Chirally constraining the nucleon light-cone wave function

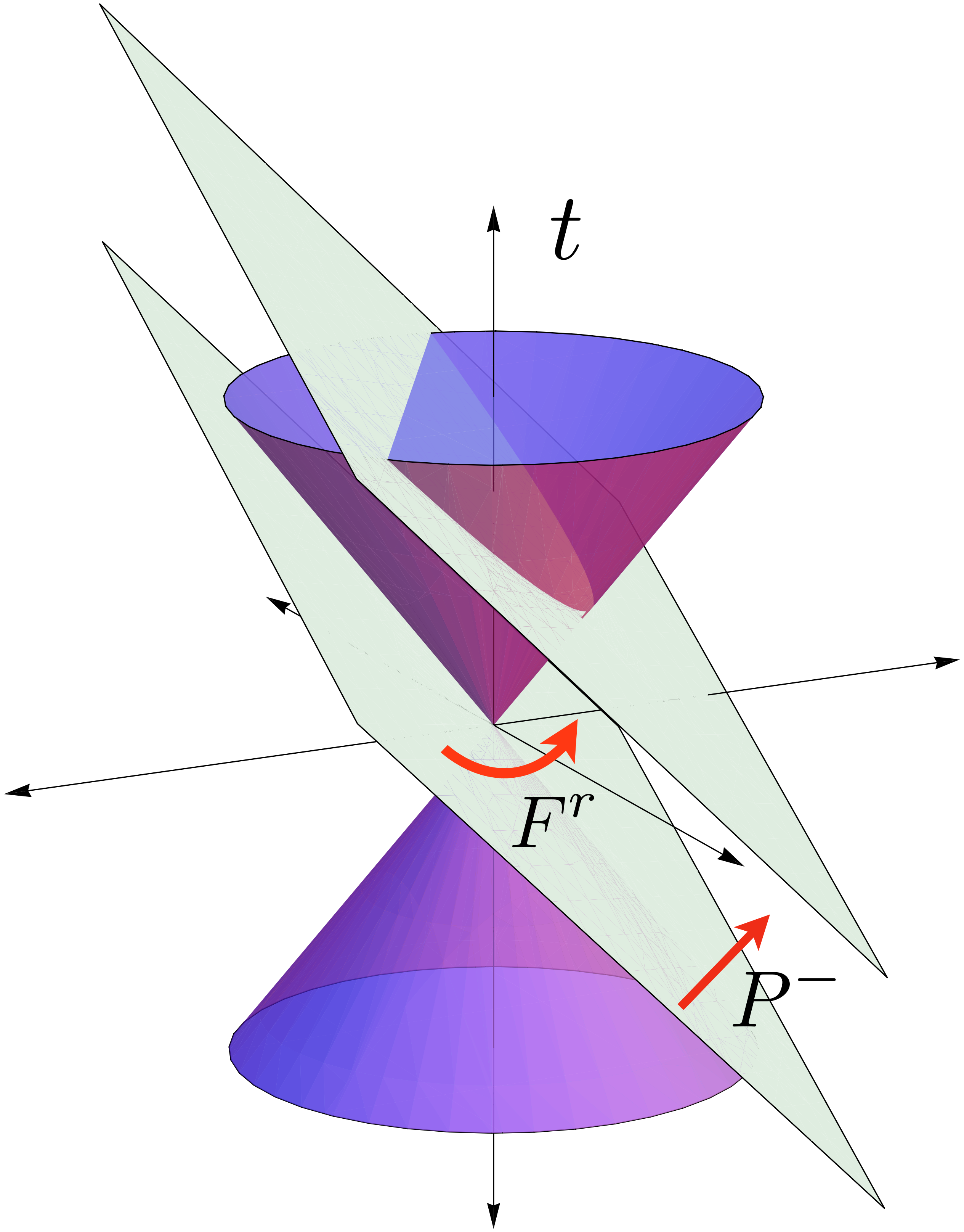
Silas Beane

LC2018 JLab 5/15/2018

Outline

- Null-plane nomenclature
- Chiral symmetry breaking
- Current algebra and sum rules
- QCD on a null plane
- The nucleon wavefunction

Null-plane Poincare′ generators



kinematical:

P^+, P_r, K_3, E_r, and J_3

$$P^+ = \frac{1}{\sqrt{2}} (P^0 + P^3)$$

$$E_r = \frac{1}{\sqrt{2}} (K_r + \epsilon_{rs} J_s)$$

Dynamical:

P^- and F_r

$$P^- = \frac{1}{\sqrt{2}} (P^0 - P^3)$$

$$F_r = \frac{1}{\sqrt{2}} (K_r - \epsilon_{rs} J_s)$$

Reduced Hamiltonians

Leutwyler, Stern, '78

✓ Using Wigner's method one can find dynamical generators valid in any frame. Useful to express as *products* of generators which satisfy $U(2)$ algebra.

Energy: M^2 **Spin:** $M\mathcal{J}_r$

$$\boxed{\begin{array}{rcl} [\,\mathcal{J}_3\,,\,M\mathcal{J}_r\,] &=& i\,\epsilon_{rs}M\mathcal{J}_s \quad , \quad [\,\mathcal{J}_3\,,\,M^2\,] = 0 \\ [\,M\mathcal{J}_r\,,\,M\mathcal{J}_s\,] &=& i\,\epsilon_{rs}M^2\mathcal{J}_3 \quad , \quad [\,M^2\,,\,M\mathcal{J}_r\,] = 0 \end{array}}$$

$$\mathcal{J}_3\,|\,p^+\,,\,\mathbf{p}_\perp\,;\lambda\,,\,n\,\rangle \;=\; \lambda\,|\,p^+\,,\,\mathbf{p}_\perp\,;\lambda\,,\,n\,\rangle$$

helicity inner variables

✓ Clean separation of kinematical and dynamical effects.

The standard picture: Nambu-Goldstone

$$G \;=\; SU(N)_L \times SU(N)_R \;\longrightarrow\; SU(N)_F$$

$$Q_\alpha |\, \Omega \,\rangle \;\neq\; 0$$

Nambu-Goldstone realization of G

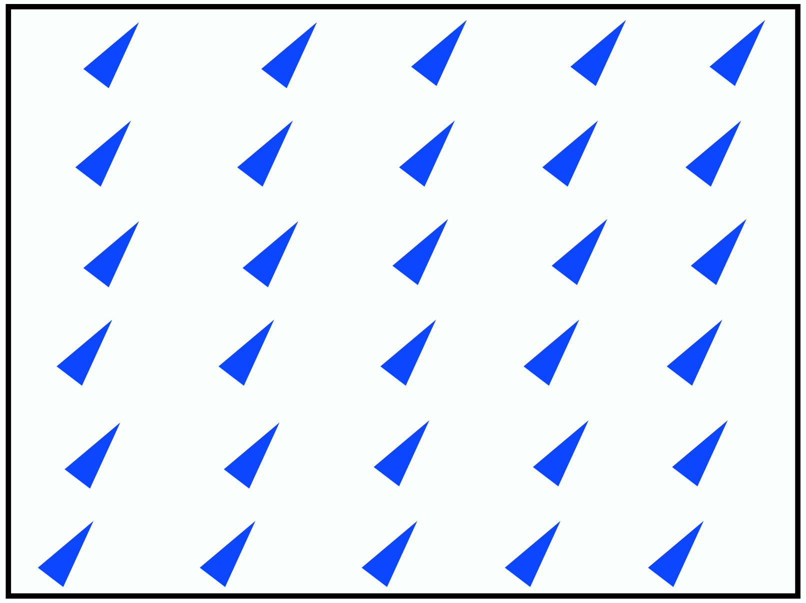
ground state is asymmetric

$$\langle\, \Omega |\, \mathcal{O}_{\not{G}} \,|\, \Omega \,\rangle \;\neq\; 0$$ "order parameter"

spectrum contains Goldstone bosons

and multiplets of $SU(N)_F$

charges are conserved



✓ But this picture is a matter of convention in *relativistic theories* of quantum mechanics.

❖ There is an <u>implicit assumption.</u>

Chiral symmetry charges

- Generally, symmetry charges are constructed from:

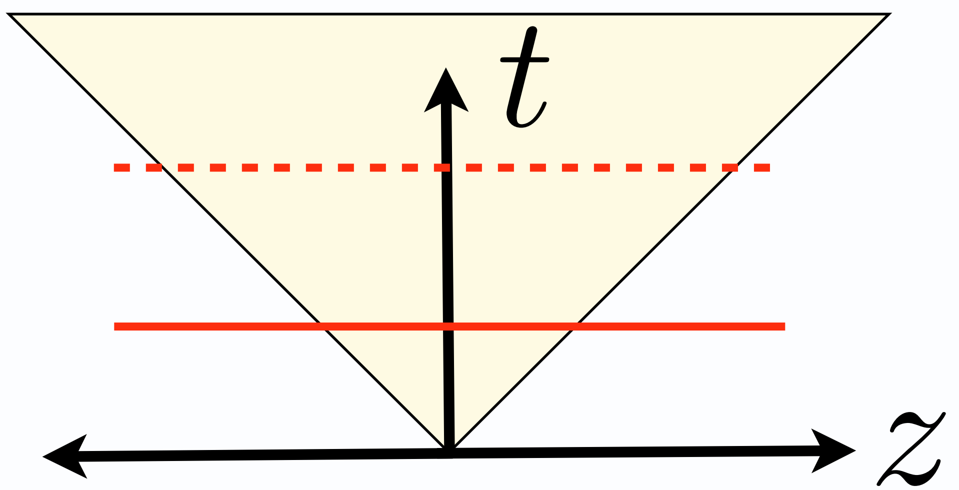
$$Q(n \cdot x) = \int d^4y \, \delta(n \cdot (x - y)) \, n \cdot J(y)$$

  - n chooses the quantization surface

$$n \cdot x = n_0 x_0 - n_i x_i \qquad i = 1, 2, 3$$

- Choose: $n^\mu = (1, 0, 0, 0)$

$$Q_\alpha = \int d^3x J^0(x)$$

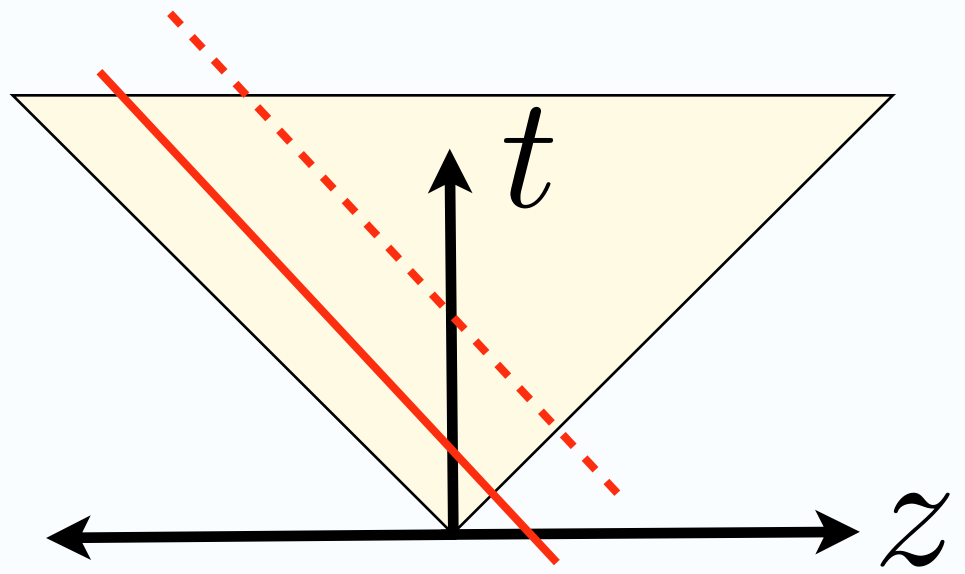


- Dynamics: evolution of parallel spaces at instants of time t

There is an alternate, physically-equivalent viewpoint

✓ Choose: $n^\mu \equiv \frac{1}{\sqrt{2}}(1,0,0,-1)$

$$\tilde{Q}_\alpha = \int d^3x \tilde{J}^+_\alpha$$



✓ Dynamics: evolution of parallel spaces tangent to light cone

$\tilde{Q}_\alpha \vert 0 \rangle = 0$
Nambu-Goldstone realization of G ground state is symmetric $\langle 0 \vert \mathcal{O}_{\not G} \vert 0 \rangle = 0$ no "order parameter" $\langle 0 \vert \mathcal{O}_{\mathbf{1}} \vert 0 \rangle \neq 0$ vacuum is non-trivial! spectrum contains Goldstone bosons and multiplets of $SU(N)_F$ charges are NOT conserved

Chiral Symmetry

- Theory with a chiral symmetry:

$$G = SU(N)_L \otimes SU(N)_R$$

- Null-plane currents from Noether procedure:

$$\tilde{J}^{\mu}_{5\alpha}(x) \quad , \quad \tilde{J}^{\mu}_{\alpha}(x)$$

$$\tilde{F}_{(5)\alpha}(x) \;=\; \tilde{F}_{(5)\alpha}(x^+, \boldsymbol{x}_\perp) \;=\; \int dx^- \; \tilde{J}^{+}_{(5)\alpha}(x)$$

- Current algebra:

$$\boxed{\begin{aligned}
[\,\tilde{F}_\alpha(x)\,,\,\tilde{F}_\beta(y)\,]|_{x^+=y^+} \;&=\; i\, f_{\alpha\beta\gamma}\, \tilde{F}_\gamma(x)\delta^2(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp) \\
[\,\tilde{F}_{5\alpha}(x)\,,\,\tilde{F}_\beta(y)\,]|_{x^+=y^+} \;&=\; i\, f_{\alpha\beta\gamma}\, \tilde{F}_{5\gamma}(x)\delta^2(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp) \\
[\,\tilde{F}_{5\alpha}(x)\,,\,\tilde{F}_{5\beta}(y)\,]|_{x^+=y^+} \;&=\; i\, f_{\alpha\beta\gamma}\, \tilde{F}_\gamma(x)\delta^2(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)
\end{aligned}}$$

✓ Symmetry generators:

$$\tilde{Q}_\alpha \;=\; \int d^2\boldsymbol{x}_\perp\, \tilde{F}^+_\alpha(x) \qquad\qquad \tilde{Q}_{5\alpha}(x^+) \;=\; \int d^2\boldsymbol{x}_\perp\, \tilde{F}^+_{5\alpha}(x)$$

$$\tilde{Q}^\alpha\,|\,0\,\rangle \;=\; \tilde{Q}^\alpha_5\,|\,0\,\rangle \;=\; 0$$

❖ Not conserved!

❖ No symmetry breaking condensate!

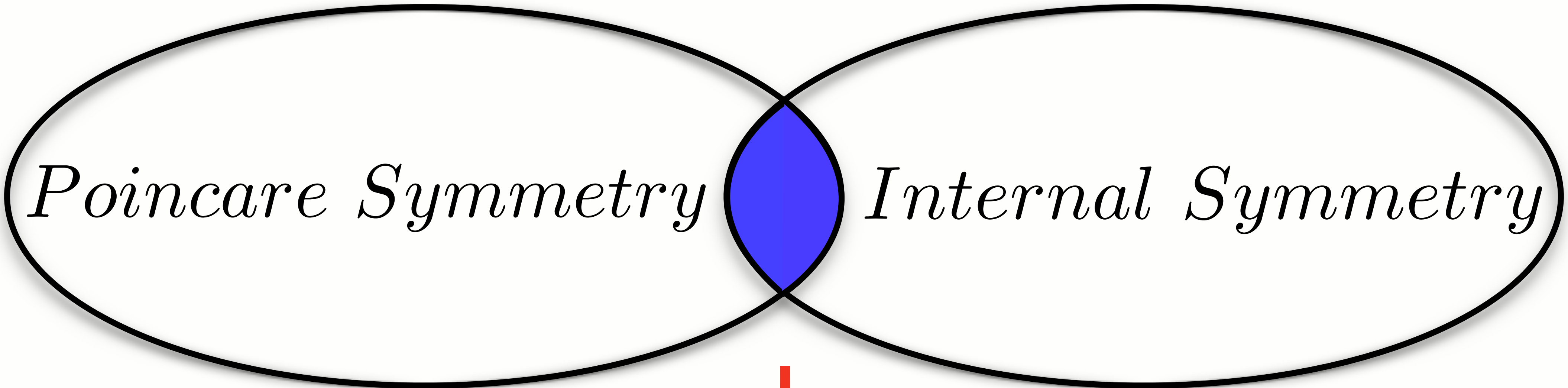
✓ Current moments:

$$\tilde{d}^r_{(5)\alpha}(x^+) \;=\; \int d^2\boldsymbol{x}_\perp\, x^r\, \tilde{F}^+_{(5)\alpha}(x)$$

$$\tilde{r}^{rs}_{(5)\alpha}(x^+) \;=\; \int d^2\boldsymbol{x}_\perp\, x^r x^s\, \tilde{F}^+_{(5)\alpha}(x)$$

$$\vdots$$

Chiral symmetry breaking on a null-plane



$$[\,\mathcal{J}_3\,,\,M\mathcal{J}_r\,] \;=\; i\,\epsilon_{rs}M\mathcal{J}_s \quad , \quad [\,\mathcal{J}_3\,,\,M^2\,] = 0$$

$$[\,M\mathcal{J}_r\,,\,M\mathcal{J}_s\,] \;=\; i\,\epsilon_{rs}M^2\mathcal{J}_3 \quad , \quad [\,M^2\,,\,M\mathcal{J}_r\,] = 0$$

$$[\,\tilde{Q}^\alpha\,,\,\tilde{Q}^\beta\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma \quad , \quad [\,\tilde{Q}_5^\alpha(x^+)\,,\,\tilde{Q}^\beta\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}_5^\gamma(x^+)$$

$$[\,\tilde{Q}_5^\alpha(x^+)\,,\,\tilde{Q}_5^\beta(x^+)\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma$$

$$SU(N)_R \otimes SU(N)_L \longrightarrow SU(N)_F$$

$$[\,\tilde{Q}_\alpha^5(x^+)\,,\,M^2] \;=\; -2i\,P^+\int dx^-\,d^2\boldsymbol{x}_\perp\,\partial_\mu \tilde{J}^\mu_{5\alpha}(x^-,\vec{x}_\perp,x^+)$$

$$[\,\tilde{Q}_\alpha^5(x^+)\,,\,M\mathcal{J}_r] \;=\; i\,\epsilon_{rs}\,P^+\int dx^-\,d^2\boldsymbol{x}_\perp\,\Gamma_s\,\partial_\mu \tilde{J}^\mu_{5\alpha}(x^-,\vec{x}_\perp,x^+)$$

$$\Gamma_s \equiv E_s - P^+ x_s$$

✓ Immediately implies Goldstone's theorem.

Current algebra sum rules

T.Hobbs,SB, '16

$$[\,\tilde{Q}_{5\alpha}(x^+)\,,\,\tilde{Q}_{5\beta}(x^+)\,] = i\,\epsilon_{\alpha\beta\gamma}\,\tilde{Q}_\gamma \quad\longrightarrow\quad \boxed{\frac{2F_\pi^2}{\pi}\int_{\bar\nu_T}^\infty \frac{d\bar\nu}{\bar\nu}\left[\sigma^{\pi^-p}(\bar\nu)-\sigma^{\pi^+p}(\bar\nu)\right] + g_A^2 = 1}$$

Adler-Weisberger

$$[\,\tilde{d}^r_\alpha(x^+)\,,\,\tilde{d}^s_\beta(x^+)\,] = i\,\epsilon_{\alpha\beta\gamma}\,\tilde{r}^{rs}_\gamma(x^+)$$

$$\longrightarrow\quad \boxed{\frac{1}{3}\langle r^2\rangle_1^V = \left(\frac{\kappa_V}{2m_N}\right)^2 + \frac{1}{2\pi^2\alpha}\int_{\bar\nu_T}^\infty \frac{d\bar\nu}{\bar\nu}\left(2\sigma^V_{1/2}(\bar\nu) - \sigma^V_{3/2}(\bar\nu)\right)}$$

Cabbibo-Radicatti

$$\longrightarrow\quad \boxed{\left(\frac{\kappa_V}{2m_N}\right)^2 = \frac{1}{2\pi^2\alpha}\int_{\bar\nu_T}^\infty \frac{d\bar\nu}{\bar\nu}\left(\sigma^V_P(\bar\nu) - \sigma^V_A(\bar\nu)\right)}$$

Gerasimov-Drell-Hearn

$$[\,\tilde{Q}_{5\alpha}(x^+)\,,\,\tilde{d}^r_\beta(x^+)\,] = i\,\epsilon_{\alpha\beta\gamma}\,\tilde{d}^r_{5\gamma}(x^+) \quad\longrightarrow\quad \boxed{g_A\frac{\kappa_V}{2m_N} = \frac{4F_\pi}{e\pi}\int_{\bar\nu_T}^\infty \frac{d\bar\nu}{\bar\nu}\mathrm{Im}\;A_1^+(\bar\nu,0)}$$

Fubini-Furlan-Rossetti I

$$[\,\tilde{Q}_{5\alpha}(x^+)\,,\,\tilde{r}^{rs}_\beta(x^+)\,] = i\,\epsilon_{\alpha\beta\gamma}\,\tilde{r}^{rs}_{5\gamma}(x^+) \quad\longrightarrow\quad \boxed{\frac{g_A}{6}\left(\langle r^2\rangle_1^V - \langle r^2\rangle^A\right) = \frac{4F_\pi}{e\pi}\int_{\bar\nu_T}^\infty \frac{d\bar\nu}{\bar\nu}\mathrm{Im}\;A_6^-(\bar\nu,0)}$$

Fubini-Furlan-Rossetti II

- Five sum rules for cross-sections!

Regge limit amplitudes

$\pi N \to \pi N$

$$\boxed{\frac{\mathcal{T}_{[\alpha\beta]}(\bar\nu,0)}{\bar\nu}\Big|_{\bar\nu\to\infty} = \frac{2}{F_\pi^2}\int \frac{dk^+ d^2\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\langle\, N;k'\,|\Big(i\epsilon_{\alpha\beta\gamma}\tilde{Q}_\gamma - [\,\tilde{Q}_{5\alpha}(x^+)\,,\,\tilde{Q}_{5\beta}(x^+)\,]\Big)\,|\,N;k\,\rangle}$$

$\gamma N \to \gamma N$

$$\boxed{\bar\nu\; T_{1[\alpha\beta]}(\bar\nu,0)\Big|_{\bar\nu\to\infty} = -\,e^2\delta_{rs}\int \frac{dk^+ d^2\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\langle\, N;k'|\Big(i\epsilon_{\alpha\beta\gamma}\tilde{r}^{rs}_\gamma(x^+) - [\tilde{d}^r_\alpha(x^+),\tilde{d}^s_\beta(x^+)]\Big)|N;k\,\rangle}$$

$\gamma N \to \gamma N$

$$\boxed{\big[T_{3(\alpha\beta)}(\bar\nu,0) + \bar\nu T_{4(\alpha\beta)}(\bar\nu,0)\big]_{\bar\nu\to\infty} = -\,\frac{ie^2}{2}\epsilon^{rs}\int \frac{dk^+ d^2\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\langle\, N;k'\,|[\,\tilde{d}^r_\alpha(x^+)\,,\,\tilde{d}^s_\beta(x^+)\,]|\,N;k\,\rangle}$$

$\gamma N \to \pi N$

$$\boxed{\mathcal{A}_{1(\alpha\beta)}(\bar\nu,0)\Big|_{\bar\nu\to\infty} = \frac{ie}{2F_\pi}e^r_\perp\int \frac{dk^+ d^2\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\langle\, N,\uparrow;k'\,|[\,\tilde{Q}_{5\alpha}(x^+)\,,\,\tilde{d}^r_\beta(x^+)\,]|\,N,\downarrow;k\,\rangle}$$

$\gamma^* N \to \pi N$

$$\boxed{\mathcal{A}_{6[\alpha\beta]}(\bar\nu,0)\Big|_{\bar\nu\to\infty} = -\,\frac{e}{4F_\pi}\delta^{rs}\int \frac{dk^+ d^2\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\langle N;k'|\Big(i\epsilon_{\alpha\beta\gamma}\tilde{r}^{rs}_{5\gamma}(x^+) - [\tilde{Q}_{5\alpha}(x^+),\tilde{r}^{rs}_\beta(x^+)]\Big)|N;k\rangle}$$

Gell-Mann-Oakes-Renner relation: instant form

✓ Match vacuum energy to chiral perturbation theory:

$$\mathbb{M}\,\frac{\partial\mathcal{E}_0}{\partial\mathbb{M}} \;=\; \mathbb{M}\,\frac{\partial\mathcal{E}_0^{\chi\mathrm{PT}}}{\partial\mathbb{M}}$$

$$\boxed{-\,\mathbb{M}\,\langle\,\Omega\,|\,\bar{\psi}\psi\,|\,\Omega\,\rangle \;=\; N\,M_\pi^2\,F_\pi^2 \;+\;\ldots}$$

- $(\bar{\mathbf{N}},\mathbf{N})\oplus(\mathbf{N},\bar{\mathbf{N}})$ breaking

How can this relation arise in the front form where there are no symmetry breaking condensates?

Chiral symmetry in null-plane QCD

- Chiral symmetry acts on dynamical quark field only:

$$\psi_+ \to e^{-i\theta_\alpha T_\alpha}\psi_+ \quad , \quad \psi_+ \to e^{-i\theta_\alpha T_\alpha \gamma_5}\psi_+$$

- Axial current and its divergence and chiral charge:

$$\begin{aligned}\tilde{J}^{\mu}_{5\alpha} &= J^{\mu}_{5\alpha} - i\bar{\psi}\gamma^\mu\gamma^+\gamma_5\, T_\alpha\, \mathbb{M}\,\frac{1}{\partial^+}\psi_+ \\ \partial_\mu \tilde{J}^{\mu}_{5\alpha} &= \bar{\psi}_+\gamma^+\gamma_5\, T_\alpha\, \mathbb{M}\,\frac{1}{\partial^+}\,(\boldsymbol{\gamma}^r g\, t^a \boldsymbol{A}^r_a)'\,\psi_+\end{aligned}$$

$$\tilde{Q}^{\alpha}_{5}\,|\,0\,\rangle = 0$$

counts quarks and antiquarks separately.

- Symmetry-breaking Hamiltonians:

$$\boxed{\begin{aligned} M^2_{(\mathbf{N},\mathbf{N})} &= -iP^+\int dx^-\, d^2\boldsymbol{x}_\perp\, \bar{\psi}_+\gamma^+\,\frac{1}{\partial^+_{\mathbb{M}}}\,(\boldsymbol{\gamma}^r g\, t^a \boldsymbol{A}^r_a)'\,\psi_+ \\ (M\mathcal{J}_r)_{(\mathbf{N},\mathbf{N})} &= i\tfrac{1}{2}\epsilon_{rs}\, P^+\int dx^-\, d^2\boldsymbol{x}_\perp\, \Gamma_s\, \bar{\psi}_+\gamma^+\,\frac{1}{\partial^+_{\mathbb{M}}}\,(\boldsymbol{\gamma}^r g\, t^a \boldsymbol{A}^r_a)'\,\psi_+ \end{aligned}}$$

- $(\bar{\mathbf{N}},\mathbf{N})\oplus(\mathbf{N},\bar{\mathbf{N}})$ breaking

$$\frac{1}{\partial^+_{\mathbb{M}}} \equiv \mathbb{M}\,\frac{1}{\partial^+}$$

Gell-Mann-Oakes-Renner relation: front form

Wu,Zhang, '04

- Match vacuum energy to chiral perturbation theory:

$$\mathbb{M}\,\langle\, 0\,|\,\frac{\partial T^{-+}}{\partial \mathbb{M}}\,|\,0\,\rangle \;=\; \mathbb{M}\,\frac{\partial \tilde{\mathcal{E}}_0}{\partial \mathbb{M}} \;=\; \mathbb{M}\,\frac{\partial \mathcal{E}_0}{\partial \mathbb{M}} \;=\; \mathbb{M}\,\frac{\partial \mathcal{E}_0^{\chi\mathrm{PT}}}{\partial \mathbb{M}}$$

$$\boxed{-\,\mathbb{M}\,\langle\, 0\,|\, i\,\bar{\psi}_+\gamma^+\frac{1}{\partial^+_{\mathbb{M}}}\psi_+\,|\,0\,\rangle \;=\; N\,M_\pi^2\,F_\pi^2 \;+\;\ldots}$$

- Chiral singlet: $(\mathbf{1},\mathbf{1})$

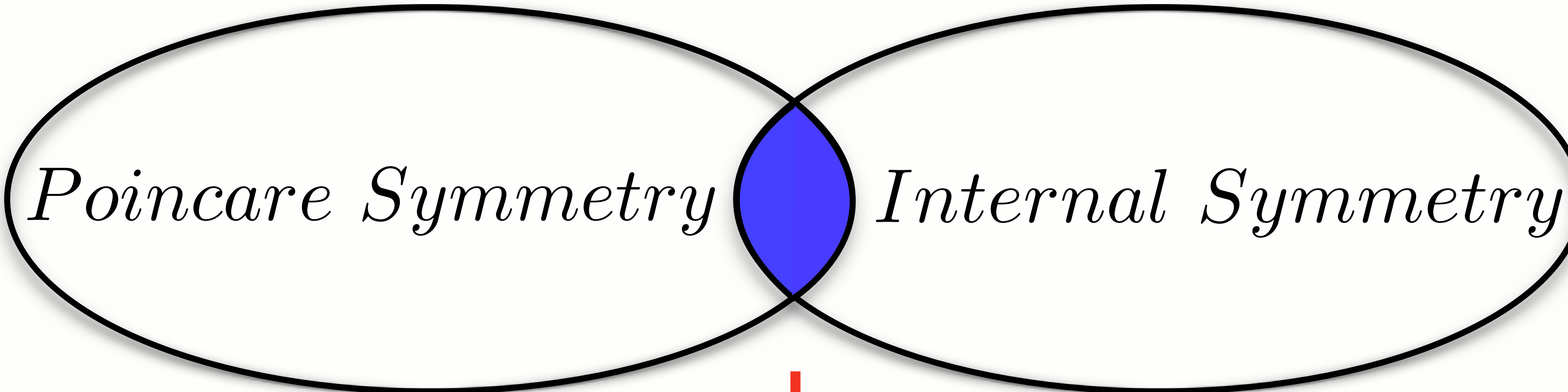
- <u>General prescription</u>:

$$[\,\tilde{Q}_5^\alpha\,,\,\psi\,] \;=\; -\gamma_5\,T^\alpha\,\psi \;-\; i\,\gamma_5\,\gamma^+\,T^\alpha\,\frac{1}{\partial^+_{\mathbb{M}}}\psi$$

$$\psi_R\,,\,\psi_L \;\in\; (\mathbf{1},\mathbf{N})\oplus(\mathbf{N},\mathbf{1}) \quad , \quad \psi_R^\dagger\,,\,\psi_L^\dagger \;\in\; (\mathbf{1},\bar{\mathbf{N}})\oplus(\bar{\mathbf{N}},\mathbf{1})$$

$$\bar{\psi}\psi \in\; (\bar{\mathbf{N}},\mathbf{N})\oplus(\mathbf{N},\bar{\mathbf{N}})\oplus(\mathbf{1},\mathbf{1})\oplus\ldots$$

Chiral symmetry breaking in null-plane QCD



$$[\,\mathcal{J}_3\,,\,M\mathcal{J}_r\,] \;=\; i\,\epsilon_{rs}M\mathcal{J}_s \quad , \quad [\,\mathcal{J}_3\,,\,M^2\,] = 0$$

$$[\,M\mathcal{J}_r\,,\,M\mathcal{J}_s\,] \;=\; i\,\epsilon_{rs}M^2\mathcal{J}_3 \quad , \quad [\,M^2\,,\,M\mathcal{J}_r\,] = 0$$

$$[\,\tilde{Q}^\alpha\,,\,\tilde{Q}^\beta\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma \quad , \quad [\,\tilde{Q}_5^\alpha(x^+)\,,\,\tilde{Q}^\beta\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}_5^\gamma(x^+)$$

$$[\,\tilde{Q}_5^\alpha(x^+)\,,\,\tilde{Q}_5^\beta(x^+)\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma$$

$$SU(N)_R \otimes SU(N)_L \longrightarrow SU(N)_F$$

$$\boxed{\mathcal{P}^{\alpha\beta;\mu\nu}\,[\tilde{Q}_5^\mu\,,\,[\tilde{Q}_5^\nu\,,\,M^2]] \;=\; \mathcal{P}^{\alpha\beta;\mu\nu}\,[\tilde{Q}_5^\mu\,,\,[\tilde{Q}_5^\nu\,,\,M\mathcal{J}_\pm]] \;=\; 0}$$

$$[\,\tilde{Q}_5^\alpha\,,\,M^2\,] \;\neq\; 0 \qquad\qquad [\,\tilde{Q}_5^\alpha\,,\,M\mathcal{J}_\pm\,] \;\neq\; 0$$

$$\mathcal{P}^{\alpha\beta;\mu\nu} \;\equiv\; \delta^{\alpha\nu}\delta^{\beta\mu} \;-\; \frac{1}{N^2-1}\,\delta^{\alpha\beta}\delta^{\mu\nu} \;-\; \frac{N}{N^2-4}\,d^{\alpha\beta\gamma}d^{\mu\nu\gamma}$$

❖ projects onto exotic channel.

The nucleon light-cone wave function

- The chiral basis:

$$\begin{aligned} \psi_{+R} &= \psi_\uparrow \in (\mathbf{1},\mathbf{2}) \\ \psi_{+L} &= \psi_\downarrow \in (\mathbf{2},\mathbf{1}) \end{aligned} \qquad (\mathcal{R}_L,\mathcal{R}_R) \in SU(2)_L\otimes SU(2)_R$$

- Baryon product states of definite quark helicity:

$$\lambda = \pm\tfrac{1}{2} \quad \begin{cases} \psi_\uparrow\psi_\uparrow\psi_\downarrow \subset (\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{3}) \\ \psi_\downarrow\psi_\downarrow\psi_\uparrow \subset (\mathbf{1},\mathbf{2})\oplus(\mathbf{3},\mathbf{2}) \end{cases}$$

$$\lambda = \pm\tfrac{3}{2} \quad \begin{cases} \psi_\uparrow\psi_\uparrow\psi_\uparrow \subset (\mathbf{1},\mathbf{2})\oplus(\mathbf{1},\mathbf{2})\oplus(\mathbf{1},\mathbf{4}) \\ \psi_\downarrow\psi_\downarrow\psi_\downarrow \subset (\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{1})\oplus(\mathbf{4},\mathbf{1}) \end{cases}$$

- General basis:

$$|\,(\mathcal{R}_\mathbf{1},\mathcal{R}_\mathbf{2})_\mathcal{R}\,\rangle\otimes|\,\lambda\,\rangle\otimes|\,\ell\,\rangle \;\equiv\; |\,(\mathcal{R}_\mathbf{1},\mathcal{R}_\mathbf{2})_\mathcal{R}\,,\,\lambda\,,\,\ell\,\rangle$$

ℓ is non valence quark helicity

✓ Three-quark Fock space expansion:

Ji,Ma,Yuan, '03

$$\boxed{|P\uparrow\rangle = |P\uparrow\rangle_{-3/2} + |P\uparrow\rangle_{-1/2} + |P\uparrow\rangle_{1/2} + |P\uparrow\rangle_{3/2}}$$

- E.g.

$$|P\uparrow\rangle_{-1/2} = \int d[1]d[2]d[3]\left((k_1^x + ik_1^y)\tilde{\psi}^{(3)}(1,2,3) + (k_2^x + ik_2^y)\tilde{\psi}^{(4)}(1,2,3)\right)$$
$$\times\frac{\epsilon^{abc}}{\sqrt{6}}\left(u^\dagger_{a\uparrow}(1)u^\dagger_{b\downarrow}(2)d^\dagger_{c\downarrow}(3) - d^\dagger_{a\uparrow}(1)u^\dagger_{b\downarrow}(2)u^\dagger_{c\downarrow}(3)\right)|0\rangle\ .$$

✓ Fock space expansion in chiral basis:

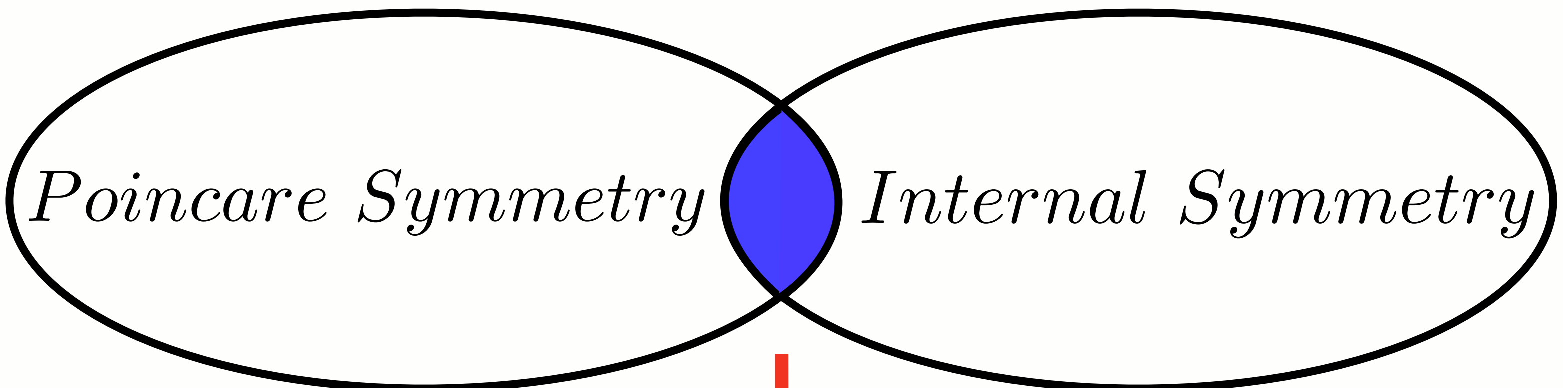
$$\boxed{\begin{aligned}|\,N\,;\,\tfrac{\mathbf{1}}{\mathbf{2}}\,\rangle \;=\; & |(\mathbf{2},\mathbf{1})\,,\,\tfrac{1}{2}\,,\,\mathbf{0}\rangle + |(\mathbf{2},\mathbf{3})_{\mathbf{2}}\,,\,\tfrac{1}{2}\,,\,\mathbf{0}\rangle + |(\mathbf{1},\mathbf{2})\,,\,-\tfrac{1}{2}\,,\,\mathbf{1}\rangle \\ & + |(\mathbf{3},\mathbf{2})_{\mathbf{2}}\,,\,-\tfrac{1}{2}\,,\,\mathbf{1}\rangle + |(\mathbf{1},\mathbf{2})\,,\,\tfrac{3}{2}\,,\,-\mathbf{1}\rangle + |(\mathbf{2},\mathbf{1})\,,\,-\tfrac{3}{2}\,,\,\mathbf{2}\rangle + \ldots\end{aligned}}$$

Relative weight of these components is not constrained by any QCD symmetry.

However, weights are constrained by the sum rules.

The Large N_c limit

- General solution to operator algebra is unknown
- Assume: $[\,\tilde{Q}_5^\alpha\,,\,M\,] \equiv \epsilon^\alpha \quad (\,\langle B'\,|\,\epsilon^\alpha\,|\,B\,\rangle \;\propto\; M_{B'} - M_B \;\sim\; 1/N_c\,)$



$$[\,\mathcal{J}_i\,,\,\mathcal{J}_j\,] = i\,\epsilon_{ijk}\,\mathcal{J}_k$$

$$[\tilde{Q}^\alpha\,,\,\tilde{Q}^\beta\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma \quad , \quad [\,\tilde{Q}_5^\alpha(x^+)\,,\,\tilde{Q}^\beta\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}_5^\gamma(x^+)$$

$$[\,\tilde{Q}_5^\alpha(x^+)\,,\,\tilde{Q}_5^\beta(x^+)\,] = i\,f^{\alpha\beta\gamma}\,\tilde{Q}^\gamma$$

$$SU(N)_R \otimes SU(N)_L \longrightarrow SU(N)_F$$

$$\mathcal{P}^{\alpha\beta;\mu\nu}\,[\tilde{Q}_5^\mu\,,\,[\tilde{Q}_5^\nu\,,\,\mathcal{J}_\pm]] \;=\; 0$$

non-trivial spin Hamiltonians

✦ Contracted spin-flavor

Weinberg, '94

✓ With $G^{\alpha 3} \equiv \tilde{Q}_5^{\alpha}$ one rewrites operator algebra as:

$$[\,G_{\alpha i}\,,\,G_{\beta j}\,] = i\,\delta_{ij}\,f_{\alpha\beta\gamma}\,\tilde{Q}_{\gamma} + \frac{2}{N}\,i\,\delta_{\alpha\beta}\,\epsilon_{ijk}\,\mathcal{J}_k + i\epsilon_{ijk}\,d_{\alpha\beta\gamma}\,G_{\gamma k}$$

$$[\,\tilde{Q}_{\alpha}\,,\,G_{\beta i}\,] = i\,f_{\alpha\beta\gamma}\,G_{\gamma i} \quad , \quad [\,\mathcal{J}_i\,,\,G_{\alpha j}\,] = i\,\epsilon_{ijk}\,G_{\alpha k}$$

$$[\,\tilde{Q}_{\alpha}\,,\,\tilde{Q}_{\beta}\,] = i\,f_{\alpha\beta\gamma}\,\tilde{Q}_{\gamma} \quad , \quad [\,\mathcal{J}_i\,,\,\mathcal{J}_j\,] = i\,\epsilon_{ijk}\,\mathcal{J}_k$$

❖ Algebra of $SU(2N)$

✓ Example: baryonic operators in $(\mathbf{2},\mathbf{3})\oplus(\mathbf{1},\mathbf{4})$

$SU(2)_L \otimes SU(2)_R$

$\lambda = 3/2$	$\psi_{+\uparrow}\psi_{+\uparrow}\psi_{+\uparrow}$	$\vert(\mathbf{1},\mathbf{4})\,,\,\frac{3}{2}\,,\,\mathbf{0}\rangle$
$\lambda = 1/2$	$\psi_{+\uparrow}\psi_{+\uparrow}\psi_{+\downarrow}$	$\vert(\mathbf{2},\mathbf{3}),\frac{1}{2},\mathbf{0}\rangle$
$\lambda = -1/2$	$\psi_{+\downarrow}\psi_{+\downarrow}\psi_{+\uparrow}$	$\vert(\mathbf{3},\mathbf{2}),-\frac{1}{2},\mathbf{0}\rangle$
$\lambda = -3/2$	$\psi_{+\downarrow}\psi_{+\downarrow}\psi_{+\downarrow}$	$\vert(\mathbf{4},\mathbf{1})\,,\,-\frac{3}{2}\,,\,\mathbf{0}\rangle$

$SU(4)$

$|\mathbf{20}\rangle$

❖ helicities are unrelated by chiral symmetry

❖ helicities are related by broken chiral symmetry!

Away from the Large N_c limit

✓ $(\mathbf{1},\mathbf{2}) \oplus (\mathbf{2},\mathbf{3}) \oplus (\mathbf{1},\mathbf{4}):$

$$\begin{pmatrix} |\, N_1 \,,\, \tfrac{\mathbf{1}}{\mathbf{2}} \,\rangle \\ |\, N_2 \,,\, \tfrac{\mathbf{1}}{\mathbf{2}} \,\rangle \end{pmatrix} = \begin{pmatrix} \sin\psi & \cos\psi \\ -\cos\psi & \sin\psi \end{pmatrix} \begin{pmatrix} |(\mathbf{1},\mathbf{2})\,,\, -\tfrac{1}{2}\,,\, \mathbf{1}\rangle \\ |(\mathbf{2},\mathbf{3})_{\mathbf{2}}\,,\, \tfrac{1}{2}\,,\, \mathbf{0}\rangle \end{pmatrix}$$

$$|\,\Delta\,,\, \tfrac{\mathbf{3}}{\mathbf{2}}\,\rangle \;=\; |(\mathbf{1},\mathbf{4})\,,\, \tfrac{3}{2}\,,\, \mathbf{0}\rangle$$
$$|\,\Delta\,,\, \tfrac{\mathbf{1}}{\mathbf{2}}\,\rangle \;=\; |(\mathbf{2},\mathbf{3})_{\mathbf{4}}\,,\, \tfrac{1}{2}\,,\, \mathbf{0}\rangle$$

✓ Axial matrix and helicity matrix:

$$\hat{G}_A \;=\; \begin{pmatrix} \tfrac{1}{3}\,(4+\cos 2\psi) & \tfrac{2}{3}\sin 2\psi \\ \tfrac{2}{3}\sin 2\psi & \tfrac{1}{3}\,(4-\cos 2\psi) \end{pmatrix} \qquad \widehat{\Delta\Sigma} \;=\; \begin{pmatrix} -\cos 2\psi & \sin 2\psi \\ \sin 2\psi & \cos 2\psi \end{pmatrix}$$

Nucleon spin decomposition

$$\tfrac{1}{2} \;=\; \tfrac{1}{2}\,\Delta\Sigma \;+\; L_q \;+\; J_q$$

$$\Delta\Sigma^N \;=\; 2\,\langle\, N \,|\, \mathcal{S}_3 \,|\, N \,\rangle \;=\; \cos 2\psi \;=\; 3\,(\, g_A \;-\; \tfrac{4}{3}\,)$$

$$L_q^N \;+\; J_q^N \;=\; \langle\, N \,|\, \mathcal{L}_3 \,|\, N \,\rangle \;=\; \sin^2\psi \;=\; \tfrac{3}{2}\,(\, \tfrac{5}{3} \;-\; g_A\,)$$

✓ Hamiltonians:

$$M^2 = \begin{array}{c|cc} & |\,(1{,}2)\,\rangle & |\,(2{,}3)\,\rangle \\ \langle\,(1{,}2)\,| & & \\ \langle\,(2{,}3)\,| & & \end{array} \begin{bmatrix} m^2_{\mathbf{1a}} & m^2_{\mathbf{22}} \\ m^2_{\mathbf{22}} & m^2_{\mathbf{1b}} \end{bmatrix} \qquad M\mathcal{J}^+ = \begin{array}{c|cc} & |\,(2{,}1)\,\rangle & |\,(3{,}2)\,\rangle \\ \langle\,(1{,}2)\,| & & \\ \langle\,(2{,}3)\,| & & \end{array} \begin{bmatrix} mj_{\mathbf{22a}} & 0 \\ 0 & mj_{\mathbf{22b}} \end{bmatrix}$$

$$M^2_{N_1}\cos^2\psi \;+\; M^2_{N_2}\sin^2\psi \;=\; M^2_\Delta$$

$$\left(J_{N_1}+\tfrac{1}{2}\right)(-1)^{J_{N_1}-1/2}\eta_{N_1}M_{N_1} \;=\; \left(J_{N_2}+\tfrac{1}{2}\right)(-1)^{J_{N_2}-1/2}\eta_{N_2}M_{N_2}$$

implies unwanted degeneracy!

Model is too simple. Minimal model is of the form:

$$|\,N\,;\,\tfrac{\mathbf{1}}{\mathbf{2}}\,\rangle \;=\; |(\mathbf{2},\mathbf{1})\,,\,\tfrac{1}{2}\,,\,\mathbf{0}\rangle \;+\; |(\mathbf{2},\mathbf{3})_\mathbf{2}\,,\,\tfrac{1}{2}\,,\,\mathbf{0}\rangle \;+\; |(\mathbf{1},\mathbf{2})\,,\,-\tfrac{1}{2}\,,\,\mathbf{1}\rangle \;+\ldots$$

- Main problem with this method is limitations of excited baryon data.

Conclusions

- On a null-plane all chiral symmetry breaking is contained in the three reduced Hamiltonians, M^2 and $M\mathcal{J}_r$.
- This property leads to a proof of Goldstone's theorem that does not rely on the presence of *symmetry-breaking* condensates. However, there must be *symmetry-preserving* condensates.
- There are five current algebra sum rules that constrain hadronic cross-sections (and many that constrain its derivatives).
- Chiral constraints on the QCD Hamiltonians place severe constraints on the nucleon light-cone wavefunction.