Collinear Distributions from Monte Carlo Global QCD Analyses

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On behalf of the JAM Collaboration

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JAM

Motivation

- Want to obtain reliable information of nonperturbative dynamics associated with hadron structure and hadronization

Factorization → separation of short and long distance physics in pQCD expressions of experimental observables, e.g.

Unpolarized deep inelastic scattering (DIS) observable

$\ell + (p,d) \to \ell' + X$

$$d\sigma(x,Q^2) \simeq \sum_f \int_x^1 \frac{d\xi}{\xi} \underbrace{f\left(\frac{x}{\xi},Q^2\right)}_{\text{Parton distribution function (PDF)}} \underbrace{d\hat{\sigma}_f(\xi,Q^2)}_{\text{Hard scattering coefficient}}$$

- Collinear factorization → distributions depend on some fraction of longitudinal momentum

- Nonperturbative distributions are typically determined empirically through global QCD analyses

→ Objects are parameterized $xf(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$

→ Parameters are optimized with a least-squares fit

$$\chi^2 = \sum_e^{N_{exp}} \sum_i^{N_{data}} \frac{(D_i^e - T_i)^2}{(\sigma_i^e)^2}$$

Motivation

- However, many observables can depend on more than one type of distribution

Polarized semi-inclusive DIS observable
$\ell + (p,d) \rightarrow \ell' + h + X$

$$A_1^h \sim \frac{g_1^h}{F_1^h} \sim \frac{\sum_{f,f'} \Delta f \otimes \Delta C_{ff'} \otimes D_{f'}^h}{\sum_{f,f'} f \otimes C_{ff'} \otimes D_{f'}^h}$$

→ Unpolarized (Polarized) PDFs: describe nucleon's momentum (spin) structure

→ Fragmentation functions (FFs) D_f: describe parton-to-hadron fragmentation

…and there are many issues with performing single chi-squared minimizations

→ Uncertainties computed by Hessian or Lagrange multiplier method introduce tolerance criteria (uncertainties inflated by arbitrary factor)

→ Parameters difficult to constrain (flat eigendirections) are typically fixed

→ Highly non-linear chi-squared function means many local minima that a single fit can be trapped in

- With a consistent theoretical framework and rigorous fitting procedure we can (more effectively):

1. Test universality
2. Explore the limits of collinear factorization
3. Study power suppressed corrections

JAM Collaboration Efforts

- Recent efforts by the JAM collaboration:

→ JAM15: Iterative Monte Carlo Analysis of spin PDFs (DIS only) – studies impact of high precision Jefferson Lab data on proton spin structure N. Sato *et. al.* Phys. Rev. D93 074005 (2016)

→ JAM16: First Monte Carlo analysis of FFs (SIA only) – preformed to obtain reliable determination of FFs and their uncertainties

N. Sato *et. al.* Phys. Rev. D94 114004 (2016)

		JAM15	JAM16	JAM17	JAM18
Process	DIS	☑	☒	☑	☑
	SIA	☒	☑	☑	☑
	SIDIS	☒	☒	☑	☑
	DY	☒	☒	☒	☑
Function	f	☒	☒	☒	☑
	Δf	☑	☒	☑	☑
	D_f^h	☒	☑	☑	☑

→ JAM17: First combined Monte Carlo analysis of polarized DIS, polarized SIDIS, and SIA data – studies impact of SIDIS on sea quark helicity distributions

JE, N. Sato, W. Melnitchouk PRL 119 132001 (2017)

→ JAM18: Universal extraction of all nonperturbative input (in progress)

N. Sato, JE, C. Andrés, W. Melnitchouk, *et. al.* (2018)

- Other JAM projects:

→ Monte Carlo extraction of transversity distribution with lattice QCD constraints

H.-W. Lin, W. Melnitchouk, A. Prokudin, N. Sato, H. Shows, PRL 120 152052 (2018)

→ Monte Carlo analysis of pion PDFs (see P. Barry's talk Tuesday p.m.)

P. C. Barry, N. Sato, W. Melnitchouk, C.-R. Ji, arXiv:1804.01965 (2018)

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JAM Fitting Methodology

- Based on Bayesian statistical methods – robust determination of "observables" O (PDFs,FFs,etc.) and their uncertainties

$$E\left[\mathcal{O}\right] = \int d^n a \mathcal{P}(\vec{a}|data)\mathcal{O}(\vec{a})$$

$$V\left[\mathcal{O}\right] = \int d^n a \mathcal{P}(\vec{a}|data)\left[\mathcal{O}(\vec{a}) - E[\mathcal{O}]\right]^2$$

- Bayes' theorem defines probability $\mathcal{P}$ as

$$\mathcal{P}(\vec{a}|data) = \frac{1}{Z}\mathcal{L}(data|\vec{a})\pi(\vec{a})$$

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Likelihood function

$\mathcal{L} = \exp\left(-\frac{1}{2}\chi^2(\vec{a})\right)$ → Gaussian form in data with $\chi^2 = \sum_{e}^{N_{exp}} \sum_{i}^{N_{data}} \frac{(D_i^e - T_i)^2}{(\sigma_i^e)^2}$

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Priors

"Evidence" $Z = \int d^n a \mathcal{L}(data|\vec{a})\pi(\vec{a})$

JAM Fitting Methodology

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$$\mathcal{P}(\vec{a}|data) = \frac{1}{Z}\mathcal{L}(data|\vec{a})\pi(\vec{a})$$

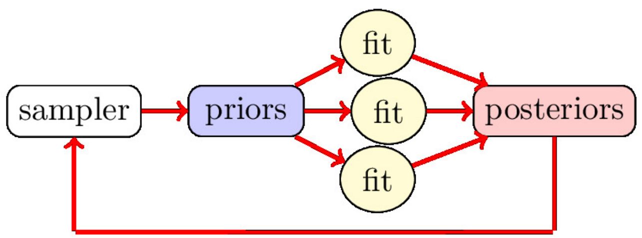
- Monte Carlo technique is used to evaluate expectation value and variance integrals

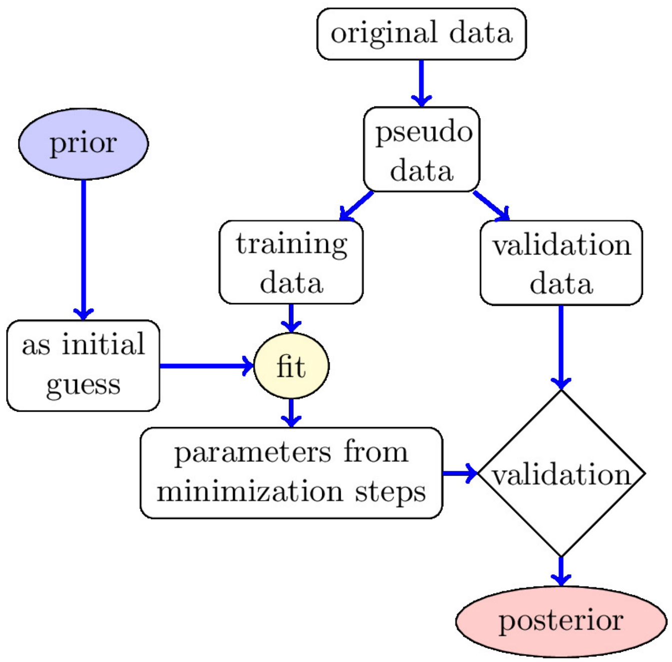
→ samples parameter space and assigns weights w_k to each parameter a_k such that

$$E[\mathcal{O}(\vec{a})] = \sum_k w_k \mathcal{O}(\vec{a}_k) \qquad V[\mathcal{O}(\vec{a})] = \sum_k w_k (\mathcal{O}\left(\vec{a}_k\right) - E[\mathcal{O}])^2$$

Iterative Monte Carlo (IMC)

(JAM17 Analysis)





- → Samples wide region of parameter space
- → Data is partitioned for cross-validation – training set is fitted via chi-square minimization
- → Posteriors used to construct sampler (multi-dimensional Guassian, kernel density estimation, etc) – where parameters are chosen for the next iteration
- → Procedure iterated until converged

$$E[\mathcal{O}] = \frac{1}{n}\sum_{k}^{n} \mathcal{O}(\vec{a}_k)$$

$$V[\mathcal{O}] = \frac{1}{n}\sum_{k}^{n} (\mathcal{O}(\vec{a}_k) - E[\mathcal{O}])^2$$

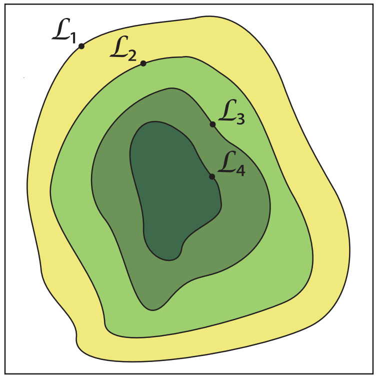
Nested Sampling

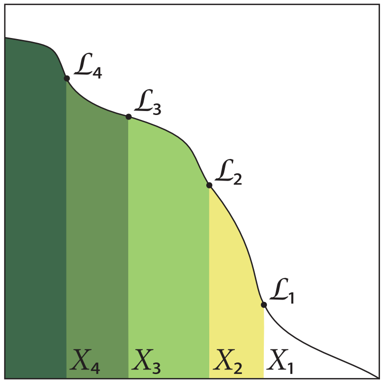
(JAM18 Analysis)

- Statistical mapping of multidimensional integral to 1-D

$$Z = \int d^n a \mathcal{L}(data|\vec{a})\pi(\vec{a}) = \int_0^1 dX \mathcal{L}(X)$$

where the *prior volume* $dX = \pi(\vec{a})d^n a$





$$Z_i \sim \sum_i \mathcal{L}_i w_i$$

where $w_i = \frac{1}{2}(X_{i-1} - X_{i+1})$

Feroz et al. arXiv:1306.2144 [astro-ph]

- Algorithm:

→ Initialize $X_0 = 1$, $L = 0$ and choose N active points $X_1, X_2, \ldots, X_N$ from prior

→ For each iteration, sample new point and remove lowest L_i, replacing with point such that L is monotonically increasing

→ Repeat until entire parameter space has been explored

Proton Spin Structure from DIS

- Typically measure longitudinal and transverse spin asymmetries

$$A_{\parallel} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} = D\left(A_1 + \eta A_2\right) \qquad A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d\left(A_2 + \zeta A_1\right)$$

→ Virtual photoproduction asymmetries: $A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1}$ $A_2 = \gamma\frac{(g_1 + g_2)}{F_1}$ $\gamma^2 = \frac{4M^2x^2}{Q^2}$

- Leading contribution to polarized structure function g_1:

$$g_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2\left[(\Delta C_q \otimes \Delta q^+)(x,Q^2) + (\Delta C_g \otimes \Delta g)(x,Q^2)\right] + \mathcal{O}\left(\frac{1}{Q}\right)$$

- First moment of polarized structure function g_1:

$$\int_0^1 dx g_1^p(x,Q^2) = \frac{1}{36}\left[8\Delta\Sigma + 3g_A + a_8\right]\left(1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right) + \mathcal{O}\left(\frac{1}{Q}\right)$$

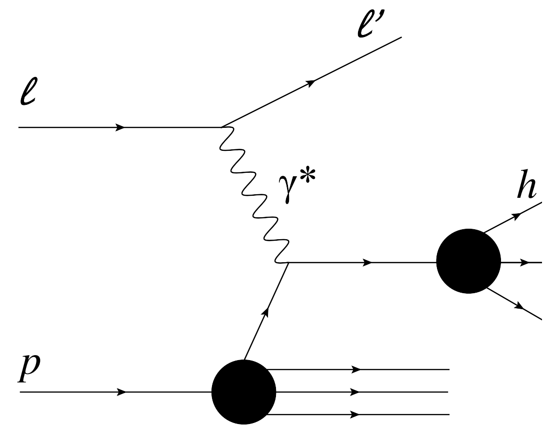
→ DIS requires assumptions about triplet and octet axial charges to extract $\Delta\Sigma$

- Assuming exact $SU(2)_f$ and $SU(3)_f$ values from weak baryon decays

$$\int dx\left(\Delta u^+ - \Delta d^+\right) = g_A \sim 1.269 \qquad \int dx\left(\Delta u^+ + \Delta d^+ - 2\Delta s^+\right) = a_8 \sim 0.586$$

$$\Delta\Sigma_{[10^{-3},0.8]} \sim 0.3$$

Proton Spin Structure from SIDIS



- Measured via longitudinal double spin asymmetries

$$A_1^h(x,z,Q^2) = \frac{g_1^h(x,z,Q^2)}{F_1^h(x,z,Q^2)}$$

- Polarized structure function at NLO defined in terms of 2-D convolution

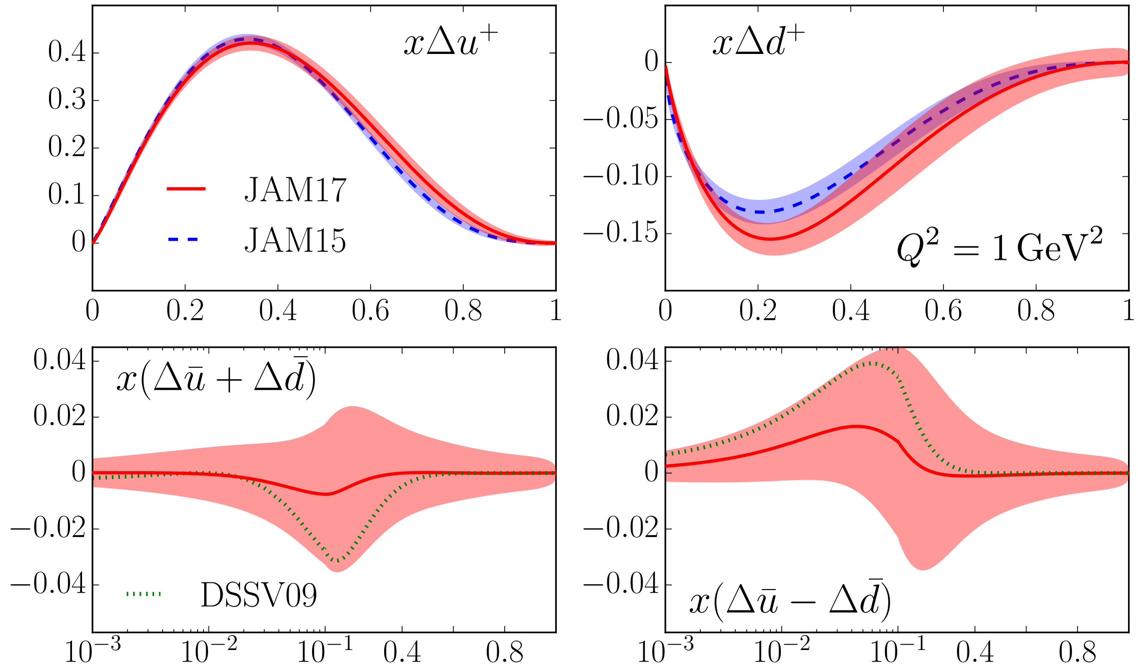
$$g_1^h(x,z,Q^2) = \frac{1}{2}\sum_q e_q^2 \Big\{ \Delta q(x,Q^2) D_q^h(z,Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \times \Big(\Delta q \otimes \Delta C_{qq} \otimes D_q^h + \Delta q \otimes \Delta C_{gq} \otimes D_g^h + \Delta g \otimes \Delta C_{qg} \otimes D_q^h \Big) \Big\}$$

- To include SIDIS observables in the JAM global analyses, fragmentation functions (FFs) must be known

→ Choice of FF parameterizations available (HKNS & DSS) differed significantly in kaon sector – strongly impacts Δs^+ extraction

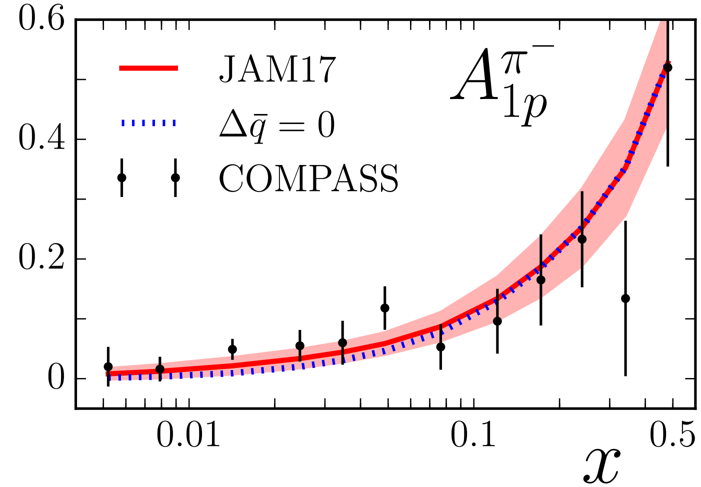
- JAM17 Analysis: first to fit simultaneously polarized PDFs + FFs and release SU(3) constraints

JAM17 Polarized PDF Distributions



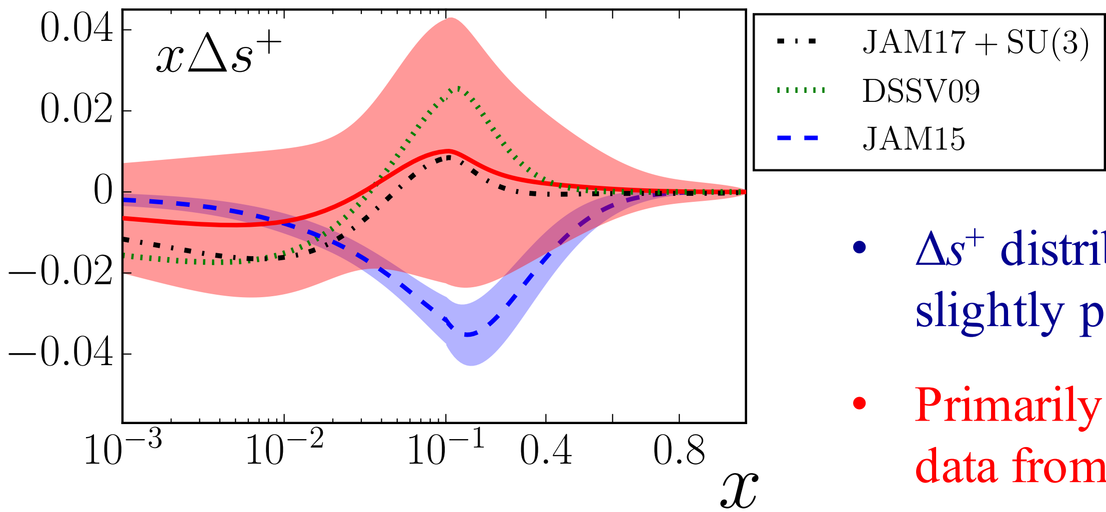
JE, N. Sato, W. Melnitchouk PRL 119 132001 (2017)

- Δu^+ consistent with previous analysis
- Δd^+ slightly larger in magnitude
  → Anti-correlation with Δs^+, which is less negative than JAM15 at $x \sim 0.2$



- Isoscalar sea distribution consistent with zero
- Isovector sea slightly prefers positive shape at low x
  → Non-zero asymmetry given by small contributions from SIDIS asymmetries

JAM17 – Resolution of the Strange Polarization

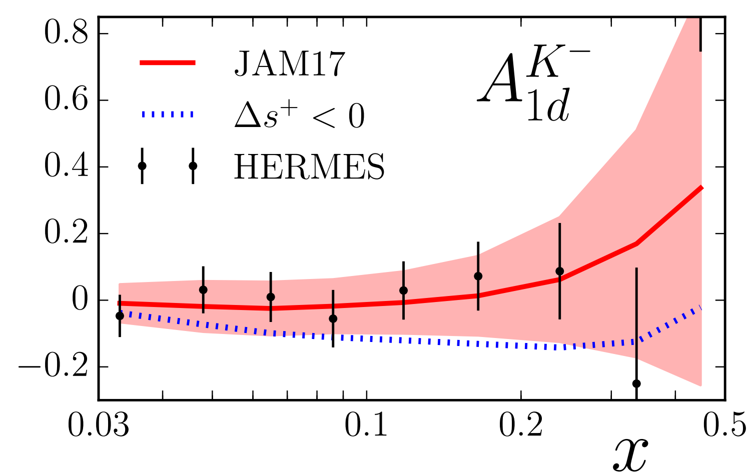


JE, N. Sato, W. Melnitchouk PRL 119 132001 (2017)

- Δs^+ distribution consistent with zero, slightly positive in intermediate x range
- Primarily influenced by HERMES K^- data from deuterium target

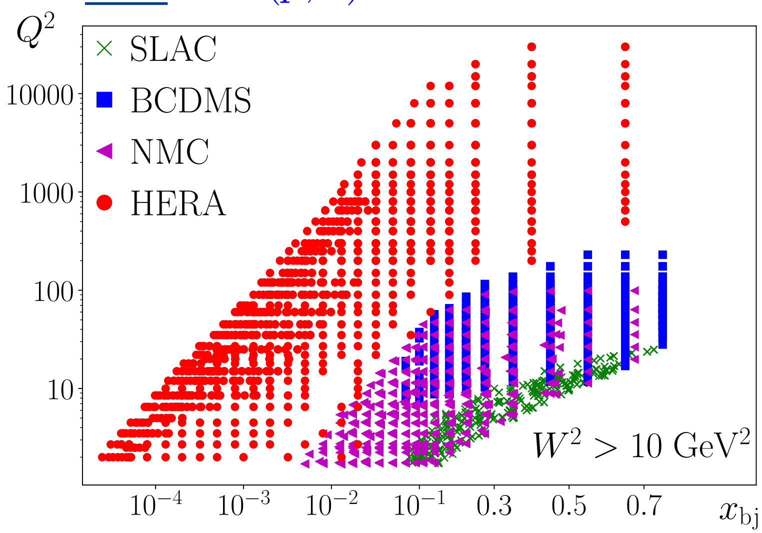
Why does DIS+SU(3) give large negative Δs^+?

- Low x DIS deuterium data from COMPASS prefers small negative Δs^+
- Negative polarization shifted to intermediate region to satisfy SU(3) constraint
- b parameter for Δs^+ typically fixed to values ~6-10, producing a peak at x~0.1

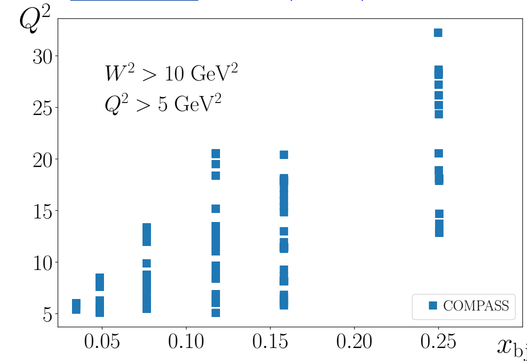


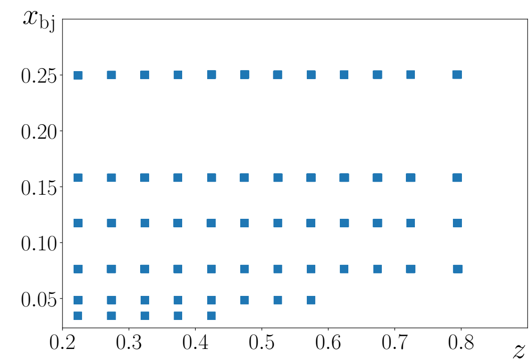
JAM18 Analysis (Preliminary)

DIS $\ell + (p,d) \rightarrow \ell' + X$

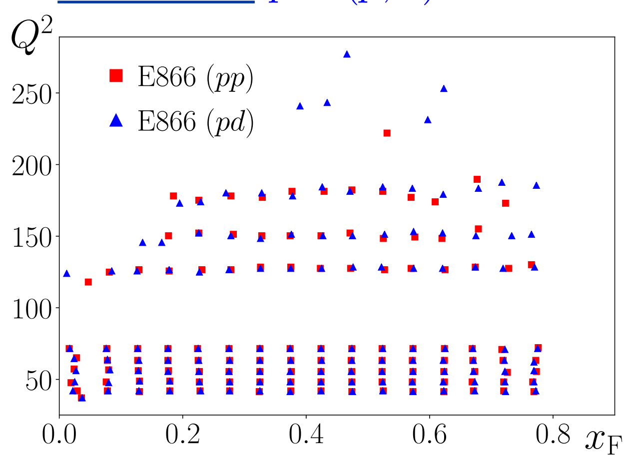


SIDIS $\ell + (p,d) \rightarrow \ell' + h + X$

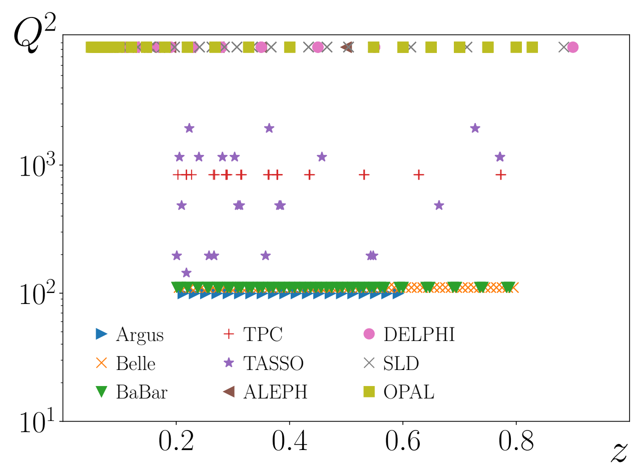




Drell-Yan $p + (p,d) \rightarrow \ell\bar{\ell} + X$



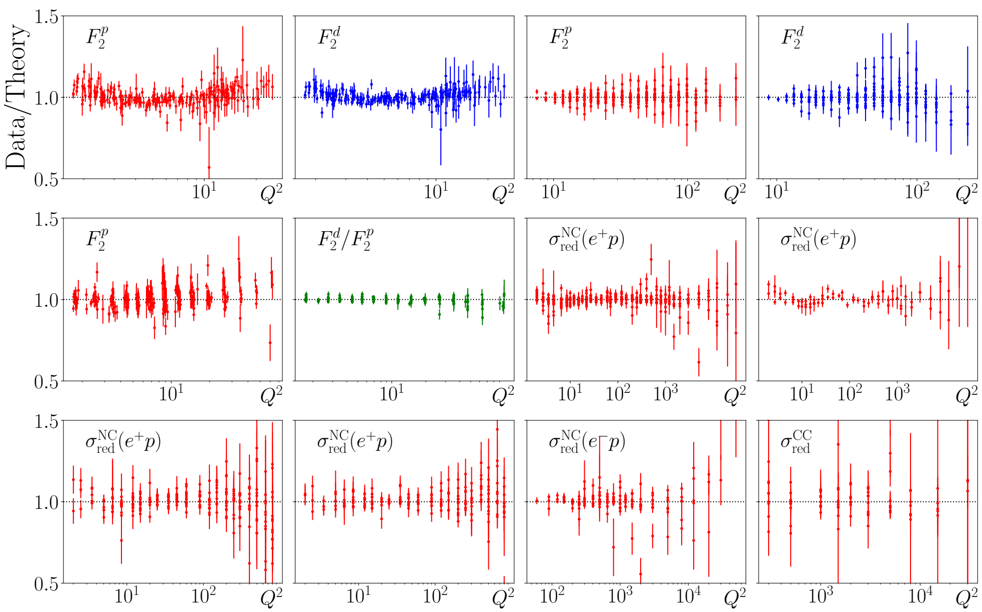
SIA $e^+ + e^- \rightarrow h + X$



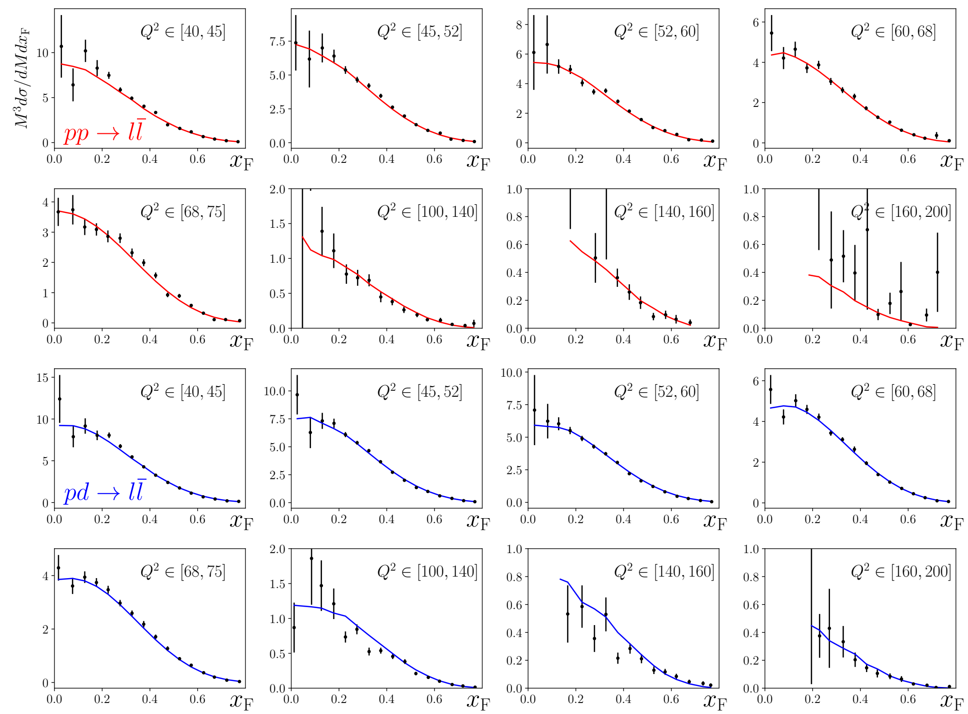
N. Sato, JE, C. Andrés, W. Melnitchouk, *et. al.* (2018)

JAM18 Data vs Theory (Preliminary)

DIS

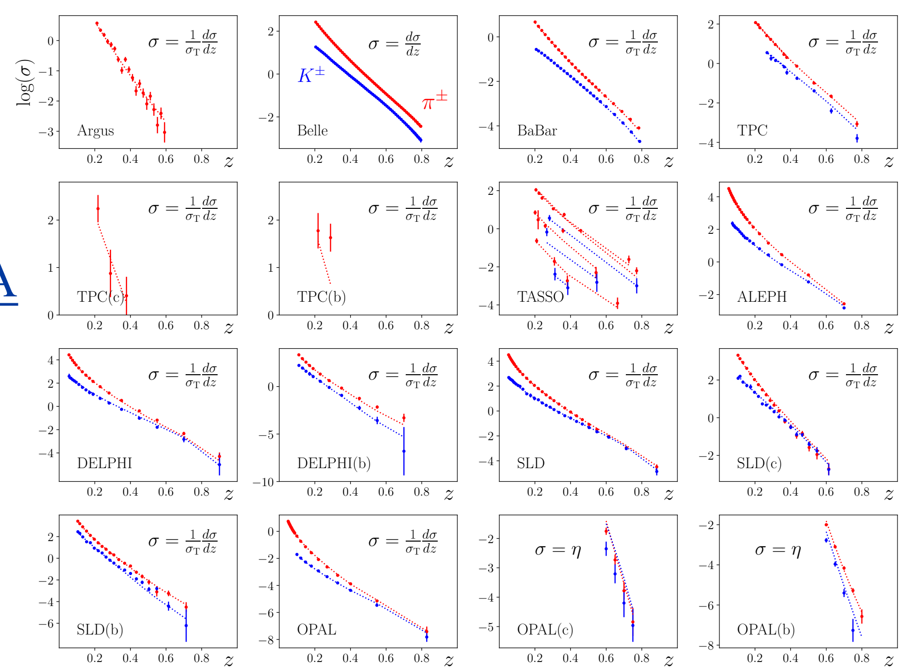


DY



N. Sato, JE, C. Andrés, W. Melnitchouk, *et. al.* (2018)

SIA

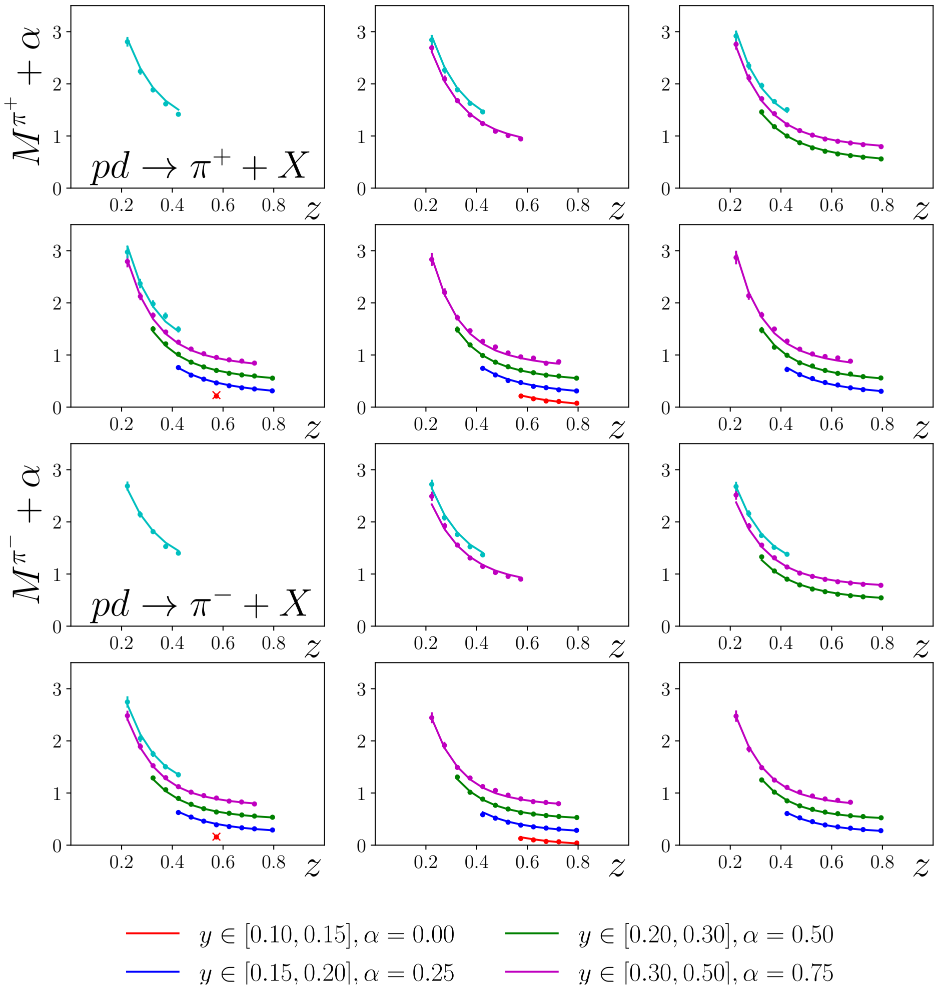


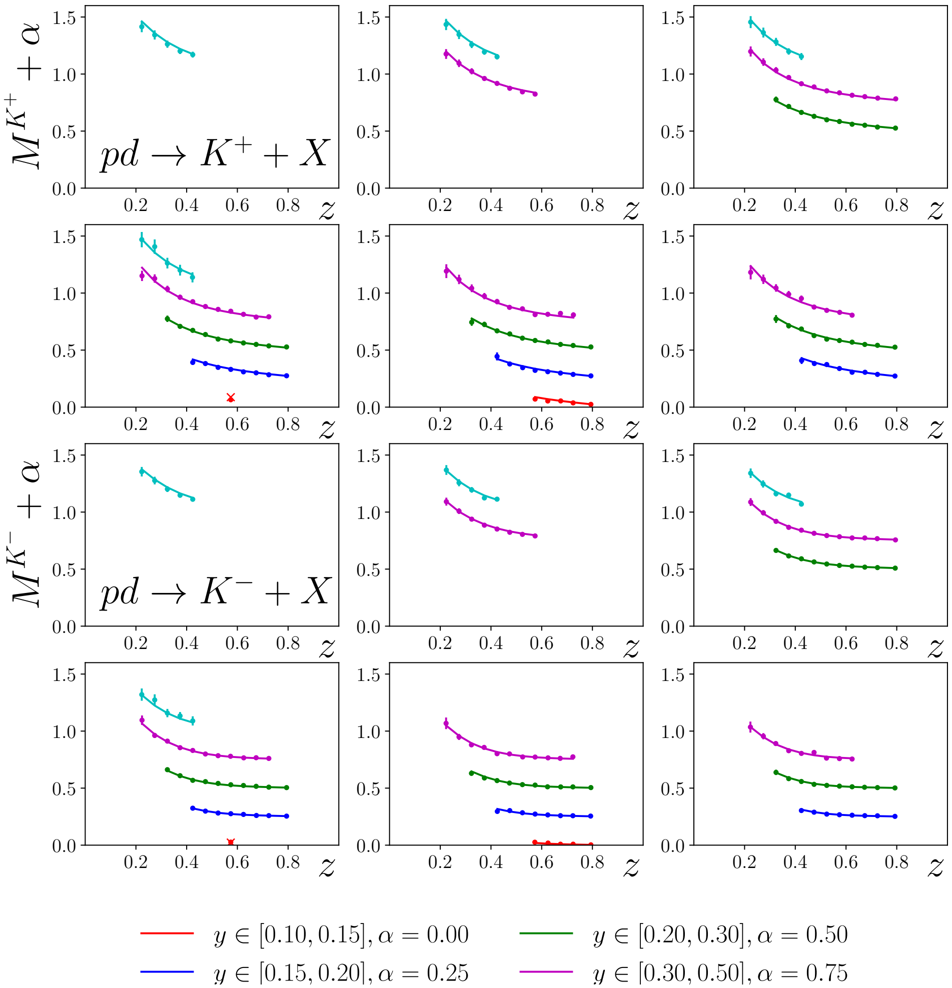
Overall agreement with DIS, DY, and SIA data

JAM18 Data vs Theory (Preliminary)

SIDIS

N. Sato, JE, C. Andrés, W. Melnitchouk, *et. al.* (2018)

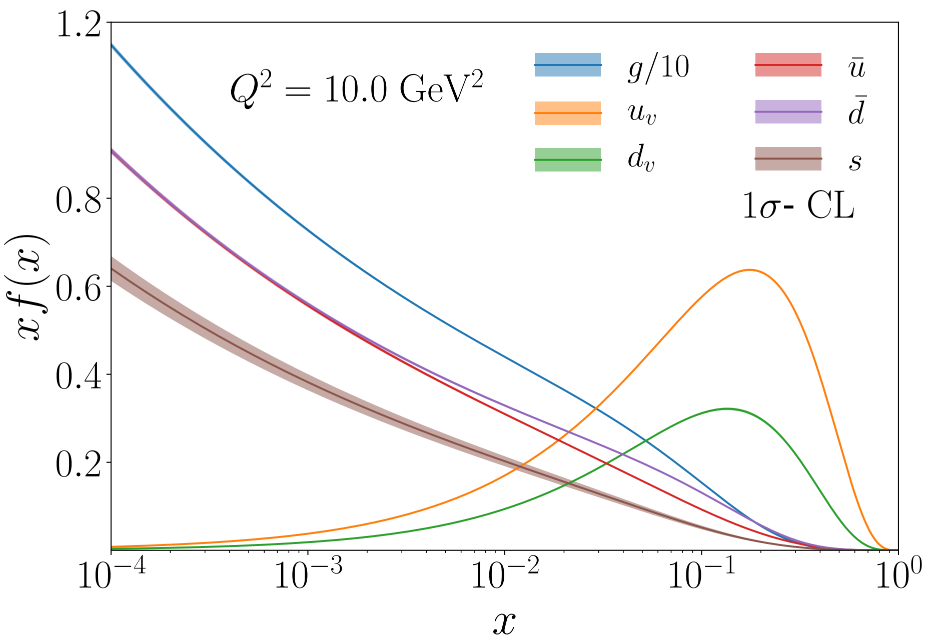




- Difficulty fitting low-Q^2 data → only $Q^2 > 5$ GeV^2 included

JAM18 Unpolarized PDFs (Preliminary)

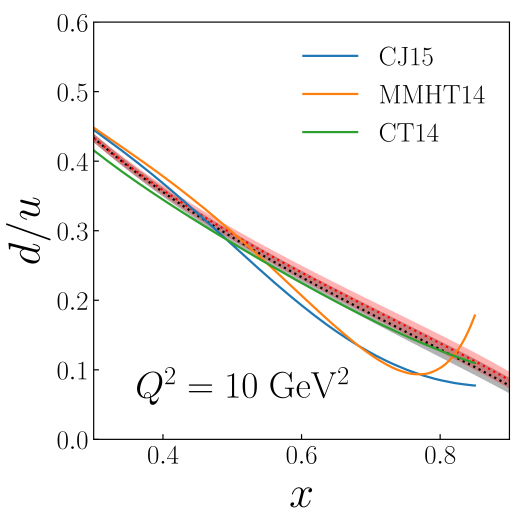
PDFs

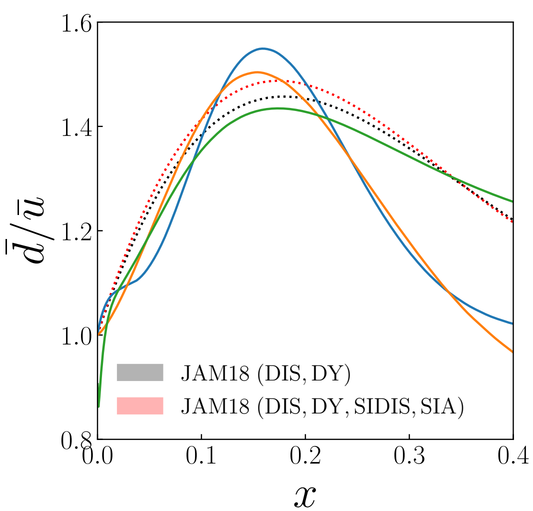


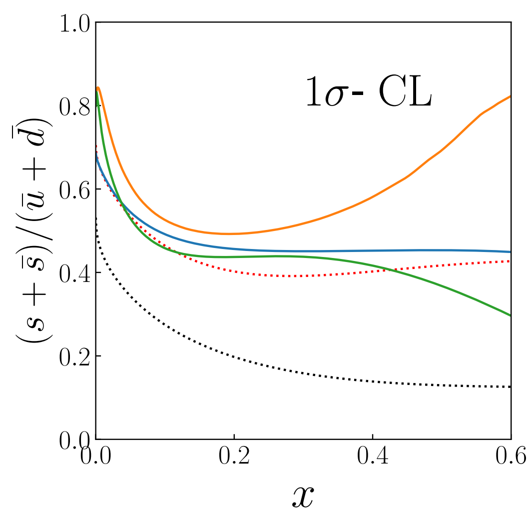
- Central value and uncertainties from maximum likelihood + data resampling method
- Distributions mostly consistent with previous analyses

  → Light sea asymmetry differs at large-x
- SIDIS supports suppression of strange distribution

N. Sato, JE, C. Andrés, W. Melnitchouk, *et. al.* (2018)

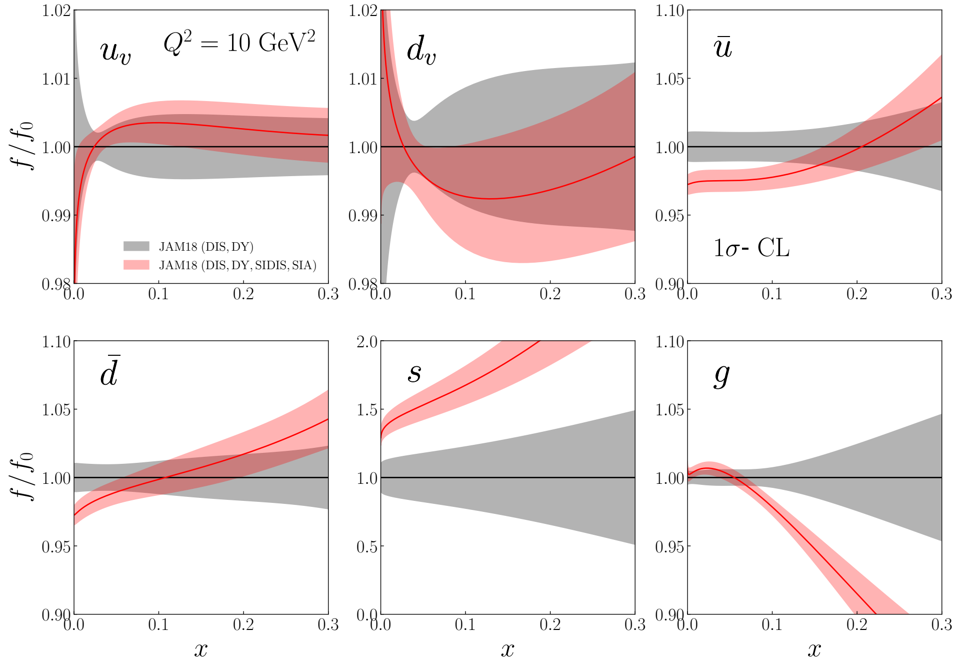






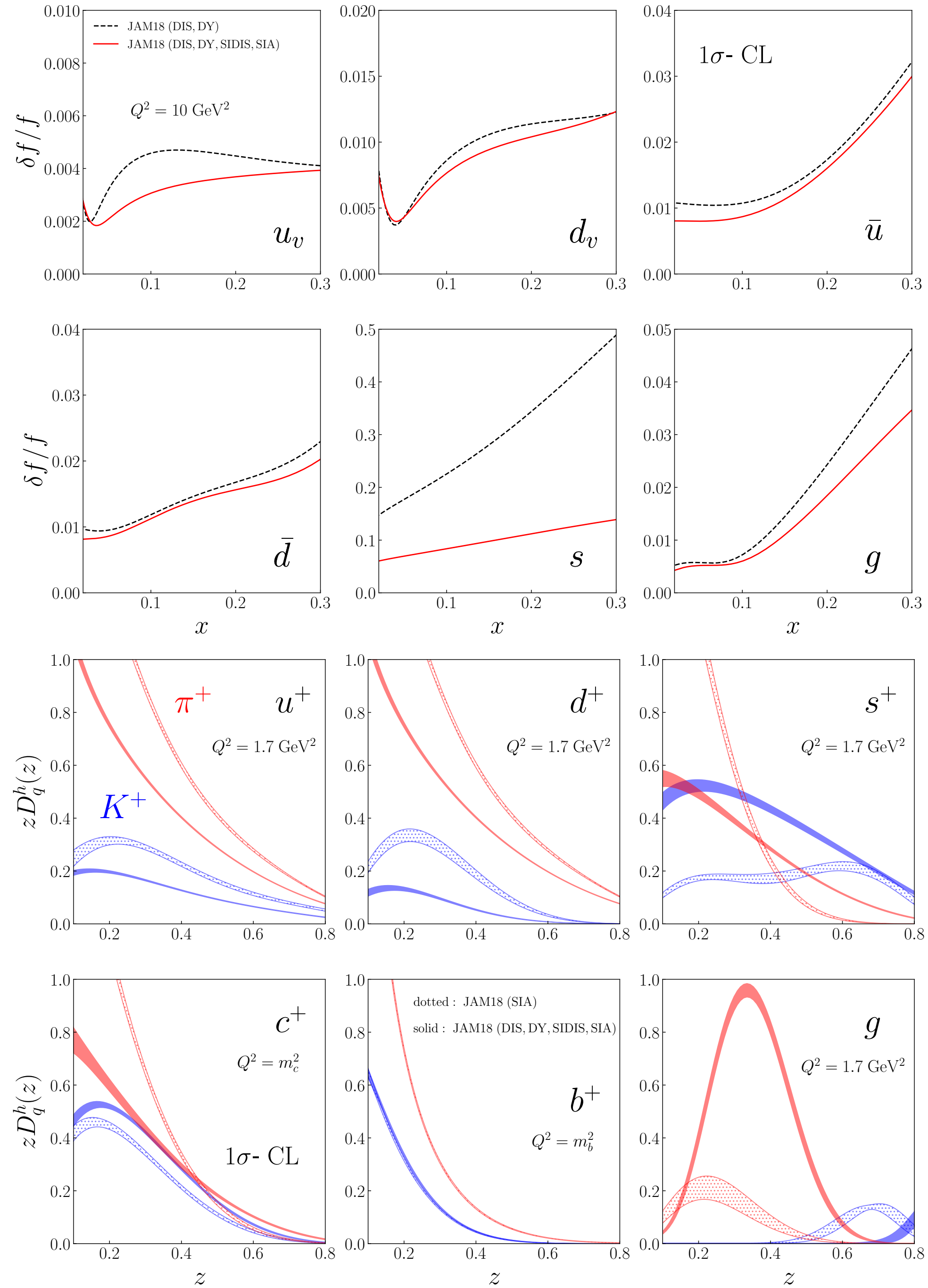
JAM18 Impact of SIDIS

N. Sato, JE, C. Andrés, W. Melnitchouk, *et. al.* (2018)



- Decrease in central value and uncertainty of strange PDF with SIDIS
- Large effect on the gluon distribution

  → Correlation with strange PDF (momentum sum rule)



Summary and Outlook

- Monte Carlo statistical methods are important for robust extractions of non-perturbative functions and their uncertainties

  → Necessary for future global QCD studies that contain large data sets and have many fit parameters (TMDs, GPDs)

- New approaches being developed:

  → Likelihood sampling methods (Nested Sampling)

  → Generalization of Gaussian likelihood (systematic treatment of incompatible data sets)

- First universal analysis of unpolarized + polarized measurements underway

  → Simultaneous extraction of all nonperturbative input

  → Strict test of universality

  → Can separate individual aligned/anti-aligned helicity distributions

- Longer term: extracting transverse momentum dependent (TMD) PDFs and FFs