quark model results for the **PDFs** and **quasi-distributions** of **nucleons** and **mesons**

Wed., May 16, 2018

Light Cone 2018, JLab

tim hobbs – **Southern Methodist University** and **CTEQ**

road map

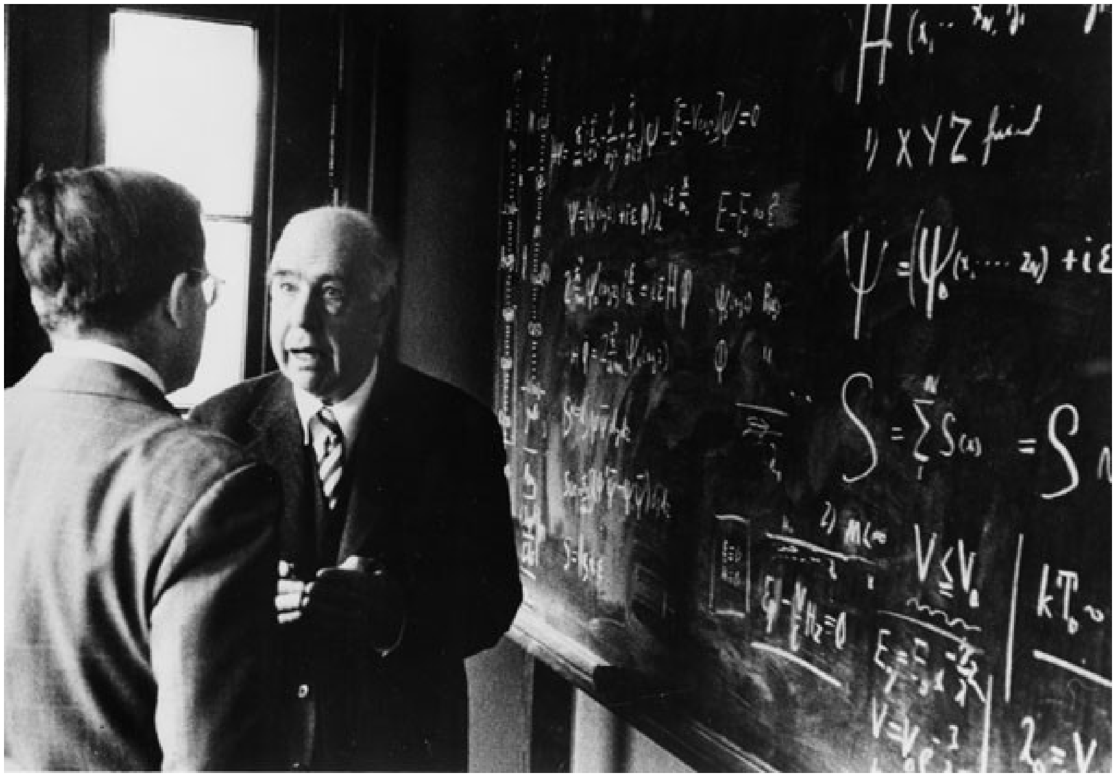
- hadronic structure from QCD global analysis
- **modeling**, especially on the light front, as a unifying tool
    - two illustrations:
        - → nonperturbative **charm** in the proton
        - → models for meson **quasi-PDFs**

- possible future directions

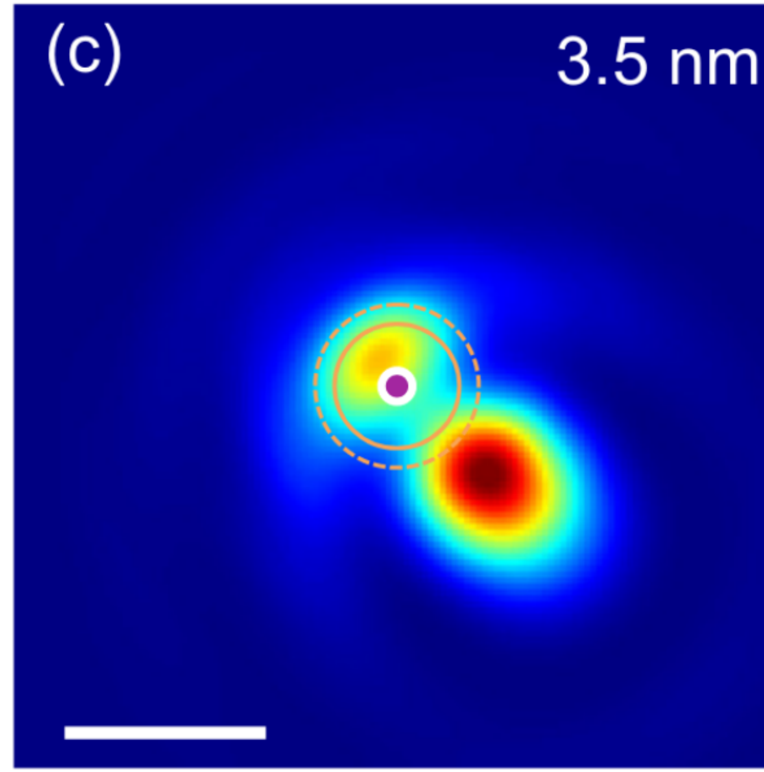
proton structure is increasingly becoming a **precision field**

- the present moment is in some ways reminiscent of progress made in atomic structure in the 20th Century:

Jeong et al., PRB93, 165140 (2016).



Niels with Aage at LANL.



Sr STEM simulated image

- much as the electronic structure of atomic matter has been mapped to high precision, we are entering an era of '**hadron tomography**'

… this is enshrined in the *2015 Nuclear Science Advisory Committee LRP*

→ AND motivation for JLab12, EIC, LHeC, collider data analyses

the 'time-honored' approach to (longitudinal) hadron structure: **QCD global fits**

- we want to determine from fits:

$$f_{q/p}(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-i\xi^- k^+} \langle p \,|\overline{\psi}(\xi^-)\gamma^+ \mathcal{U}(\xi^-,0)\psi(0)\,|p\rangle$$

- for single-particle hadroproduction of gauge bosons at, e.g., LHC, factorization gives

$$\sigma(AB \to W/Z+X) \;=\; \sum_n \alpha_s^n(\mu_R^2) \sum_{a,b} \int dx_a dx_b$$

$$\times\; f_{a/A}(x_a,\mu^2)\, \hat{\sigma}^{(n)}_{ab\to W/Z+X}\left(\hat{s},\, \mu^2, \mu_R^2\right) f_{b/B}(x_b,\mu^2)$$

PDFs determined by fits to data

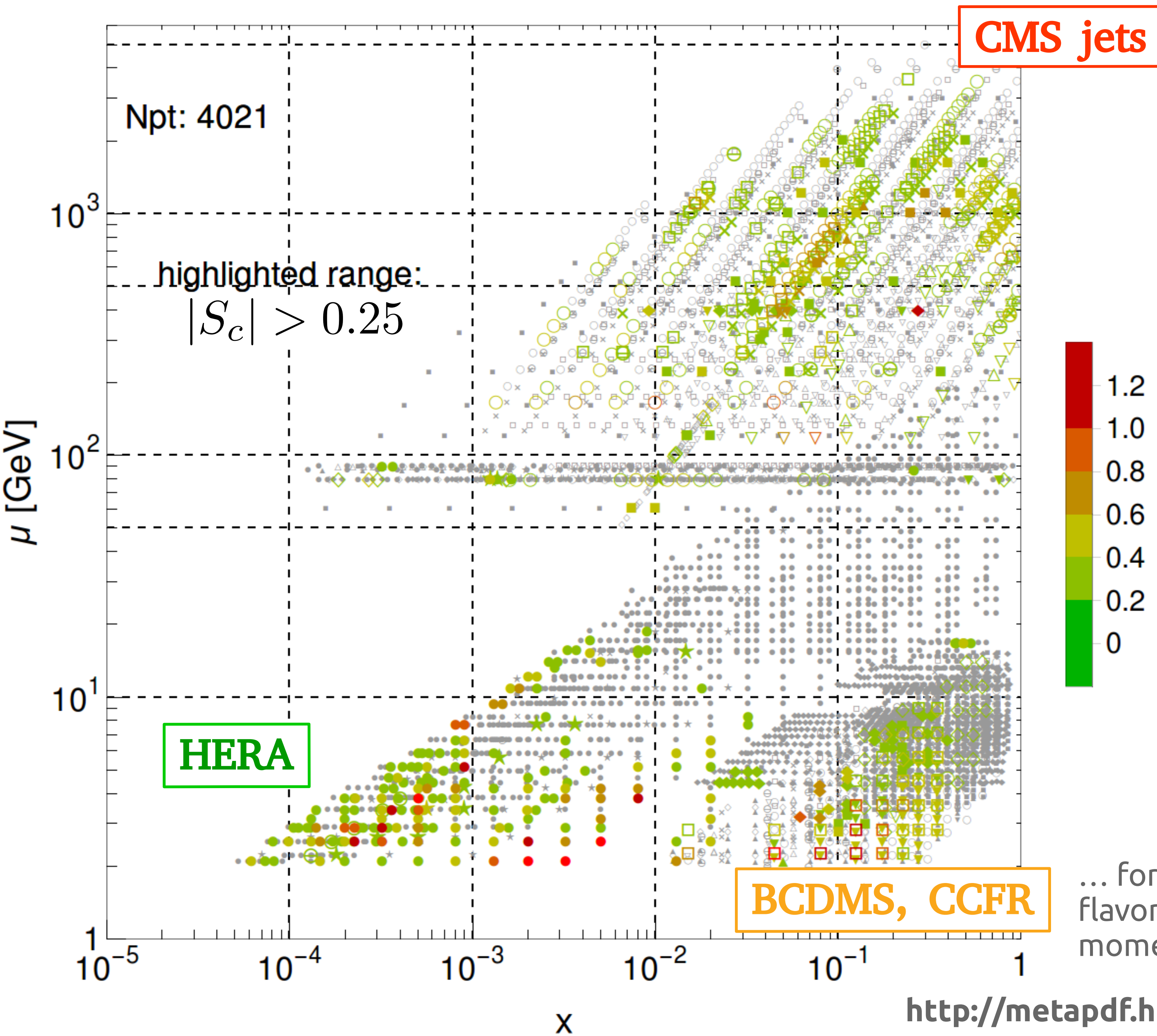
pQCD matrix elements – specified by theoretical formalism in a given fit

- constrain a flexible PDF parametrization:

$$f_{q/h}(x,Q_0^2) = a_{q0}\, x^{a_{q1}} (1-x)^{a_{q2}}\, P[x,\{a_{qn-3}\}]$$

| S_f | for c(x,μ), CT14HERA2NNLO

Wang, TJH, Doyle, Gao, Hou, Nadolsky, Olness,
arXiv:1803.02777 [hep-ph].



- in QCD global analyses, empirical data 'teach us' the PDFs!
- PDFSense: a tool to quantify and visualize the sensitivity of measured data to the PDFs (or PDF-dependent quantities)
- e.g., can identify those data that most strongly determine our knowledge of the charm PDF

… for many other results (PDF flavors, combinations, moments, cross sections), visit:

http://metapdf.hepforge.org/PDFSense/

- so, QCD analyses offer precision and robustness

→ in actuality, however, there are also limits to the insights one might gain concerning nonperturbative structure from global analyses

---

for instance, parametrizations are generally chosen with maximum flexibility so as to follow empirical data:

NNPDF : feed-forward neural network

CT : Regge theory, counting rules-inspired form

. . . .

models (esp. light-front wave functions) can help us negotiate this landscape

→ **illuminate connections** among (wave function dependent) observables **(1st ex.)**

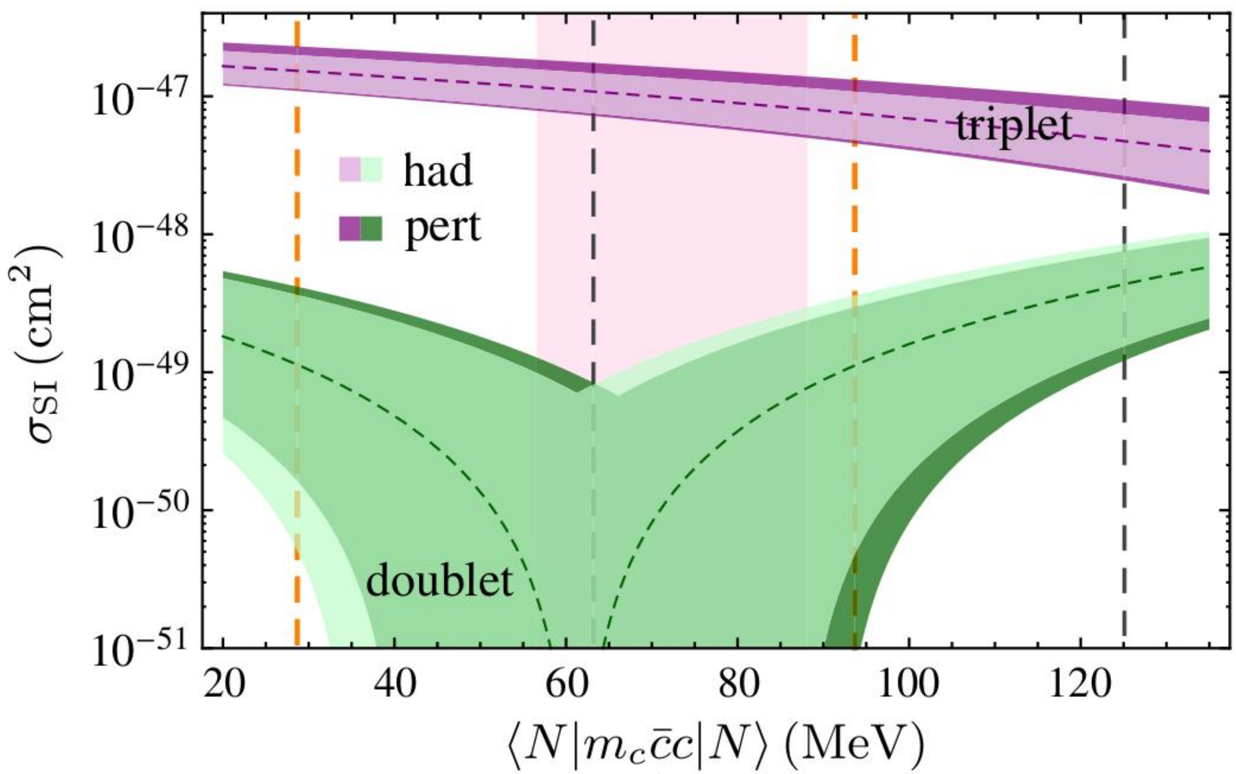
→ **guide** measurements, analyses (parametrizations), lattice calculations/interpretation **(2nd ex.)**

1st example : **modeling** nucleon charm

- the **heavy quark sigma term** (esp. for charm) is important to WIMP direct searches :

$$\mathcal{L}_{\chi_\nu} = \frac{1}{M_W^3}\overline{\chi}_\nu \chi_\nu \left(\sum_q c_q^{(0)} \left(m_q \bar{q} q\right) + c_g^{(0)} \left(G^A_{\mu\nu}\right)^2 \right)$$

Hill and Solon, Phys. Rev. Lett. **112**, 211602 (2014).



- meanwhile, models and QCD global analyses of DIS data seek to constrain the nucleon's *intrinsic charm* in terms of

$$\langle x \rangle_{c+\bar{c}} \equiv \int_0^1 dx\, \left[c+\bar{c}\right](x)$$

at $Q^2 = m_c^2$

... various determinations:

$$\langle x \rangle_{c+\bar{c}} \lesssim 0.2\%^{*};\ \lesssim 2\%^{*}$$

* P. Jimenez-Delgado, TJH, J. T. Londergan and W. Melnitchouk; PRL **114**, no. 8, 082002 (2015).

* T. J. Hou et al., arXiv: **1707.00657** [hep-ph].

- **can these objects be correlated?**

... need models for *both* the charm PDF *and* $\sigma_{c\bar{c}}$

- **light-front wave functions (LFWFs)** are one such approach
- they deliver a **frame-independent** description of hadronic bound state structure

  - the light front represents physics *tangent* to the light cone:

$$x^\mu = (x^0, \mathbf{x}) \longrightarrow (x^+, x_\perp, x^-)$$

$$x^\pm = x^0 \pm x^3, \quad x_\perp = (x^r)\,; \quad r = \{1, 2\}$$

- with them, many matrix elements (GPDs, TMDs) are calculable via the same **universal** objects:

$$c(x) \sim \langle \bar{c}\,\gamma^+ c \rangle \longleftrightarrow \sigma_{c\bar{c}} \;=\; m_c \, \langle p \big| \bar{c}c \big| p \rangle$$

- in fact, had already developed this technology for **nucleon strangeness**!

TJH, M. Alberg, and G. A. Miller; PRC91, 035205 (2015).

we build a model for the charm wave func^n... 1st the PDF

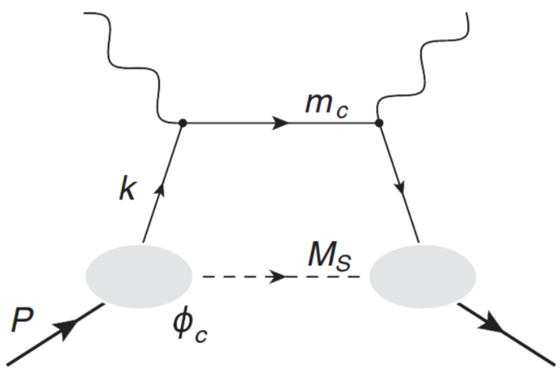
- use a scalar spectator picture; details in helicity wave func^{n}s :

$$|\Psi_P^\lambda(P^+, \mathbf{P}_\perp)\rangle = \frac{1}{16\pi^3} \sum_{q=c,\bar{c}} \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}}$$

$$\times \psi_{q\lambda_q}^\lambda(x, \mathbf{k}_\perp) \, |q; xP^+, x\mathbf{P}_\perp + \mathbf{k}_\perp\rangle$$

TJH, Alberg and Miller,
Phys. Rev. D**96** (2017) no.7, 074023.

(see Stan's talk, 05.15.2018)



$$F_2^{c\bar{c}}(x, Q^2 = m_c^2) = \frac{4x}{9}\left(c(x) + \bar{c}(x)\right)$$

$$c(x) = \frac{1}{16\pi^2} \int \frac{dk_\perp^2}{x^2(1-x)} \left[\frac{k_\perp^2 + (m_c + xM)^2}{(M^2 - s_{cS})^2}\right] \left|\phi_c(x, k_\perp^2)\right|^2$$

use a power-law (γ=3) covariant vertex function, $\phi_c(x, k_\perp^2) = \sqrt{g_c}\left(\frac{\Lambda_c^2}{t_c - \Lambda_c^2}\right)^\gamma$

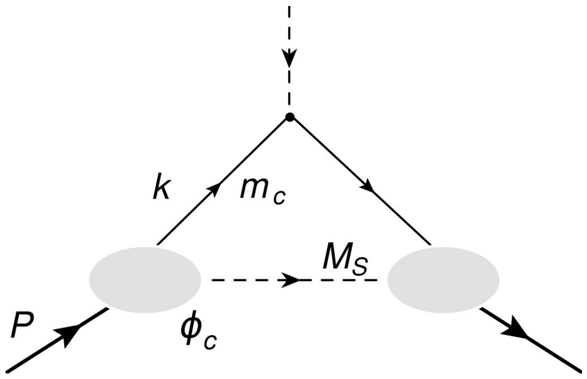
$$s_{cS}\left(x, k_\perp^2\right) = \frac{1}{x\,(1-x)}\left(k_\perp^2 + (1-x)\, m_c^2 + x\, M_S^2\right)$$ **invariant mass**

$$t_c\left(x, k_\perp^2\right) = \frac{1}{1-x}\left(-k_\perp^2 + x\left[(1-x)M^2 - M_S^2\right]\right)$$ **covariant k^2**

then, a covariant formalism gives the **sigma term**:

- IF the LFWFs can be constrained with information from the DIS sector, we may evaluate $\sigma_{c\bar{c}}$

$$\sigma_c = \frac{ig_c}{2M}\int\frac{d^4k}{(2\pi)^4}\,\overline{u}(p)\left(\frac{1}{\not{k}-m_c+i\epsilon}\right)[m_c\mathcal{I}_4]\left(\frac{1}{\not{k}-m_c+i\epsilon}\right)u(p)$$
$$\times\left(\frac{1}{[p-k]^2-M_S^2+i\epsilon}\right)\left(\frac{\Lambda^2}{k^2-\Lambda_c^2+i\epsilon}\right)^{2\gamma}$$



$$\sigma_{c\bar{c}} = \sigma_c + \sigma_{\bar{c}}$$

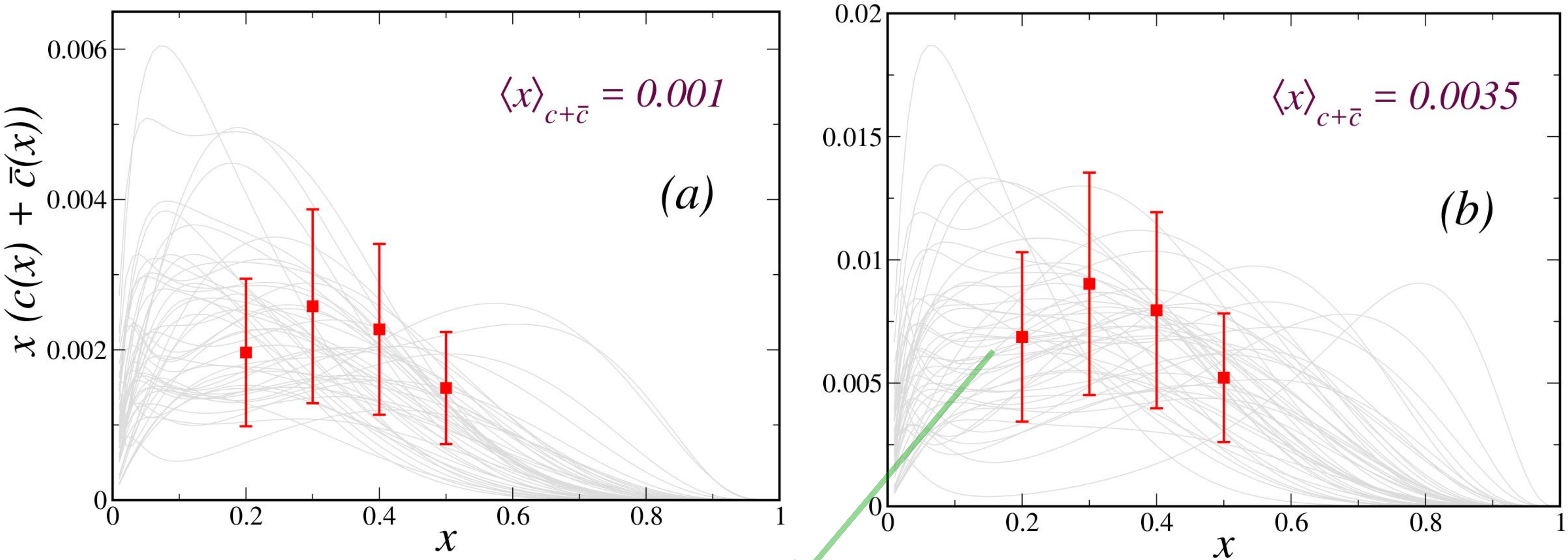
…we determine **probability distribution functions** (p.d.f.s) for this quantity

- this formalism is required because the LFWFs contain noncovariant parts:

$$i\frac{\sum_\lambda u_\lambda(k)\,\overline{u}_\lambda(k)}{k^2-m^2+i\epsilon} = \frac{i}{\not{k}-m+i\epsilon} - i\frac{\gamma^+}{2k^+}$$

- **it remains to determine the (free) *parameters* of the light-front model,**

$$\left(g_c,\ m_c,\ \Lambda_c,\ \Lambda_{\bar{c}},\ M_S,\ M_{\bar{S}}\right)$$



- we constrain the model with hypothetical **pseudo-data** (taken from the \`confining' MBM) of a given $\langle x \rangle_{\mathrm{IC}} \pm 50\%$

(input data normalizations are inspired by the just-described global analysis)

$$\begin{cases} \langle x \rangle_{\mathrm{IC}} = 0.001 \\ \langle x \rangle_{\mathrm{IC}} = 0.0035 \end{cases}$$

[upper limit tolerated by the full fit/dataset]

[central value preferred by EMC data alone]

- rather than traditional χ^2 minimization, the model space is instead explored using **Bayesian methods**

model simulations with **m**arkov **c**hain **m**onte **c**arlo (MCMC)

- specifically, use a **D**elayed-**R**ejection **A**daptive **M**etropolis (DRAM) algorithm

Haario et al., Stat. Comput. (2006) **16**: 339–354.

construct a Markov chain consisting of $n_{sim} \approx 10^5 - 10^6$ simulations, sampling the ***joint posterior distribution***

$$p(\vec{\theta}\,|x) \sim p(x|\vec{\theta}\,)\, p(\vec{\theta}\,)$$

x : input data

$\vec{\theta}$: parameters

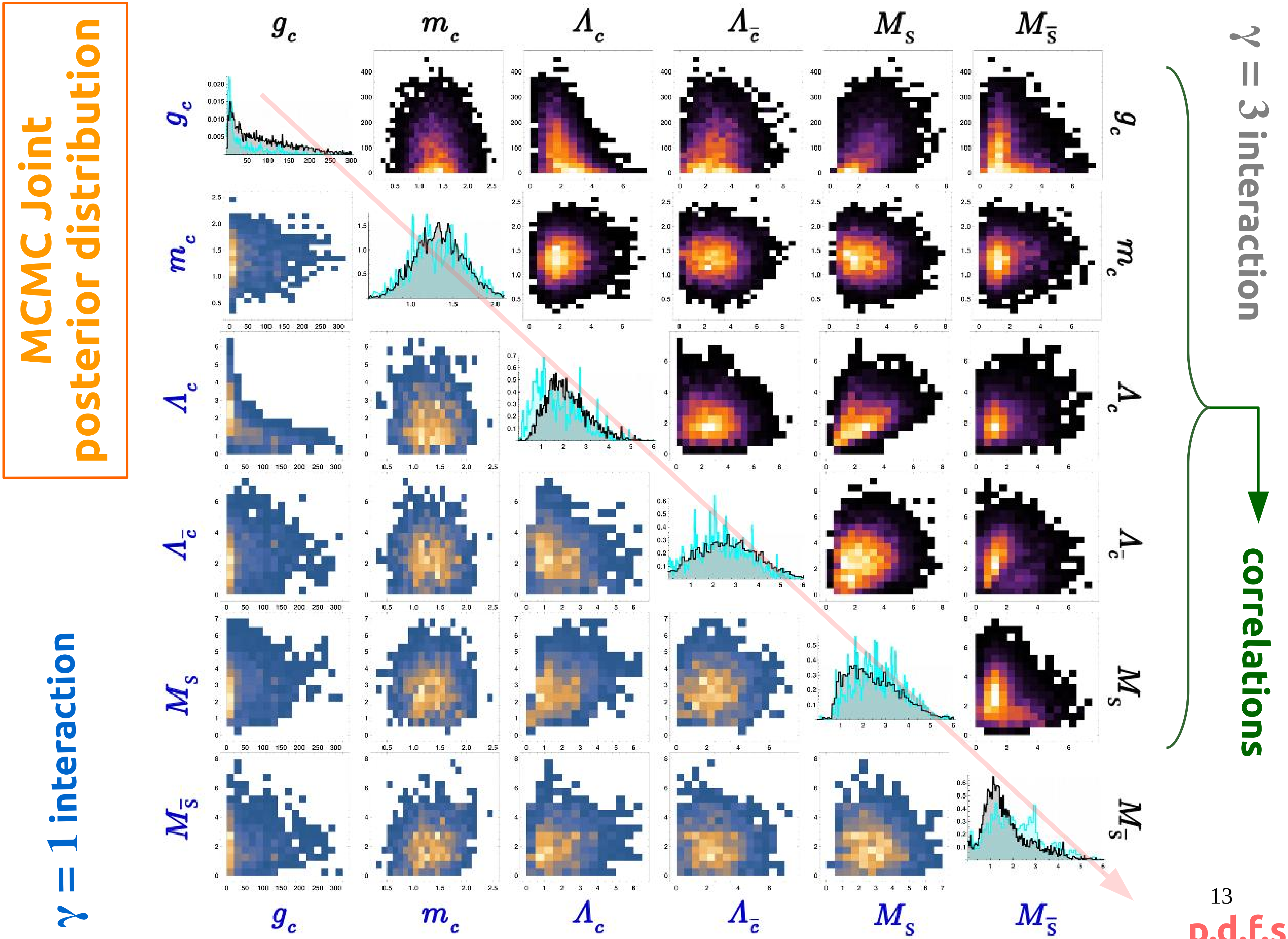
BROAD gaussian priors

likelihood function $p(x|\vec{\theta}\,) = \exp\left(-\chi^2/2\right)$

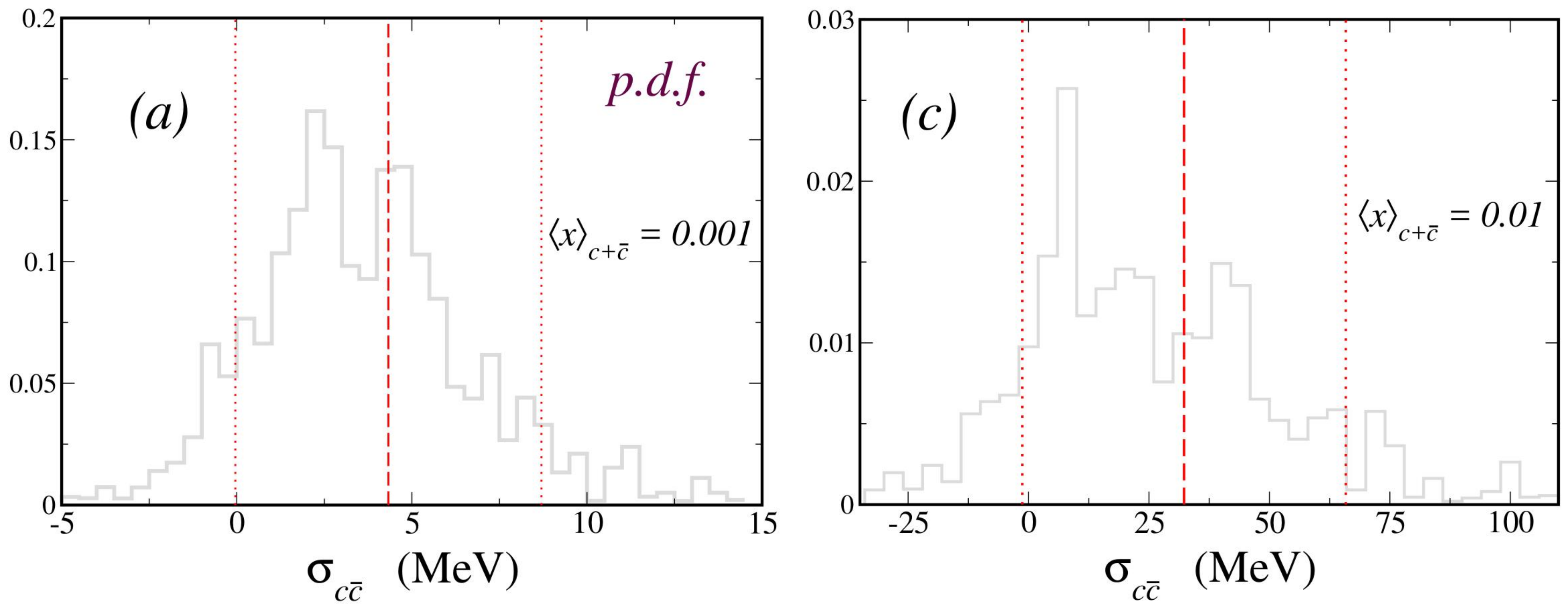
$$\chi^2 = \sum_i \left(\frac{1}{\sigma_i^{data}}\right)^2 \left|\, F_2^{c\bar{c}}(x_i,\vec{\theta}\,) - F_2^{c\bar{c},\,data}(x_i)\right|^2$$

- asymptotically, the MCMC chain fully explores the joint posterior distribution

✔ from this, we extract **probability distribution functions (p.d.f.s)** for the model parameters and derived quantities, **including** $\sigma_{c\bar{c}}$



MCMC Joint posterior distribution
g_c m_c Λ_c $\Lambda_{\bar{c}}$ M_S $M_{\bar{S}}$
$\gamma = 3$ interaction
correlations
$\gamma = 1$ interaction
p.d.f.s



$$\sigma_{c\bar{c}} = 4.3 \pm 4.4\,\mathrm{MeV} \quad (\,\gamma = 3 \text{ interaction}\,) \quad \sigma_{c\bar{c}} = 32.3 \pm 33.6\,\mathrm{MeV}$$

- we find better concordance cf. existing **lattice determinations**, for somewhat larger IC magnitudes; also, close correlation with the DIS sector –

$$\sigma_{c\bar{c}} = 94\,(31)\,\mathrm{MeV}$$ (χQCD)[1]

$$= 67\,(34)\,\mathrm{MeV}$$ (MILC)[2]

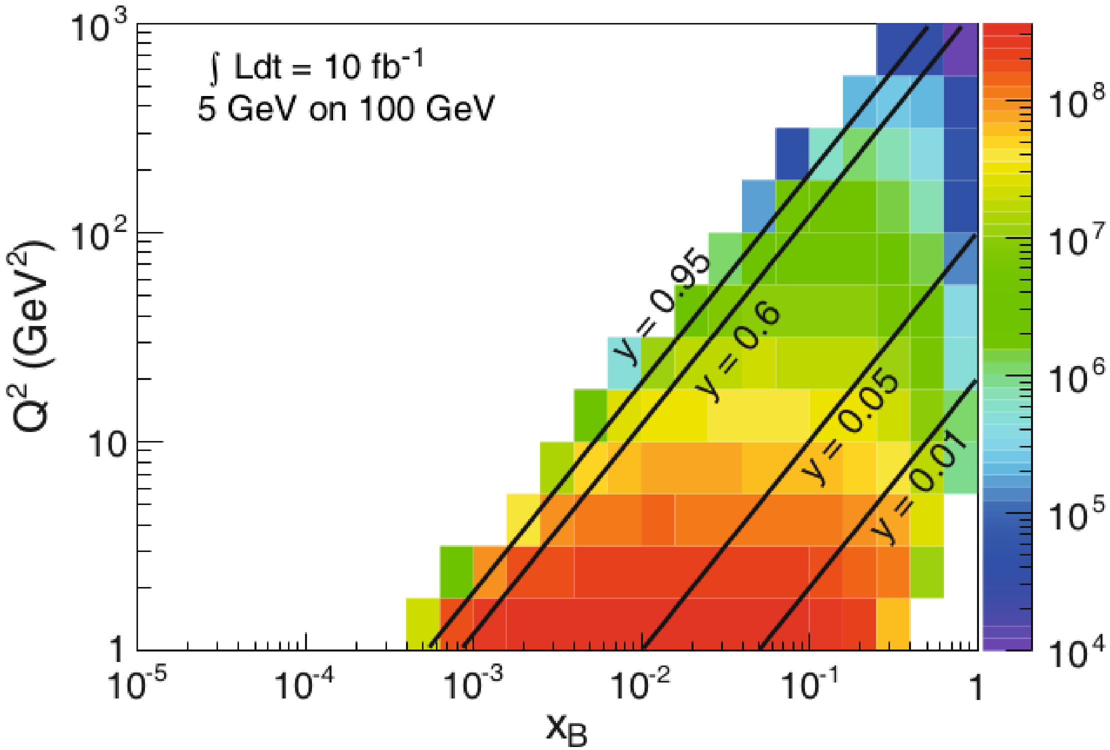
$$\sigma_{c\bar{c}} = 79\,(21)\binom{12}{8}\,\mathrm{MeV}$$ (AR)[3]

$\mathcal{O}\left(\alpha_s^3\right)$ pQCD is similar…

[1]Gong et al., Phys. Rev. **D88**, 014503 (2013).

[2]Freeman and Toussaint, Phys. Rev. **D88**, 054503 (2013).

[3]Abdel-Rehim et al., Phys. Rev. Lett. **116**, 252001 (2016).



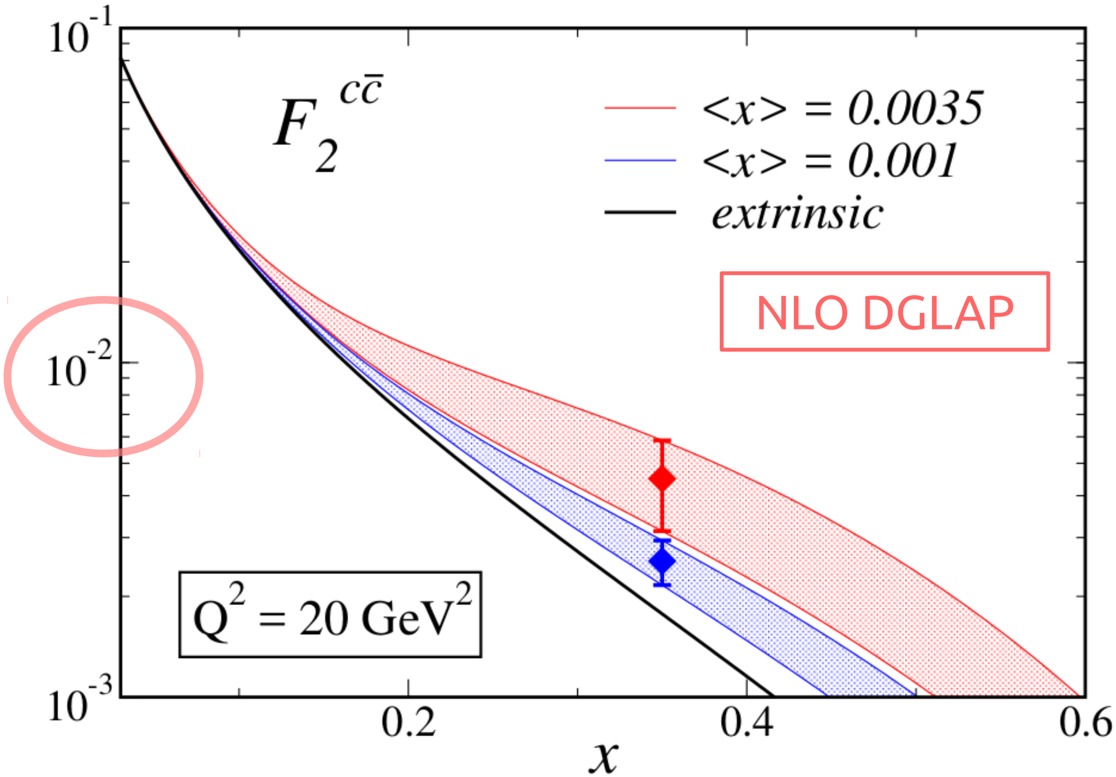
EIC Whitepaper, Eur. Phys. J. A (2016) **52**: 268

- e.g., MEIC-like scenario:

$$\sqrt{s} = 45\,\text{GeV}$$

- a definitive measurement would simply **reprise the EMC observation of $F^{c\bar{c}}_2$**

- still, considerable precision will be needed to be sensitive at the necessary level

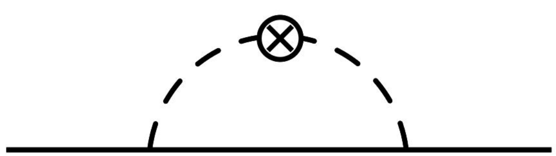


a future, unified description of the proton wave function may have the potential to provide the charm PDF and sigma term within a more comprehensive tomography

2nd example : LaMET and the pion structure function

- via the **Sullivan process**, the structure of the interacting nucleon receives important contributions from pionic modes – the *pion cloud*:

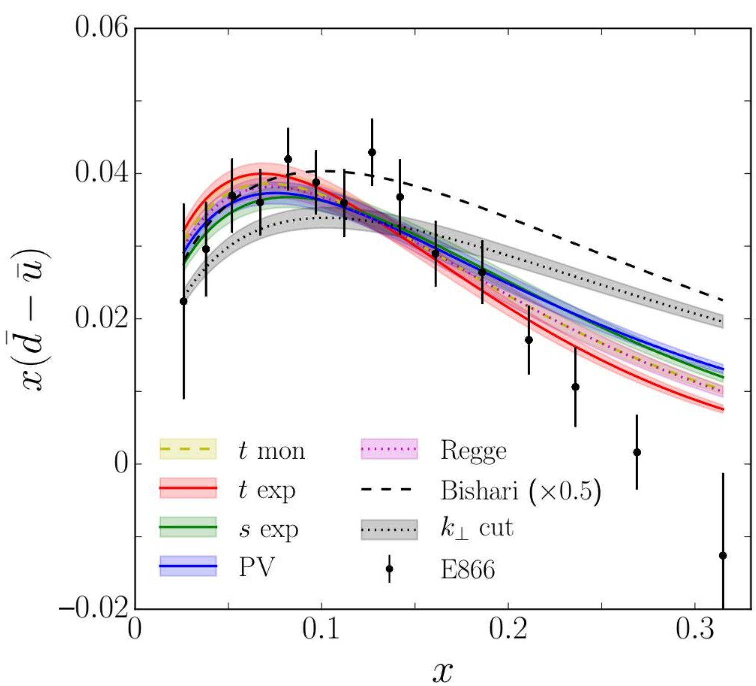
$$Z_{bare} \sim 1 - P_{N\pi} - P_{\Delta\pi} \qquad \begin{cases} P_{N\pi} \sim 0.20 - 0.25 \\ P_{\Delta\pi} \sim 0.05 - 0.10 \end{cases}$$



S. Theberge *et al.*, Phys. Rev. D**22**, 2838 (1980).

- this has implications for parton distributions; esp., light **flavor asymmetries**:

J. R. McKenney *et al.*, Phys. Rev. **D93**, 054011 (2016).



$$\bar{d} - \bar{u} = \left(f_{\pi^+ n} - \frac{2}{3} f_{\pi^- \Delta^{++}} \right) \otimes \bar{q}_v^{\pi}$$

hadronic splittings, à la χ_{PT}

pion valence PDF

- **can we better determine the pion structure funcⁿ?**
  - new **measurements** (SeaQuest, JLab TDIS) → (A. Tadepalli)
  - expanded QCD **global analyses** → (P. Berry)
  - direct calculation on the **Lattice** → (many!/this talk)

formalism: LaMET and the pion structure function

- knowledge of the pion structure function is crucial to unraveling the nucleon's light quark sea (e.g., $\bar{d} - \bar{u}$); LaMET techniques may open this quantity to Lattice QCD

TJH, Phys. Rev. D**97** (2018) no.5, 054028.

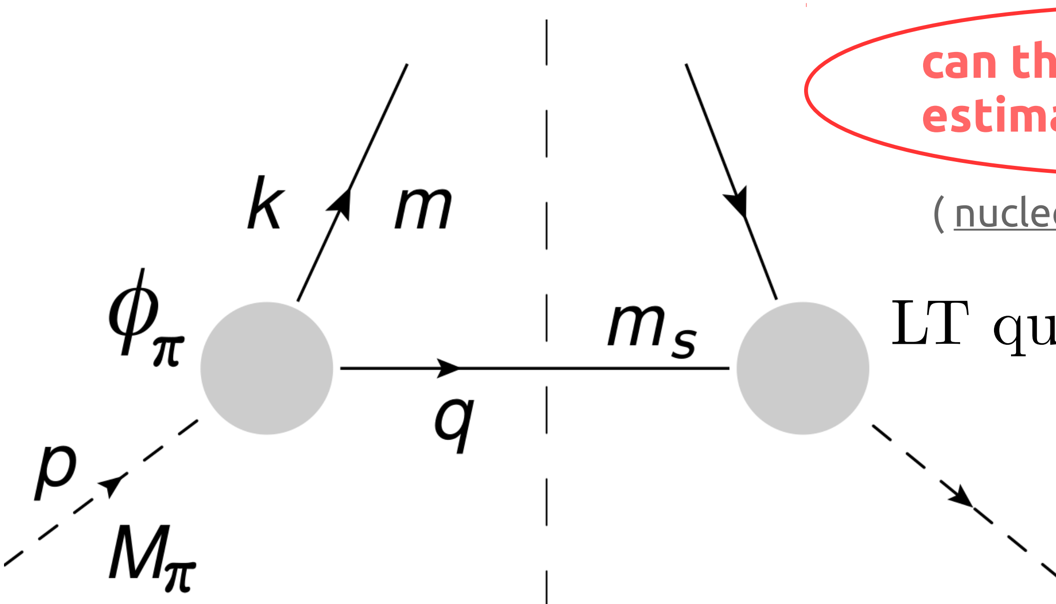
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-i\xi^- k^+} \langle p \left| \overline{\psi}(\xi^-)\gamma^+ \mathcal{U}(\xi^-,0)\psi(0) \right| p \rangle$$

...while matrix elements for lightlike correlations are not accessible on a Euclidean Lattice, *quasi-PDFs* are:

Ji, PRL**110**, 262002 (2013).

$$\widetilde{q}(x,\mu^2,p_z) = \int \frac{d\xi_z}{4\pi} e^{-i\xi_z k_z} \langle p \left| \overline{\psi}(\xi_z)\gamma^z \mathcal{U}(\xi_z,0)\psi(0) \right| p \rangle$$

these differ from the exact PDFs by power-suppressed corrections of order $\mathcal{O}\left(\frac{\Lambda^2}{p_z^2}, \frac{M^2}{p_z^2}\right)$



can these effects be estimated / controlled ?

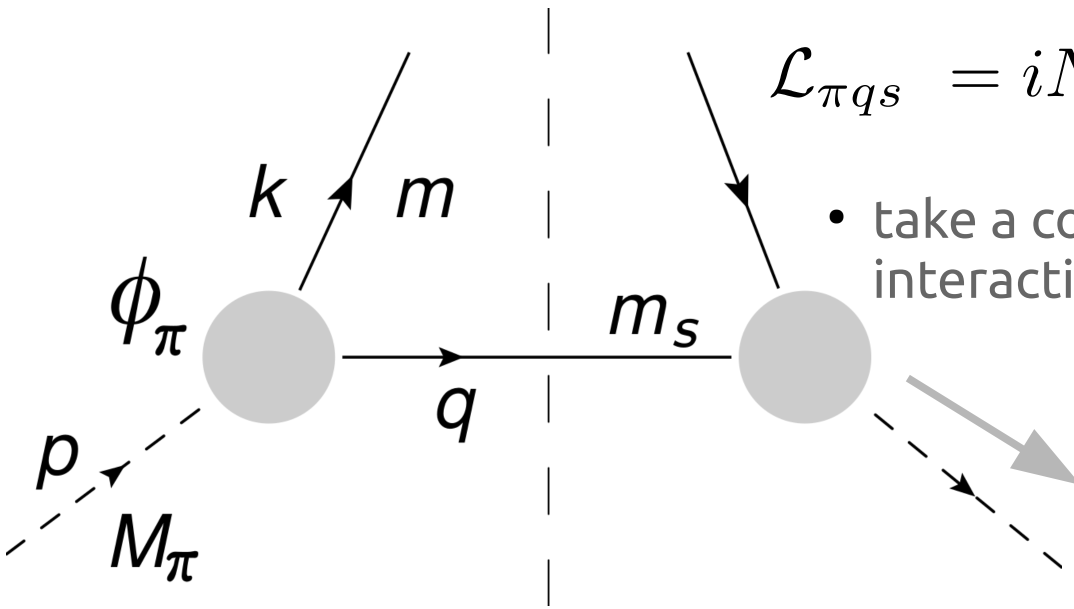
(nucleon: Gamberg et al., PLB743 (2015) 112.)

$$\text{LT quark-line cut} \longrightarrow \frac{\gamma^+}{2p^+}\delta\left(x - \frac{k^+}{p^+}\right)$$

$$\frac{\gamma^z}{2p_z}\delta\left(x - \frac{k_z}{p_z}\right)$$

the “exact” pion light-front PDF via a constituent quark model

- first evaluate the LF pion valence PDF using a minimal model that couples the pion to its constituent quarks



$$\mathcal{L}_{\pi q s} = i N_\pi^{1/2}\, \overline{\psi}_q \gamma_5 \varphi_\pi \psi_s + \text{h.c.}$$

- take a covariant vertex factor for the quark-pion interaction consistent with high x DSE results:

$$q_v(x) \sim (1-x)^2$$

$$\left[\phi_\pi(k^2)\right]^2 \equiv \left[\Lambda_\pi^2 \Big/ \left(k^2 - \Lambda_\pi^2\right)\right]^{3/2}$$

$$q_\pi^{\mathrm{LF}}(x) = \frac{N_\pi}{2\,(2\pi)^4} \int dk^+ dk^- d^2 k_\perp \left(\frac{1}{2p^+}\right) \delta\left(x - \frac{k^+}{p^+}\right)$$

$$\times\ tr\Big(\gamma_5\,(\not{k}+m)\,\gamma^+(\not{k}+m)\gamma_5(-\not{q}+m_s)\Big)\ 2\pi\,\delta\big(q^2 - m_s^2\big) \left[\frac{\phi_\pi(k^2)}{(k^2-m^2)}\right]^2$$

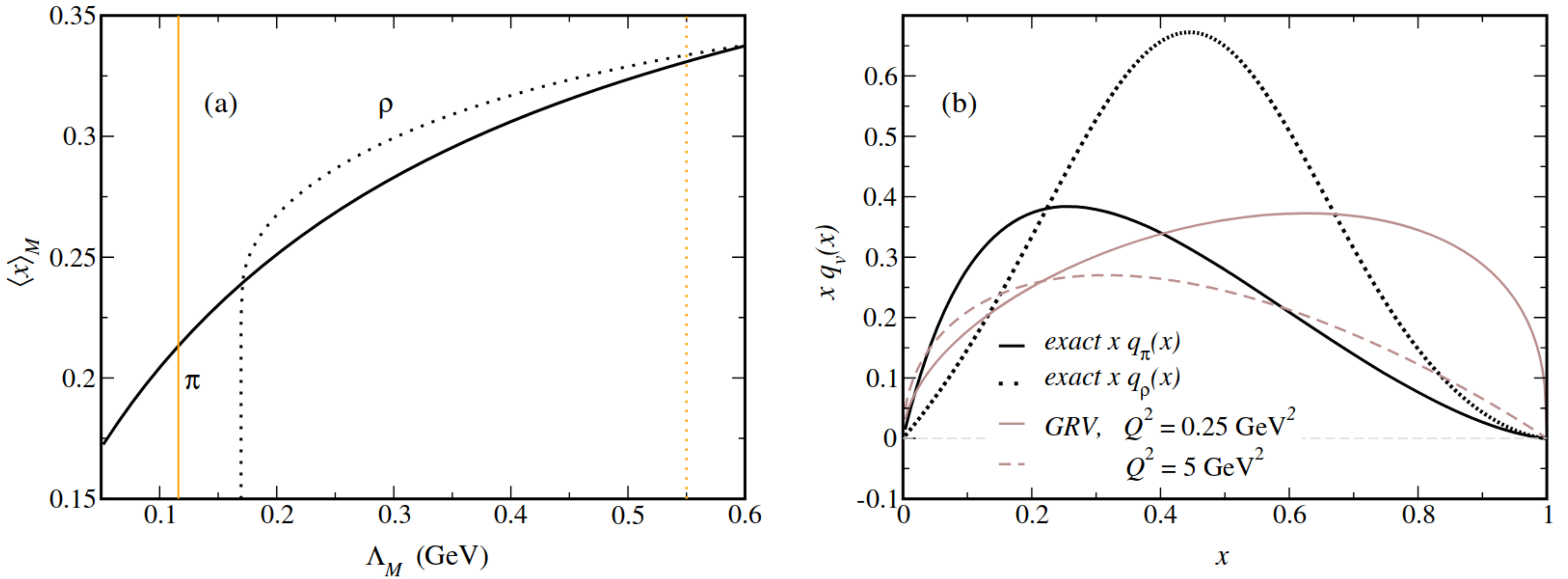
$$q = p - k$$

$$q_\pi^{\mathrm{LF}}(x) = \frac{N_\pi}{8\pi^2} \int \frac{dk_\perp^2}{x^2(1-x)} \left\{k_\perp^2 + \big(x m_s + (1-x)\, m\big)^2\right\} \left[\frac{\phi_\pi(t_\pi)}{(M_\pi^2 - \hat{s})}\right]^2$$

determining the pion SF model parameters

- for the pion, masses can be fixed to physical or constituent values:

$$M_\pi = 0.139\,\mathrm{GeV},\ \ m = M/3 \approx 0.33\,\mathrm{GeV}$$



$$\langle x \rangle_\pi \;=\; \int_0^1 dx\, x q_\pi^{\mathrm{LF}}(x) \;=\; 0.214(15)\left({}^{+12}_{-9}\right)^{*} \quad\longrightarrow\quad \Lambda_\pi \;=\; 0.116\,\mathrm{GeV}$$

*LQCD 1st moment calculation: Abdel-Rehim et al., PRD**92**, 114513 (2015).

- the overall strength is set by a **normalization condition** such that the model is then completely determined

$$N_\pi \;=\; 1 \Big/ \int dx\, q_\pi^{\mathrm{LF}}(x)$$

the corresponding pion quasi-PDF may then be found:

$$\tilde{q}_\pi(x,p_z) \;=\; \frac{N_\pi}{(2\pi)^4}\int dk^0 dk_z\, d^2k_\perp \left(\frac{1}{2p_z}\right)\,\delta\left(x-\frac{k_z}{p_z}\right)$$

$$\times\; tr\Big(\gamma_5\,(\not{k}+m)\,\gamma^z(\not{k}+m)\gamma_5(-\not{q}+m_s)\Big)\;2\pi\,\delta\big(q^2-m_s^2\big)\left[\frac{\phi_\pi(k^2)}{(k^2-m^2)}\right]^2$$

- now, integrating delta functions introduces explicit dependence on p_z ——

$$\delta\left(q^2-m_s^2\right) \;=\; \frac{1}{2\left(p^0-k^0\right)}\,\delta\left(p^0-k^0-\sqrt{m_s^2+k_\perp^2+(1-x)^2p_z^2}\right)$$

$$\tilde{q}_\pi(x,p_z) \;=\; \frac{N_\pi}{4\pi^2}\int\frac{dk_\perp^2}{2(1-x)\mu_s}\;\left\{2x\left(mm_s+(\tilde{q}\cdot\tilde{k}_\pi)\right)+(m^2-\tilde{k}_\pi^2)\,(1-x)\right\}$$

$$\times\left[\frac{\phi_\pi(\tilde{k}_\pi^2)}{(M_\pi^2+m_s^2-m^2+2(1-x)\big(1-\mu_\pi\,\mu_s\big)}\right]^2$$

(the main result for the pion quasi-PDF)

$$\mu_\pi \;\equiv\; \sqrt{1+\frac{M_\pi^2}{p_z^2}}$$

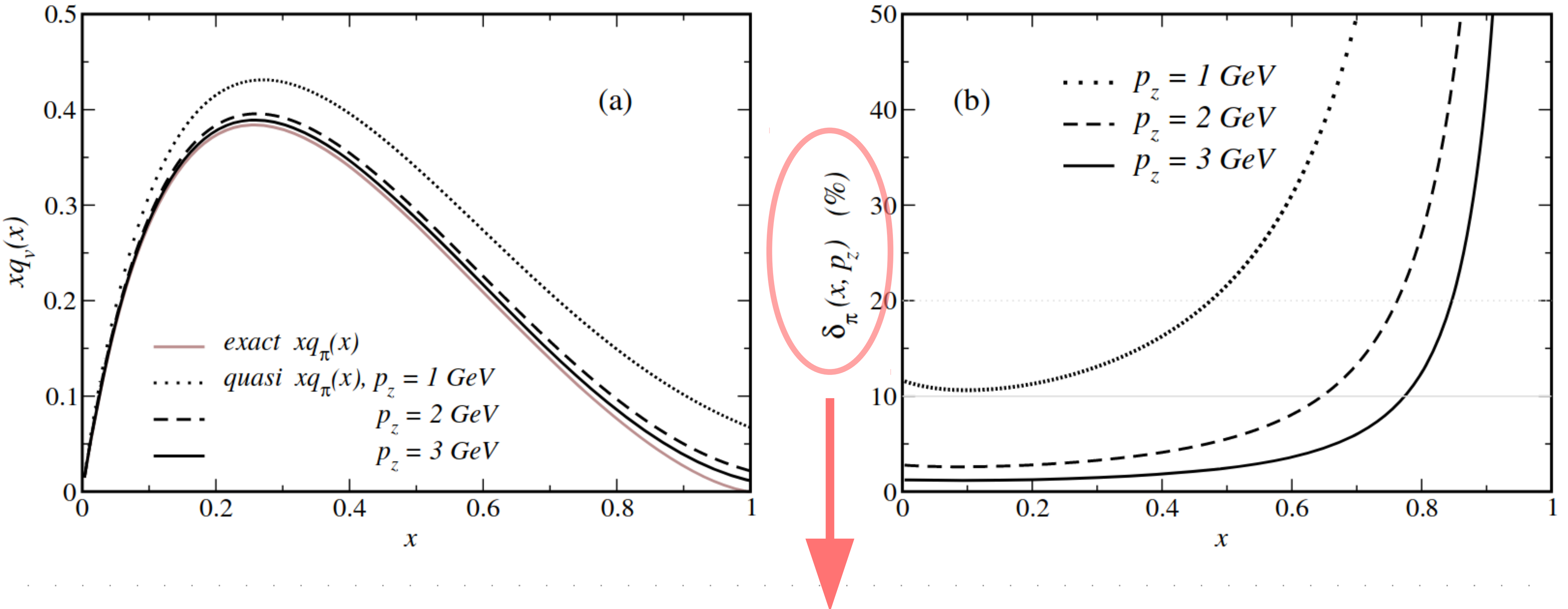
$$\mu_s \;\equiv\; \sqrt{1+\frac{m_s^2+k_\perp^2}{(1-x)^2p_z^2}}$$

$$\tilde{k}_\pi^2 \;=\; M_\pi^2+m_s^2+2\,(1-x)\big(1-\mu_\pi\,\mu_s\big)\,p_z^2$$

→ compare π quasi-/PDFs for several p_z

- we observe the expected behavior: at infinite boost, meson quasi-PDFs match onto the exact result,

$$\lim_{p_z\to\infty} \widetilde{q}_M(x,p_z) \;=\; q_M^{\rm LF}(x)$$



- away from this limit, we compute the LaMET deviations from the LF PDF:

$$\delta_M(x,p_z) \;\equiv\; \frac{\widetilde{q}_M(x,p_z)}{q_M^{\rm LF}(x)} - 1$$

→ even at fairly modest p_z these corrections can be $\lesssim 10\%$!

conclusions and future directions

- understanding the nucleon's structure remains *the* challenge for hadronic physics, but **light-front methods** can help
  - → can construct interpolating models that access the flavor structure of the proton wave function
  - → this can be extended to charm!
- we have established a close connection between $F_{2,\mathrm{IC}}^{c\bar{c}}$ and $\sigma_{c\bar{c}}$
  - → to exploit this connection, **more experimental information** is required, but diverse channels are/will be available (e.g., at EIC)
- LaMET techniques hold promise for computing the valence **quasi-distributions** of the pion, $\widetilde{q}_\pi(x)$, and *models can give guidance*

▪ **more broadly, modeling provides *a tool* as we confront nucleon PDFs, cf.**

choice/motivation of **input parametrizations** and fit results

possible constraints from **novel channels** (e.g., elastic information)

potential for **new measurements** to inform global analyses

THANKS