What ET Thinks of the LF Vacuum

Work done in collaboration with M. Burkardt and S. Chabysheva

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Outline

- LF as a limit of ET
  - shifted free scalar
  - ϕ^4 theory
  - divergent vacuum
- LF compared to ET
  - missing tadpole
  - ϕ^4 critical coupling
  - compute LF/ET mass shift
- summary

ET to LF interpolation

- K. Hornbostel, Phys. Rev. D **45**, 3781 (1992)
- $x^{\pm} = \frac{1}{\sqrt{2}}[\sqrt{1 \pm c}t \pm \sqrt{1 \mp c}z]$
  - ET: $c = 1$, $x^{\pm} = t, -z$
  - LF: $c = 0$, $x^{\pm}$ as usual, modulo $\sqrt{2}$
- $p_{\pm} = \frac{1}{\sqrt{2}}[\sqrt{1 \pm c}E \mp \sqrt{1 \mp c}p_z]$
- $p \cdot x = p_+ x^+ + p_- x^-$
- $\mu^2 = E^2 - p_z^2 = cp_+^2 - cp_-^2 + 2sp_+p_-$, $s \equiv \sqrt{1 - c^2}$
  - positive root $\rightarrow p_+ = [\sqrt{p_-^2 + c\mu^2} - sp_-]/c$

$$p_+ \rightarrow \begin{cases} \sqrt{p_z^2 + \mu^2}, & c = 1 \\ \frac{\mu^2}{2p_-} = \frac{\mu^2}{2p^+}, & c = 0, p_- > 0 \\ \frac{\mu}{\sqrt{c}}, & c = 0, p_- = 0 \\ \frac{2|p_-|}{c}, & c = 0, p_- > 0 \end{cases}$$

Free scalar field

- Lagrangian $\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2$

  $\rightarrow\ \mathcal{L}_0 = \frac{1}{2}c[(\partial_+\phi)^2 - (\partial_-\phi)^2] + s\partial_+\phi\partial_-\phi - \frac{1}{2}\mu^2\phi^2$

  with $s = \sqrt{1-c^2}$
- Hamiltonian $\mathcal{P}_+^0 = \int dx^-(\pi\partial_+\phi - \mathcal{L}_0)$ with $\pi = c\partial_+\phi + s\partial_-\phi$
- mode expansion: $\phi = \int_{-\infty}^{\infty}\frac{dp^-}{\sqrt{4\pi w_p}}[a(p_-)e^{-ip\cdot x} + a^\dagger(p_-)e^{ip\cdot x}]$

  with $w_p \equiv \sqrt{p_-^2 + c\mu^2}$, $[a(p_-), a^\dagger(p'_-)] = \delta(p^- - p'_-)$

  $\mathcal{P}_+^0 = \int_{-\infty}^{\infty} dp^- \frac{w_p - sp_-}{c} a^\dagger(p_-)a(p_-)$,

  $\mathcal{P}_- = \int_{-\infty}^{\infty} dp_-\, p_-\, a^\dagger(p_-)a(p_-)$

Discretization

- box: $-L < x^- < L$
- periodic BC $\rightarrow$ $p_- = \frac{n\pi}{L}$
- mode expansions:

$\phi = \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{4\pi w_n}} [a_n e^{-in\pi x^-/L} + a_n^\dagger e^{in\pi x^-/L}]$

with $w_n \equiv \sqrt{n^2 + c\tilde{L}^2}$, $\tilde{L} \equiv \mu L/\pi$, and $[a_n, a_m^\dagger] = \delta_{nm}$

$\frac{1}{\mu}\mathcal{P}_+^0 = \frac{1}{\tilde{L}} \sum_{n=-\infty}^{\infty} \frac{w_n - sn}{c} a_n^\dagger a_n$

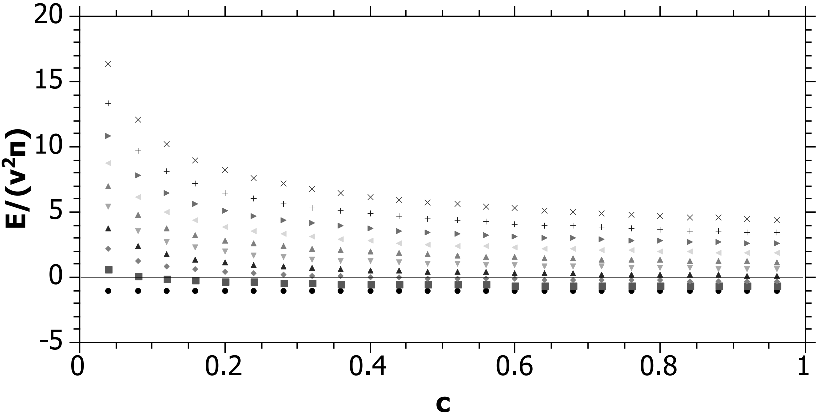
Simple shift: $\phi \to \phi + v$

- Lagrangian: $\mathcal{L} = \mathcal{L}_0 - \mu^2 v\phi - \frac{1}{2}\mu^2 v^2$
- Hamiltonian: $\mathcal{P}_+ = \mathcal{P}_+^0 + \mathcal{P}_+^I$ (drop constant)

$$\mathcal{P}_+^I = \int_{-L}^{L} dx^- \mu^2 v\phi = \mu \frac{v\sqrt{\tilde{L}\pi}}{c^{1/4}}[a_0 + a_0^\dagger]$$

- eigenstate is a coherent state: $|\mathrm{vac}\rangle = e^{-\sqrt{\tilde{L}\pi\sqrt{c}}v(a_0^\dagger - a_0)}|0\rangle$
- eigenenergy: $-\frac{1}{2}\mu^2 v^2(2L)$, which is the dropped constant
- $|\mathrm{vac}\rangle \to |0\rangle$ when $c \to 0$ but energy independent of c
- can solve numerically in finite basis $(a_0^\dagger)^n|0\rangle$
  - plot scaled spectrum vs c
  - truncated to $n = 10$

Shifted free field spectrum



Lowest state's energy clearly independent of c, while higher states' energies diverge as $c \to 0$.

ϕ^4 theory

Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4.$$

Discrete interaction Hamiltonian from the ϕ^4 term:

$$\mathcal{P}_+^I = \int_{-L}^{L} dx^- \frac{\lambda}{4!} : \phi^4 : .$$

Define dimensionless coupling $g \equiv \lambda/(4\pi\mu^2)$ and use same mode expansion: $\phi = \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{4\pi w_n}}[a_n e^{-in\pi x^-/L} + a_n^\dagger e^{in\pi x^-/L}]$

with $w_n \equiv \sqrt{n^2 + c\tilde{L}^2}$, $\tilde{L} \equiv \mu L/\pi$, and $[a_n, a_m^\dagger] = \delta_{nm}$

ϕ^4 interaction Hamiltonian

$$\frac{1}{\mu}\mathcal{P}_+^I = \frac{g\tilde{L}}{4}\sum_{n_1\dots n_4}\frac{1}{\sqrt{w_{n_1}\cdots w_{n_4}}}\Bigg[\frac{1}{12}(a_{n_1}\cdots a_{n_4} + a_{n_1}^\dagger\cdots a_{n_4}^\dagger)\delta_{n_1+\cdots+n_4,0}$$
$$+\frac{1}{3}(a_{n_1}^\dagger a_{n_2}a_{n_3}a_{n_4} + a_{n_2}^\dagger a_{n_3}^\dagger a_{n_4}^\dagger a_{n_1})\delta_{n_1,n_2+n_3+n_4}$$
$$+\frac{1}{2}a_{n_1}^\dagger a_{n_2}^\dagger a_{n_3}a_{n_4}\delta_{n_1+n_2,n_3+n_4}\Bigg]$$

Look for even and odd states with total $P_- = 0$,

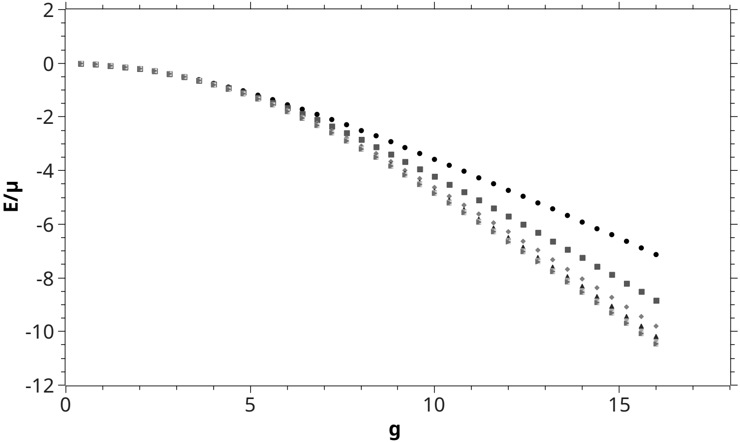
$$|\mathrm{even}\rangle = \psi_0|0\rangle + \sum_n \psi_2(n)a_n^\dagger a_{-n}^\dagger|0\rangle + \cdots$$

$$|\mathrm{odd}\rangle = \psi_1 a_0^\dagger|0\rangle + \sum_{n_1,n_2}\psi_3(n_1,n_2)a_{n_1}^\dagger a_{n_2}^\dagger a_{-n_1-n_2}^\dagger|0\rangle + \cdots$$

as eigenstates of $(\mathcal{P}_+^0 + \mathcal{P}_+^I)|\psi\rangle = E|\psi\rangle$.

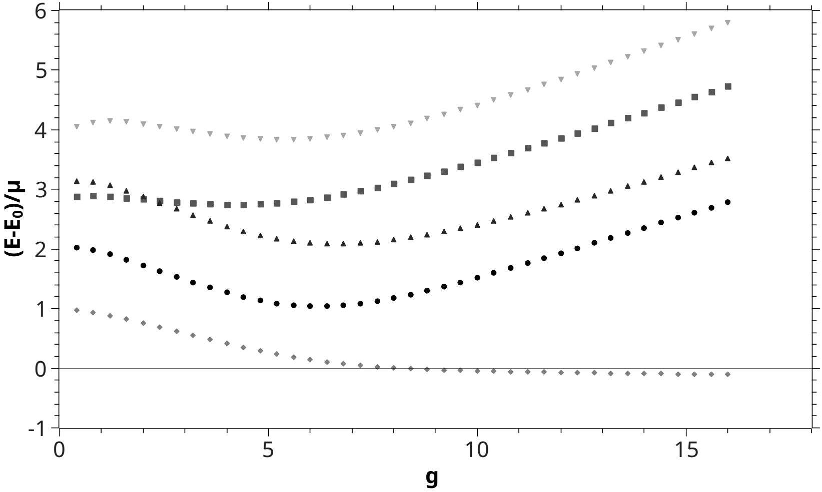
Even ET vacuum

$E_{\max} = 20\mu$, $\tilde{L} = 1$, $c = 1$, # of constituents varied up to 20

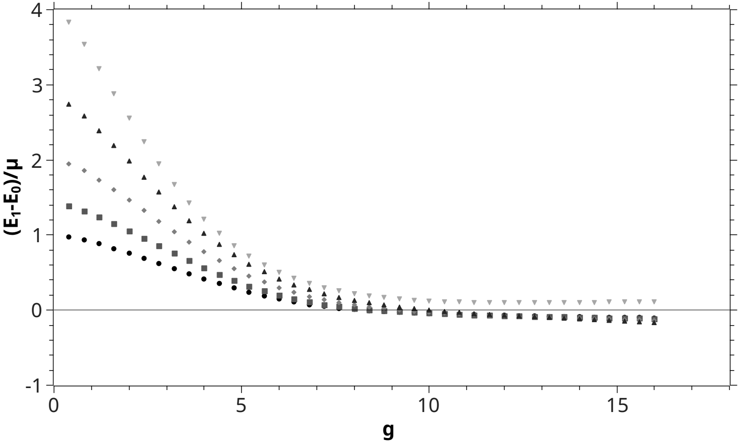


Compare Rychkov and Vitale, PRD **91**, 085001 (2015)
with $g = 6g_{RV}/\pi$, $\tilde{L} = L_{RV}/(2\pi)$

Subtracted ET spectrum

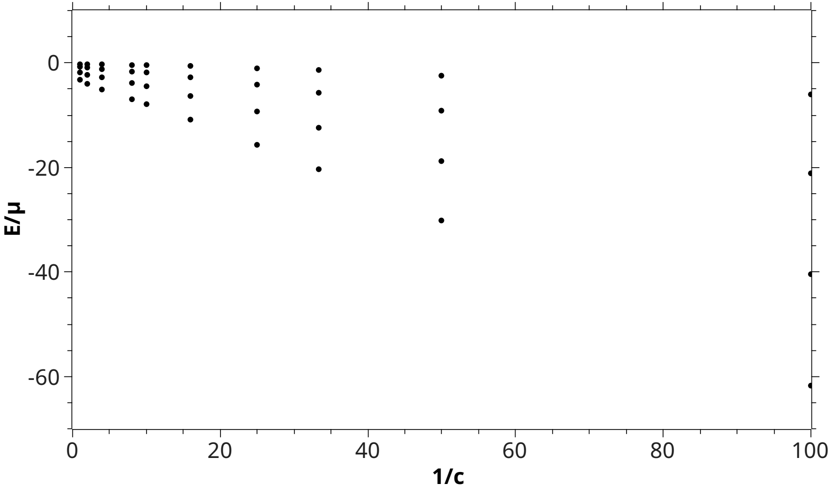


Difference for $c = 1, 0.5, 0.25, 0.125, 0.0625$



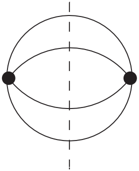
Qualitatively consistent with ET critical coupling.

Lowest state vs c for $g = 2, 4, 6, 8$



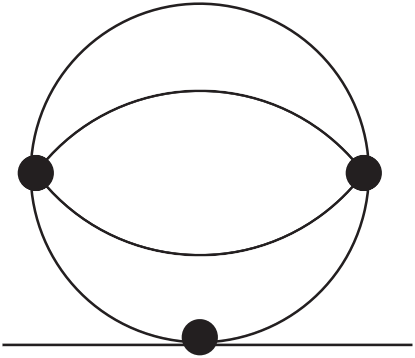
Divergence at $c = 0$

- spectrum diverges as $c \to 0$
    - simplest contribution from 'basketball' graph with all zero modes



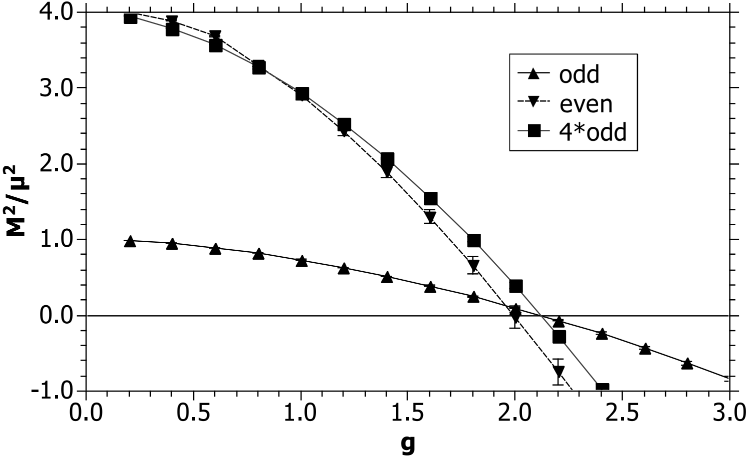
- $\Delta E \sim \frac{1}{w_0^2} \frac{1}{E_0 - 4w_0/c} \frac{1}{w_0^2}$
- $w_0 = \tilde{L}\sqrt{c} \Rightarrow \Delta E \sim c^{-3/2}$
- cannot graft onto LF calculation of $P^+ = P_- \neq 0$
- must consider $P_- \neq 0$ for sequence of finite c and subtract before limit

Related tadpole graph



Contributes to ET self-energy, but not LF.

LF Mass squared for unbroken phase



Critical coupling at $g = 2.1 \pm 0.05$.

Critical couplings compared: $\bar{g} \equiv \frac{\pi}{6} g$

Method	$\bar{g}_c$	Reported by
LF sym. polys.	1.1 ± 0.03	Burkardt, SSC & JRH
DLCQ	1.38	Harindranath & Vary
Quasi-sparse eigenvector	2.5	Lee & Salwen
Density matrix	2.4954(4)	Sugihara
Lattice	$2.70\left\{\begin{array}{l}+0.025\\-0.013\end{array}\right.$	Schaich & Loinaz
	2.79 ± 0.02	Bosetti *et al.*
Uniform matrix product	2.766(5)	Milsted *et al.*
Renorm. H trunc.	2.97(14)	Rychkov & Vitale

Systematic difference between LF (top) and ET (bottom).

Mass renormalization

Bare mass renormalized by tadpole contributions in ET quantization but not in LF quantization
[M. Burkardt, PRD **47**, 4628 (1993)]

$$\mu_{\rm LF}^2 = \mu_{\rm ET}^2 + \lambda \left[\langle 0 | \frac{\phi^2}{2} | 0 \rangle - \langle 0 | \frac{\phi^2}{2} | 0 \rangle_{\rm free} \right] .$$

The vev's of ϕ^2 resum the tadpole contributions; the subscript *free* indicates the vev with $\lambda = 0$.
$\rightarrow$ need to calculate vev's
[MB, SSC, and JRH, PRD **94**, 065006 (2016)]

$$\langle 0 | \frac{\phi^2}{2} | 0 \rangle \rightarrow \frac{1}{2} \langle 0 | \phi(\epsilon^+, \epsilon^-) \int_0^\infty dP \sum_n |\psi_n(P)\rangle\langle\psi_n(P)| \phi(0,0) | 0 \rangle .$$

$$\phi(\epsilon^+, \epsilon^-) = e^{i\mathcal{P}^- \epsilon^+/2} \phi(0, \epsilon^-) e^{-i\mathcal{P}^- \epsilon^+/2} .$$

Mass shift

$$\langle 0|\tfrac{\phi^2}{2}|0\rangle - \langle 0|\tfrac{\phi^2}{2}|0\rangle_{\rm free} \\ = \sum_n \tfrac{|\psi_{n1}|^2}{4\pi}\left[K_0(M_n\sqrt{-\epsilon^2+i\eta}) - K_0(\mu\sqrt{-\epsilon^2+i\eta})\right].$$

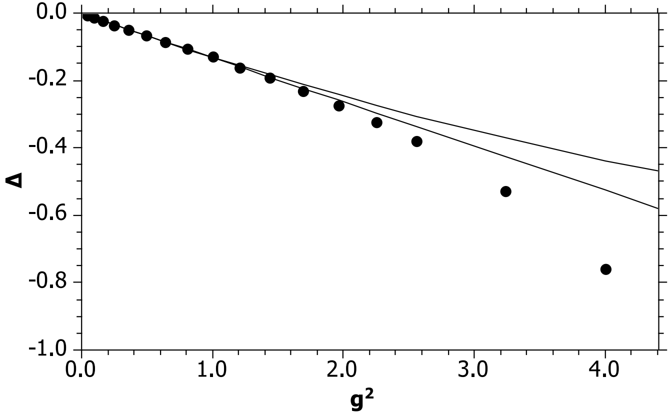
η is from a convergence factor; $K_0(z) \to -\ln(z/2) - \gamma$

$$\langle 0|\frac{\phi^2}{2}|0\rangle - \langle 0|\frac{\phi^2}{2}|0\rangle_{\rm free} = -\sum_n \frac{|\psi_{n1}|^2}{4\pi}\ln\frac{M_n}{\mu_{\rm LF}} \equiv -\Delta/4\pi,$$

$$\mu_{\rm LF}^2 = \mu_{\rm ET}^2 - \frac{\lambda}{4\pi}\Delta \quad \text{or} \quad \frac{\mu_{\rm ET}^2}{\mu_{\rm LF}^2} = 1 + g_{\rm LF}\Delta.$$

$$g_{\rm ET} = \frac{g_{\rm LF}}{\mu_{\rm ET}^2/\mu_{\rm LF}^2} = \frac{g_{\rm LF}}{(1+g_{\rm LF}\Delta)} \quad \text{and} \quad \frac{M^2}{\mu_{\rm ET}^2} = \frac{1}{1+g_{\rm LF}\Delta}\frac{M^2}{\mu_{\rm LF}^2}$$

Plot of shift



Points obtained as extrapolations in the basis size.
Lines are linear and quadratic fits to shifts below $g = 1$, extrapolated to the region of the critical coupling.

LF/ET consistency

- divergence from failure of $|\psi_{11}|$ to go to zero with M_1
  - the product $|\psi_{11}|^2 \ln(M_1/\mu_{\rm LF})$ then diverges
- can use extrapolations from below $g = 1$ to estimate Δ at the critical coupling
  - $\Delta(g = 2.1) = -0.47 \pm 0.12$
  - value is from the higher-order extrapolation, with the lower-order extrapolation used to indicate the error.
- from the latest equal-time value for the critical coupling [Rychkov & Vitale], $g_{\rm ETc} = \frac{6}{\pi}2.97 = 5.67$, we extract a shift of $(g_{\rm LFc}/g_{\rm ETc} - 1)/g_{\rm LFc} = -0.30$
- consistent with the estimated value of the shift

Summary

- can make transition from ET to LF as a limiting process
  - limit diverges but can study energy differences
  - the LF limit of $E_1 - E_0$ consistent with ET
  - qualitatively reproduced known ET results
- can understand difference between ET and LF by taking different mass renormalizations into account
  - the apparent difference between ET and LF ϕ^4 critical couplings is understood
  - the difference is related to the vacuum graphs that diverge in the LF ($c \to 0$) limit
- however, cannot graft $c \to 0$ limit of ET onto a first-principles LF calculation
  - vacuums disagree