Using Kinematic Fitting in CLAS EG6: Beam-Spin Asymmetry of Exclusive Nuclear DVMP

Frank Thanh Cao

Advisor: K. Joo
Co-Advisor: K. Hafidi

University of Connecticut

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Motivation

After particle identification, we are left with a set of particles and we want to know whether they are part of the same process of interest. Usually, we rely on forming exclusivity variables from the measured 4-momenta of the positively identified particles.

We must confront the fact that 4-vectors coming from detectors are not perfect and it may be possible to do better.

This presentation will outline kinematic fitting as an answer to this and some surprising results when applying it to the relatively rare process of DVπ^0P off ${}^4\mathrm{He}$ in CLAS EG6.

Outline

Kinematic Fitting in a Nutshell

Kinematic fitting takes measure values and allows them to move within the measured values' errors and are directed by a set of constraints.

This is perfectly applicable for taking a set of measured 3-momenta and allowing each to move simultaneously, within detector resolutions, to satisfy energy and momentum conservation.

Formalism

Let $\vec{\boldsymbol{\eta}}$ be a vector of n-measured variables. Then the true vector of the n-variables, $\vec{\boldsymbol{y}}$, will be displaced by n-variables, $\vec{\varepsilon}$. They are related simply by:

$$\vec{\boldsymbol{y}} = \vec{\boldsymbol{\eta}} + \vec{\varepsilon}$$

If there are, say m, unmeasured variables too, then they can be put in a vector, $\vec{\boldsymbol{x}}$.

The two vectors, $\vec{\boldsymbol{x}}$ and $\vec{\boldsymbol{y}}$, are then related by r constraint equations, indexed by k:

$$f_k\left(\vec{\boldsymbol{x}}, \vec{\boldsymbol{y}}\right) = 0$$

Suppose $\vec{\boldsymbol{x}}^0$ and $\vec{\boldsymbol{y}}^0$ are our best guess (measurements) of the vectors $\vec{\boldsymbol{x}}$ and $\vec{\boldsymbol{y}}$, respectively. Then Taylor expanding to first order each $f_k(\vec{\boldsymbol{x}}, \vec{\boldsymbol{y}})$ about $\vec{\boldsymbol{x_0}}$ and $\vec{\boldsymbol{y_0}}$ gives:

$$\begin{aligned} f_k(\vec{\boldsymbol{x}}, \vec{\boldsymbol{y}}) \approx f_k\left(\vec{\boldsymbol{x}}^0, \vec{\boldsymbol{y}}^0\right) &+ \sum_{i=0}^{m} \left(\frac{\partial f_k}{\partial x_i}\right)\Bigg|_{\left(\vec{\boldsymbol{x}}^{\mathbf{0}}, \vec{\boldsymbol{y}}^{\mathbf{0}}\right)} \left(\vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}^0\right)_i \\ &+ \sum_{j=0}^{n} \left(\frac{\partial f_k}{\partial y_j}\right)\Bigg|_{(\vec{\boldsymbol{x}}^0, \vec{\boldsymbol{y}}^0)} \left(\vec{\boldsymbol{y}} - \vec{\boldsymbol{y}}^0\right)_j \end{aligned} \tag{1}$$

where $\left(\vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}^0\right)_i$ and $\left(\vec{\boldsymbol{y}} - \vec{\boldsymbol{y}}^0\right)_j$ denote the i-th and j-th components of vector differences, respectively.

For convenience, let's introduce

$$
\begin{aligned}
A_{ij} &:= \left(\frac{\partial f_i}{\partial x_j}\right)\Bigg|_{(\vec{\boldsymbol{x}}^0,\vec{\boldsymbol{y}}^0)} \\
B_{ij} &:= \left(\frac{\partial f_i}{\partial y_j}\right)\Bigg|_{(\vec{\boldsymbol{x}}^0,\vec{\boldsymbol{y}}^0)} \\
c_i &:= f_i\left(\vec{\boldsymbol{x}}^0,\vec{\boldsymbol{y}}^0\right)
\end{aligned}
\qquad , \tag{2}
$$

and

$$
\begin{aligned}
\vec{\boldsymbol{\xi}} &:= \vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}^0 \\
\vec{\boldsymbol{\delta}} &:= \vec{\boldsymbol{y}} - \vec{\boldsymbol{y}}^0
\end{aligned}
\qquad .
$$

Then, since $f_k(\vec{\boldsymbol{x}}, \vec{\boldsymbol{y}}) \equiv 0 \ \ \forall k$, **Eq. 1** can be written in matrix form as:

$$\vec{\mathbf{0}} \equiv A\vec{\boldsymbol{\xi}} + B\vec{\boldsymbol{\delta}} + \vec{\boldsymbol{c}} \tag{3}$$

where A and B are $(r \times n)$ and $(r \times m)$ matrices with components a_{ij} and b_{ij}, respectively, as defined by **Eqn.'s 2**.

Kinematic fitting can be done iteratively to get the best* value of $\vec{\boldsymbol{y}}$ and $\vec{\boldsymbol{x}}$ as possible.
Let ν be the index that denotes the ν-th iteration. Then, we have

$$\vec{\boldsymbol{\xi}} \to \vec{\boldsymbol{\xi}}^{\nu} = \vec{\boldsymbol{x}}^{\nu} - \vec{\boldsymbol{x}}^{\nu-1}$$
$$\vec{\boldsymbol{\delta}} \to \vec{\boldsymbol{\delta}}^{\nu} = \vec{\boldsymbol{x}}^{\nu} - \vec{\boldsymbol{x}}^{\nu-1}$$

and

$$A \to A^{\nu}$$
$$B \to B^{\nu}$$
$$\vec{\boldsymbol{c}} \to \vec{\boldsymbol{c}}^{\nu}$$

Finally, we introduce the overall difference:

$$\vec{\boldsymbol{\epsilon}}^{\nu} := \vec{\boldsymbol{y}}^{\nu} - \vec{\boldsymbol{y}}^{0} \tag{4}$$

*We can quantify best by introducing and minimizing χ^2.

Constructing χ^2

If we have a really good understanding of the correlations between our initial measured values, in $\vec{\boldsymbol{\eta}} \equiv \vec{\boldsymbol{y}}^0$, then we can construct a covariance matrix, C_η:

$$C_\eta = \vec{\boldsymbol{\sigma}}_\eta{}^T \rho_\eta \vec{\boldsymbol{\sigma}}_\eta$$

where $\vec{\boldsymbol{\sigma}}_\eta$ is a vector of the resolution errors of η and ρ_η is a symmetric correlation matrix whose components, $\rho_{ij} \in [-1, 1]$, house pairwise correlations coefficients, between η_i and η_j $(\Rightarrow \rho_{ii} = 1)$.

Consider χ^2, generalized to include correlations between measurements, to be:

$$\left(\chi^2\right)^\nu = \left(\vec{\boldsymbol{\epsilon}}^\nu\right)^T C_\eta^{-1} \vec{\boldsymbol{\epsilon}}^\nu \tag{5}$$

Then, if there are no correlations, ρ_η is the unit matrix and so the covariance matrix is just a diagonal matrix of the variances of η. In this case, the χ^2 becomes the recognizable:

$$\left(\chi^2\right)^\nu = \sum_{i=0}^{m} \frac{\left(y_i^\nu - y_i^0\right)^2}{\left(\sigma_\eta\right)_i^2} = \sum_{i=0}^{m} \frac{\left(\epsilon_i^\nu\right)^2}{\left(\sigma_\eta\right)_i^2}$$

Now that we have a χ^2 to minimize, we can introduce a Lagrangian, $\mathcal{L}$, with Lagrange multipliers $\vec{\boldsymbol{\mu}}$ such that:

$$\mathcal{L} = \left(\vec{\boldsymbol{\epsilon}}^{\nu}\right)^T C_\eta^{-1} \vec{\boldsymbol{\epsilon}}^{\nu} + 2\left(\vec{\boldsymbol{\mu}}^{\nu}\right)^T \left(A^\nu \vec{\boldsymbol{\xi}}^{\nu} + B^\nu \vec{\boldsymbol{\delta}}^{\nu} + \vec{\boldsymbol{c}}^{\nu}\right) \tag{6}$$

is to be minimized.

Minimization conditions are then:

$$\vec{\mathbf{0}} \equiv \frac{1}{2}\frac{\partial \mathcal{L}}{\partial \vec{\boldsymbol{\delta}}^{\nu}} = C_\eta^{-1} \vec{\boldsymbol{\epsilon}}^{\nu} + \left(B^\nu\right)^T \vec{\boldsymbol{\mu}}^{\nu} \tag{7}$$

$$\vec{\mathbf{0}} \equiv \frac{1}{2}\frac{\partial \mathcal{L}}{\partial \vec{\boldsymbol{\mu}}^{\nu}} = A^\nu \vec{\boldsymbol{\xi}}^{\nu} + B^\nu \vec{\boldsymbol{\delta}}^{\nu} + \vec{\boldsymbol{c}}^{\nu} \tag{8}$$

$$\vec{\mathbf{0}} \equiv \frac{1}{2}\frac{\partial \mathcal{L}}{\partial \vec{\boldsymbol{\xi}}^{\nu}} = \left(A^\nu\right)^T \vec{\boldsymbol{\mu}}^{\nu} \quad . \tag{9}$$

Solving for such $\vec{\boldsymbol{\xi}}^{\nu}, \vec{\boldsymbol{\mu}}^{\nu}, \vec{\boldsymbol{\delta}}^{\nu}$ that satisfy these conditions result in:

$$\begin{aligned}
\vec{\boldsymbol{\xi}}^{\nu} &= -C_x^{\nu} \left(A^{\nu}\right)^T C_B^{\nu} \vec{\boldsymbol{r}}^{\nu} \\
\vec{\boldsymbol{\mu}}^{\nu} &= C_B^{\nu} \left(A^{\nu} \vec{\boldsymbol{\xi}}^{\nu} + \vec{\boldsymbol{r}}^{\nu}\right) \\
\vec{\boldsymbol{\delta}}^{\nu} &= -C_{\eta} \left(B^{\nu}\right)^T \vec{\boldsymbol{\mu}}^{\nu} - \vec{\boldsymbol{\epsilon}}^{\nu-1}
\end{aligned} \quad . \tag{10}$$

where C_B^{ν} is conveniently defined as

$$\begin{aligned}
C_B^{\nu} &:= \left[B^{\nu} C_{\eta} \left(B^{\nu}\right)^T\right]^{-1} \\
C_x^{\nu} &:= \left[\left(A^{\nu}\right)^T C_B^{\nu} A^{\nu}\right]^{-1} \\
\vec{\boldsymbol{r}}^{\nu} &:= \vec{\boldsymbol{c}}^{\nu} - B^{\nu} \vec{\boldsymbol{\epsilon}}^{\nu-1}
\end{aligned}$$

With these new incremental vectors that satisfies the minimization condition, we can finally form our new fitted vectors $\vec{\boldsymbol{x}}^{\nu}$ and $\vec{\boldsymbol{y}}^{\nu}$:

$$\begin{aligned} \vec{\boldsymbol{x}}^{\nu} &= \vec{\boldsymbol{x}}^{\nu-1} + \vec{\boldsymbol{\xi}}^{\nu} \\ \vec{\boldsymbol{y}}^{\nu} &= \vec{\boldsymbol{y}}^{\nu-1} + \vec{\boldsymbol{\delta}}^{\nu} \end{aligned} \tag{11}$$

with new covariance matrices:

$$\begin{aligned} C_x &= \left(\frac{\partial \vec{\boldsymbol{x}}}{\partial \vec{\boldsymbol{\eta}}}\right) C_\eta \left(\frac{\partial \vec{\boldsymbol{x}}}{\partial \vec{\boldsymbol{\eta}}}\right)^T \\ &= \left(A^T C_B A\right)^{-1} \\ C_y &= \left(\frac{\partial \vec{\boldsymbol{y}}}{\partial \vec{\boldsymbol{\eta}}}\right) C_\eta \left(\frac{\partial \vec{\boldsymbol{y}}}{\partial \vec{\boldsymbol{\eta}}}\right)^T \\ &= C_\eta - C_\eta \left(B^T C_B B\right) C_\eta + C_\eta \left(B^T C_B \left[A C_x A^T\right] C_B B\right) C_\eta \end{aligned}$$

.

Quality of Fit

To check on the quality of the fit, we look to two sets of distributions: The **Confidence levels** and the **Pull distributions**.

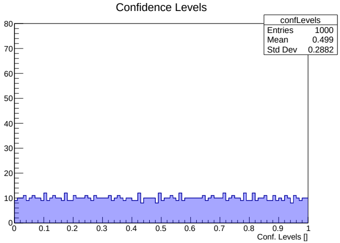
Confidence Levels

Since $\chi^2 := \vec{\epsilon}^T C_\eta^{-1} \vec{\epsilon}$ will produce an χ^2 distribution for N degrees of freedom, let's define the confidence level, CL as:

$$CL := \int_{x=\chi^2}^{\infty} f_N(x) \, \mathrm{d}x,$$

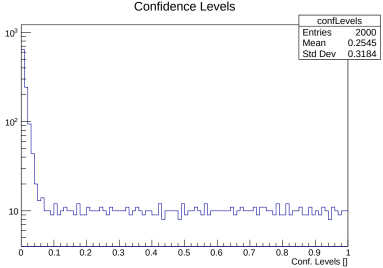
where $f_N(x)$ is the χ^2 distribution for N degrees of freedom. The fit is then referred to as a NC-fit.

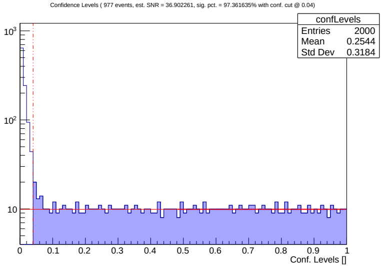
- **Characteristics**
  - If there is no background in the fit, the distribution is uniform and flat.



Confidence Levels

- **Characteristics**
  - In the presence of background, there will be a sharp rise as $CL \to 0$.





Cutting out the sharp rise as $CL \to 0$ will cut out the much of the background while keeping much of the signal intact.

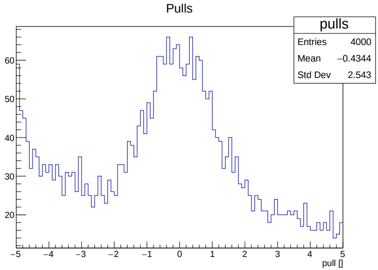
Pull Distributions

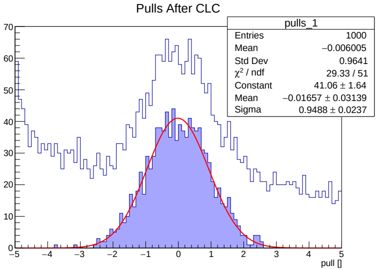
To see if the covariance matrix is correctly taking into account all pairwise correlations between the variables, we look to the pull distributions. Let's define $\vec{z}$ to house the pulls, z_i, defined as

$$z_i := \frac{y_i - \eta_i}{\sqrt{\sigma_{y_i}^2 - \sigma_{\eta_i}^2}}$$

Pull Distributions

- **Characteristics**
  Since these are normalized differences, the distributions should be normally distributed with
  - mean 0 and
  - width 1.



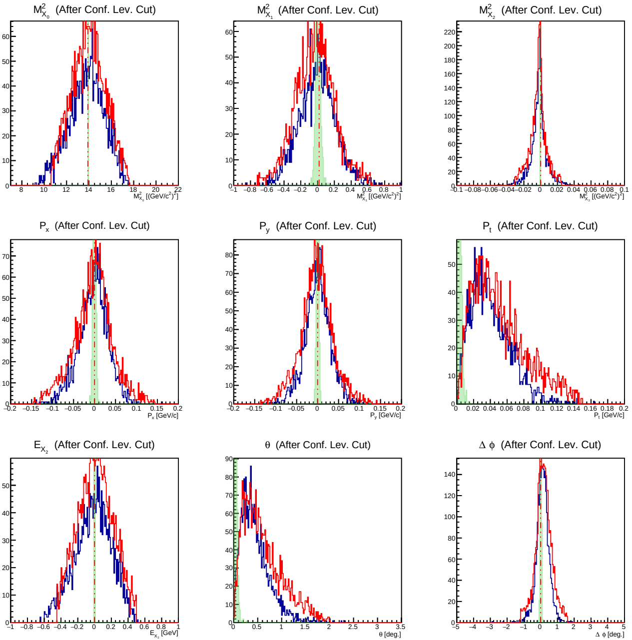


Kinematic Fit Applied to EG6: DVCS $4C$-fit Validation

Exclusivity Variable Distributions

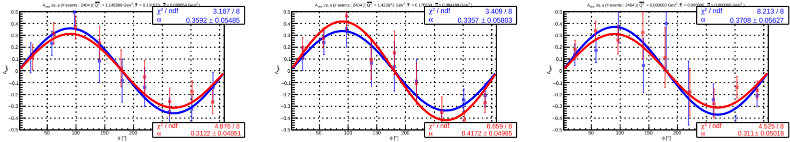
<ins>Measured values from</ins>:
Red: Exclusivity Cuts
Blue: Kinematic Fit

<ins>Fitted values from</ins>:
Green: Kinematic Fit



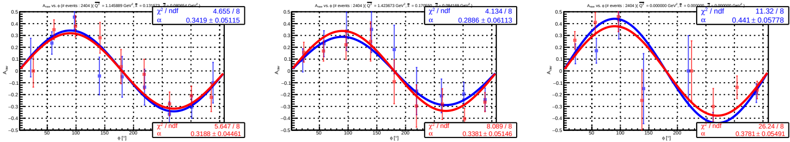
Beam-Spin Asymmetries

Bins in Q^2

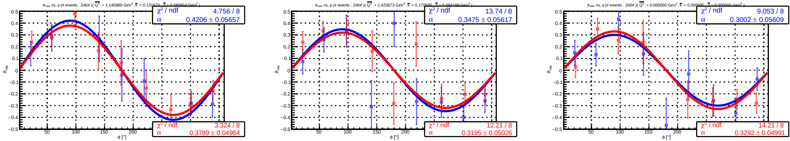


Bins in x

Measured values from:
Red: Exclusivity Cuts
Blue: Kinematic Fit



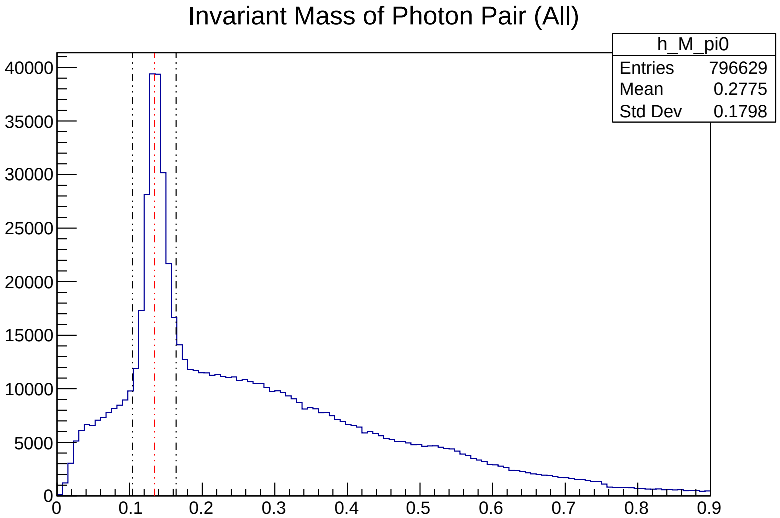
Bins in $-t$



Kinematic Fit Applied to EG6: $4C$-fit on DVπ^0P

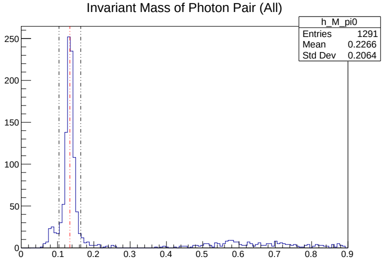
Motivation

Even with the detected e in CLAS and $^4\mathrm{He}$ in the RTPC, we still have to sift all combinations of photon pairs formed from both the IC and EC:

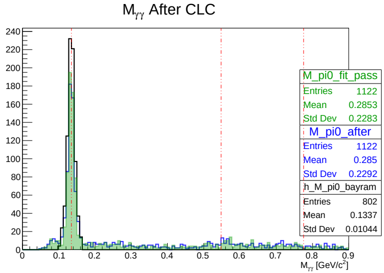


Motivation

[1]Exc. Cuts



$4C$ Kin. Fit



When applying the $4C$ kinematic fit, we see that the invariant mass distribution has a clear π^0-peak with very little background (and maybe a broader, shallower η peak [$M_\eta \approx 0.55$ GeV/c^2]). Note: Nowhere in the implementation is the nominal value of M_{π^0} used!

[1]For a fair comparison, additional π^0 cuts includes a photon distance cut ($|\Delta x_{\gamma\gamma} - 5\text{cm}| < 2\text{cm}$) and a momentum cut ($p_{\pi^0} > 3\text{GeV}/c$).

Kinematic Fit Applied to EG6: $5C$-fit on DVπ^0P

Setting up Kinematic Input Vectors

For convenience, let's first introduce some 4-vectors before defining our input vectors for kinematic fitting:

$$\begin{aligned} P_{X_{\pi^0}} &:= \left(\vec{\boldsymbol{p}}_{\gamma_1} + \vec{\boldsymbol{p}}_{\gamma_2} \ , \ \sqrt{\|\vec{\boldsymbol{p}}_{\gamma_1} + \vec{\boldsymbol{p}}_{\gamma_2}\|^2 + M_{\pi^0}^2} \right) \\ P_{fin} &:= P_e + P_{^4\mathrm{He}} + P_{X_{\pi^0}} \\ P_{init} &:= P_{\text{Beam}} + P_{\text{Targ}} \end{aligned}$$

and constraint 4-momentas for exclusivity and decay

$$\begin{aligned} P_{Exc.} &:= P_{init} - P_{fin} \\ P_{Decay} &:= P_{X_{\pi^0}} - (P_{\gamma_1} + P_{\gamma_2}) \end{aligned}$$

respectively.

Setting up Kinematic Inputs

Then,

$$\vec{\boldsymbol{y}}^0 = \begin{bmatrix} p_e \\ \theta_e \\ \phi_e \\ p_{^4\mathrm{He}} \\ \theta_{^4\mathrm{He}} \\ \phi_{^4\mathrm{He}} \\ p_{\gamma_1} \\ \theta_{\gamma_1} \\ \phi_{\gamma_1} \\ p_{\gamma_2} \\ \theta_{\gamma_2} \\ \phi_{\gamma_2} \end{bmatrix}, \quad \vec{\boldsymbol{x}}^0 = \begin{bmatrix} p_{\pi^0} \\ \theta_{\pi^0} \\ \phi_{\pi^0} \end{bmatrix}, \quad \vec{\boldsymbol{c}}^0 = \begin{bmatrix} (P_{Exc.})_x \\ (P_{Exc.})_y \\ (P_{Exc.})_z \\ (P_{Exc.})_E \\ (P_{Decay})_x \\ (P_{Decay})_y \\ (P_{Decay})_z \\ (P_{Decay})_E \end{bmatrix}, \qquad (12)$$

Setting up Kinematic Input Matrices

Before writing matrices A^0 and B^0 out, let's define D_β, where β represents the particle, $\beta \in \{e, {}^4\mathrm{He}, \gamma_1, \gamma_2, \pi^0\}$:

$$D_\beta := (-1)\begin{bmatrix} \sin\theta_\beta\cos\phi_\beta & p_\beta\cos\theta_\beta\cos\phi_\beta & -p_\beta\sin\theta_\beta\sin\phi_\beta \\ \sin\theta_\beta\sin\phi_\beta & p_\beta\cos\theta_\beta\sin\phi_\beta & p_\beta\sin\theta_\beta\cos\phi_\beta \\ \cos\theta_\beta & -p_\beta\sin\theta_\beta & 0 \\ \frac{p_\beta}{E_\beta} & 0 & 0 \end{bmatrix}. \tag{13}$$

The convention of the -1 emphasizes that these are final state particles. Then,

$$B^0 = \begin{bmatrix} D_e & D_{^4\mathrm{He}} & 0 & 0 \\ 0 & 0 & D_{\gamma_1} & D_{\gamma_2} \end{bmatrix}, \quad A^0 = \begin{bmatrix} D_{\pi^0} \\ -D_{\pi^0} \end{bmatrix}. \tag{14}$$

Setting up Covariance Matrix

Now, we set up the covariance matrix. Let's start with a simple, uncorrelated matrix:

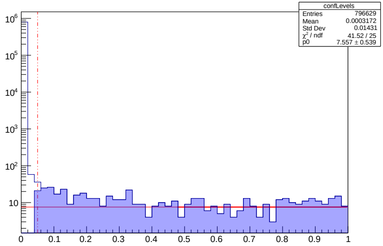
$$C_\eta = \mathrm{diag}\left(\sigma^2_{p_e}, \sigma^2_{\theta_e}, \sigma^2_{\phi_e}, \ldots, \sigma^2_{p_{\gamma_2}}, \sigma^2_{\theta_{\gamma_2}}, \sigma^2_{\phi_{\gamma_2}}\right) \tag{15}$$

$$= \begin{bmatrix} \sigma^2_{p_e} & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & \sigma^2_{\theta_e} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \sigma^2_{\theta_{\gamma_2}} & 0 \\ 0 & \ldots & \ldots & \ldots & 0 & \sigma^2_{\phi_{\gamma_2}} \end{bmatrix} \tag{16}$$

where the σ's are the widths extracted from previous Monte-Carlo studies that each depend on different combinations of measured p, θ, ϕ.

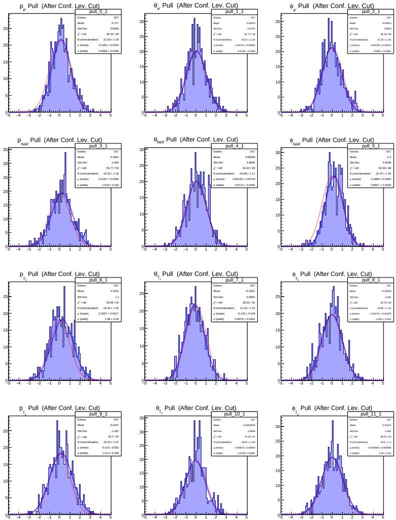
Fit Outputs

Confidence Level Distribution



$$\text{CLC} = 5 \times 10^{-2}\%$$

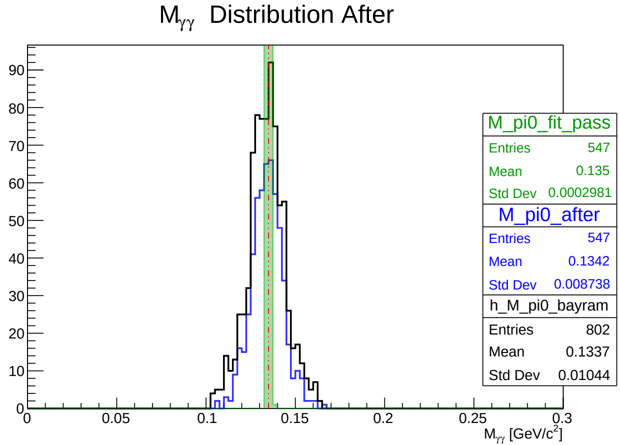
Pull Distributions



Invariant Mass Distribution for $\gamma\gamma$

Measured values from:
Black: Exclusivity Cuts
Blue: Kinematic Fit

Fitted values from:
Green: Kinematic Fit



Comparison to Exclusivity Cuts

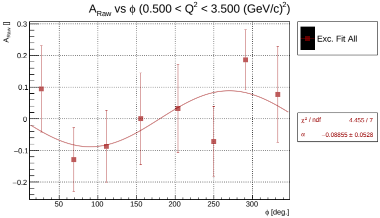
Results

For the EG6 experiment, the BSA for the coherent DVMP process

$$e\,^4\mathrm{He} \rightarrow e'\,^4\mathrm{He}'\,\pi^0 \tag{17}$$

is obtained from two different event selection methods:

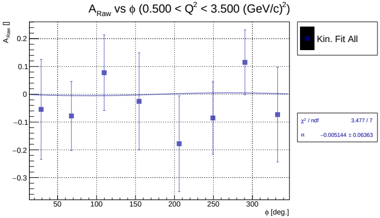
Exclusivity Cuts



BSA = -8.9±5.3 %

(800 events)

Kinematic Fit

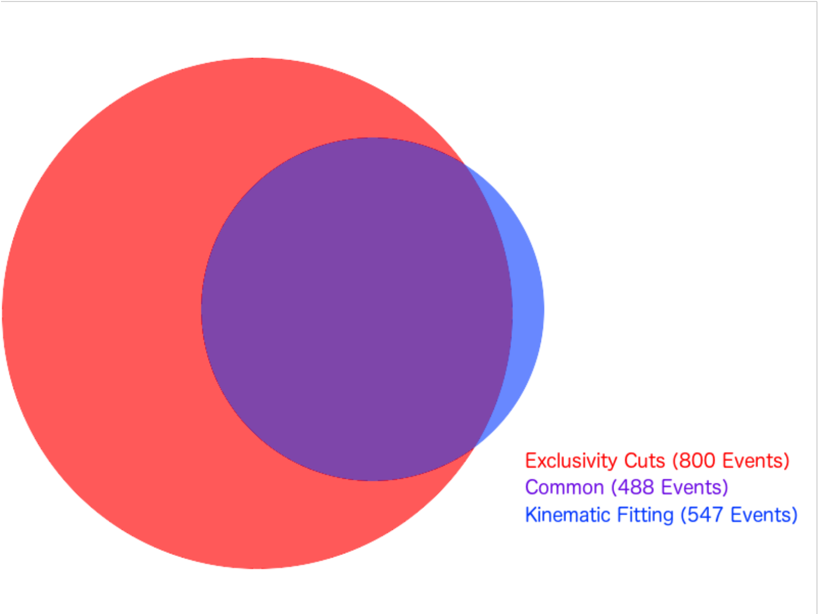


BSA = -0.5±6.3 %

(537 events)

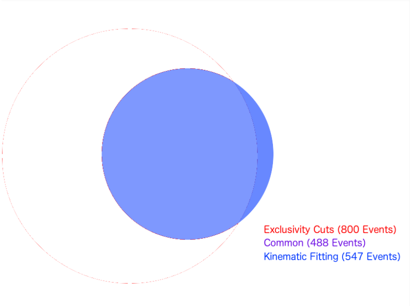
Datasets

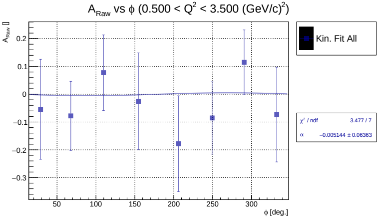
Consider the Venn diagram of the datasets:



Beam Spin Asymmetries

Beam spin asymmetries for all $5C$-fitted events :



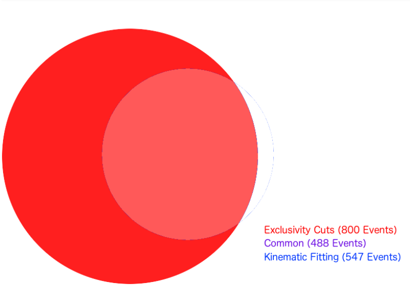


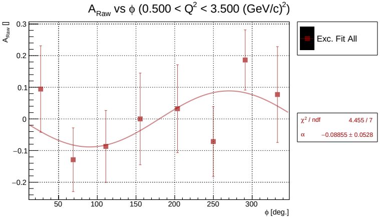
(537 Events)

(BSA = - 0.5±6.4 %)

Beam Spin Asymmetries

Beam spin asymmetries for all events passing exclusivity cuts :



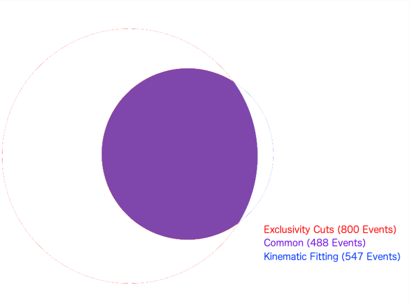


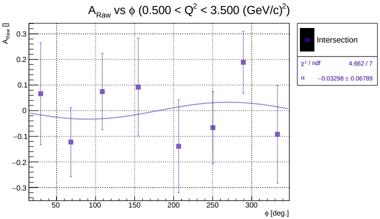
(800 events)

(BSA = - 8.9±5.3 %)

Beam Spin Asymmetries

Beam spin asymmetries for events passing only $5C$-fit :



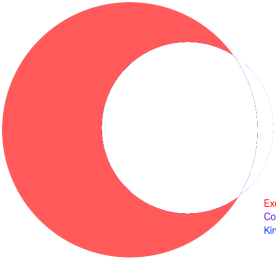


(488 Events)

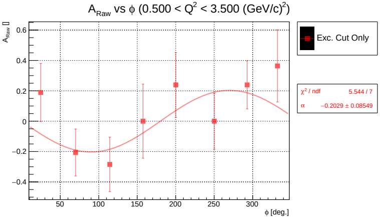
(BSA = - 3.3±6.8 %)

Beam Spin Asymmetries

Beam spin asymmetries for events only passing exclusivity cuts :

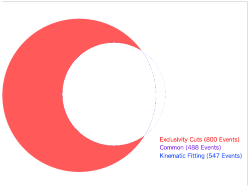


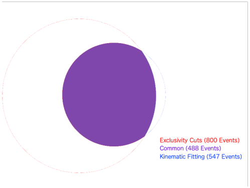
(312 Events)

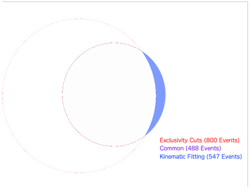


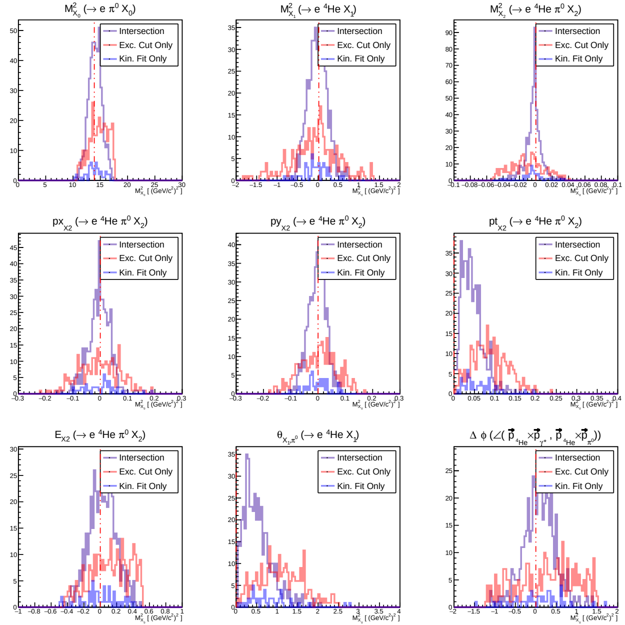
(BSA = - 20.3$\pm$8.5 %)

Exclusivity Variable Distributions



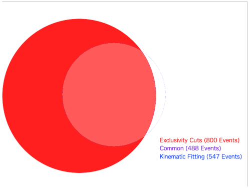




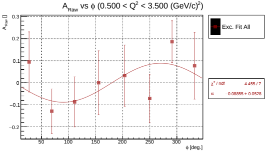


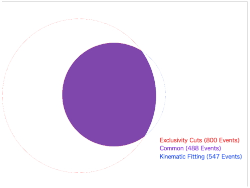
Beam Spin Asymmetries

Beam spin asymmetries summary:

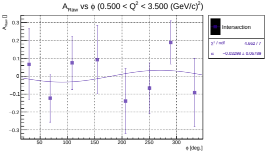


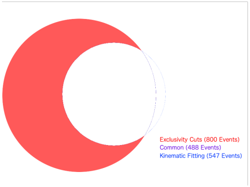
(800 events, BSA = -8.9±5.3%)



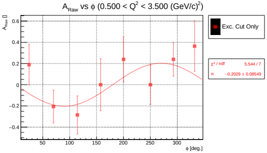


(488 events, BSA = -3.3±6.8%)





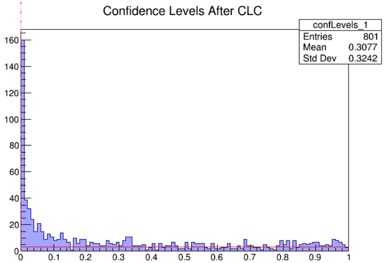
(312 events, BSA = -20.3±8.5%)



Sliding Down CLC to Match Statistics

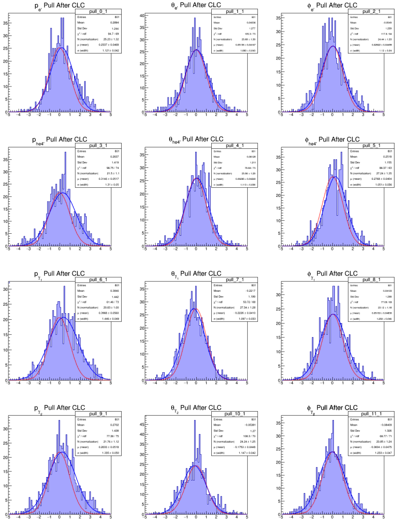
Fit Outputs

Confidence Level Distribution

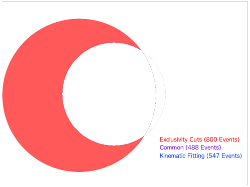


$$\text{CLC} = 6 \times 10^{-4}$$

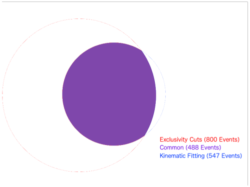
Pull Distributions



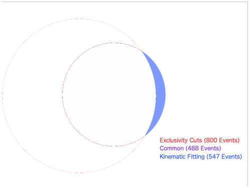
Exclusivity Variable Distributions



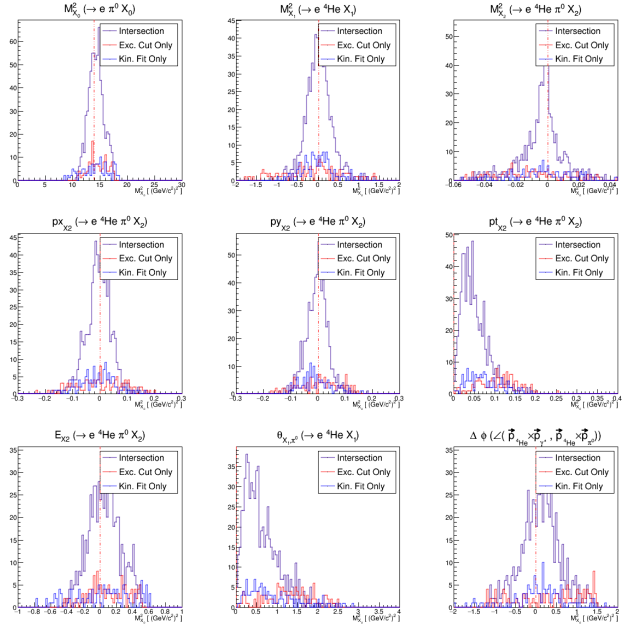
131 Events



669 Events



129 Events



Conclusion

The kinematic fit has a surprising effect of partitioning the previous 800 coherent π^0 events into 312 events with asymmetry ($\approx$20%) and 488 events without asymmetry ($\approx$ 3%).

Although it is not clear what this extra asymmetry is coming from, it is clear that events passing both the kinematic fit and the exclusivity cuts is diluting this larger asymmetry from seemingly background events.

Kinematic Fitting allows to clean events using both detector resolutions and conservation law constraints. Some of these events cannot be accessed by any obvious series of cuts.

Cuts for event selection require some extra insight and/or some cleverness.

Questions?

Backup Slides

Exclusivity Variables Definitions

Missing M^2, E, p, etc. definitions corresponding to distributions:

$M^2_{X_0}$	$M^2_{X_1}$	$M^2_{X_2}$
px_{X_2}	py_{X_2}	pt_{X_2}
E_{X_2}	θ_{X_0,π^0}	$\Delta\phi$

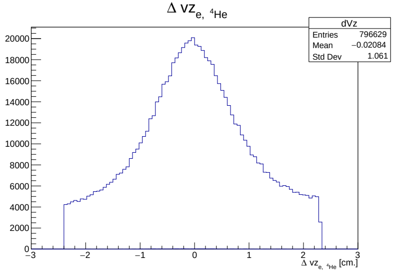
where

$$X_0 : \quad e\,^4\mathrm{He} \rightarrow e'\,^4\mathrm{He}' X_0$$
$$X_1 : \quad e\,^4\mathrm{He} \rightarrow e'\pi^0 X_1$$
$$X_2 : \quad e\,^4\mathrm{He} \rightarrow e'\,^4\mathrm{He}'\pi^0 X_2$$

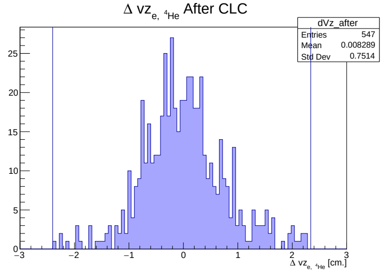
Sanity Check Distributions

Motivation: Vertex Coincidence

Before CLC

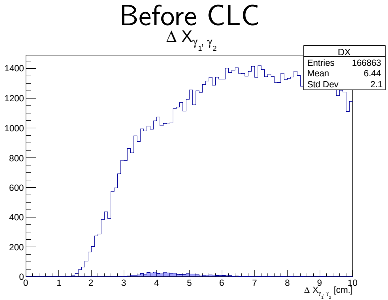


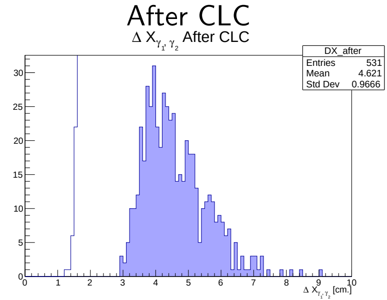
After CLC



Line Distribution: All Measured Events
Filled Distribution: Measured Events After CLC

Sanity Check: Photon Distance



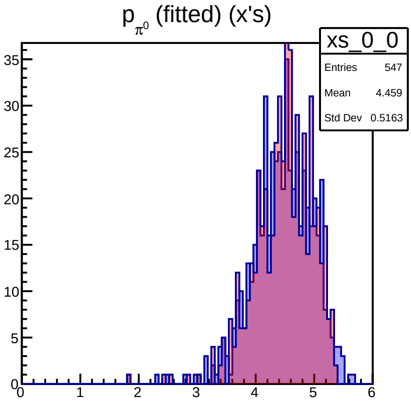


Line Distribution: All Measured Events
Filled Distribution: Measured Events After CLC

The $5C$-fit has no knowledge of the vertex coincidence between the helium in the RTPC and the electron in CLAS but produces a clean distribution of their distance.

B. Torayev's Cut : $\Delta X \in [3, 7]$ cm

Sanity Check: π^0 Momentum Distribution

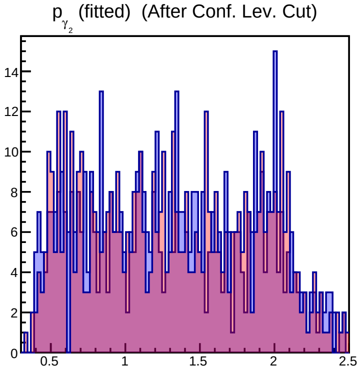


Red: Fitted After CLC
Blue: Measured After CLC

The $5C$-fit has no cut on the π^0 momentum but the distribution shows that the minimum momentum is around $3GeV/c$.

B. Torayev's Cut : $P_{\pi^0} > 3$ GeV$/c$

Sanity Check: γ_2 Momentum Distribution



Red: Fitted After CLC
Blue: Measured After CLC

The $5C$-fit has no cut on the γ_2 but the distribution shows that the minimum momentum is around $0.3 GeV/c$.

B. Torayev's Cut : $P_{\gamma_2} > 0.4 GeV/c$

Detector Resolutions

Table: Detector Resolutions

	δp (%)	$\delta\theta$ (deg.)	$\delta\phi$ (deg.)	δx (cm)
DC (Electron)	3.40	2.50	4.00	–
IC (Photon)	1.33	–	–	1.20
RTPC (Helium)	10.00	4.00	4.00	–
	δp (%)	$\delta\theta$ (rad.)	$\delta\phi$ (rad.)	δx (cm)
EC (Photon)	–	0.004	0.004	–

Let $\oplus$ denote the square-root quadrature sum:

$$a \oplus b \oplus c \oplus \ldots := \sqrt{a^2 + b^2 + c^2 + \ldots}$$

Then with these resolutions, we can calculate the widths that were extracted from simulation particle-by-particle. The explicit forms of the widths are shown in the following subsections. For the following, all input momenta are in GeV/c, all input angles are in units denoted by the subscripts, and resolutions are in units given by **Table 1**.

Detector Errors: DC

Table: Parameters for DC widths

Index i	A_i	B_i	C_i	D_i	E_i
	Parameter				
p	3375	35	0.7	0.0033	0.0018
θ	1000	0.55	1.39	–	–
ϕ	1000	3.73	3.14	–	–

$$
\begin{aligned}
\sigma_{p_e}[\mathrm{GeV}] &= \frac{A_p}{I_{\mathrm{beam}}}\left(\frac{\theta_{\mathrm{deg.}}}{B_p}\right)^{C_p} p\delta p\left[(D_p p)\oplus\frac{E_p}{\beta}\right] \\
\sigma_{\theta_e}[\mathrm{rad}] &= \frac{\delta\theta}{A_\theta}\left[B_\theta\oplus\frac{C_\theta}{p\beta}\right] \\
\sigma_{\phi_e}[\mathrm{rad}] &= \frac{\delta\phi}{A_\phi}\left[B_\phi\oplus\frac{C_\phi}{p\beta}\right]
\end{aligned}
\tag{18}
$$

where $I_{\mathrm{beam}} = 1900\mathrm{A}$, $\beta = pc/E$, and parameters A_i through E_i are listed in **Table 2**.

Detector Errors: IC

Table: Parameters for IC widths

Index i	Parameter A_i	B_i	C_i
p	0.024	0.0033	0.0019
θ	0.003	0.013	–
ϕ	0.003	–	–

$$
\begin{aligned}
\sigma_{p_\gamma}[\text{GeV}] &= p\delta p \left[A_p \oplus \frac{B_p}{\sqrt{p}} \oplus \frac{C_p}{p} \right] \\
\sigma_{\theta_\gamma}[\text{rad}] &= \delta x \left[\frac{A_\theta}{\sqrt{p}} \oplus (B_\theta \theta_{\text{rad.}}) \right] \\
\sigma_{\phi_\gamma}[\text{rad}] &= \delta x \left[\frac{A_\phi}{\sqrt{p}} \right]
\end{aligned}
\tag{19}
$$

Detector Errors: EC

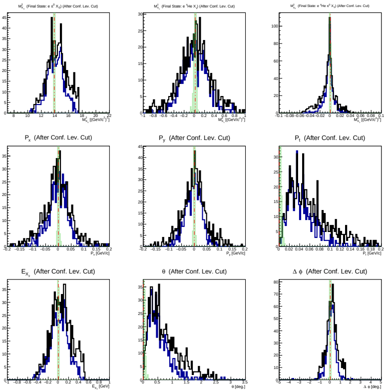
$$
\begin{aligned}
\sigma_{p_\gamma}[\mathrm{GeV}] &= A_p\sqrt{p} \\
\sigma_{\theta_\gamma}[\mathrm{rad}] &= \delta\theta_{EC} \\
\sigma_{\phi_\gamma}[\mathrm{rad}] &= \delta\phi_{EC}
\end{aligned} \tag{20}
$$

where the parameter $A_p = 0.116$.

Detector Errors: RTPC

$$\begin{aligned} \sigma_{p_{^4\mathrm{He}}}[\mathsf{GeV}] &= p\delta p \\ \sigma_{\theta_{^4\mathrm{He}}}[\mathsf{rad}] &= \delta\theta_{\mathsf{rad}} \\ \sigma_{\phi_{^4\mathrm{He}}}[\mathsf{rad}] &= \delta\phi_{\mathsf{rad}} \end{aligned} \tag{21}$$

Sanity Check: Exclusivity Variable Distributions



Black: B. Torayev's Distributions
Blue: Measured After CLC
Green: Fitted After CLC

B. Torayev's Cuts:

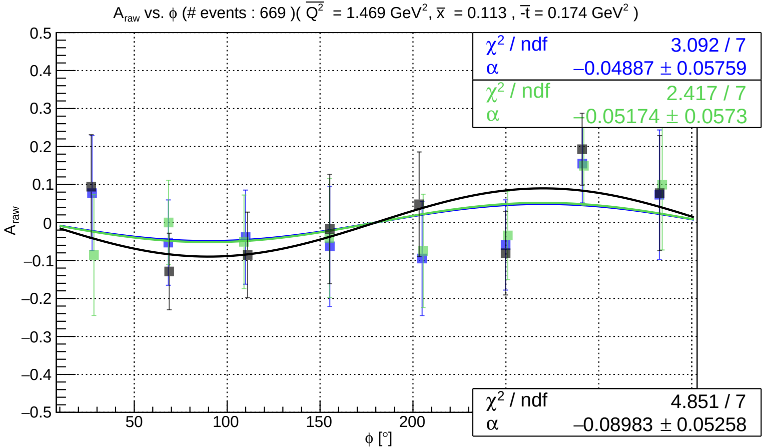
$$|M^2_{X_2} - 0.005| < 0.048\,(\text{GeV}/c^2)^2$$
$$|\Delta\phi - 0.16| < 0.138 \text{ deg.}$$

$$|\theta_{\pi^0,X_1} - 2.5| < 0.03 \text{ deg.}$$
$$|M^2_{X_0} - 14.079| < 0.03\,(\text{GeV}/c^2)^2$$

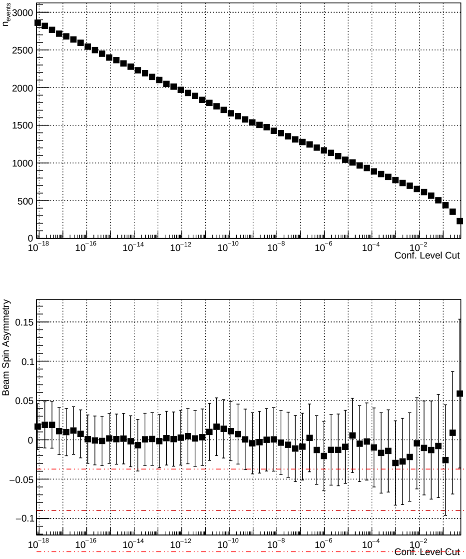
The 5C-fit has no cuts on any of the exclusivity variables but they are essentially within the previous cuts.

Common Events' BSA for Equally Statistic Events

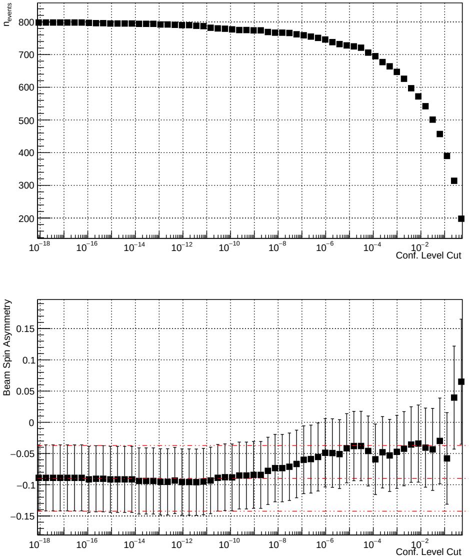
800 Kin. Fit Events with CLC @ 6×10^{-4} overlap with 800 Exc. Cut events. The overlap is 669 events.



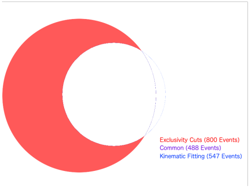
BSA vs. Conf. Level Cut: Full Dataset

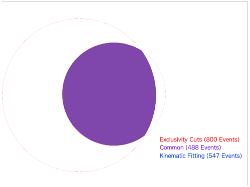


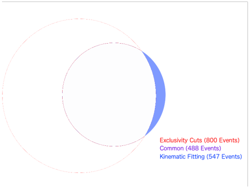
BSA vs. Conf. Level Cut: Exclusivity Selected Events

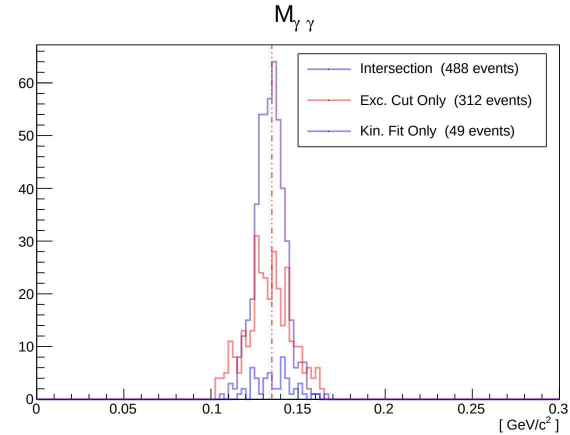


Invariant Mass Distributions



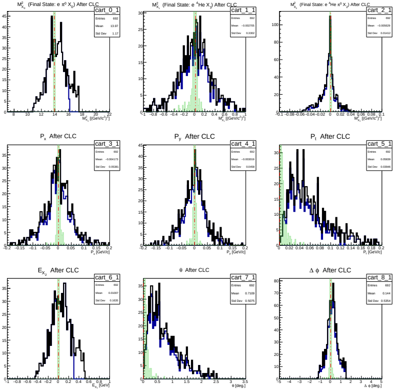




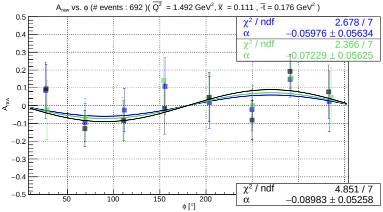


Adding One Exclusivity Cut: *E* Cut

Exclusivity Variable Distributions

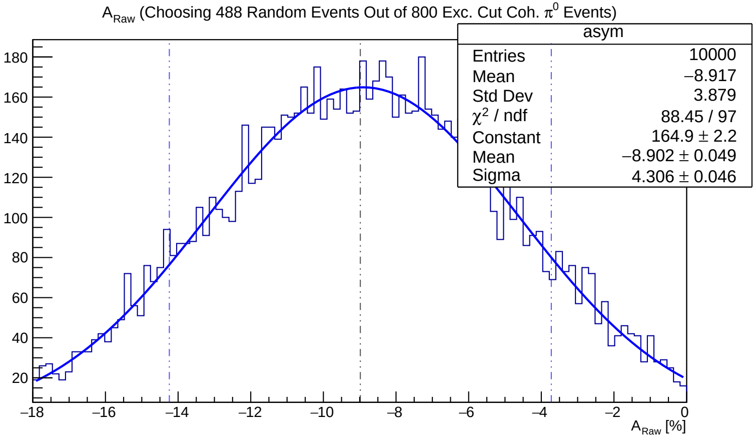


Beam Spin Asymmetry

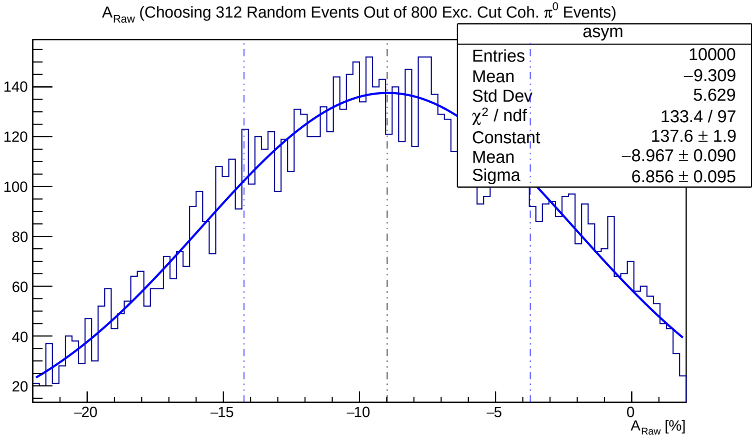


(692 events, BSA $=$ -6.4$\pm$5.6%)

Likelihood of Selecting 488 out of 800 events having $A_{Raw} = -3.3\%$

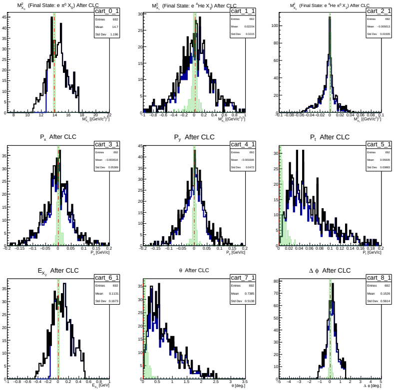


Likelihood of Selecting 312 out of 800 events having $A_{Raw} = -20.3\%$

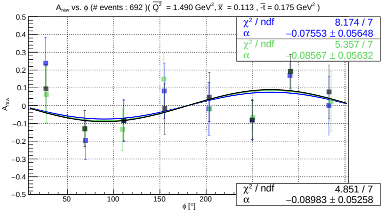


Adding One Exclusivity Cut: E Cut

Exclusivity Variable Distributions



Beam Spin Asymmetry



(692 events, BSA = -7.8±5.6%)

Likelihood of 692/800 events having 33% Less Asymmetry