A few remarks concerning the triviality/simplicity of the LF vacuum

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Main argument for the general statement $|vac\rangle = |0\rangle$ in the light-front (LF) QFT : positivity of the spectrum of the KINEMATICAL Poincaré generator P^+ in addition to the dynamical energy operator P^-

this together with conservation of P^+ implies NO terms in the LF Hamiltonian with only creation or only annihilation operators, the only exception: dynamical field modes carrying $p^+ = 0$

SSB also associated with the CONSTRAINED zero modes (solutions (approximate) of the non-linear operator equations that express the (non-dynamical) zero mode in terms od the rest (normal) field modes - J. Hiller, S.Pinsky...(1994-5))

attempts to derive the non-trivial structure of the LF vacuum (Kalloniatis and Robertson (1996) - theta vacuum in the bosonized massless Schwinger model: the gauge-field zero mode treated as a QM coordinate realizing large gauge transformations (non-trivial topology), also non-abelian 2D models)

"INTERMEDIATE" SCENARIO:

The LF vacuum (in an interacting theory!) $=$ Fock vacuum (no quanta) is "almost" true, an excellent first approximation which, depending on the details of the dynamics or symmetry of the given model, can further be improved/enriched/transformed - still a BIG simplification wrt conventional (instant/SL form of QFT) where the physical vacuum is a dynamical problem - needs a solution of the interacting theory (Bogoliub tr'n for soluble models)

Few examples

- Simple dynamics (Yukawa model, e.g.): no additional mechanism seems to exist, $|vac\rangle_{LF} = |0\rangle$ true
  Question, however: how is this compatible with nonvanishing vacuum bubbles in LFPT (J. Collins, LM's talk) ??
- LF description of the broken symmetry phase (double-well/mexican hat potential): shift operator $U(b) = \exp\big[-2ib\int_{-L}^{+L} dx^- \Pi_\phi(x^-)\big]$, $b = v = \frac{\mu}{\sqrt{\lambda}}$ determined from minimum of the LF energy - semiclassical description of the non-trivial LF ground state, coherent-state type

$$|v\rangle = U(v)|0\rangle = e^{-8iv\phi(L)}|0\rangle = \exp\big(v\sum_{n=1}^{\Lambda}\tilde{c}_n(a_n^\dagger - a_n)\big)|0\rangle. \tag{1}$$

- large gauge transformations (in the second-quantization version) in the (massive) Schwinger model (LM 2001): residual symmetry realized on the quantum level, unitary operators $\hat{Z}_1$ and $\hat{F}_1$ implementing the symmetry transform the LF Fock vacuum, gauge-field zero mode plus a fermionic structure of the physical vacuum (in terms of an infinite set of degenerate vacua labeled by a topological index, as well as in terms of non-trivial particle content of each individual vacuum state)

$$|\theta\rangle = \sum_{\nu=-\infty}^{+\infty} e^{i\nu\theta}|\nu\rangle, \quad |\nu\rangle = (\hat{T}_1)^\nu|0\rangle, \quad \hat{T}_1 = \hat{Z}_1\hat{F}_1, \quad \hat{Z}_1 = e^{\frac{1}{\sqrt{2}}(a_0^\dagger - a_0)}. \tag{2}$$

- fermionic zero modes, generators of axial-vector transformations transform the Fock vacuum into an infinite set of degenerate vacuum states labeled by the continuous parameter β, Goldstone theorem along usual SL arguments (LM and JPV (2001))

  $O(2)$-symmetric σ-model with fermions, finite volume, $Q^5 = Q^5_0 + Q^5_N$,

$$|vac;\beta\rangle = e^{-i\beta Q^5}|0\rangle, \qquad |vac;\beta\rangle = \exp\Big(-i\beta\sum_{s,p_\perp} 2s\big[b_0^\dagger(s,p_\perp)d_0^\dagger(-s,-p_\perp) + H.c.\big]\Big)|0\rangle. \qquad (3)$$

Similar construction: axiomatic approach F. Coester, W. Polyzou (1994)

The realization that vacuum amplitudes are non-zero in the LF perturbation theory is in agreement with this scenario