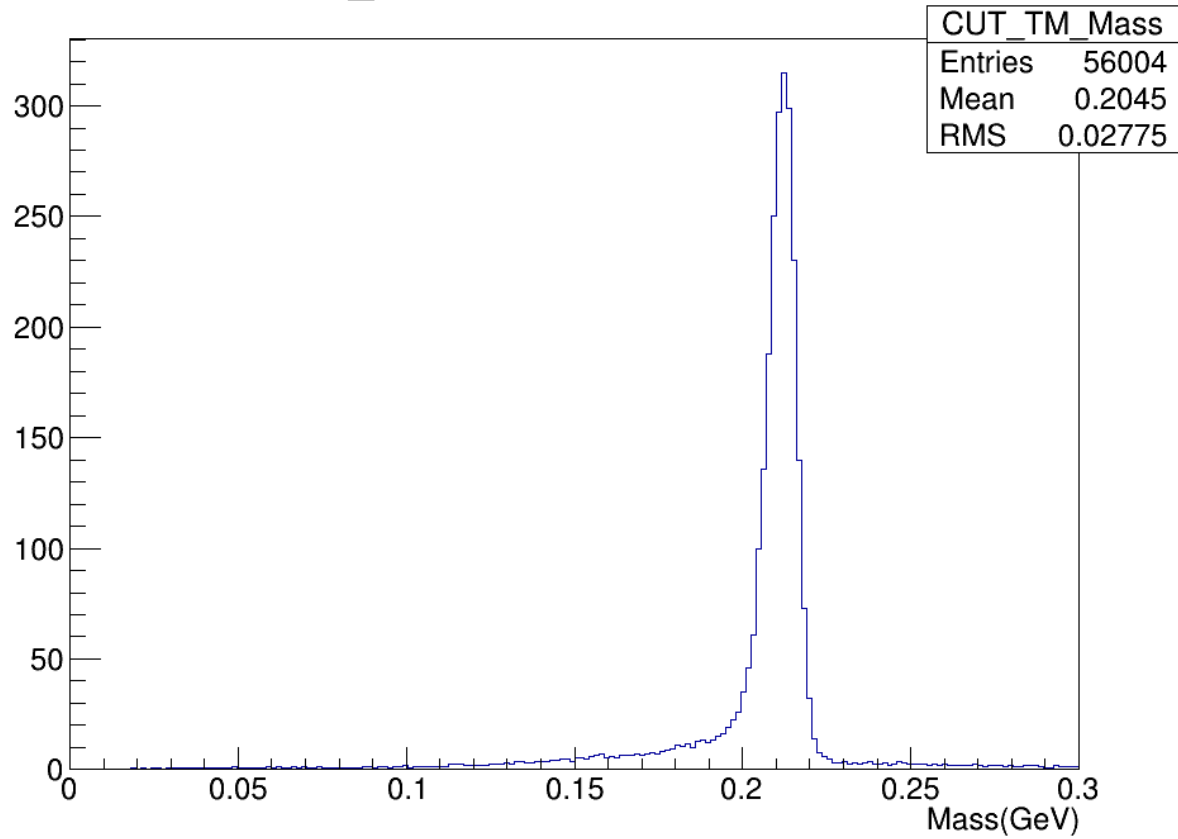


## V0 Momentum vs. E (Fiducial)

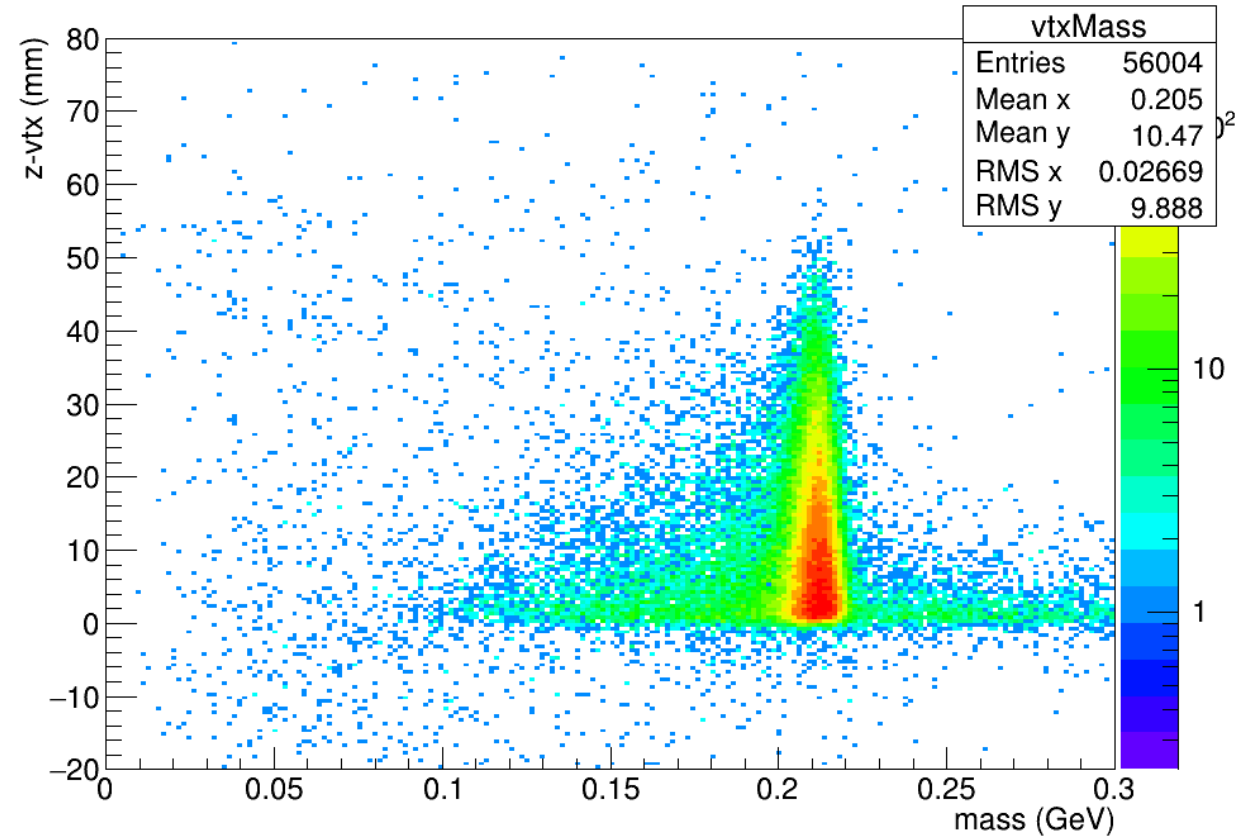


# Invariant Mass

TM\_Candidate Invariant Mass

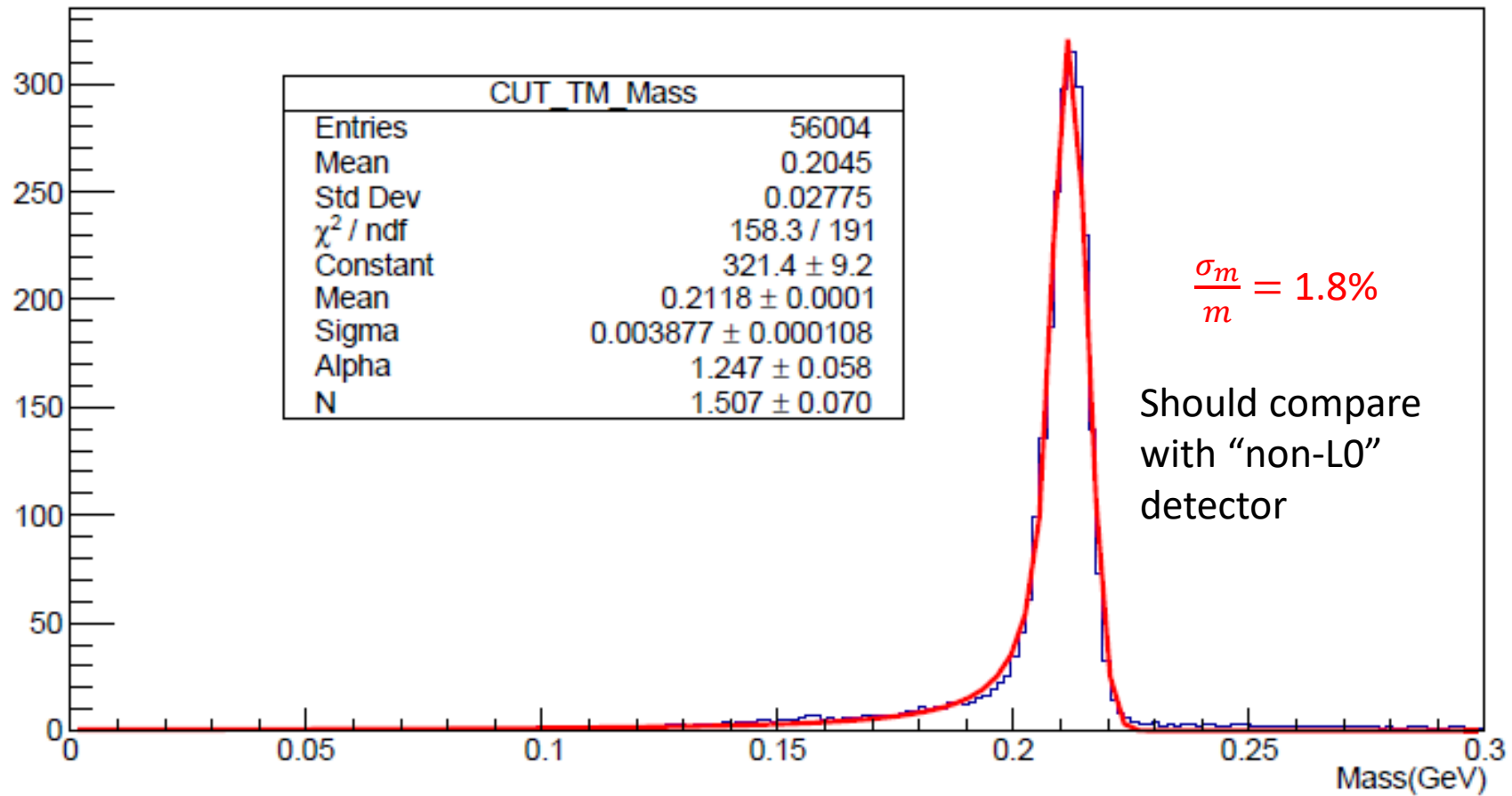


z-Vtx vs. Mass



# (Very) preliminary 6.6 GeV MC resolution

TM\_Candidate Invariant Mass





# Estimated TM Yield @ 6.6 GeV

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Events generated/month:

Radiative 1S only: 33.13 events

Radiative + 3g: 168.36 events

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True Muonium trigger acceptance  
 $91355/470000 = 19.43\%$

+ Tracking/Vertexing acceptance  
 $56004/470000 = \mathbf{11.91\%}$

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True Muonium trigger acceptance  
 $91355/470000 = 19.43\%$

+ Tracking/Vertexing acceptance  
 $56004/470000 = \mathbf{11.91\%}$

**Number of analyzable TM events/month:**

Radiative 1S only: **3.94 events**

Radiative + 3g: up to **20.06 events**

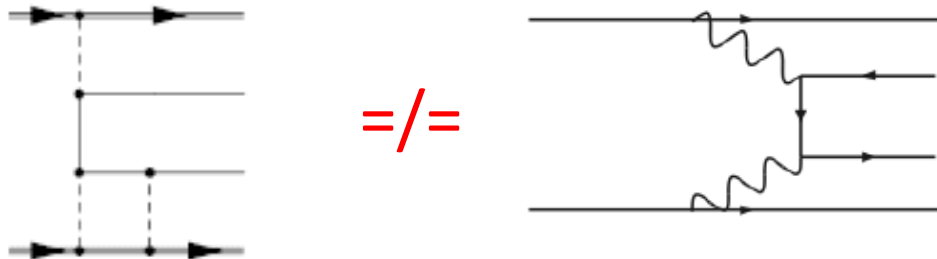
This assumes a similar acceptance for 3-photon events, which we can't actually know without a generator...

# Summary and future plans

- The bad news
  - $< 4$  events/month accepted, if only looking for the radiative 1S1 state
    - We are currently only sensitive to the radiative 1S1 state.
  - This doesn't even include analysis cuts
  - We don't have a generator for the 3-photon diagram
- The good news
  - Since the mass is known, the search itself becomes much easier
    - Analysis can be tuned to a single mass
    - No “look-elsewhere effect”
    - More efficient background studies
      - Generator mass cut can be exploited
  - The 2S and 2P states can possibly add a few events
    - only differ by decay length (8x larger lifetime)
  - Probing the BH (low  $x$ ) region would cement the feasibility of the TM search

# Summary and future plans

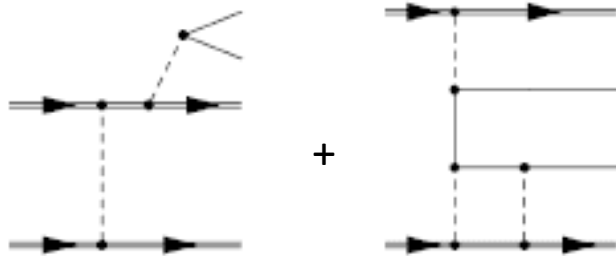
- What is needed
  - 6.6 GeV background to analyze, mix, find cuts, etc.
  - See if the hodoscope geometry picks up additional events
- What is probably also needed
  - A way to conduct a search in the Bethe-Heitler region
  - simulation of 3-photon events (not trivial)



# Bonus Slides

# True Muonium Production

- Muon-antimuon bound state
  - Two spin  $\frac{1}{2}$  fermions, total spin can add to either 0 or 1
- Two mechanisms for production:
  - Odd # of photons, spin-1 triplet states, “ortho” ( $TM \rightarrow e^+ e^-$ )



- 1-photon: “radiative-like”, 3-photon: “Bethe-Heitler-like” kinematics
- Branching ratio beyond 3-photon diagram makes those yields negligible.

# Primary Production Cross Sections

- Banburski Schuster - arXiv:1206.3961v1 [hep-ph] 18 Jun 2012

Radiative (1-photon) mechanism:

$$d\sigma = \frac{1}{4n^3} \frac{Z^2 \alpha^7}{m_\mu^2} \frac{x(1-x)(1-x + \frac{1}{3}x^2) dx}{[1-x + (m_e/m_{\mu\bar{\mu}})^2]^2} \times \left( \ln \left[ \frac{(E_{beam}/m_\mu)^2 (1-x)^2}{1-x + (m_e/m_{\mu\bar{\mu}})^2} \right] - 1 \right)$$

3-photon mechanism:

$$\frac{d\sigma}{dx} = \frac{1}{4n^3} Z^2 \frac{\alpha^7}{m_\mu^2} \left( \frac{Z\alpha\Lambda}{m_\mu} \right)^2 \frac{4B}{x} \times \left( \left( 1-x + \frac{x^2}{2} \right) \text{Log} \left[ \frac{(1-x)(m_\mu)^2}{x^2(m_e)^2} \right] - 1 + x \right)$$

Additional  $Z^2$  dependence





# Expected True Muonium Rate (1S)

- Using the production and dissociation cross sections for the 1S state

$$\frac{dN_{1S}}{dz} = N_e(z) \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} - N_{1S}$$

(dissociation length, to all final states)

Where  $z = \frac{l}{l_{1S \rightarrow X}}$

(average distance traveled before breakup)

$$l_{1S \rightarrow X} = \frac{1}{N\sigma_b} = \frac{1}{(6.306 * 10^{22} \text{ cm}^{-3})(5.67937 * 10^{-20} \text{ cm}^2)} = 2.792 \mu\text{m}$$

(atomic density) (breakup cross section)

$8 \mu\text{m}$  W HPS target is  
~3x the 1S dissociation length

2S and 2P dissociate more easily in target,  
by an order of magnitude.

$$N_e(z) \approx N_e = \frac{5625 \text{ electrons}}{2 \text{ ns}}$$

450 nA for 6.6 GeV

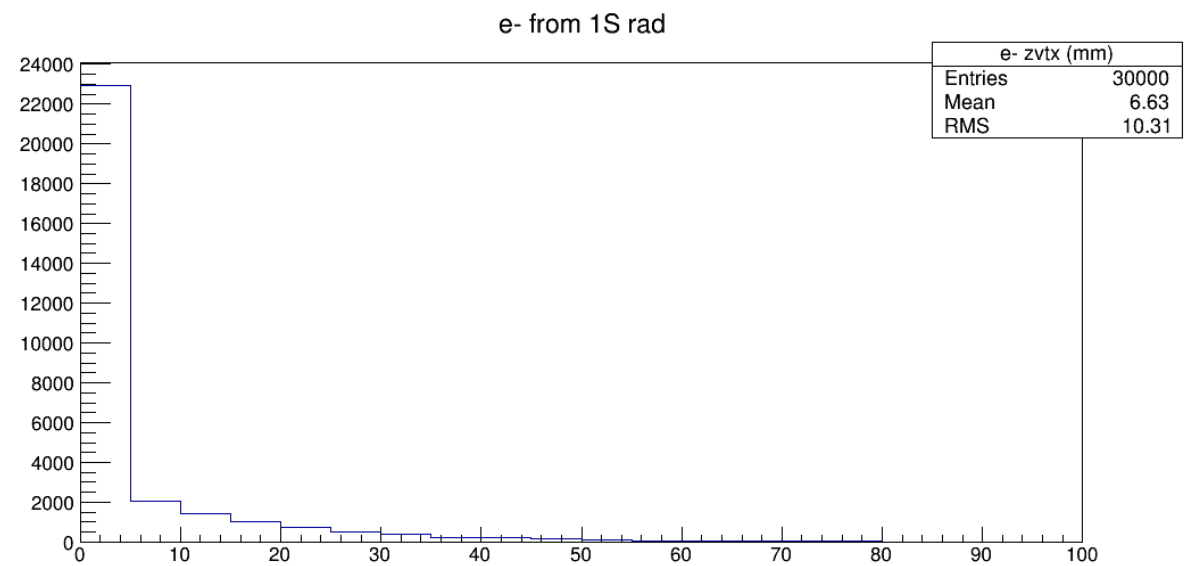
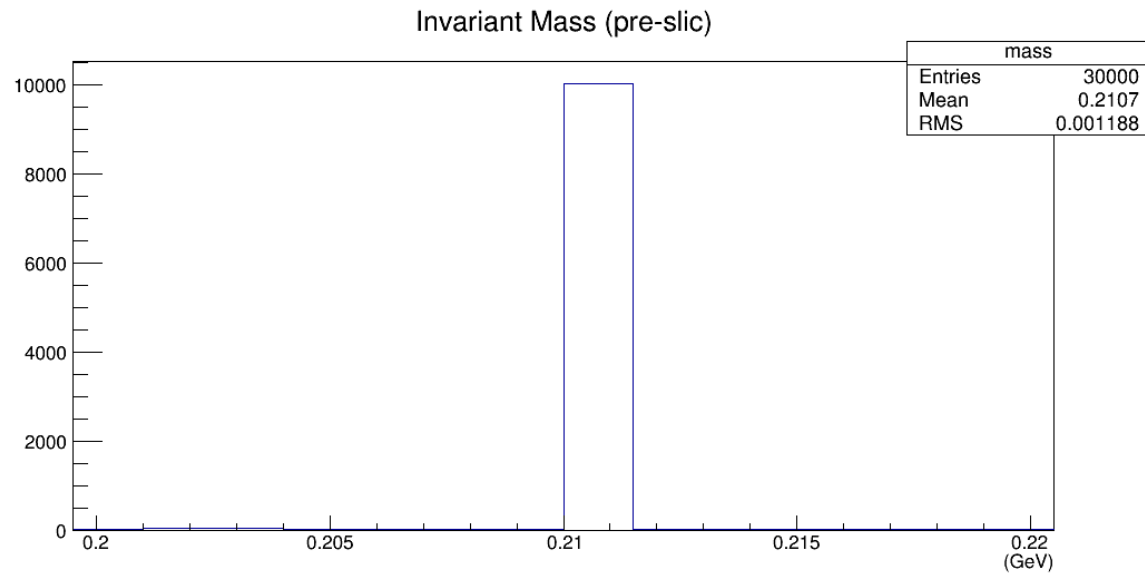
# Generated True Muonium $1^3S$ events

*MadGraph4 A' generator ( $> e^+e^-$ ), Alpha correction*

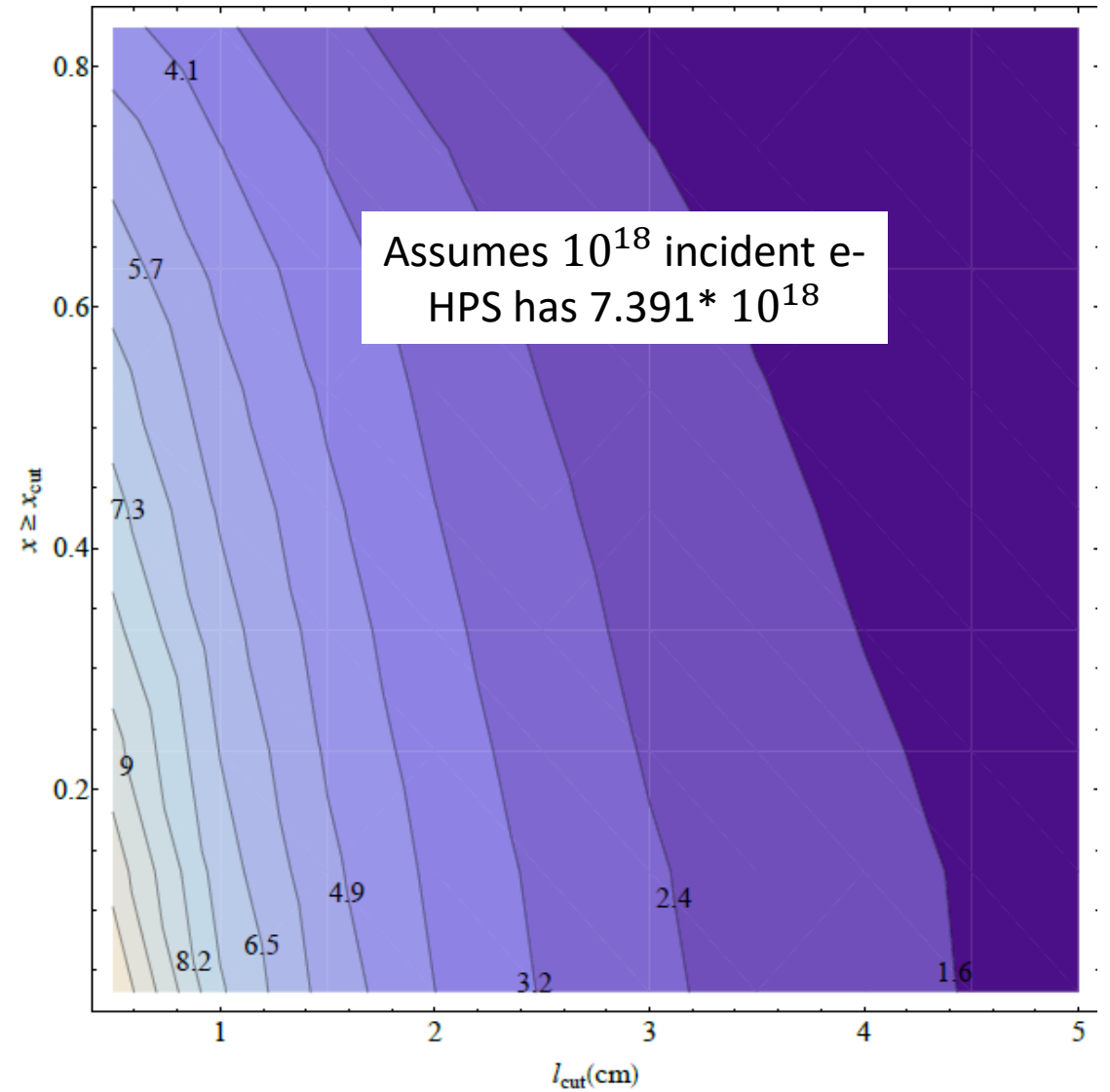
*Mass = 0.211 GeV*

*: Width =  $3.627 * 10^{-13}$  GeV =  **$4.9349 \times 10^{17}$  pb***

*Epsilon = 1*



# Banburski-Schuster TM Yield (corrected)




# Secondary Production/Dissociation (target interaction)

- Transfer matrix elements

$$F^{nlm,n'l'm'}(q) = \int_0^\infty \int_0^\pi \int_0^{2\pi} x^2 \sin(\theta) e^{iqx \cos(\theta)} \times \\ \psi^{n'l'm'}(x, \theta, \phi)^* \psi^{nlm}(x, \theta, \phi) d\phi d\theta dx$$


- Transition cross section (to all other bound states + dissociation)

$$d\sigma_{\text{tot}}^{nl} = Z^2 \frac{\alpha^2}{\pi} \left( 1 - F^{nl0,n'l'0}(q) \right) \\ \times \frac{1}{a^2} |\Delta(q, Z)|^2 q dq. \quad \int_0^\infty \text{diffTotE1Ground}[Z, q] dq \\ := \int_0^\infty Z^2 \alpha^2 / \pi (1 - F[2,0,0,2,0,0,q]) 1/a_{TM}^2 \Delta[q,Z] \Delta[q,Z] q dq = \mathbf{4.90741 \times 10^{17} \text{ pb}}$$

2S state -> all states + dissoc. 

*MadGraph4 A' generator (> e<sup>+</sup>e<sup>-</sup>), Alpha correction*

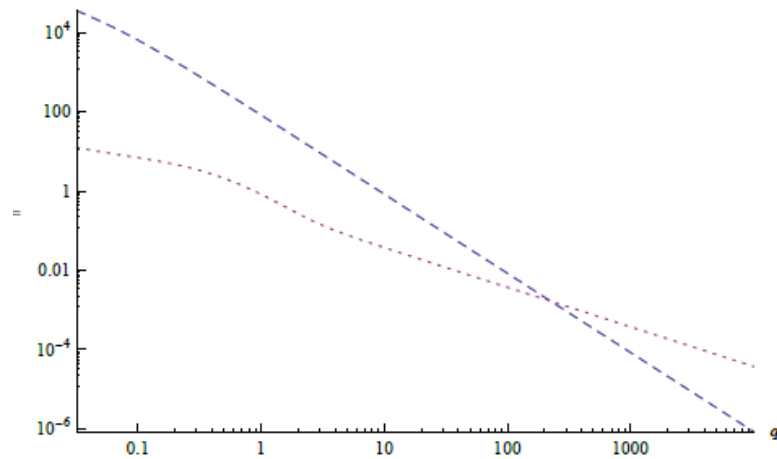
*Mass = 0.211 GeV*

*: Width = 3.627 \* 10<sup>-13</sup> GeV* | = **4.9349 × 10<sup>17</sup> pb**  Similar for 4.4 and 6.6 GeV

*Epsilon = 1*

# Thomas-Fermi-Molier vs. HPS G2 Factors

- Form Factors



--- Molier  
 ..... HPS

Thomas-Fermi-Molier:

$$\Delta(q, Z) = 4\pi \sum_{i=1}^3 \frac{\alpha_i}{q^2 + \beta_i^2},$$

$$\Delta[\mathbf{q}_-, \mathbf{z}_-] := 4\pi \left( \frac{0.1}{q^2/a^2 + \beta_1[\mathbf{z}]^2} + \frac{0.55}{q^2/a^2 + \beta_2[\mathbf{z}]^2} + \frac{0.35}{q^2/a^2 + \beta_3[\mathbf{z}]^2} \right)$$

$\alpha_1 = 0.10, \alpha_2 = 0.55, \alpha_3 = 0.35$

HPS G2 elastic form factor (used by MG4,  $\propto d\sigma/dx$ ):

```
Anuc = 184;
aval := 111 / (0.0005111 * 74^(1/3));
dval = 0.164 / Anuc^(2/3);
apval = 773 / (0.0005111 * 74^(2/3))
ΔHPS[q_, Z_] :=
  ((Z^2) * (aval^4) * (q^2) / ((1 + aval^2 * q) * (1 + q / dval))^2 + Z * (apval^4) *
  q^2 * (1 + 1.9276 * q)^2 / ((1 + apval^2 * q) * (1 + 1.40845 * q)^4)^2)^0.5;
```

# 2S and 2P True Muonium States

- Unlike the “primarily produced” 1S state, which survives the target, the 2S and 2P dissociate far more easily in the target.
- Their primary production yields are also an order of magnitude lower.
- The displaced 2S and 2P states will then come from a 1S state transitioning into them within the target (“secondary production”).
- While these states are difficult to detect, the 2P state is able to transition to additional 1S states that may survive the target, contributing to its overall yield.

# Secondary Production/Dissociation (target interaction)

- Transfer matrix elements

$$F^{nlm,n'l'm'}(q) = \int_0^\infty \int_0^\pi \int_0^{2\pi} x^2 \sin(\theta) e^{iqx \cos(\theta)} \times \\ \psi^{n'l'm'}(x, \theta, \phi)^* \psi^{nlm}(x, \theta, \phi) d\phi d\theta dx$$

- Transition cross section (to all other bound states + dissociation)

$$d\sigma_{\text{tot}}^{nl} = Z^2 \frac{\alpha^2}{\pi} \left( 1 - F^{nl0,n'l'0}(q) \right) \int_0^\infty \text{diffTotE1Ground}[Z, q] dq := \\ \int_0^\infty Z^2 \alpha^2 / \pi (1 - F[1,0,0,1,0,0,q]) 1/a_{TM}^2 \Delta[q,Z] \Delta[q,Z] q dq = 5.67937 \times 10^{16} \text{ pb} \\ \times \frac{1}{a^2} |\Delta(q, Z)|^2 q dq. \int_0^\infty Z^2 \alpha^2 / \pi (1 - F[1,0,0,1,0,0,q]) 1/a_0^2 \Delta_0[q,Z] \Delta_0[q,Z] \\ q / (127.9/137)^2 q dq \\ = 6.88469 \times 10^{18} \text{ pb}$$

# Secondary Production/Dissociation (target interaction)

- Transfer matrix elements

$$F^{nlm,n'l'm'}(q) = \int_0^\infty \int_0^\pi \int_0^{2\pi} x^2 \text{Sin}(\theta) e^{iqx \text{Cos}(\theta)} \times \\ \psi^{n'l'm'}(x, \theta, \phi)^* \psi^{nlm}(x, \theta, \phi) d\phi d\theta dx$$

- Transition cross section (to all other bound states + dissociation)

$$d\sigma_{\text{tot}}^{nl} = Z^2 \frac{\alpha^2}{\pi} \left( 1 - F^{nl0,n'l'0}(q) \right) \int_0^\infty \text{diffTotE1Ground}[Z, q] dq := \\ \int_0^\infty Z^2 \alpha^2 / \pi (1 - F[1,0,0,1,0,0,q]) 1/a_{TM}^2 \Delta[q,Z] \Delta[q,Z] q dq = 5.67937 \times 10^{16} \text{ pb} \\ \times \frac{1}{a^2} |\Delta(q, Z)|^2 q dq. \int_0^\infty Z^2 \alpha^2 / \pi (1 - F[1,0,0,1,0,0,q]) 1/a_0^2 \Delta_0[q,Z] \Delta_0[q,Z] \\ * \Delta\text{HPS}[q,Z] / (127.9/137)^2 q dq \\ = 4.13904 \times 10^{18} \text{ pb}$$



# Secondary Production/Dissociation (target interaction)

- Transfer matrix elements

$$F^{nlm,n'l'm'}(q) = \int_0^\infty \int_0^\pi \int_0^{2\pi} x^2 \sin(\theta) e^{iqx \cos(\theta)} \times \\ \psi^{n'l'm'}(x, \theta, \phi)^* \psi^{nlm}(x, \theta, \phi) d\phi d\theta dx$$

- Transition cross section (to all other bound states + dissociation)

$$d\sigma_{\text{tot}}^{nl} = Z^2 \frac{\alpha^2}{\pi} \left( 1 - F^{nl0,n'l'0}(q) \right) \int_0^\infty \text{diffTotE1Ground}[Z, q] dq := \\ \int_0^\infty Z^2 \alpha^2 / \pi (1 - F[1,0,0,1,0,0,q]) 1/a_{TM}^2 \Delta[q,Z] \Delta[q,Z] q dq = 5.67937 \times 10^{16} \text{ pb} \\ \times \frac{1}{a^2} |\Delta(q, Z)|^2 q dq. \int_0^\infty Z^2 \alpha^2 / \pi (1 - F[1,0,0,1,0,0,q]) 1/a_{TM}^2 \Delta[q,Z] \Delta[q,Z] \\ * \Delta\text{HPS}[q,Z] / (127.9/137)^2 q dq \\ = 2.34739 \times 10^{17} \text{ pb}$$


# Secondary Production/Dissociation (target interaction)

- Transfer matrix elements

$$F^{nlm,n'l'm'}(q) = \int_0^\infty \int_0^\pi \int_0^{2\pi} x^2 \sin(\theta) e^{iqx \cos(\theta)} \times \\ \psi^{n'l'm'}(x, \theta, \phi)^* \psi^{nlm}(x, \theta, \phi) d\phi d\theta dx$$


- Transition cross section (to all other bound states + dissociation)

$$d\sigma_{\text{tot}}^{nl} = Z^2 \frac{\alpha^2}{\pi} \left( 1 - F^{nl0,n'l'0}(q) \right) \\ \times \frac{1}{a^2} |\Delta(q, Z)|^2 q dq. \quad \int_0^\infty \text{diffTotE1Ground}[Z, q] dq \\ := \int_0^\infty Z^2 \alpha^2 / \pi (1 - F[1,0,0,1,0,0,q]) 1/a_{TM}^2 \Delta[q,Z] \Delta[q,Z] q dq = \mathbf{5.67937 \times 10^{16} \text{ pb}}$$

1S state -> all states + dissoc. 

*MadGraph4 A' generator (> e<sup>+</sup>e<sup>-</sup>), Alpha correction*

*Mass = 0.211 GeV*

*: Width = 3.627 \* 10<sup>-13</sup> GeV* | = **4.9349 × 10<sup>17</sup> pb**  Similar for 4.4 and 6.6 GeV

*Epsilon = 1*

# TM Theory vs. MadGraph A' generator XS

- If MadGraph assumes no state transitions, only 1S production and dissociation of a hydrogen-like atom, correcting for  $\alpha$  in MadGraph4:

$$\frac{MG4 \text{ generated}}{TM \text{ Theory } (1S \rightarrow e^+e^-)} = \frac{4.9349 * 10^{17} \text{ pb}}{2.21044 * 10^{16} \text{ pb}} = 22.32$$

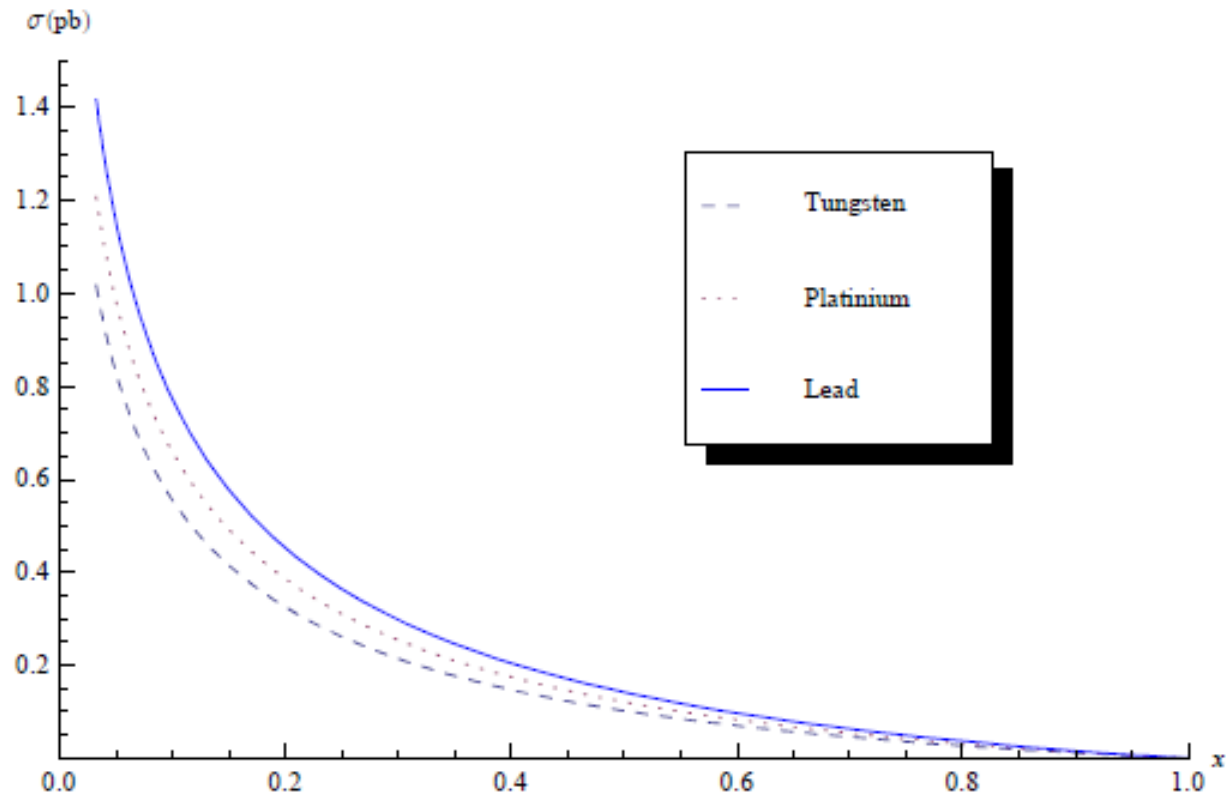
- If also using HPS's G2 form factor for A' production:

$$\frac{MG4 \text{ generated}}{TM \text{ Theory } (1S \rightarrow e^+e^-) + HPS \text{ FF}} = \frac{4.9349 * 10^{17} \text{ pb}}{2.0005 * 10^{17} \text{ pb}} = 2.466$$

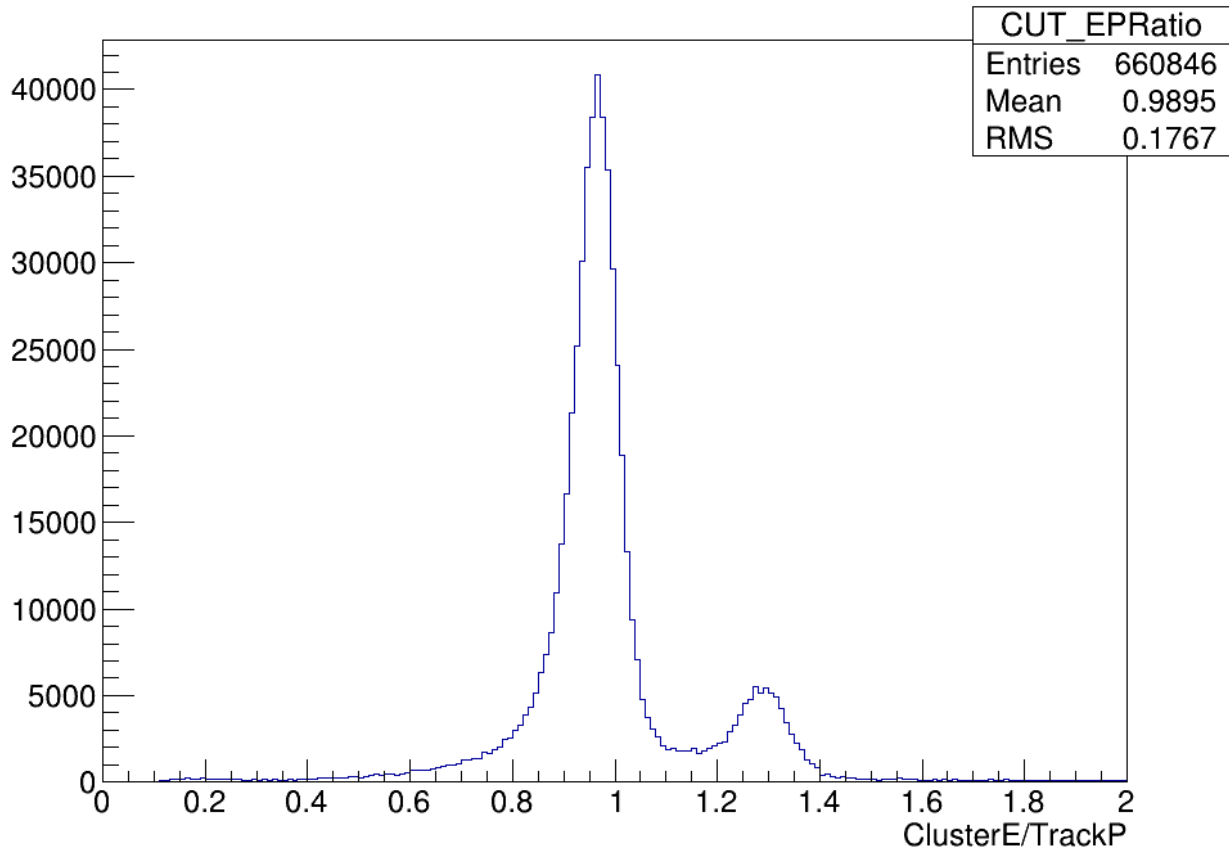
- If Assuming all state transitions + dissociation, not just 1S

$$\frac{MG4 \text{ generated}}{TM \text{ Theory}(All \rightarrow e^+e^-) + HPS \text{ FF}} = \frac{4.9349 * 10^{17} \text{ pb}}{2.34739 * 10^{17} \text{ pb}} = 2.10$$

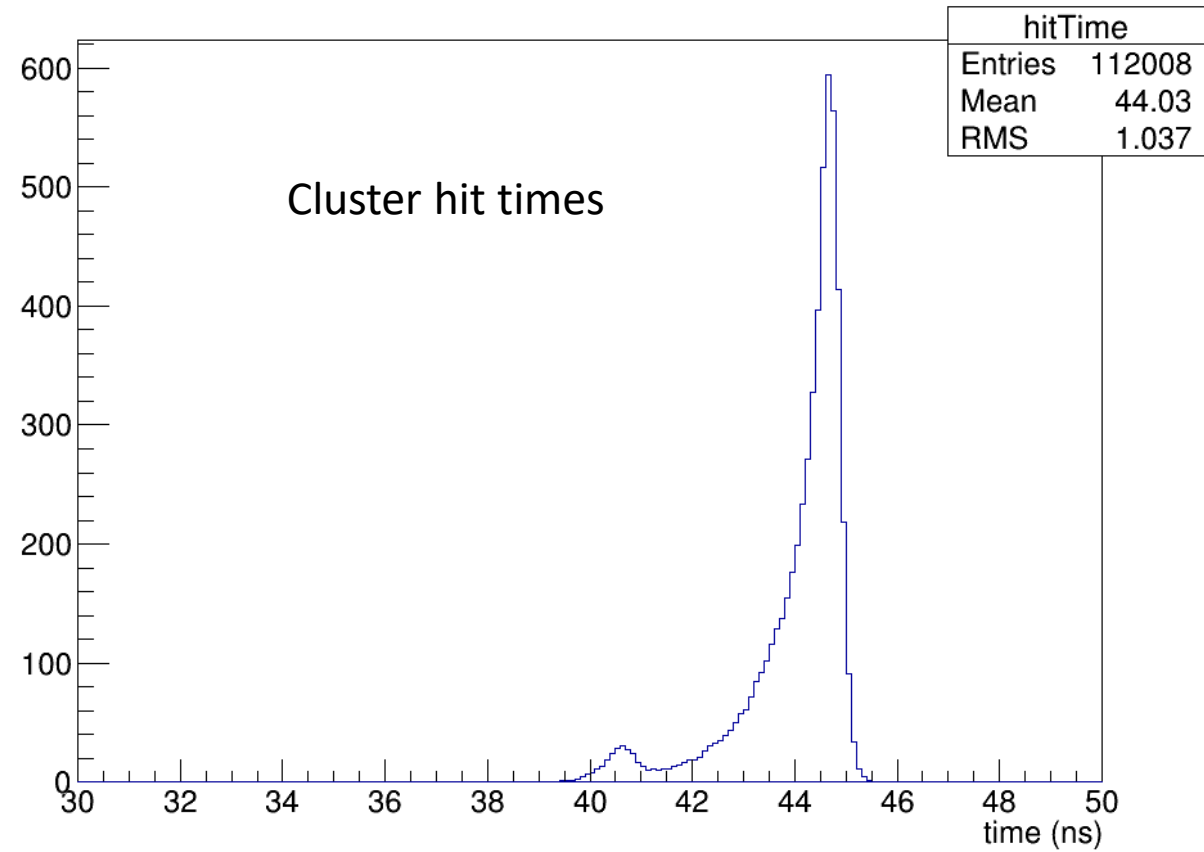
# Integrated Production Cross Section (radiative)

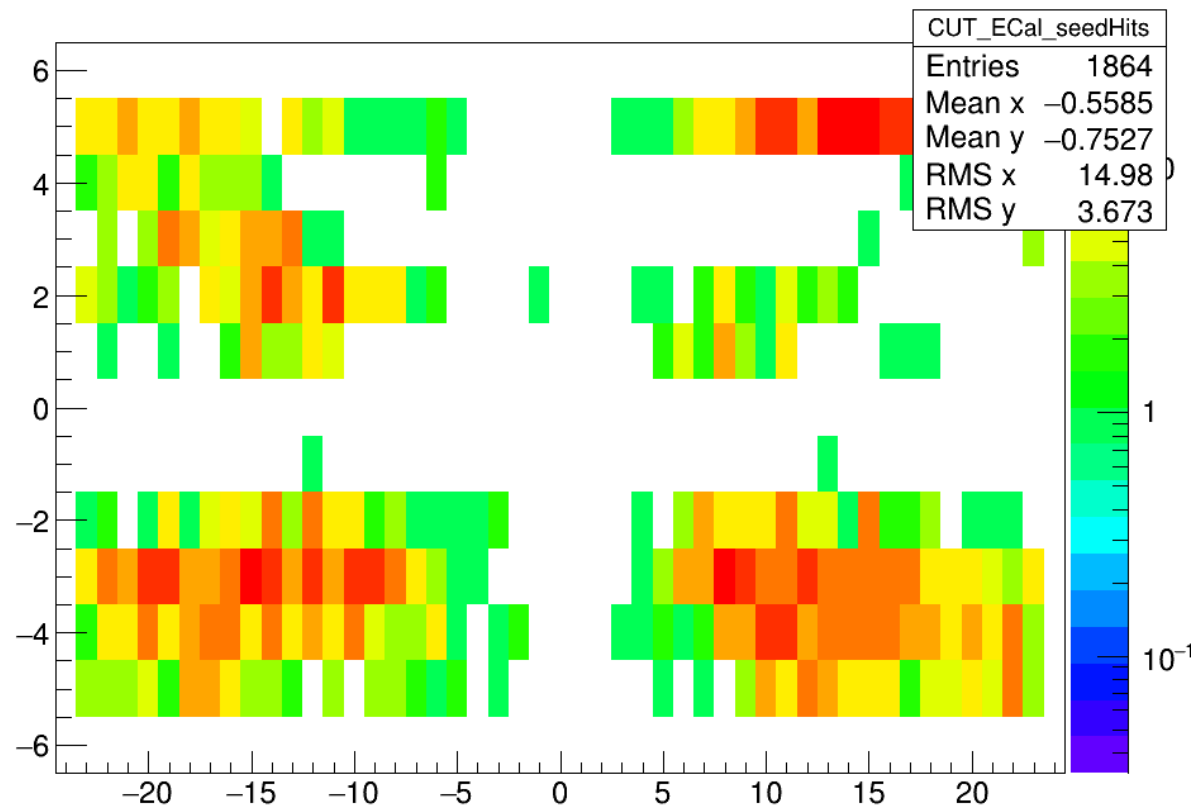
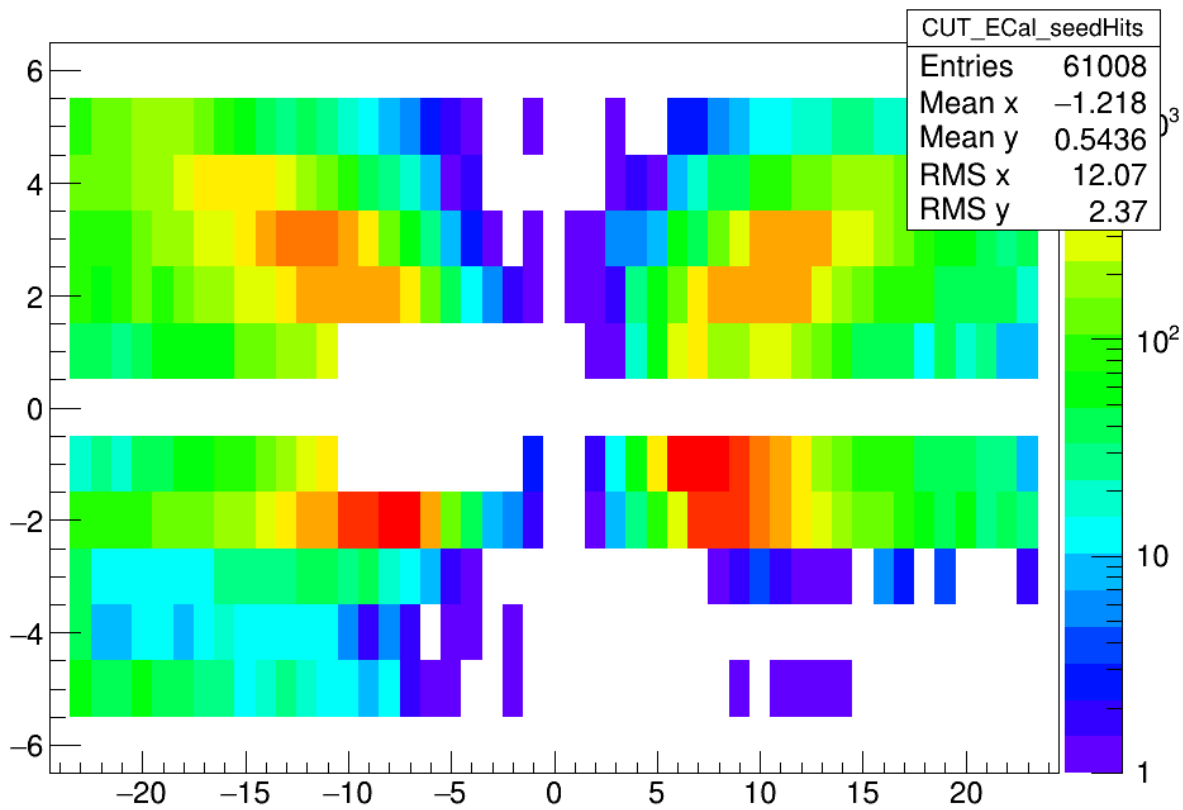


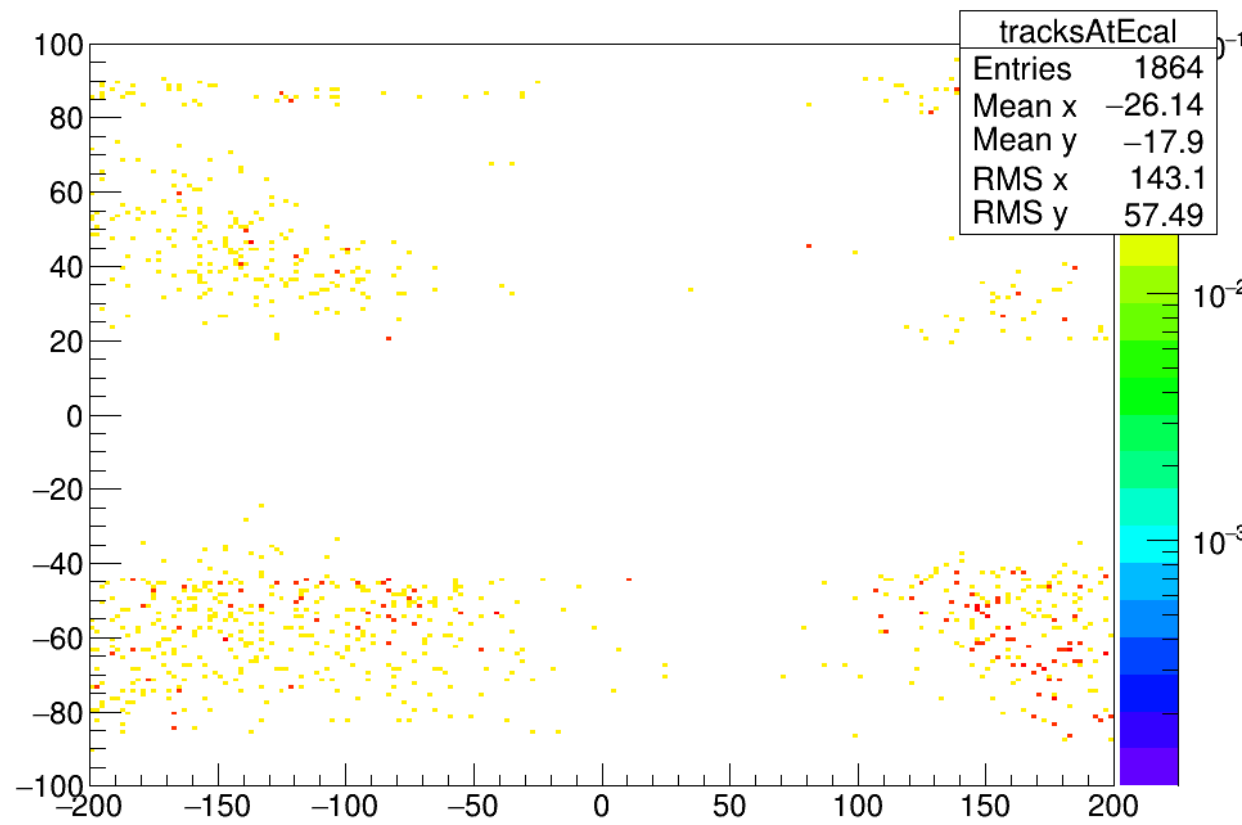
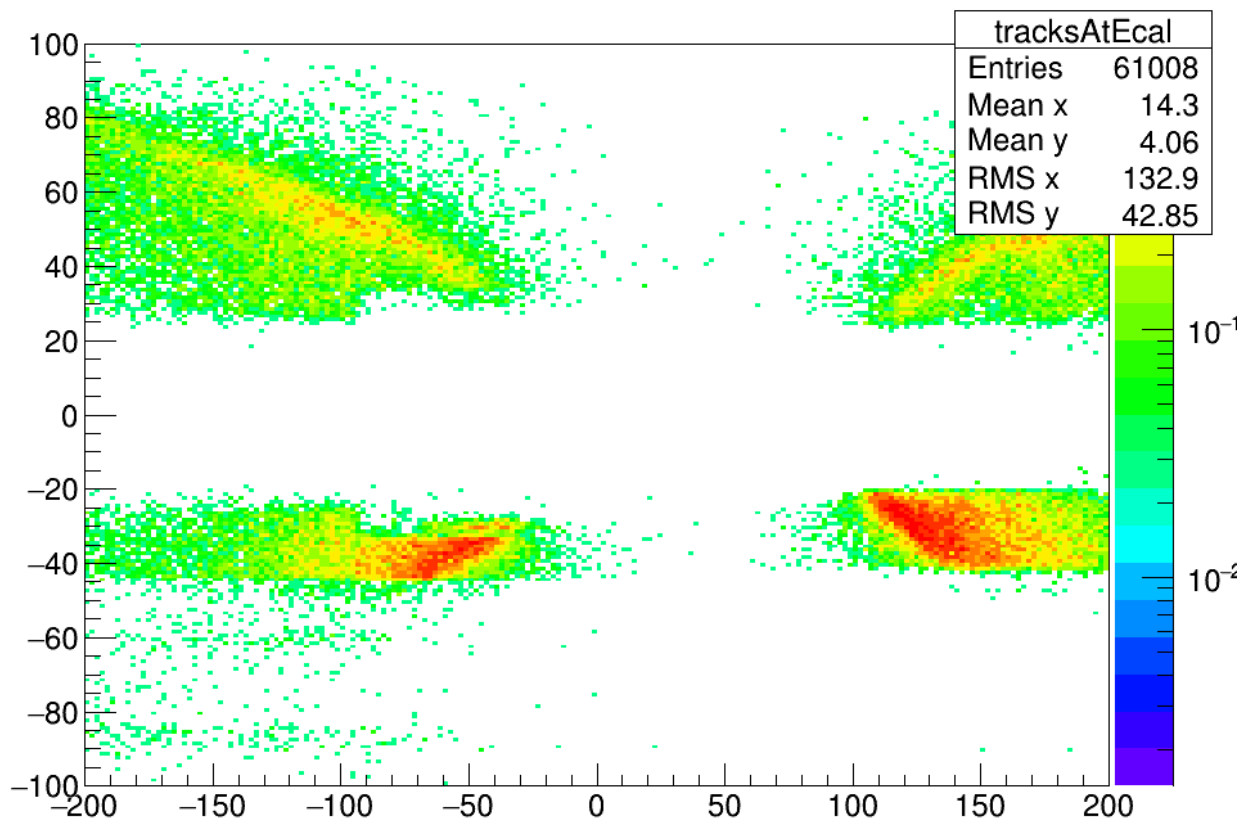
### E/P Ratio



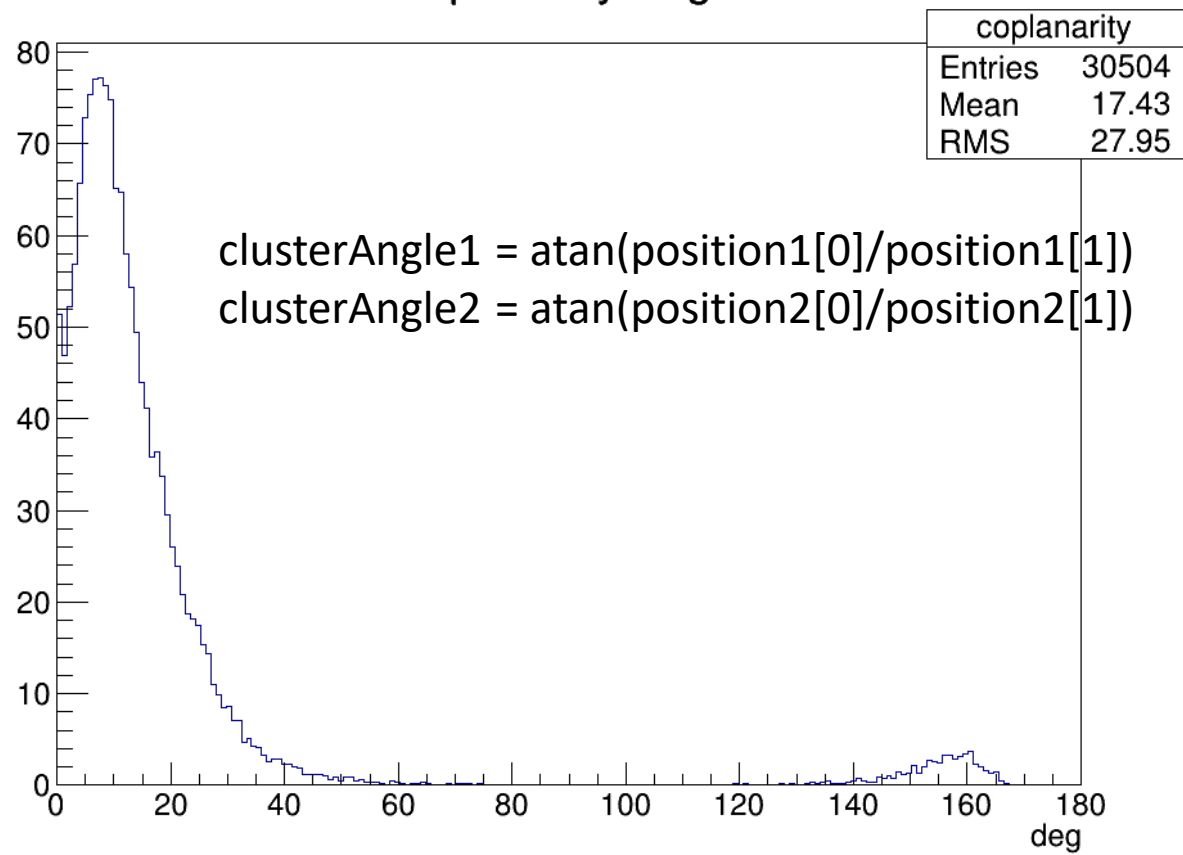
### Cluster hit times







### Coplanarity Angle



### Coplanarity Angle

