Beam Spin Asymmetries for Positively Charged Kaons

David Riser, University of Connecticut
in collaboration with Kyungseon Joo, Nick Markov

- Motivation
- Measurement of BSA
- Analysis of BSA Moments
- Results, Conclusion, Outlook

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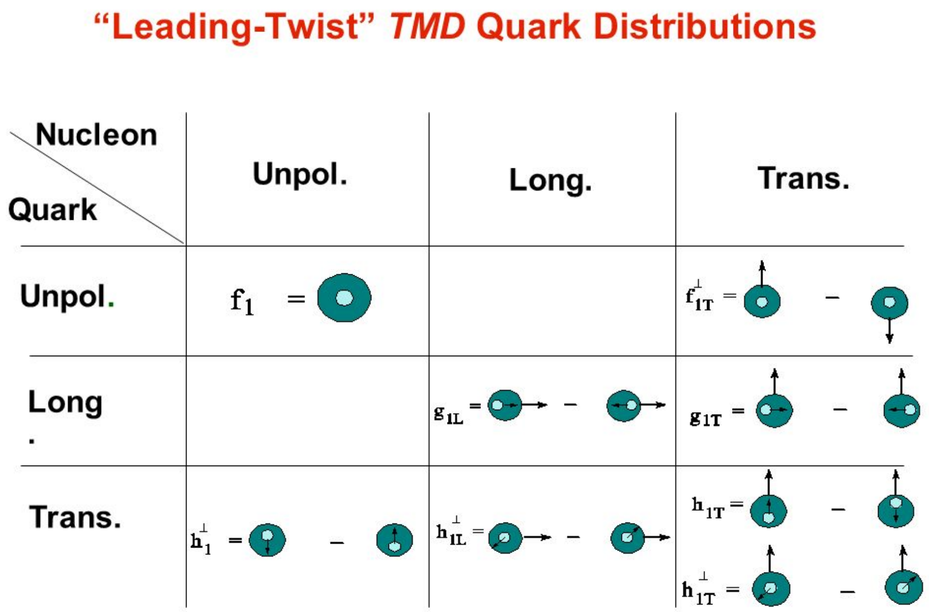
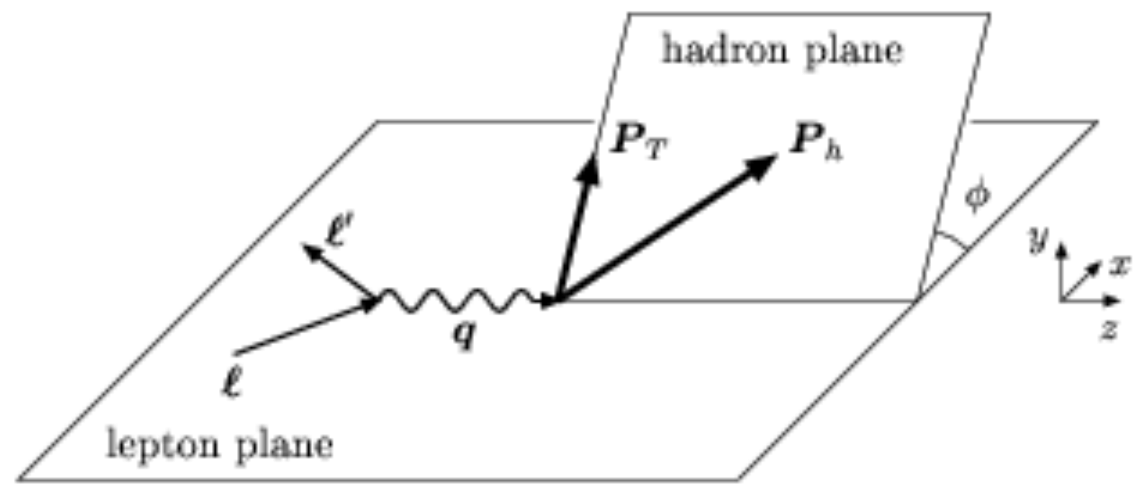
Currently there is big interest in 3-D nucleon structure in the form of **GPDs** and **TMDs**





The **SIDIS** cross section can be expressed in terms of model independent structure functions

$$\frac{d\sigma}{dx_B\, dQ^2\, dz\, d\phi_h\, dp_{h\perp}^2} = K(x,y,Q^2)\Big\{F_{UU,T} + \varepsilon F_{UU,L}$$
$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}\Big\}$$

(without polarized target)

Mulders, Tangerman (1995) Complete tree-level result for polarized deep-inelastic lepto-production.

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TMD and Fragmentation Functions

$$C[\omega f D] = x \sum_a e_a^2 \int d^2\vec{p}_\perp d^2\vec{k}_\perp \delta^{(2)} \left(\vec{p}_\perp - \vec{k}_\perp - \vec{P}_{h\perp}/z\right) \omega(\vec{p}_\perp, \vec{k}_\perp) f^a(x, p_\perp^2) D^a(z, k_\perp^2)$$

$$F_{LU}^{\sin\phi} = \frac{2M}{Q}\mathcal{C}\left(-\frac{\hat{\mathbf{h}}\cdot\mathbf{k_T}}{M_h}\left(xeH_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p_T}}{M}\left(xg^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z}\right)\right)$$

TMD and Fragmentation Functions

$$C[\omega f D] = x \sum_a e_a^2 \int d^2\vec{p}_\perp d^2\vec{k}_\perp \delta^{(2)} \left(\vec{p}_\perp - \vec{k}_\perp - \vec{P}_{h\perp}/z\right) \omega(\vec{p}_\perp, \vec{k}_\perp) f^a(x, p_\perp^2) D^a(z, k_\perp^2)$$

$$F_{LU}^{\sin\phi} = \frac{2M}{Q}\mathcal{C}\left(-\frac{\hat{\mathbf{h}}\cdot\mathbf{k_T}}{M_h}\left(x e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p_T}}{M}\left(x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z}\right)\right)$$

twist-3 pdf

Collins FF

unpolarized dist. function

twist-3 FF

twist-3 t-odd dist. funcion

Boer-Mulders

twist-3 FF

Structure function has a **twist-3** piece in every term

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Beam Spin Asymmetry (BSA) measurements are a good tool for extracting "moments".

$$BSA = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \frac{A_{LU}^{\sin\phi}\sin\phi}{1 + A_{UU}^{\cos\phi}\cos\phi + A_{UU}^{\cos(2\phi)}\cos(2\phi)}$$

Experimentally, they are measured by selecting your event of interest and calculating the quantity defined below.

$$BSA_i = \frac{1}{P_e}\frac{N_i^+ - N_i^-}{N_i^+ + N_i^-}$$

If acceptance corrections don't depend on helicity, the correction cancels.

For more on acceptance cancellations see thesis of Wes Gohn.

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Experimental Details

Run period: E1-F

- Beam energy 5.5 GeV
- Torus field run at 60% (2250 A)
- Negatively charged particles - Inbending
- Over 1B event triggers
- LH_2 target, 5 cm

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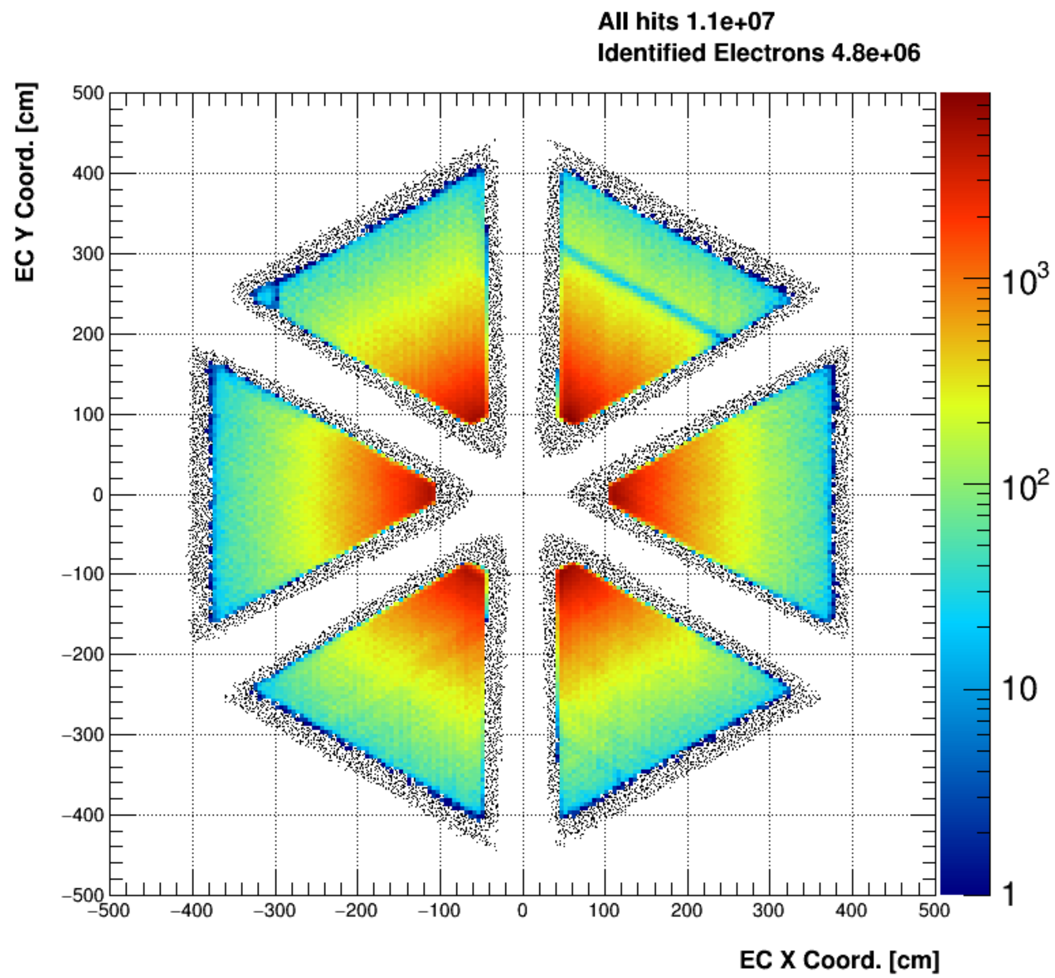

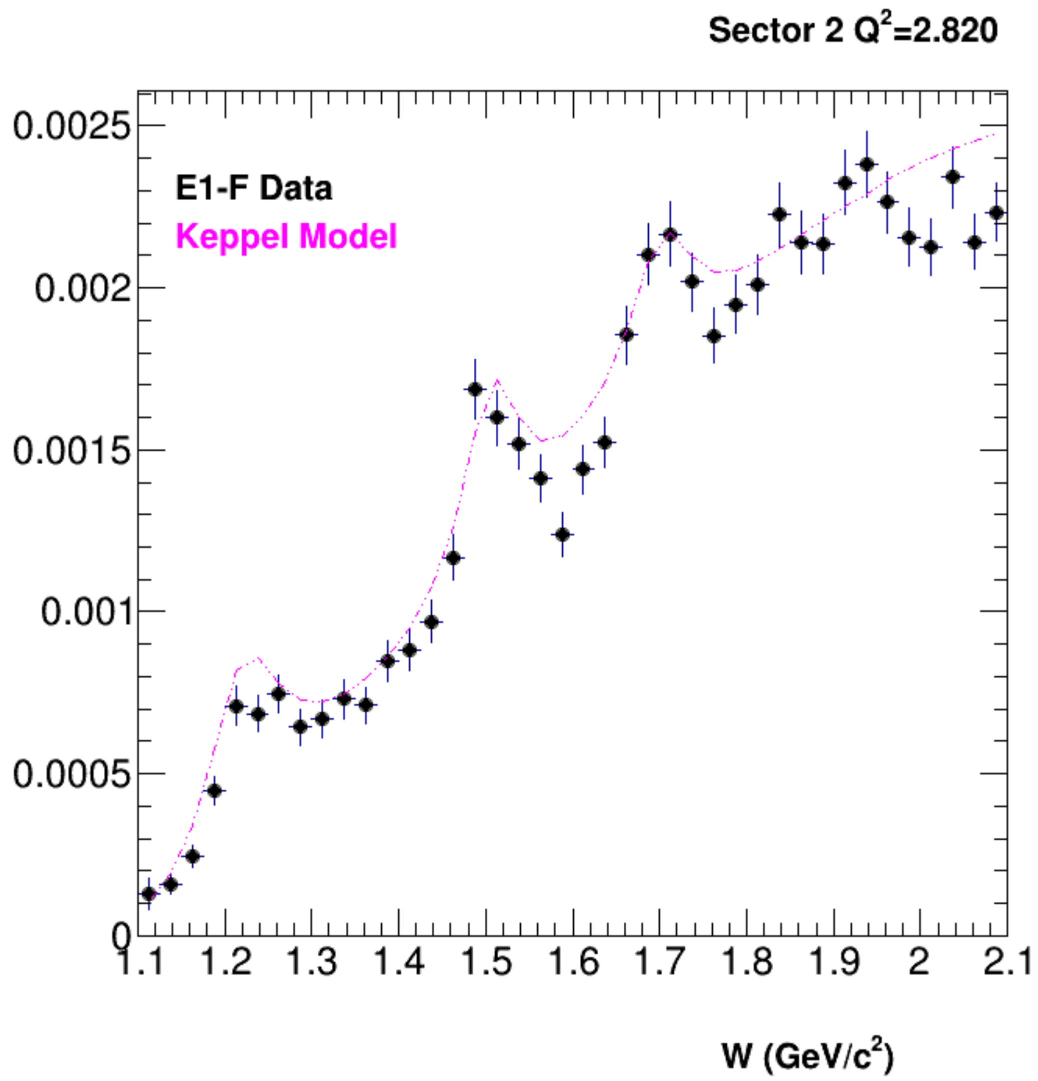

A brief overview of electron identification.

- Sampling fraction cut (momentum dependent)
- Geometrical fiducial cuts (DC, EC, CC)
- Cherenkov PMT matching cuts (theta, phi)
- EC energy deposition cut
- Z-vertex cut
- Vertex and kinematic corrections done after ID.

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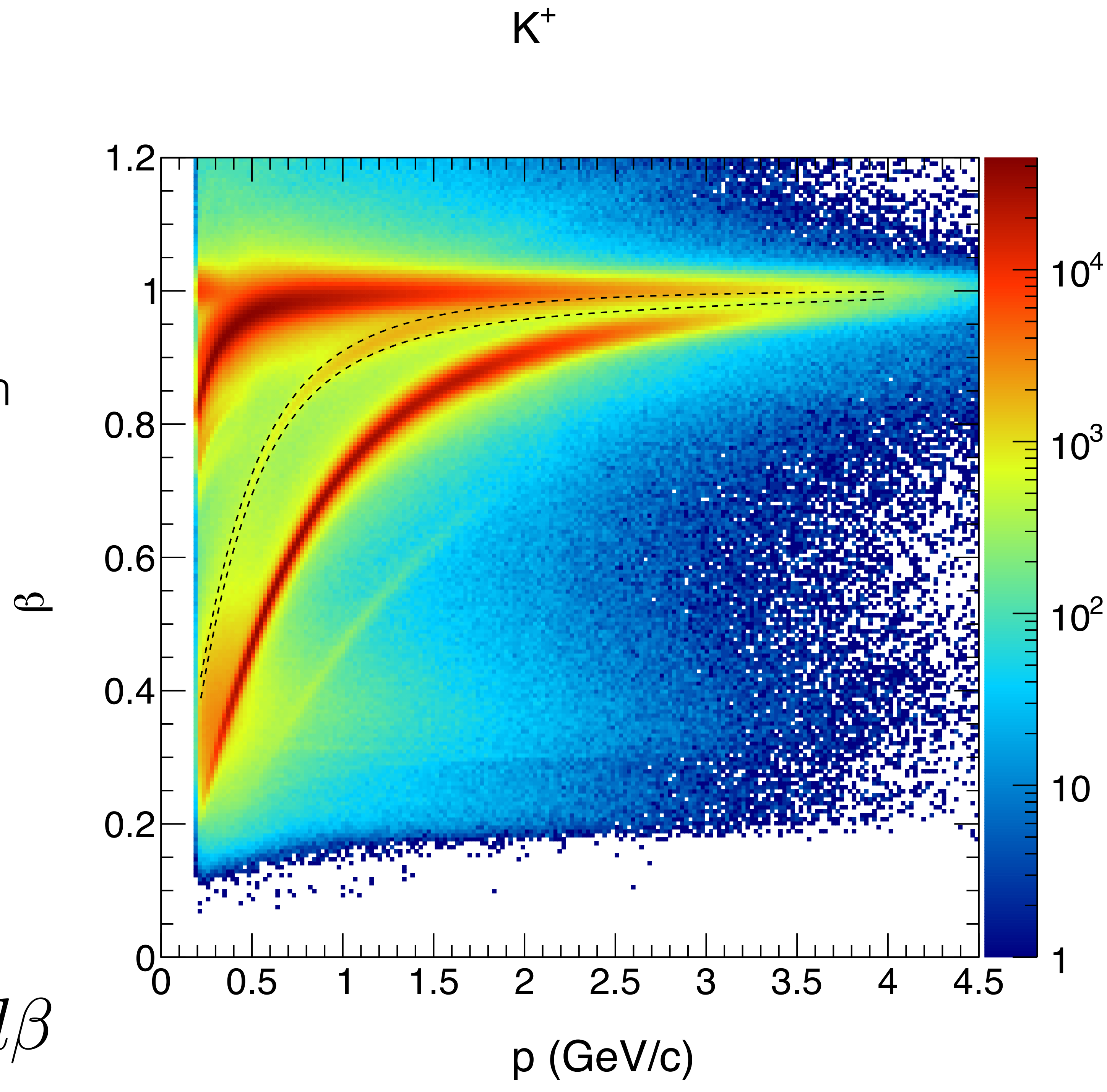
A brief overview of kaon identification.

- Drift chamber region 1 fiducial cut
- Electron vertex difference cut
- Likelihood ratio maximization condition
- Significance level cut

$$\frac{\mathcal{L}_h}{\mathcal{L}_\pi + \mathcal{L}_K + \mathcal{L}_p}$$

$$\alpha = 1 - \int_{\mu-\beta_{obs}}^{\mu+\beta_{obs}} P(\beta; p, h) d\beta$$



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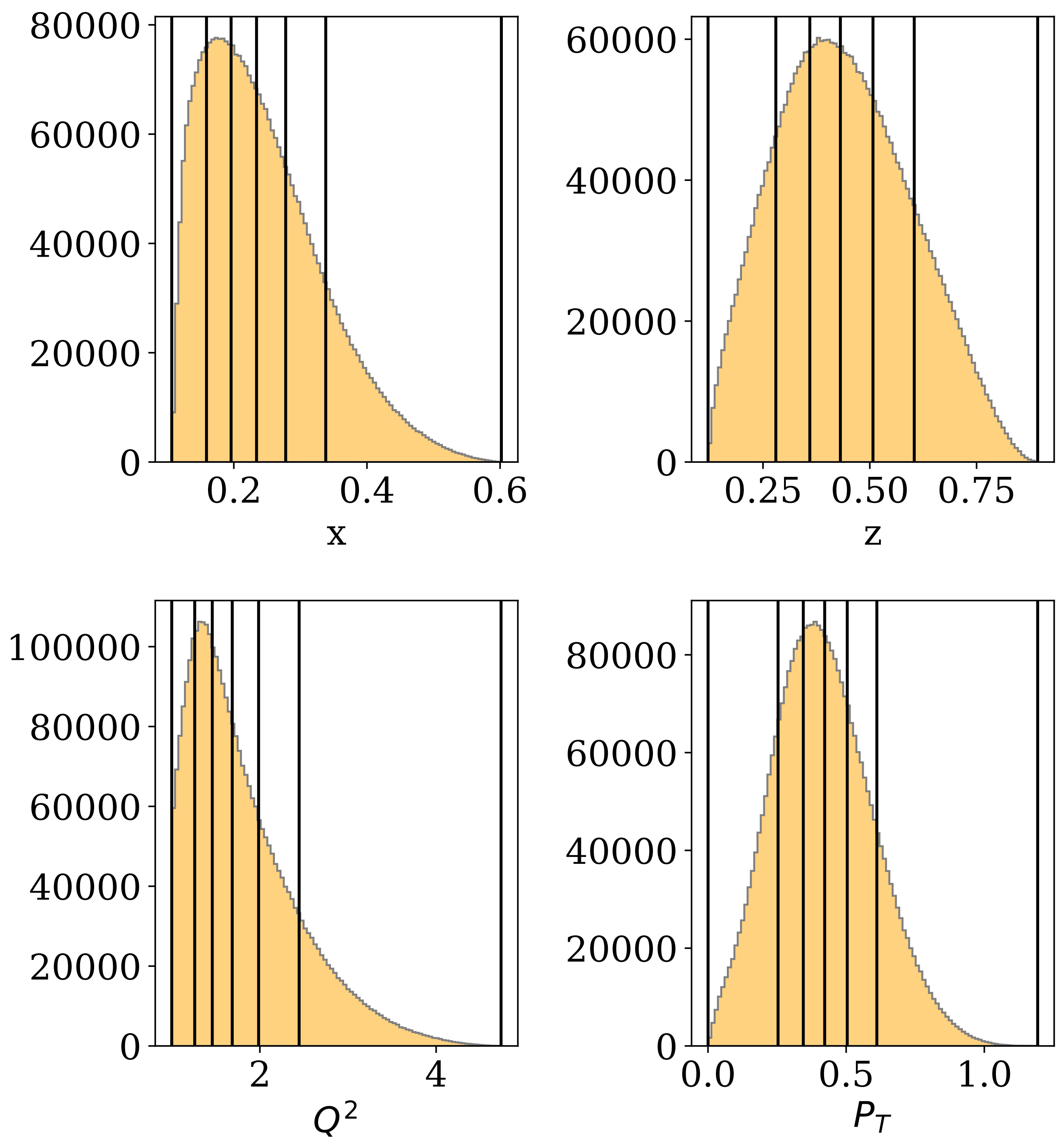
Simple binning for 4 axes of interest
The z-range is restricted in non-z axes (0.25, 0.75).



$$Q^2 > 1,\ W > 2.0$$

$$M_X(ep \to eK^+X) > 1.27$$

12 bins in phi
6 bins in other axes

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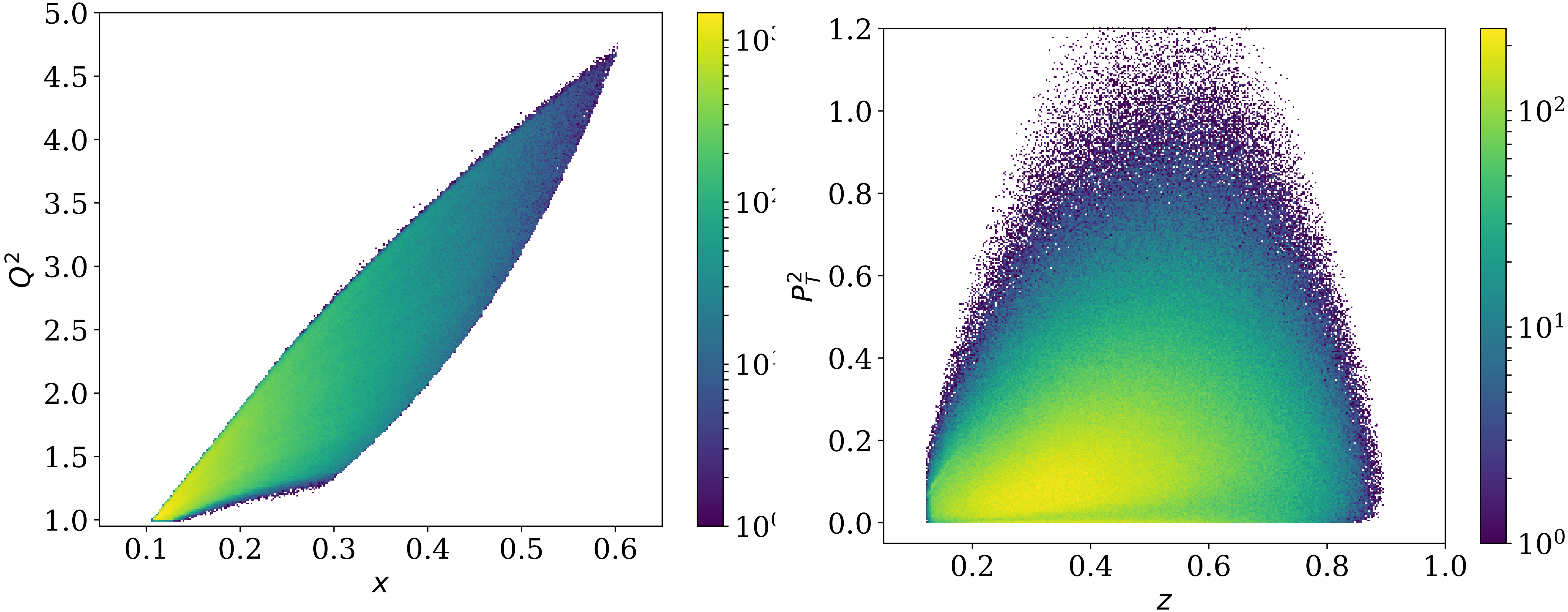
More kinematic distributions



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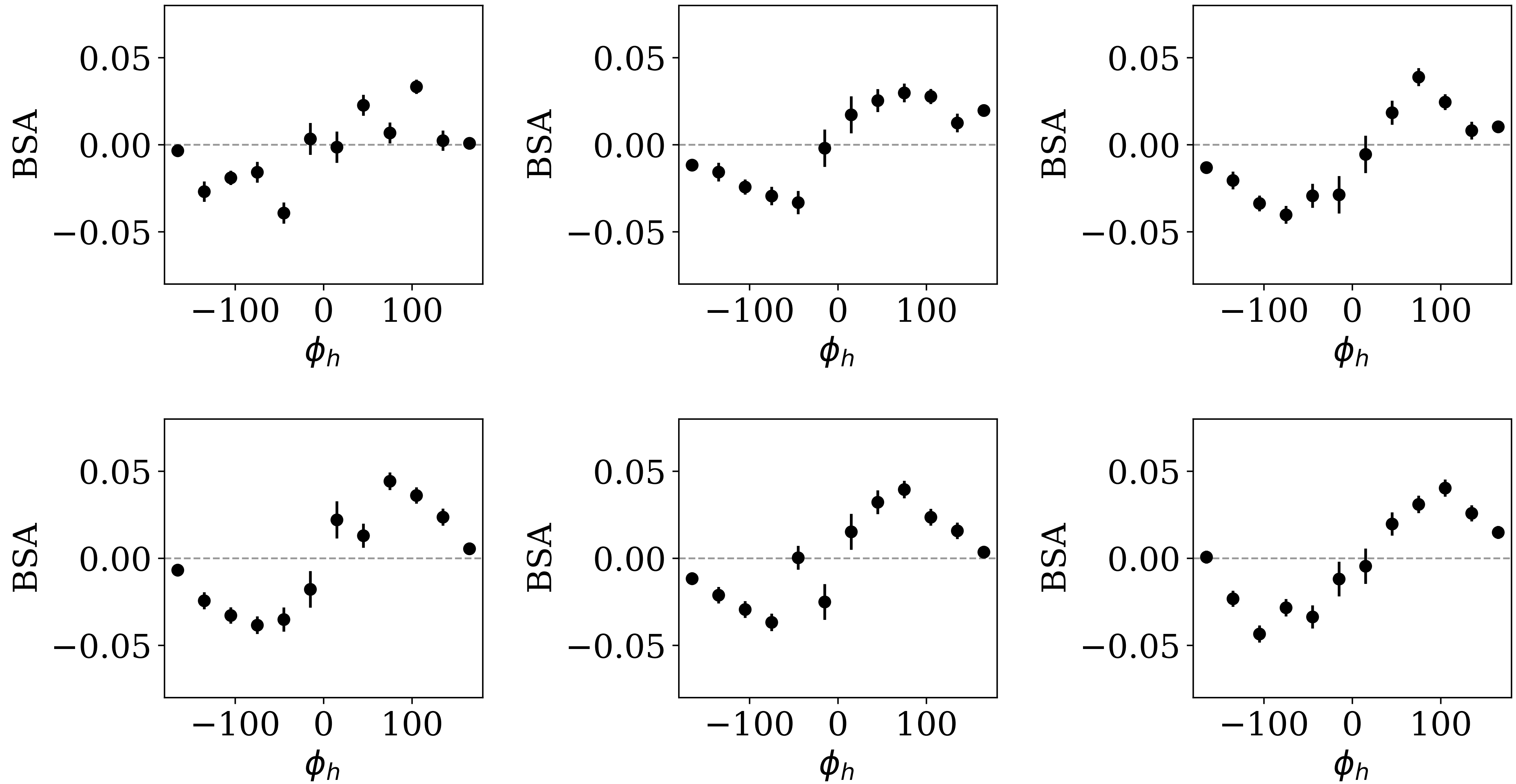
Result of measurement with statistical uncertainties

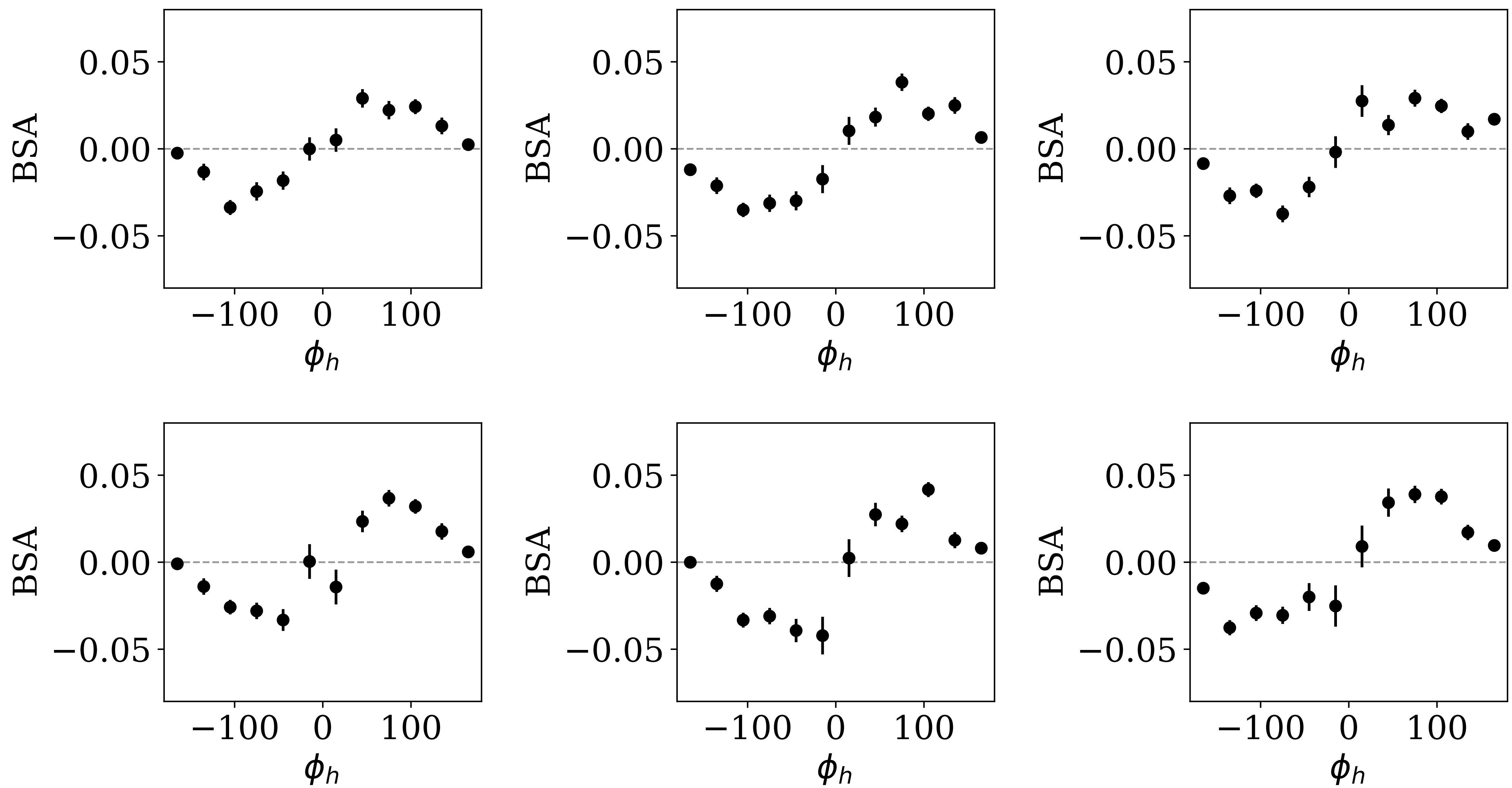
$$A_i = \frac{1}{P_e} \frac{N_i^+ - N_i^-}{N_i^+ + N_i^-} \qquad \sigma_{A_i}^2 = \frac{1 - A_i^2}{N_i^+ + N_i^-}$$



Shown above: Different x bins.

Result of measurement with statistical uncertainties

$$A_i = \frac{1}{P_e}\frac{N_i^+ - N_i^-}{N_i^+ + N_i^-} \qquad \sigma^2_{A_i} = \frac{1 - A_i^2}{N_i^+ + N_i^-}$$



Shown above: Different z bins.

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Systematic uncertainty - Result of uncertainty in estimate of systematic effects arising from selection bias, backgrounds, calibrations etc.

Source	Cut type
Kaon ID SL	Min
CC theta match	Min/Max
DC R1 Fid	Min
DC R3 Fid	Min
Sampling frac.	Min/Max
EC Edep	Min
EC-U	Min/Max
EC-V	Max
EC-W	Max
z-vertex	Min/Max

Source	Size
Beam polarization	3%
Kaon momentum	TBD

In the case that none of the parameters are correlated,

$$\rho_{ij} = \delta_{ij}$$

$$(\delta\mathcal{O})^2 = \sum_{i=1}^{N} (\mathcal{O}(\theta_i + \delta\theta_i/2) - \mathcal{O}(\theta_i - \delta\theta_i/2))^2$$

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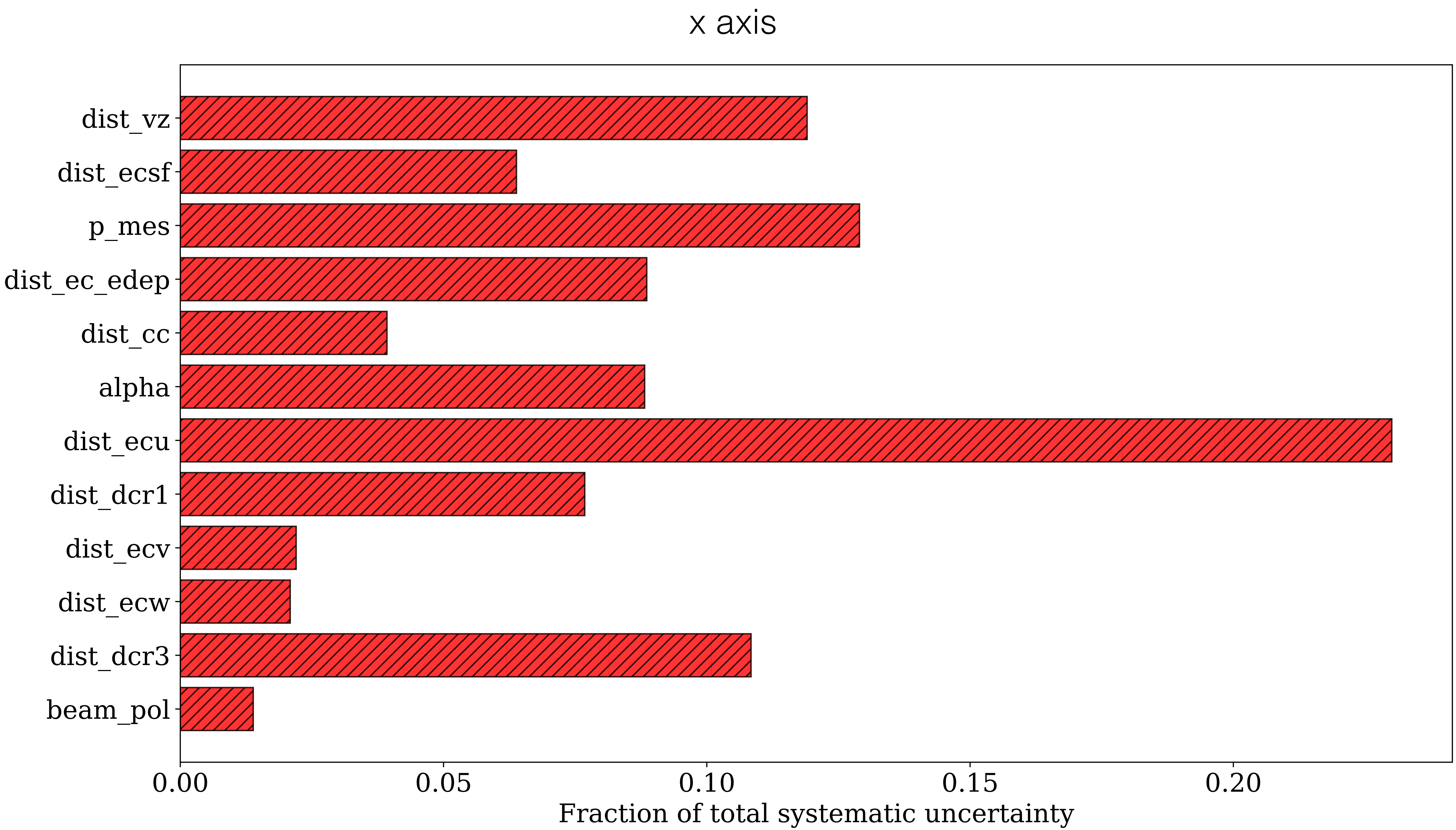

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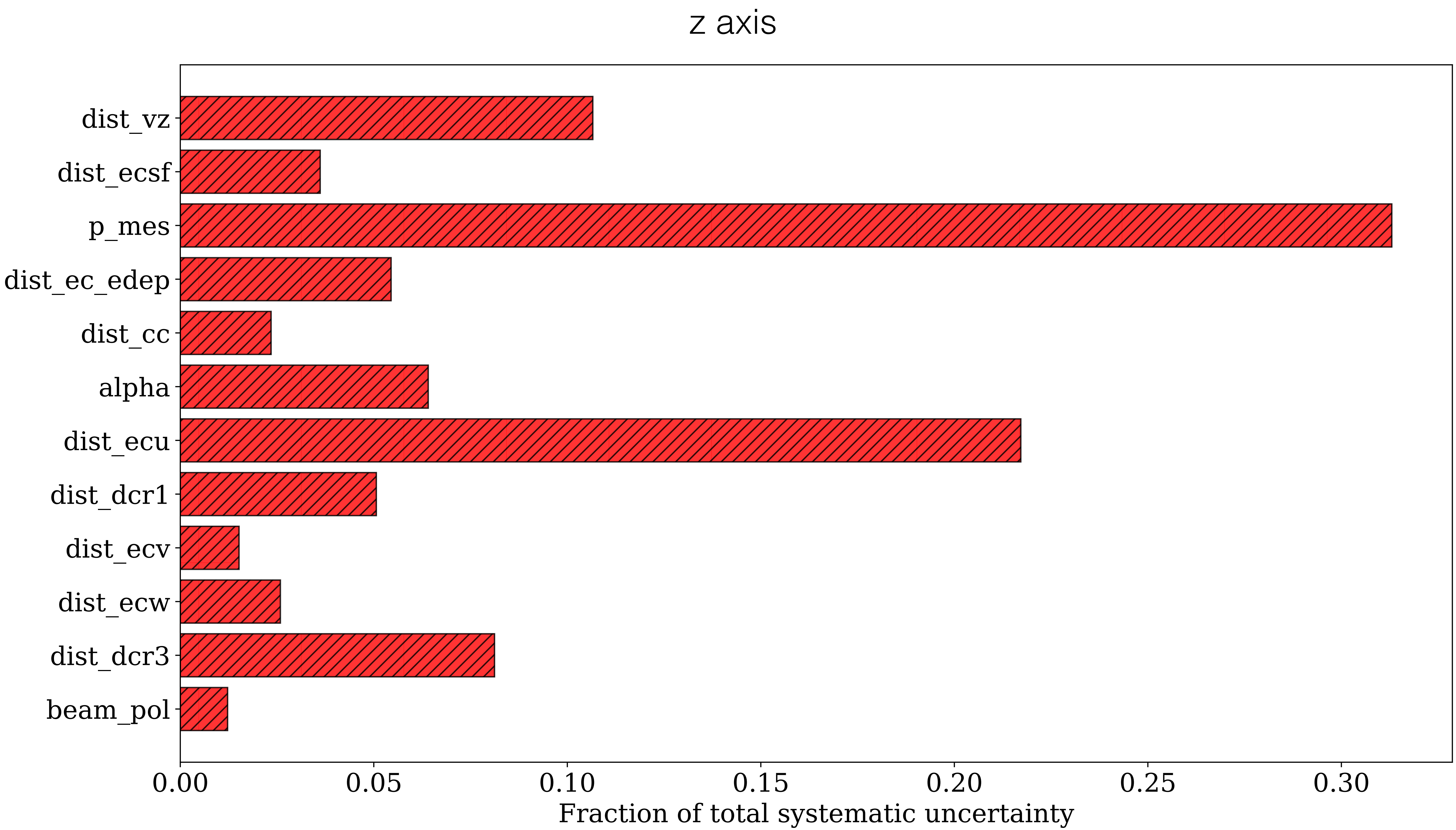

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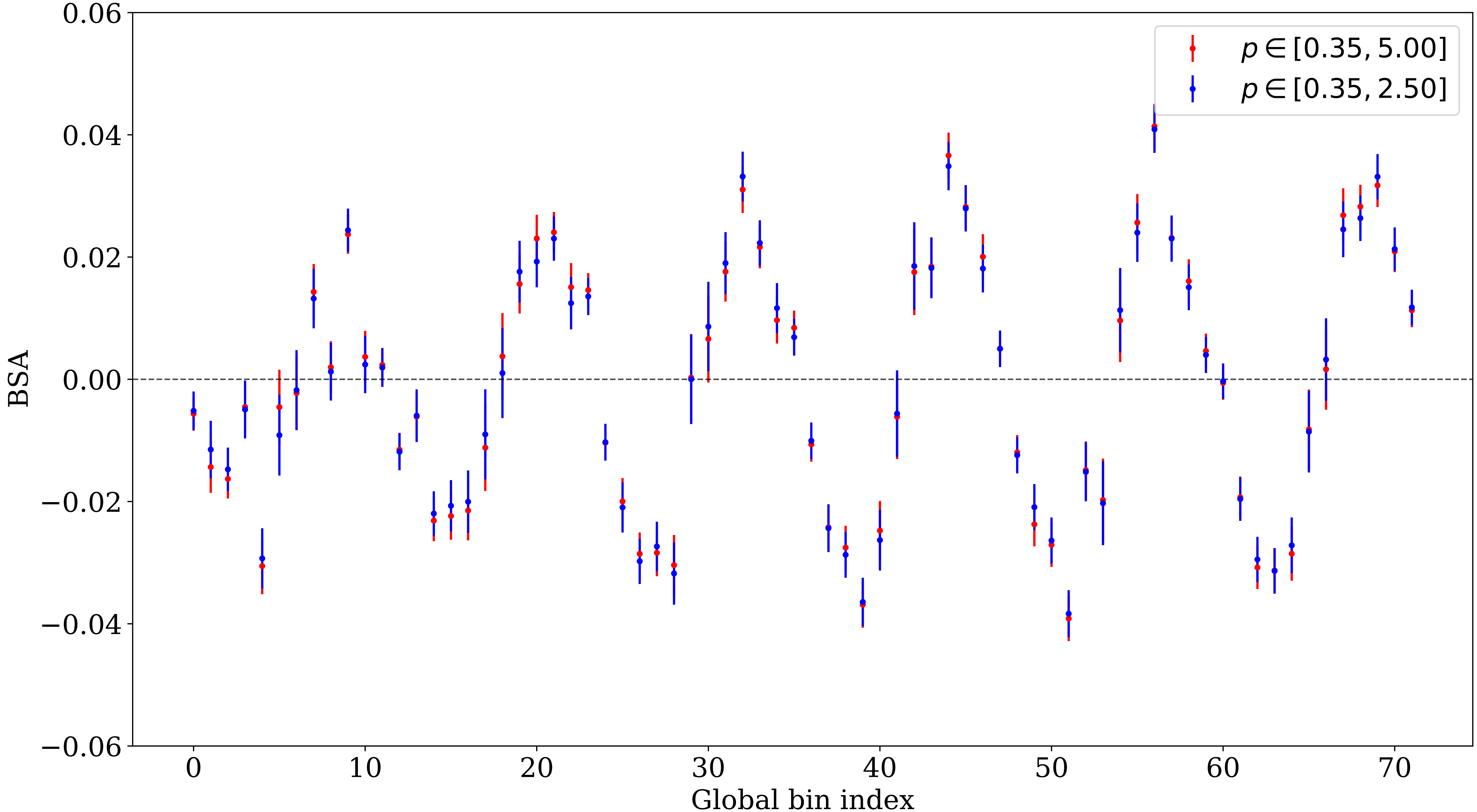
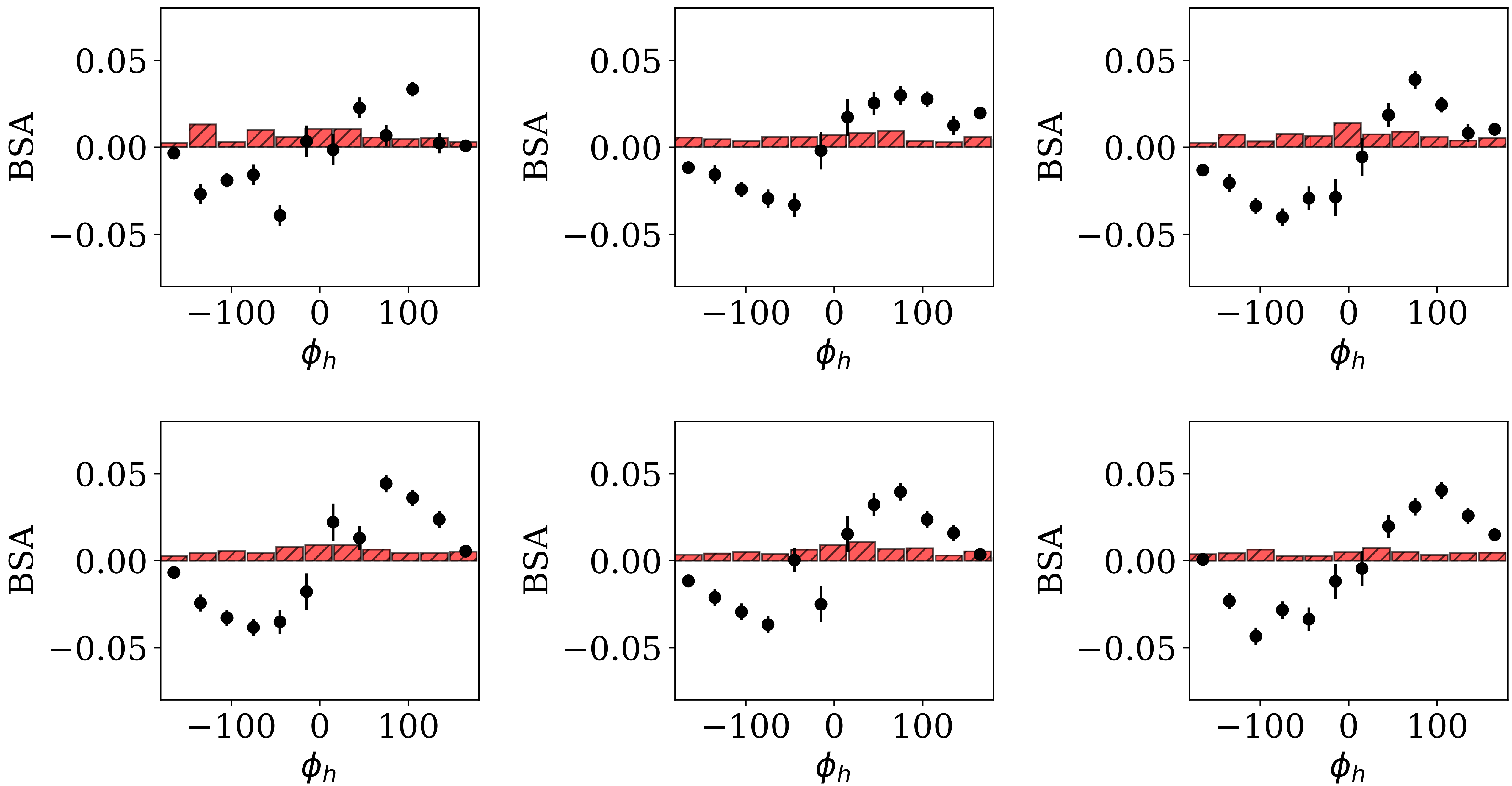
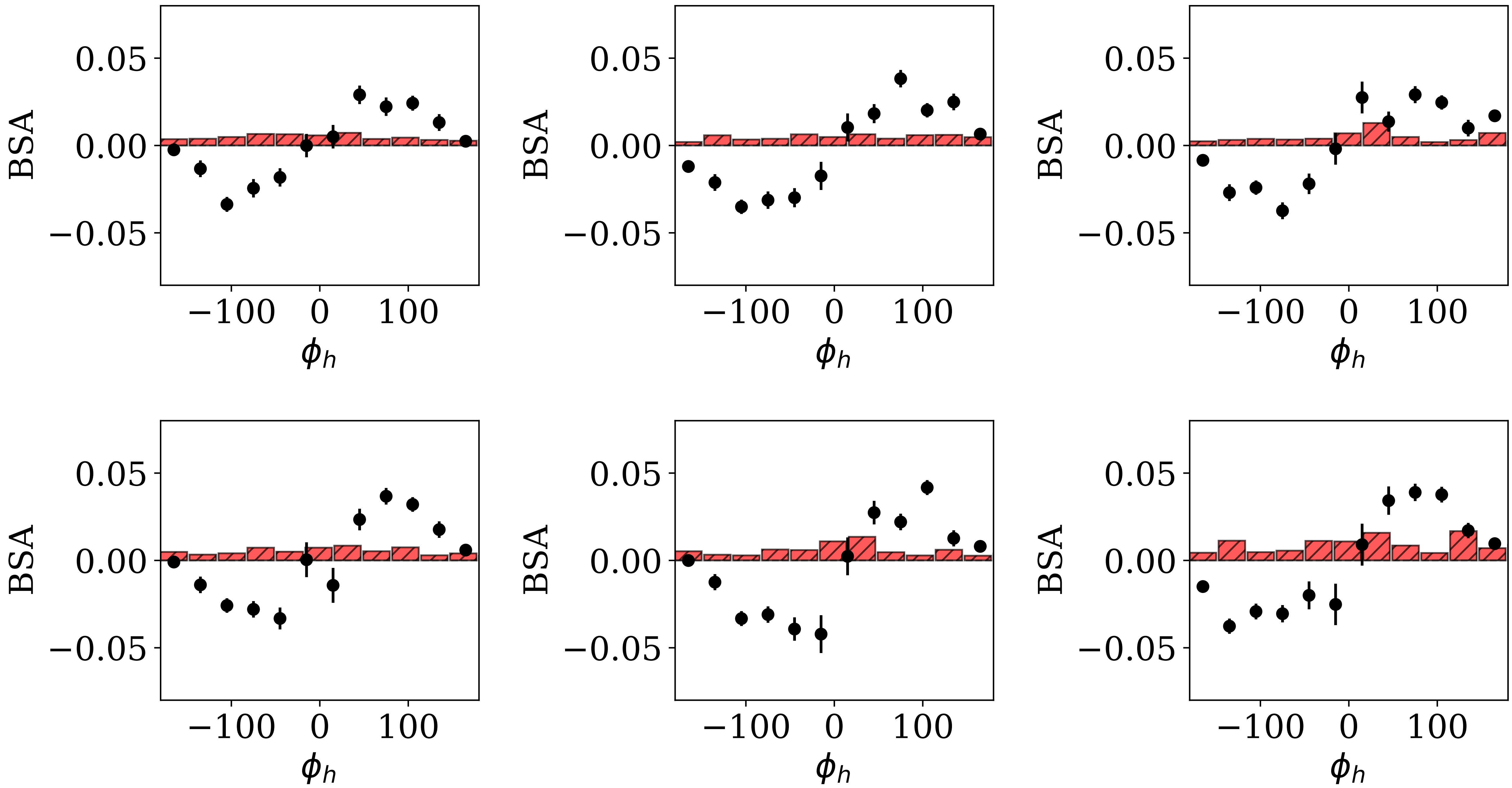
Momentum range of accepted kaon is varied to check impact of contamination at higher momentum. Difference is included as systematic.





Shown above: Different x bins.



Shown above: Different z bins.

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Extraction of Moments

Recall what we are measuring

$$BSA = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \frac{A_{LU}^{\sin\phi}\sin\phi}{1 + A_{UU}^{\cos\phi}\cos\phi + A_{UU}^{\cos(2\phi)}\cos(2\phi)}$$

We minimize the total chi-2 (with respect to the parameters a) for each kinematic bin by gradient descent

$$\chi^2 = \sum_{i=1}^{n_\phi} \frac{(A_{obs}(\phi_i) - A_{pred}(\phi_i, \vec{a}))^2}{\sigma_{stat}^2 + \sigma_{sys}^2}$$

One would be very happy to extract all 3 moments, in some cases it may be possible.

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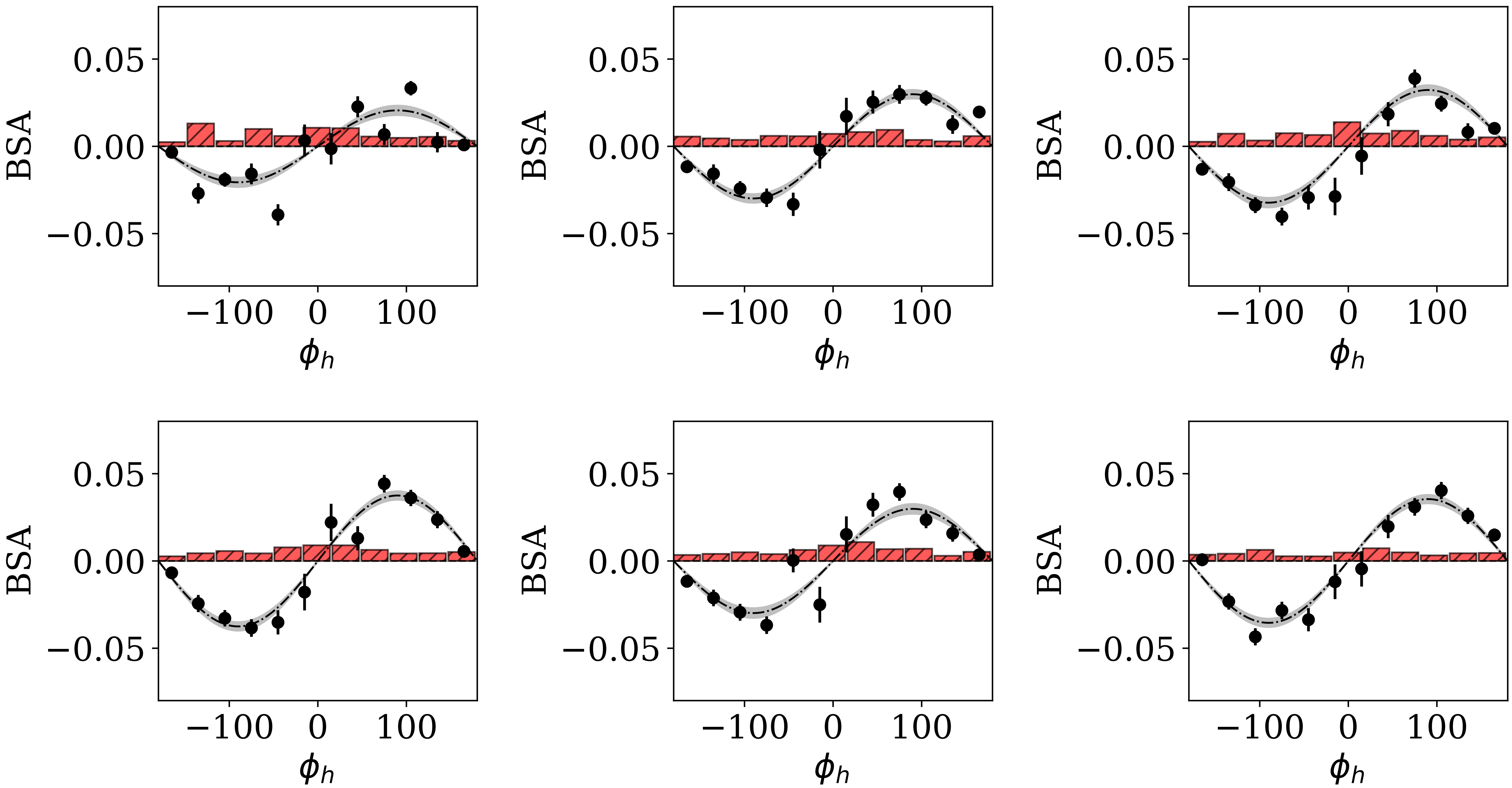
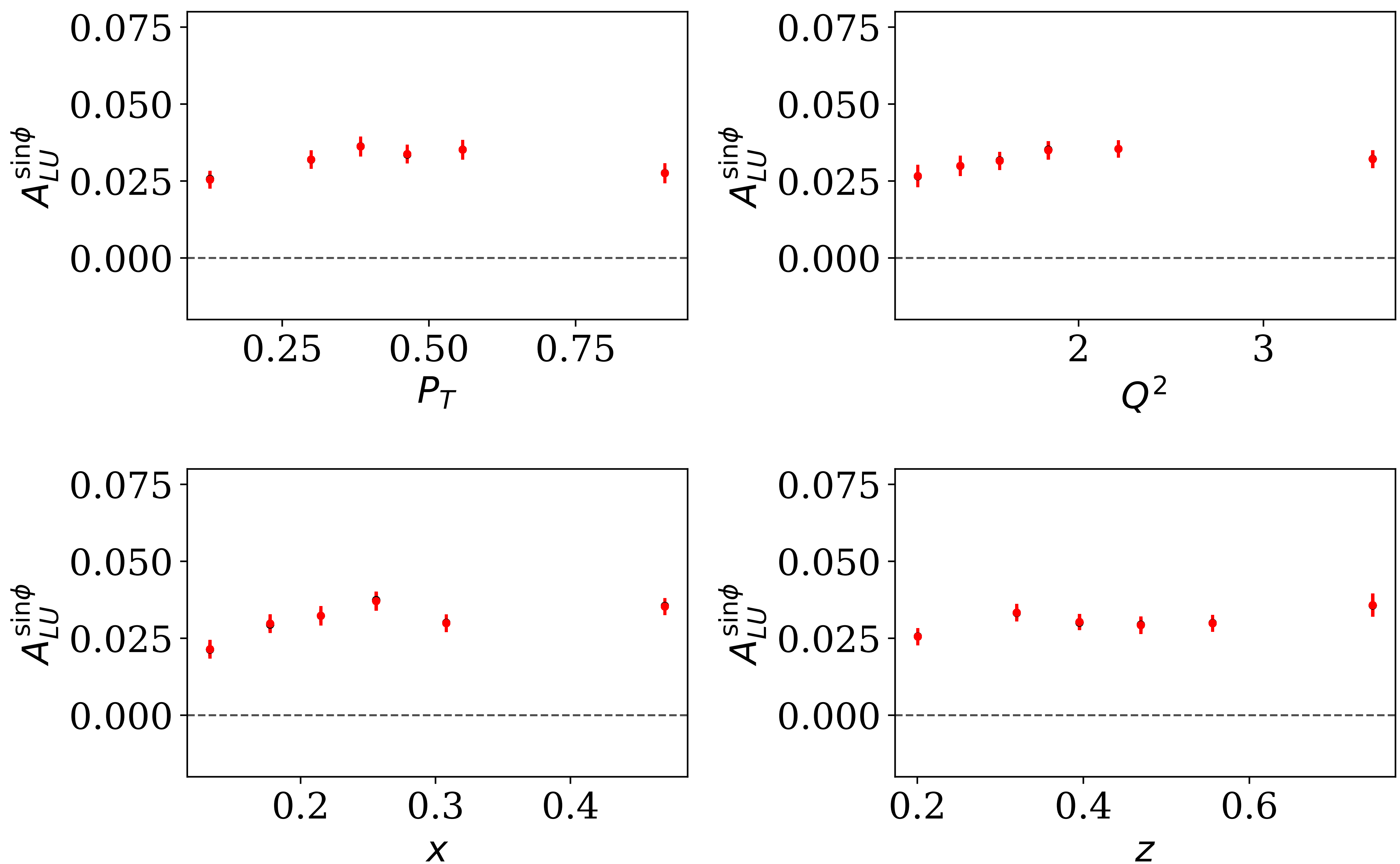

BSA
0.05
0.00
−0.05
−100
0
100
ϕ_h

(Sine, Full)
Results almost identical.



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Points of conclusion

- Measured BSA for 4 kinematic variables on a sample of SIDIS kaons (+), statistical and systematic uncertainty is calculated for each point.
    - Most systematic uncertainty comes from kaon ID at high momentum, but it is still reasonably small.
- An analysis of moments has been performed for each kinematic point.
- Theoretical predictions are in progress for the kinematic dependence of this measurement based on TMD PDF/FF functions

Thank you for your time.

Thanks to the following people for helping with this analysis:

Kyungseon Joo, Harut Avakian, Nick Markov, Andrey Kim, Nobuo Sato,
Nathan Harrison, Kemal Tezgin, Frank Cao, and Brandon Clary

For various parts of the analysis I thank previous workers:
Wes Gohn, Marco Mirazita, Nathan Harrison

Extra Slides

Parameter Estimation

Single fit - Minimize the chi-2 between your data points and the predictions from your model.

$$\chi^2 = \sum_{i=1}^{n_\phi} \frac{(A_{obs}(\phi_i) - A_{pred}(\phi_i, \vec{a}))^2}{\sigma^2_{stat} + \sigma^2_{sys}}$$

This is done using gardient descent. The errors and correlations on/between the fit parameters a are calculated by using the topology of the chi-2 function around the parameter minima. More explicity the inverse Hessian matrix.

$$\sigma_{ij} = H^{-1}_{ij} = C_{ij}$$

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Parameter Estimation (2)

Give a probabilistic interpretation of our measurement and give a probabilistic interpretation to the parameters of our model for the result (using Bayes Theorem).

$$P(\vec{a}|A_i) = \frac{P(A_i|\vec{a})P(\vec{a})}{P(A_i)}$$

Use Markov chain MC (MCMC) to explore the parameter space {a} (Metropolis algorithm).

$$P_{Accept} = \frac{P(A_i|\vec{a}_{n+1})P(\vec{a}_{n+1})}{P(A_i|\vec{a_n})P(\vec{a_n})}$$

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Max likelihood ratio method for charged hadrons

$$\mathcal{L}_h = \prod_{i=1}^{N} P_i(x_i; p, h)$$

$$\frac{\mathcal{L}_h}{\mathcal{L}_\pi + \mathcal{L}_K + \mathcal{L}_p}$$

Simple case of one distribution used

$$P(\beta; p, h) = \frac{1}{\sqrt{2\pi}\sigma_\beta(p, h)} exp\left\{-\frac{1}{2}\left(\frac{\beta - \mu_\beta(p, h)}{\sigma_\beta(p, h)}\right)^2\right\}$$

"Significance", use to tighten/loosen cut on beta, discard low likelihood events (positron example).

$$\alpha = 1 - \int_{\mu-\beta_{obs}}^{\mu+\beta_{obs}} P(\beta; p, h) d\beta$$

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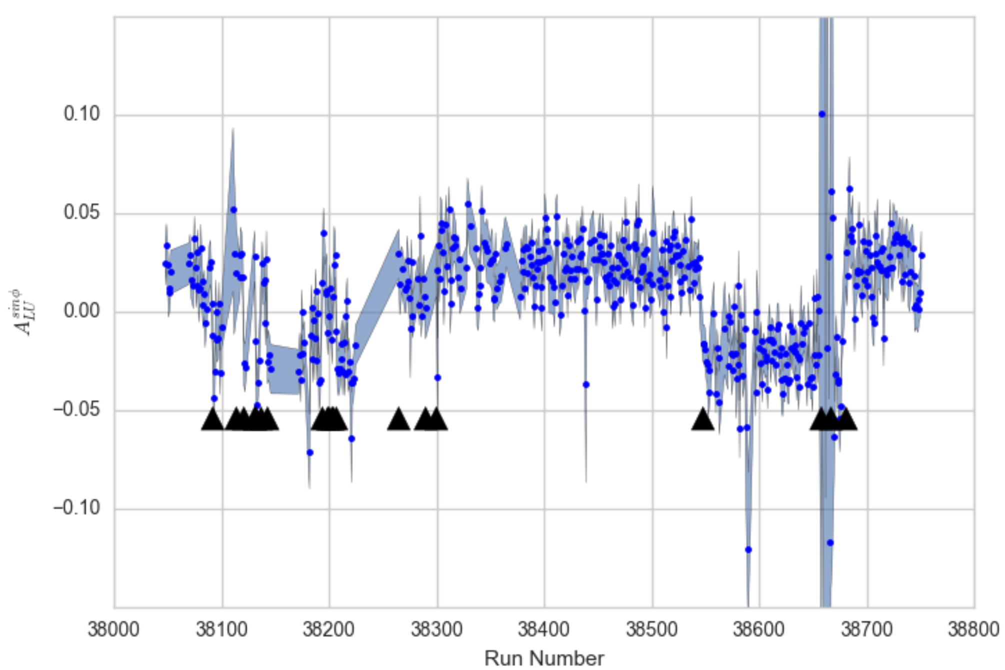
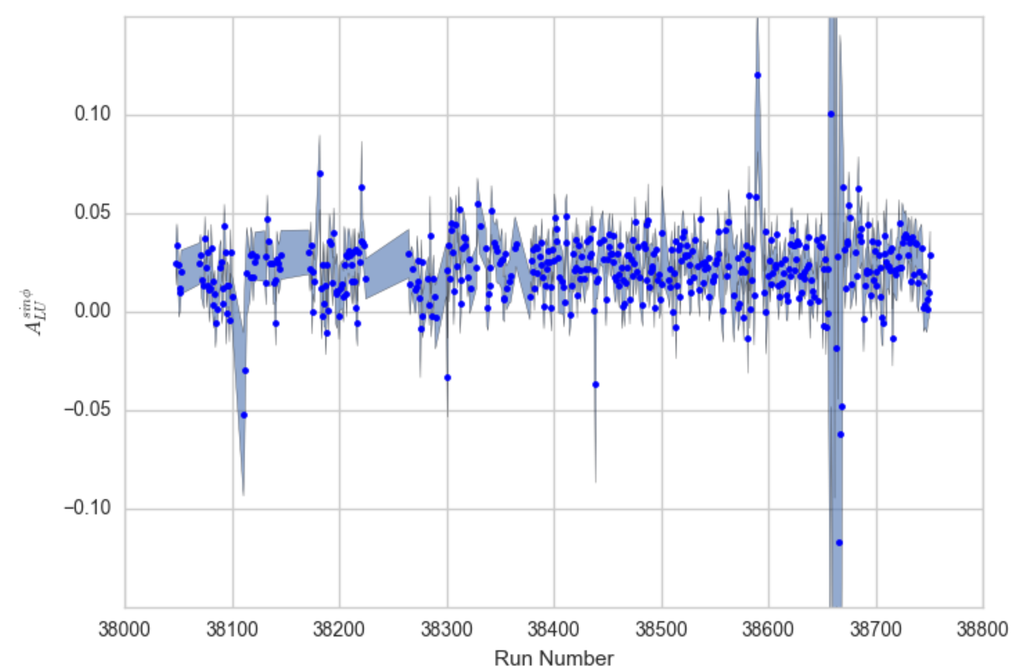
Measurement of the beam spin asymmetry is done experimentally by recording events with different electron helicity states and counting the ratio below. Helicity flipping occurs at high enough frequency that acceptance effects are expected to cancel.

$$A_{LU}^{\sin\phi} = \frac{1}{P_e}\frac{N^{+} - N^{-}}{N^{+} + N^{-}}$$

The **average beam polarization** was determined to be (75 +/- 3) %, and the wave-plate position was determined as a function of run number by analyzing the sine phi moment for positive pions.





Treatment of systematic uncertainties

Analysis depends on some set of parameters, cut values, calibration values, etc.

$$\vec{\theta} = (\theta_i, \theta_2, ..., \theta_N)$$

The values of these parameters impact the outcome of the measurement of your observable. The standard formula for "error propagation" is,

$$(\delta\mathcal{O})^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial\mathcal{O}}{\partial\theta_i}\frac{\partial\mathcal{O}}{\partial\theta_j}\rho_{ij}\delta\theta_i\delta\theta_j$$

However it is not analytically possible to find in most cases the relationship between the parameter and the observable.

$$\frac{\partial(BSA)}{\partial(ECU\ Cut\ Value)} = ?$$

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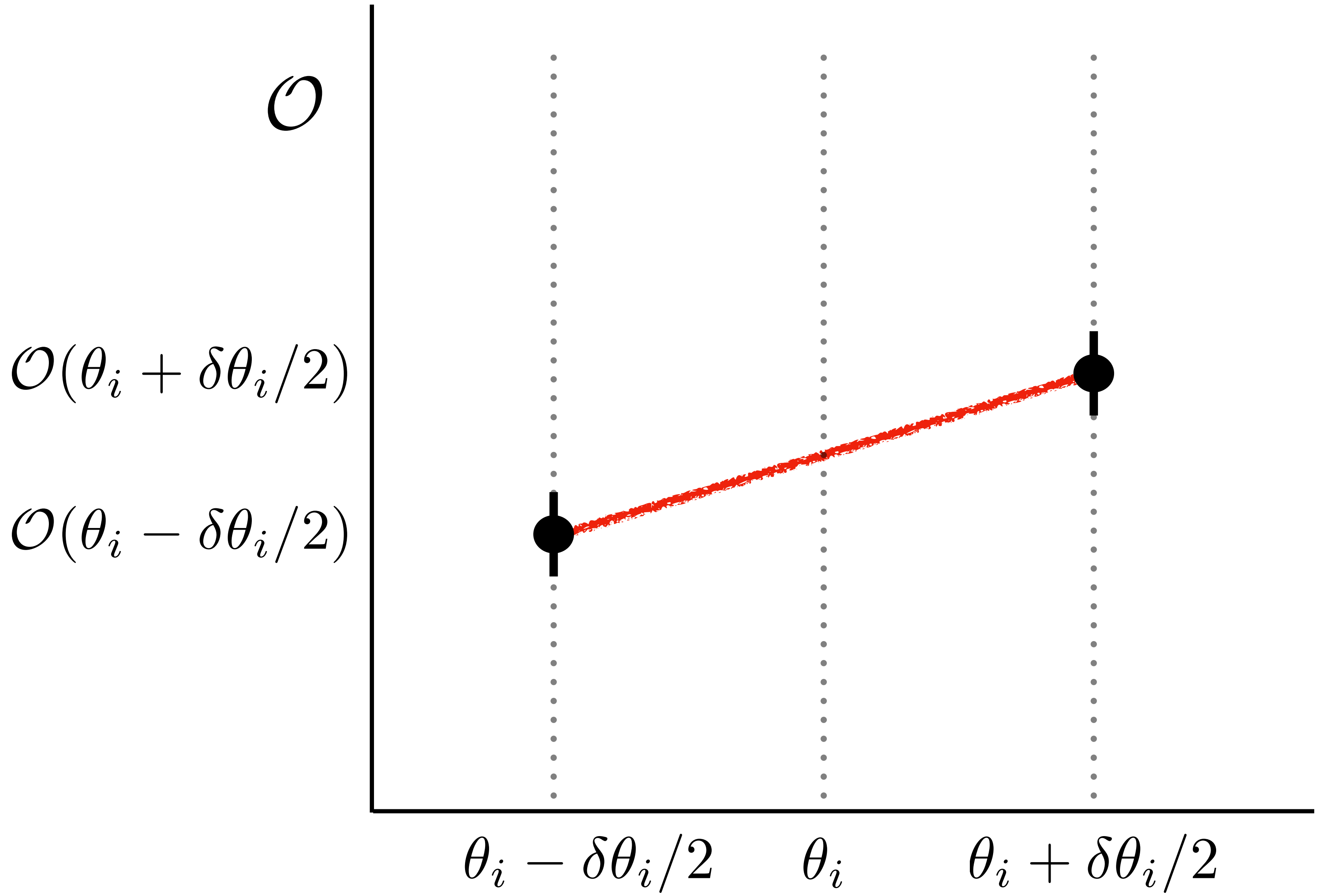
Numerically find the derivative around the nominal parameter value

$$\frac{\partial \mathcal{O}}{\partial \theta_i} \approx \frac{\mathcal{O}(\theta_i + \delta\theta_i/2) - \mathcal{O}(\theta_i - \delta\theta_i/2)}{\delta\theta_i}$$



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$$\frac{\partial \mathcal{O}}{\partial \theta_i}\frac{\partial \mathcal{O}}{\partial \theta_j}\rho_{ij}\delta\theta_i\delta\theta_j \approx (\mathcal{O}(\theta_i + \delta\theta_i/2) - \mathcal{O}(\theta_i - \delta\theta_i/2))$$
$$\times(\mathcal{O}(\theta_j + \delta\theta_j/2) - \mathcal{O}(\theta_j - \delta\theta_j/2))\rho_{ij}$$

In the case that none of the parameters are correlated, $\rho_{ij} = \delta_{ij}$

$$(\delta\mathcal{O})^2 = \sum_{i=1}^{N}(\mathcal{O}(\theta_i + \delta\theta_i/2) - \mathcal{O}(\theta_i - \delta\theta_i/2))^2$$

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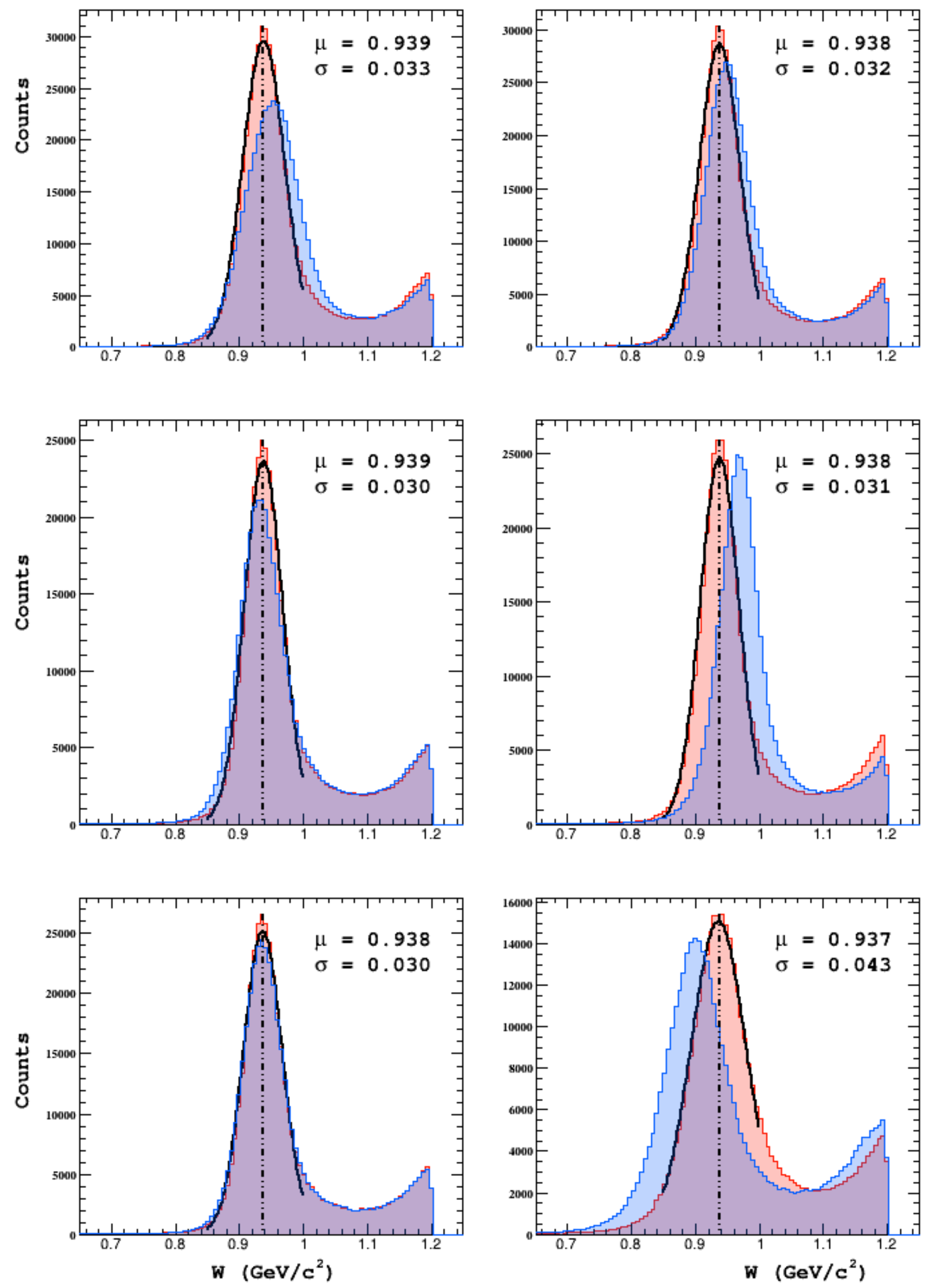
Electron Momentum Corrections Before and After



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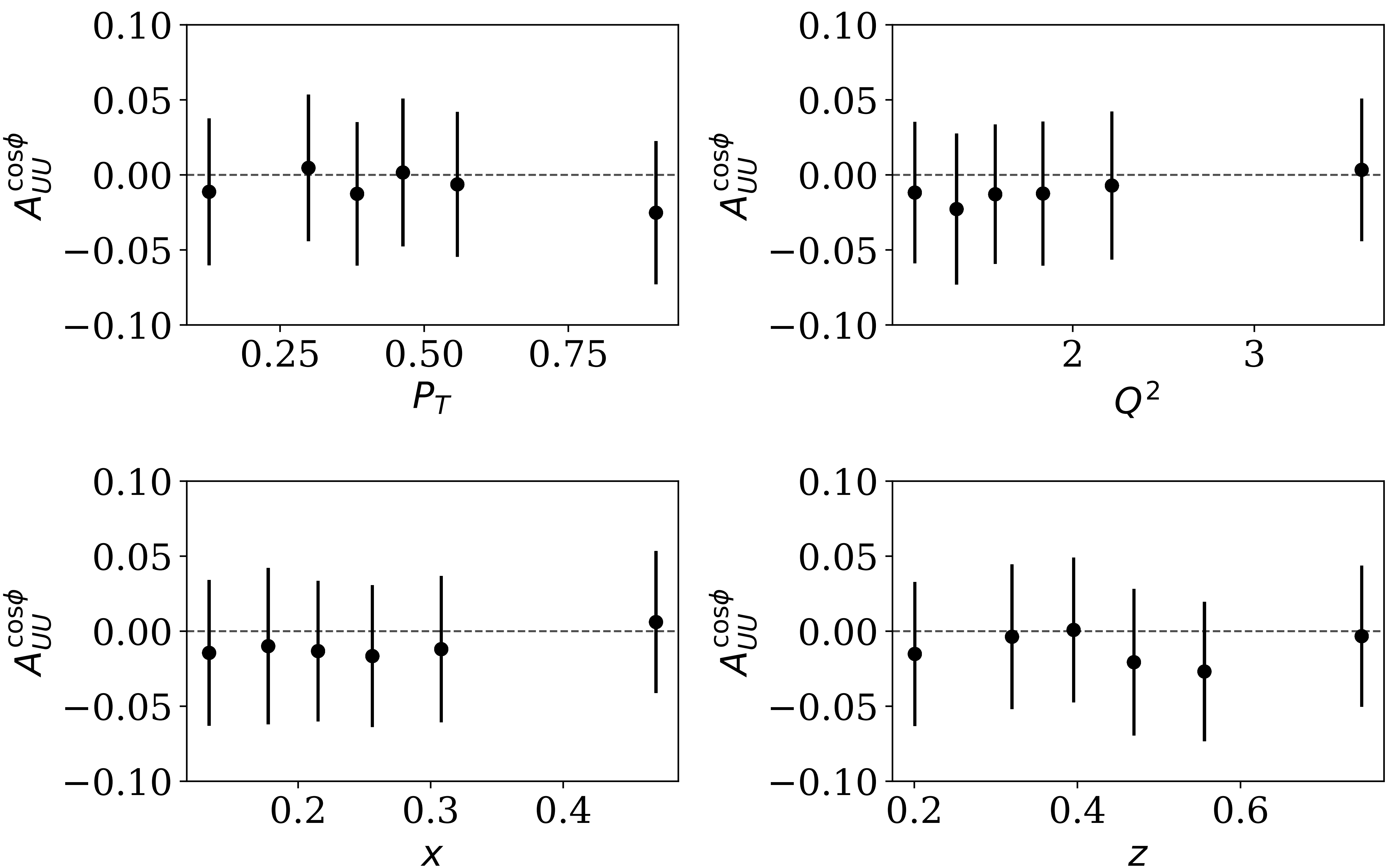
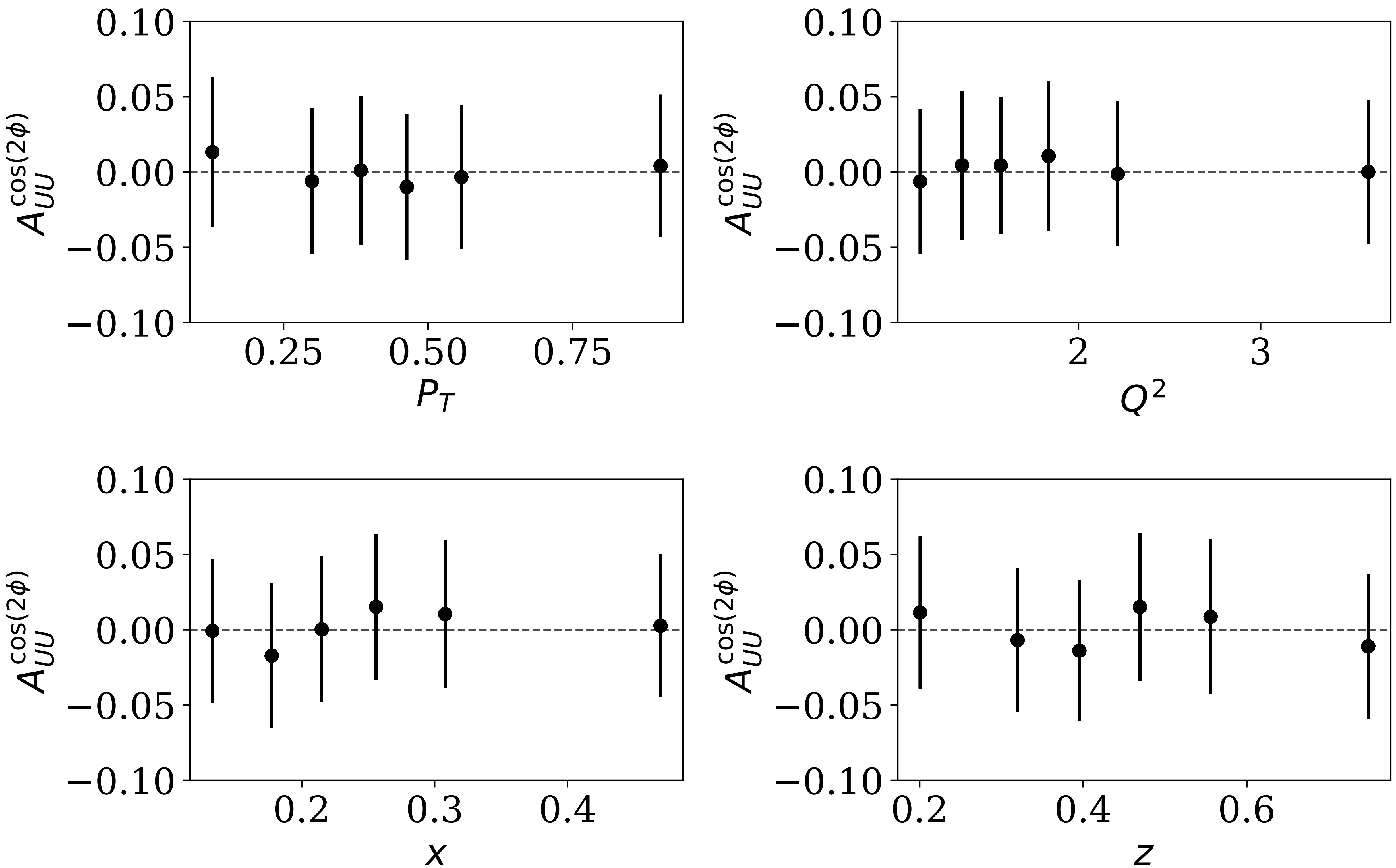
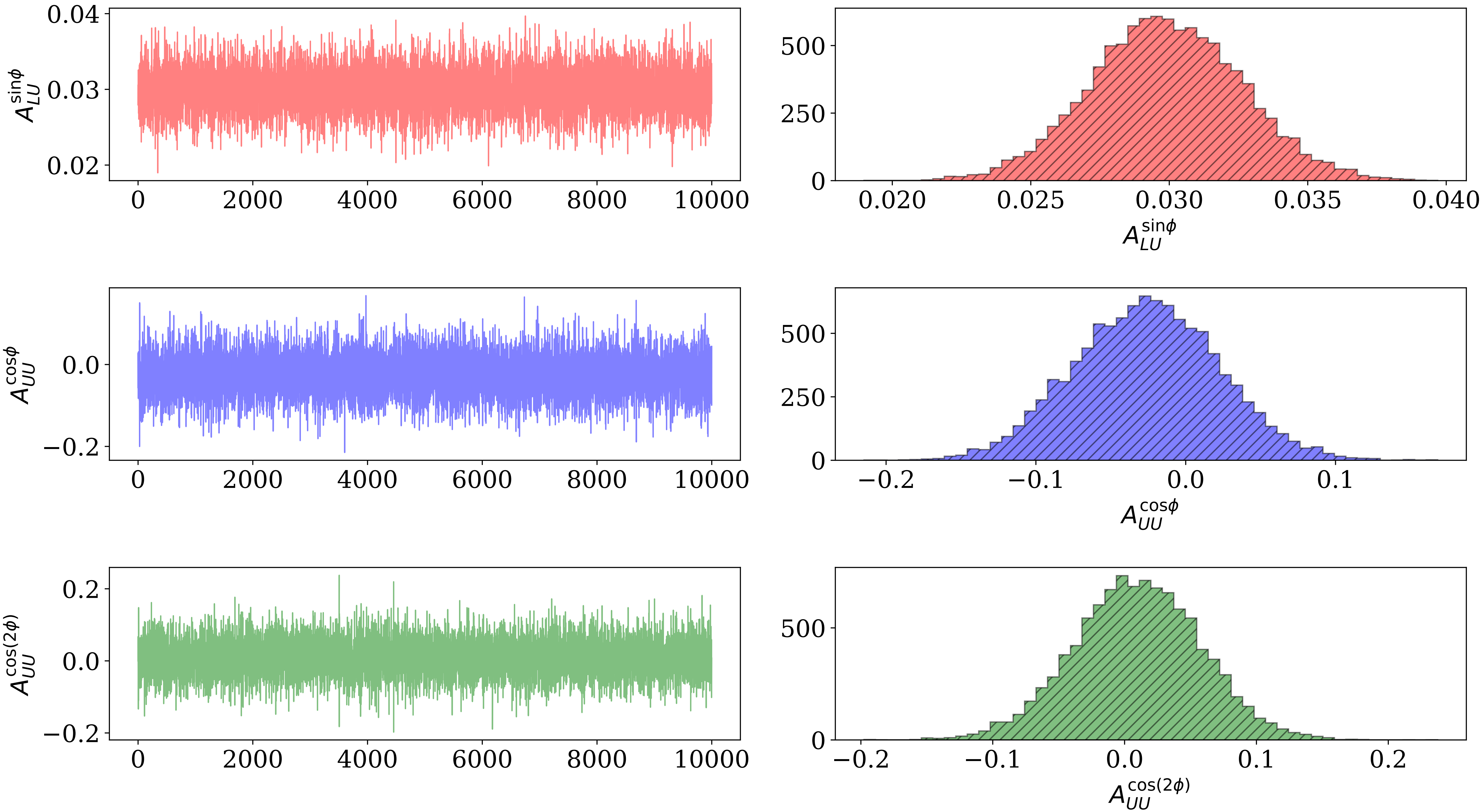

0.10
0.05
0.00
−0.05
−0.10
$A_{UU}^{\cos\phi}$
0.25
0.50
0.75
P_T
2
3
Q^2
0.2
0.3
0.4
x
0.6
z


0.10
0.05
0.00
−0.05
−0.10
$A_{UU}^{\cos(2\phi)}$
0.25
0.50
0.75
P_T
2
3
Q^2
0.2
0.3
0.4
x
0.2
0.4
0.6
z


0.04
0.03
0.02
$A_{LU}^{\sin\phi}$
0
2000
4000
6000
8000
10000
500
250
0
0.020
0.025
0.030
0.035
0.040
$A_{LU}^{\sin\phi}$
0.0
−0.2
$A_{UU}^{\cos\phi}$
−0.2
−0.1
0.0
0.1
$A_{UU}^{\cos\phi}$
0.2
0.0
−0.2
$A_{UU}^{\cos(2\phi)}$
0.2
0.1
−0.1
$A_{UU}^{\cos(2\phi)}$