Partonic quasi- and pseudo-distributions of the pion from a chiral quark model

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Light Cone 2018, 14-18 May 2018, Jefferson Lab

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Supported by grant 2015/19/B/ST2/00937

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Outline

I. Modeling:

- Various parton distributions of the pion in chiral quark models
- Comparison of the model distributions to experimental and lattice data

II. Exploration of the Radyushkin relation between quasi-distributions and transverse-momentum distributions in the continuum:

- Sum rules for x and k_T moments
- Finite P_3 results interesting in their own right
- Evolution of TMD (in the continuum) $\rightarrow$ evolution of QDF (breaking of the transverse-longitudinal factorization, ...)

Ji's quasi-distributions

[see talks by Monahan, Radyushkin, Steffens, Liuti, Karpie]

Parton quasi-distributions (quarks in the pion)

[Ji 2013]

Parton Quasi-Distribution Function (QDF):

$$\tilde{q}(y;P_3) = \int \frac{dz^3}{4\pi} e^{iyP^3 z^3} \, \langle P|\bar{\psi}(0)\gamma^3 \, [0,z]\psi(z)|P\rangle\big|_{z^0=0, z^\perp=0}$$

Parton Quasi-Distribution Amplitude (QDA):

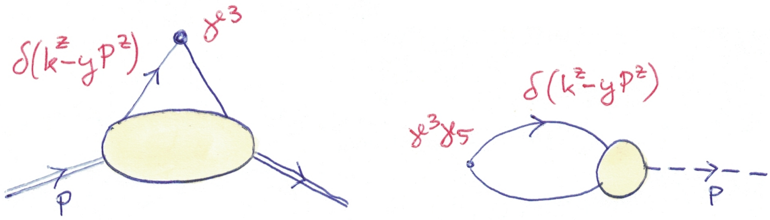
$$\tilde{\phi}(y;P_3) = \frac{i}{F_\pi} \int \frac{dz^3}{2\pi} e^{i(y-1)P^3 z^3} \, \langle P|\bar{\psi}(0)\gamma^+\gamma_5 \, [0,z]\psi(z)|\mathrm{vac}\rangle\big|_{z^0=0, z^\perp=0}$$

y - fraction of pion's P_3 carried by the quark, analogy to DF and DA

$$\lim_{P_3\to\infty} \tilde{q}(x;P_3) = q(x), \quad \lim_{P_3\to\infty} \tilde{\phi}(x;P_3) = \phi(x)$$

Interesting in their own right!

QDF and QDA in the momentum representation

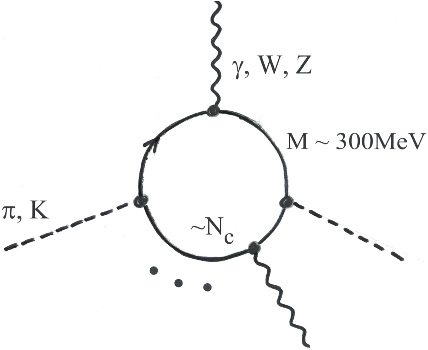


Constrained longitudinal momenta, but $y \in (-\infty, \infty)$
(partons can move "backwards")

Simple to evaluate at the one-quark-loop level

Chiral quark models

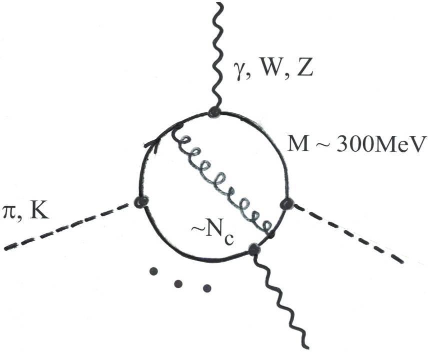
Chiral quark models



- χSB breaking $\rightarrow$ massive quarks
- Point-like interactions
- Soft matrix elements with pions (and photons, W, Z)
- One-quark loop, regularization:
  1) Pauli-Villars (PV)
  2) Spectral Quark Model (SQM) - implements VMD via complex spectral representations

Quantities evaluated at the quark model scale
(where constituent quarks are the only degrees of freedom)

Chiral quark models



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Need for evolution

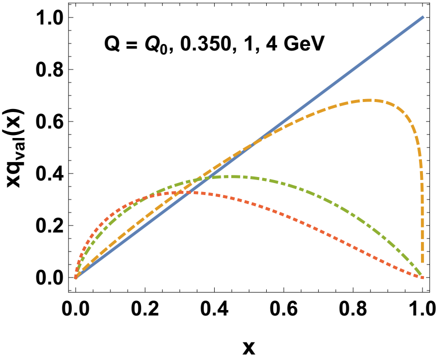
Gluon dressing, renorm-group improved

Scale and evolution

QM provide non-perturbative result at a low scale Q_0

$$F(x, Q_0)|_{\text{model}} = F(x, Q_0)|_{\text{QCD}}\,, \quad Q_0 - \text{the matching scale}$$

Quarks carry 100% of momentum at Q_0. One adjusts Q_0 in such a way that when evolved to $Q = 2$ GeV, the quarks carry the experimental value of 47% (radiative generation of gluons and sea quarks)



LO DGLAP evolution:

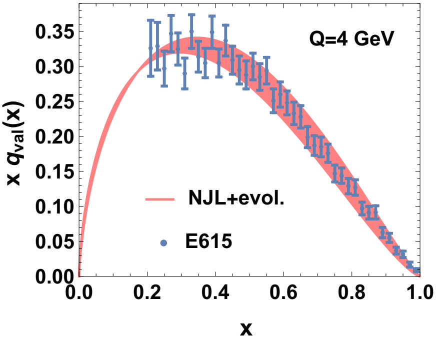
$$Q_0 = 313^{+20}_{-10}\ \text{MeV}$$

NLO close to LO [Davidson, Arriola 1995]:

$$q_{\text{val}}(x; Q_0) = 1$$

$$\sim (1-x)^{p+\frac{4C_F}{\beta_0}\log\frac{\alpha(Q_0)}{\alpha(Q)}}$$

Pion valence quark DF, QM vs E615

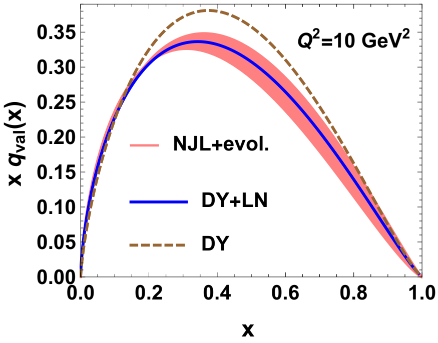


points: Fermilab E615
Drell-Yan, $\pi^{\pm} W \to \mu^{+}\mu^{-} X$

band: QM + LO DGLAP
from $Q_0 = 313^{+20}_{-10}$ MeV to
$Q = 4$ GeV

Pion valence quark DF, QM vs JAM analysis

[P. C. Barry, N. Sato, W. Melnitchouk, C.-R. Ji, arXiv:1804.01965]



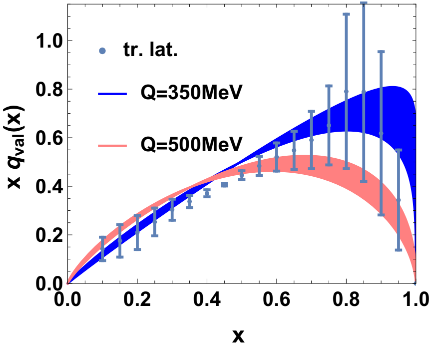
curves: JAM data analysis

band: QM + LO DGLAP from $Q_0 = 313^{+20}_{-10}$ MeV to $Q^2 = 10\ \mathrm{GeV}^2$

[Barry's talk on Tuesday]

Pion quark DF, QM vs transverse lattice

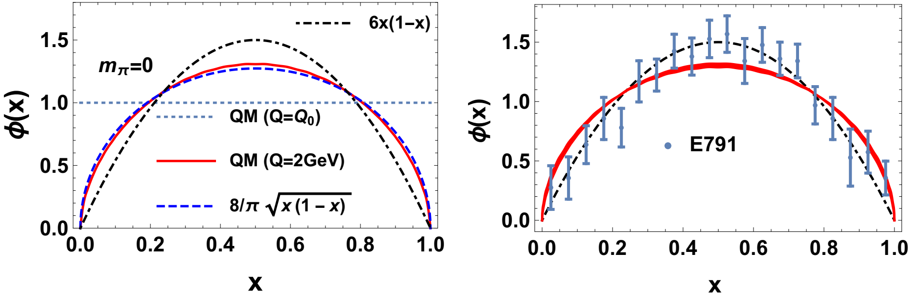
Hamiltonian lattices, LF dynamics (!)



points: transverse lattice [Dalley, van de Sande 2003]

bands: QM + LO DGLAP

Pion DA, QM vs E791



points: E791 data from dijet production in $\pi + A$
lines: QM + ERBL, $Q_0 = 313^{+20}_{-10}$ MeV

At $Q = 2$ GeV the model happens to be very close to the Brodsky-de Teramond AdS/CFT formula

Quasi-distributions from QM

Analytic formulas (in the chiral limit)

SQM (at the QM scale):

$$\tilde{\phi}(y, P_z) = V(y, P_z) = \frac{1}{\pi}\left[\frac{2m_\rho P_z y}{m_\rho^2 + 4{P_z}^2 y^2} + \text{arctg}\left(\frac{2P_z y}{m_\rho}\right)\right] + (y \to 1-y)$$

(similar simplicity for PV NJL)

Satisfy the proper normalization

$$\int_{-\infty}^{\infty} dy\, \tilde{\phi}(y, P_z) = \int_{-\infty}^{\infty} dy\, \tilde{q}(y, P_z) = 1, \quad \int_{-\infty}^{\infty} dy\, 2y\tilde{q}(y, P_z) = 1$$

and the limit

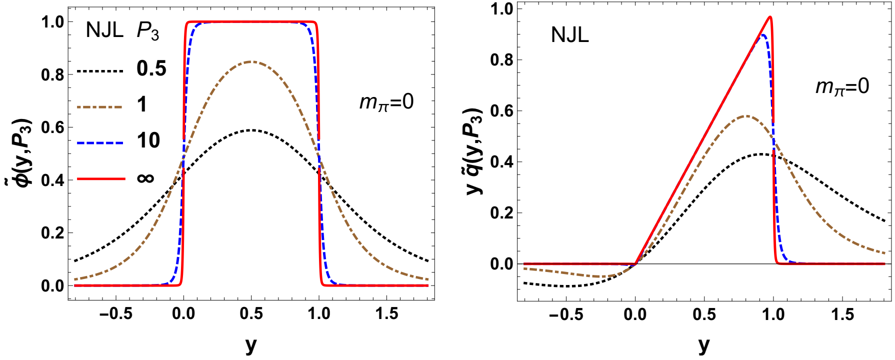
$$\lim_{P_z \to \infty} \tilde{\phi}(y, P_z) = \lim_{P_z \to \infty} \tilde{q}(y, P_z) = \theta[y(1-y)] = \phi(x) = q(x), \quad (y = x)$$

(all needed features are satisfied with a proper regularization)

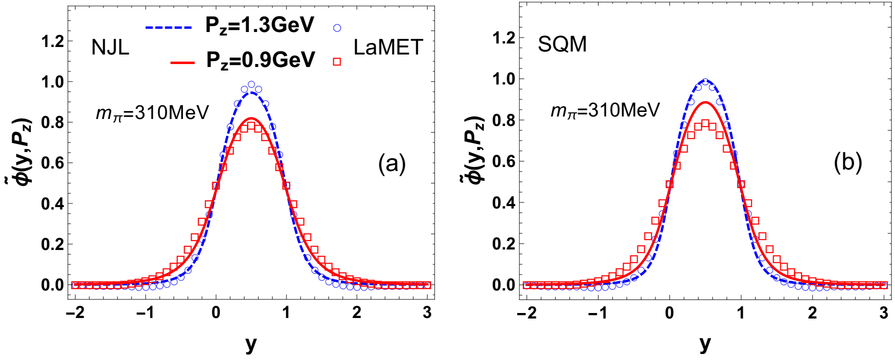
Analytic formulas at QM scale, $m_\pi = 0$

name symbol	PV Nambu–Jona-Lasinio	Spectral Quark Model
PDA, PDF $\phi(x)$, $q(x)$	$\theta[x(1-x)]$	$\theta[x(1-x)]$
QDA, QDF $\tilde{\phi}(y,P_3)$, $\tilde{q}(y,P_3)$	$\frac{N_c M^2}{4\pi^2 f^2}\,\mathrm{sgn}(y)\ln\frac{P_3\lvert y\rvert+\sqrt{M^2+P_3^2y^2}}{M}\Big\rvert_{\mathrm{reg}}$ $+(y\leftrightarrow 1-y)$	$\frac{1}{\pi}\left[\frac{2m_\rho P_3 y}{m_\rho^2+4{P_3}^2y^2}+\mathrm{arctg}\left(\frac{2P_3 y}{m_\rho}\right)\right]$ $+(y\leftrightarrow 1-y)$
LCWF, TMD $\Psi(x,k_T^2)$, $q(x,k_T^2)$	$\frac{N_c M^2}{4\pi^2 f^2}\,\frac{1}{k_T^2+M^2}\Big\rvert_{\mathrm{reg}}\,\theta[x(1-x)]$	$\frac{6m_\rho^3}{\pi\left(4k_\perp^2+m_\rho^2\right)^{5/2}}\theta[x(1-x)]$
pseudo-DA, DF $\mathcal{P}(x,\lvert\boldsymbol{z}\rvert)$, $\hat{q}(x,\lvert\boldsymbol{z}\rvert)$	$\frac{N_c M^2}{4\pi^3 f^2}K_0(M\lvert\boldsymbol{z}\rvert)\Big\rvert_{\mathrm{reg}}\,\theta[x(1-x)]$	$\frac{1}{2}e^{-\frac{m_\rho\lvert\boldsymbol{z}\rvert}{2}}\left(m_\rho\lvert\boldsymbol{z}\rvert+2\right)\theta[x(1-x)]$
IDA, IDF $\mathcal{M}(\nu,\lvert\boldsymbol{z}\rvert)$	$\frac{N_c M^2}{2\pi^3 f^2}\frac{\sin\left(\frac{\nu}{2}\right)}{\nu}K_0(M\lvert\boldsymbol{z}\rvert)\Big\rvert_{\mathrm{reg}}$	$\frac{\sin\left(\frac{\nu}{2}\right)}{\nu}e^{-\frac{m_\rho\lvert\boldsymbol{z}\rvert}{2}}\left(m_\rho\lvert\boldsymbol{z}\rvert+2\right)$
VDA, VDF $\Phi(x,\mu)$	$\frac{N_c M^2}{4\pi^2 f^2}\mu\ e^{-\mu M^2}\Big\rvert_{\mathrm{reg}}\,\theta[x(1-x)]$	$\frac{\mu^{5/2}m_\rho^3 e^{-\frac{1}{4}\mu m_\rho^2}}{4\sqrt{\pi}}\theta[x(1-x)]$

QDA and QDF from chiral quark models at QM scale

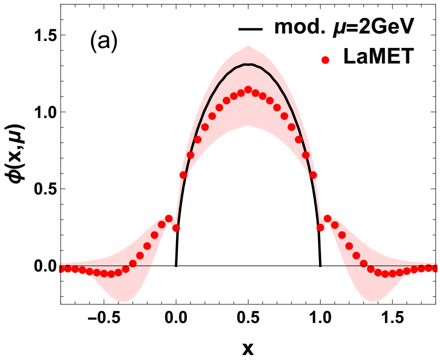


Comparison of QDA to Euclidean lattice



Quark QDA of the pion in NJL (a) and SQM (b) ($m_\pi = 310$ MeV, $P_z = 0.9$ and 1.3 GeV), plotted vs y and compared to the lattice at $Q = 2$ GeV (LaMET [Zhang et al. 2017])

Comparison of DA to Euclidean lattice



Evolution of QDF (in the continuum)

Radyushkin relations to k_T-unintegrated quantities: TMA (a.k.a. LC wave function), TMD

Relations follow from Lorentz covariance

[see talk by Radyushkin]

$$\tilde{\phi}(y, P_z) = \int dk_1 \int dx\, P_z \Psi(x, k_1^2 + (x-y)^2 P_z^2).$$

QDA can be obtained from TMA/LCWF via a double integration

Analogously

$$\tilde{q}(y, P_z) = \int dk_1 dx\, P_z \hat{q}(x, k_1^2 + (x-y)^2 P_z^2).$$

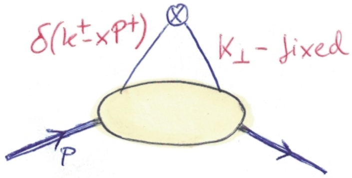
(QDF from TMD)

Inverse relation for quasi-DF:

$$\hat{q}(x, |\boldsymbol{z}|) = z_2 \int dy \int dP_z e^{i(y-x)|\boldsymbol{z}|P_z} \tilde{q}(y, P_z)$$

Evolution of unintegrated DF (in continuum)

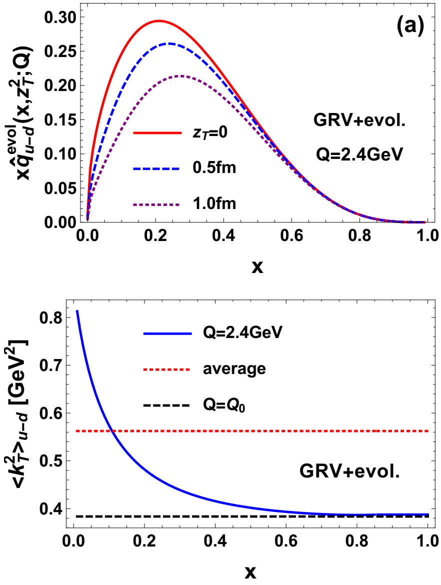
UDF or TMD or quasi-DF



Evolution: Kwieciński's method [2003], based on one-loop CCFM ("interpolates" between DGLAP and BFKL), diagonal in b-space conjugate to k_T. For the non-singlet case:

$$Q^2 \frac{\partial \hat{q}(x, z_T; Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 d\xi \, P_{qq}(\xi) \Big[\Theta(\xi - x) \times J_0[(1-\xi) Q z_T] \, \hat{q}\left(\frac{x}{\xi}, z_T; Q\right) - \hat{q}(x, z_T; Q) \Big]$$

k_T-spreading (here for the nucleon)

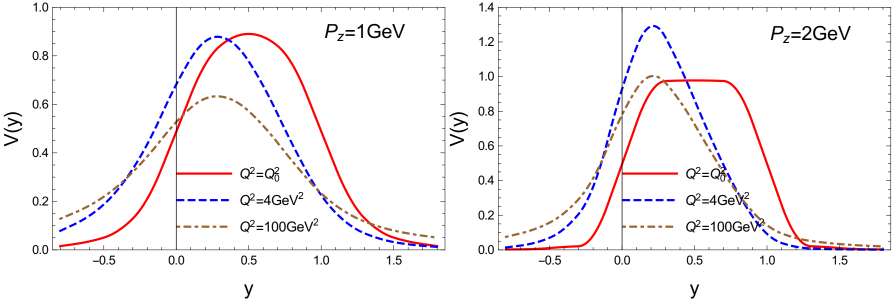


low x: $\langle k_T^2 \rangle \sim \sqrt{\log \frac{1}{x}}$ - integrable

Results of evolution of pion valence QDF in Q at fixed P_z

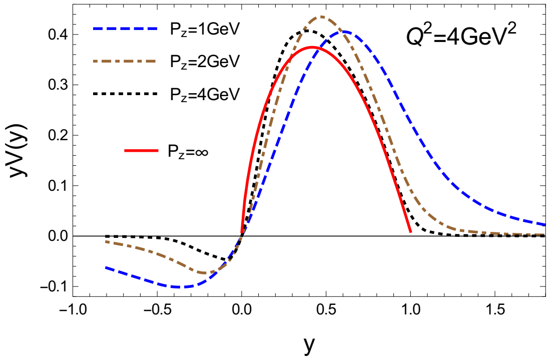
$$\tilde{q}(y, P_z; Q) = \int dk_1 dx\, P_z \hat{q}(x, k_1^2 + (x-y)^2 P_z^2; Q)$$

(notation: $V(z, P_3) \equiv \tilde{q}_{\rm val}(z, P_3)$ – QDF)



Strength moved to lower y as Q increases

Changing P_z at fixed Q



$P_z \to \infty$ limit (PDF) achieved fastest at $y \sim 0.6 - 0.9$

Sum rules

Sum rules from Ioffe-Time Distributions

$$h(-P_3 z_3, -z_3^2) = \int dx\, e^{iP_3 z_3 x} \hat{q}(x, -z_3^2) = \int_{-\infty}^{\infty} dy\, e^{iP_3 z_3 y} \tilde{q}(y, P_3)$$

Use $\nu = P_3 z_3$ - the Ioffe time (strictly speaking, $\tau_I = \nu/M$)

Differentiation wrt. ν at the origin $\rightarrow$

Slope:

$$\frac{d}{d\nu} h\left(-\nu, -\frac{\nu^2}{P_3^2}\right)\Big|_{\nu=0} = i\langle x\rangle = i\langle y\rangle$$

Curvature:

$$\frac{d^2}{d\nu^2} h\left(-\nu, -\frac{\nu^2}{P_3^2}\right)\Big|_{\nu=0} = -\langle x^2\rangle - \frac{1}{2P_3^2}\int dx \langle k_T^2(x)\rangle q(x) = -\langle y^2\rangle(P_3)$$

(similarly for gluon distributions)

Sum rules (contd.)

$$\langle x \rangle = \langle y \rangle$$

$$\langle x^2 \rangle + \frac{N_q \overline{\langle k_T^2 \rangle}}{2P_3^2} = \langle y^2 \rangle (P_3)$$

where

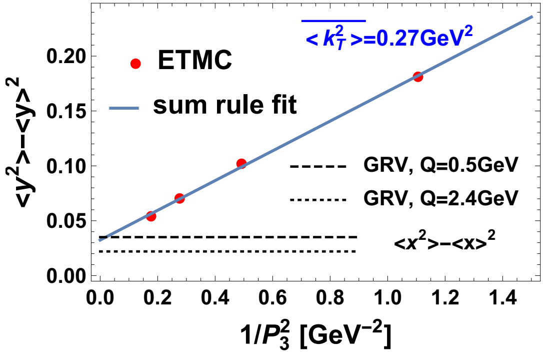
$$\overline{\langle k_T^2 \rangle} = \int dx \langle k_T^2 \rangle (x) q(x) / N_q$$

Exists for $\langle k_T^2 \rangle (x) \sim \sqrt{\log \frac{1}{x}}$

Comparison to lattice data

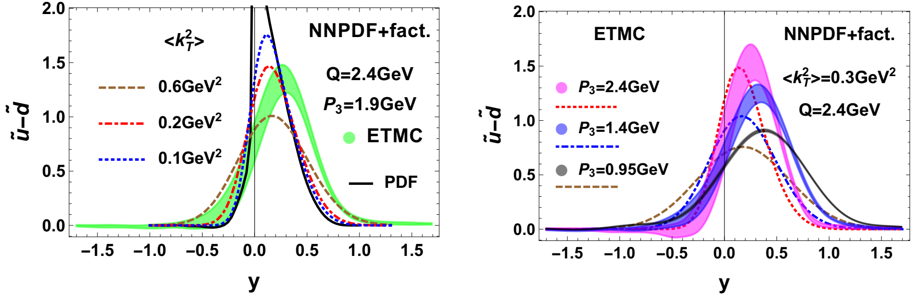
Sum rules with the ETMC data for $u - d$ in the nucleon

$$\langle y^2 \rangle - \langle y \rangle^2 = \langle x^2 \rangle - \langle x \rangle^2 + \frac{\overline{\langle k_T^2 \rangle}}{2P_3^2} = A + \frac{B}{P_3^2}$$



(ETMC data at $m_\pi = 370$ MeV from K. Cichy)
Another determination by [Musch et al. 2011] via Gaussian fit in k_T yields $\overline{\langle k_T^2 \rangle} = (0.16(1)\ \mathrm{GeV})^2$ at $m_\pi = 600$ MeV

QDF from PDF for the nucleon



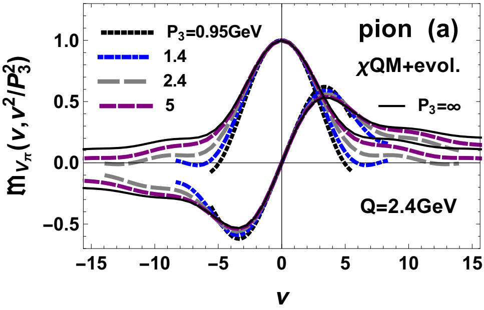
[Alexandrou, Cichy et al. 2015-2017] use a correlator retaining a sub-leading structure $\sim z^{\mu}$, mixing with a twist-3 scalar, $m_\pi = 370$ MeV

[Orginos et al. 2017] extraction from the (quenched, $m_\pi = 600$ MeV) lattice [see the talk by Karpie for full-QCD results]

Lattice QDFs visibly to the right from phenomenology, first moments too large

Factorization breaking by evolution

Factorization breaking from the QCD Kwieciński evolution (for the pion)



In QM the evolution goes over a large span in Q, which leads to significant factorization breaking seen in the reduced IDFs – not a single universal curve!

Conclusions

Conclusions

- Model evaluation of quasi-distributions of the pion at the QM scale, simple analytic results, all consistency conditions met, illustration and check of definitions and methods, need for evolution
- QDFs at finite and not necessarily large P_3 interesting on their own, can be used to verify models and methods
- Sum rules, relating y moments of QDFs, and x and k_T moments of TMDs/pseudo DF – work encouragingly well with the ETMC data
- Longitudinal-transverse factorization breaking from evolution, can be significant when the evolution ratio is large, seen in reduced IDFs

Extras

CCFM

[for a summary see Golec-Biernat et al. 2007]:

CCFM:

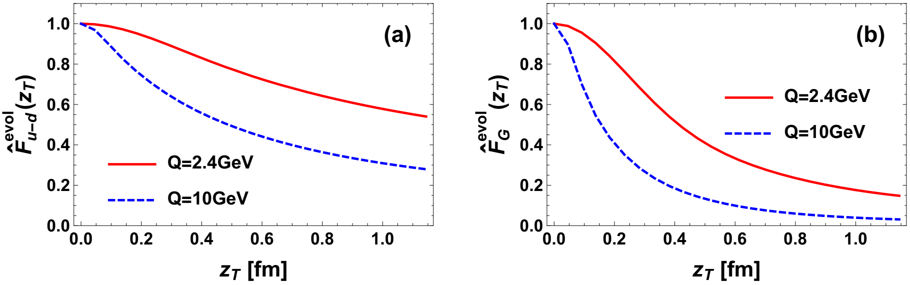
- for the unintegrated gluon distribution
- all-loop approximation – angular ordering (coherence) for both large and small values of x
- a new non-Sudakov form factor that sums virtual corrections for small x

Kwieciński:

- one-loop approximation – angular ordering at small x and the corresponding virtual corrections not included
- coherence only in the real parton emissions for large x
- at small x the standard DGLAP transverse momentum ordering
- both quark and gluon unintegrated distributions

Evolution-generated form factor

$$\hat{F}^{\mathrm{evol}}(z_T;Q) = \int dx\, \hat{q}^{\mathrm{evol}}(x, z_T; Q)$$



asymptotically quarks $\sim z_T^{-4\frac{C_F}{\beta_0}\log\frac{\alpha_s(Q_0)}{\alpha_s(Q)}}$, gluons $\sim z_T^{-4\frac{N_c}{\beta_0}\log\frac{\alpha_s(Q_0)}{\alpha_s(Q)}}$

$(C_F = 4/3,\ N_c = 3,\ \beta_0 = 9)$ [WB, ERA 2004]

Lorentz covariance

$$\langle P|\bar{\psi}(0)\gamma^\mu U[0,z]\psi(z)|P\rangle = P^\mu h(P\cdot z, z^2) + z^\mu h_z(P\cdot z, z^2)$$

($\nu = -P\cdot z$ is referred to as the Ioffe time)

$$\tilde{q}(y, P_3) = P_3 \int \frac{dz_3}{2\pi} e^{-iyP_3 z_3} h(-P_3 z_3, -z_3^2) \qquad (*)$$

$$q(x) = P_+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} h(P_+ z_-, 0)$$

Transverse momentum Distribution (TMD):

$$\hat{q}(x, \boldsymbol{z_T}) = P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} h(P_+ z_-, -\boldsymbol{z_T}^2) \qquad (**)$$

or:

$$q(x, \boldsymbol{k_T}) = P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} \int \frac{dz_T^2}{(2\pi)^2} e^{i\boldsymbol{k_T}\cdot\boldsymbol{z_T}} h(P_+ z_-, -\boldsymbol{z_T}^2)$$

Sum rules from reduced IDFs

Reduced IDFs [Munsch et al. 2011, Orginos et al. 2017]

$$\mathfrak{M}(\nu, z_3^2) = \frac{h(-\nu, -z_3^2)}{h(0, -z_3^2)} = \frac{\int dx\, e^{i\nu x} \hat{q}(x, z_3^2)}{\int dx\, \hat{q}(x, z_3^2)} =$$

$$= (\text{factorization}) = \frac{\hat{F}(z_3) \int dx\, e^{i\nu x} q(x)}{\hat{F}(z_3) \int dx\, q(x)} = \int dx\, e^{i\nu x} q(x) = h(-\nu, 0)$$

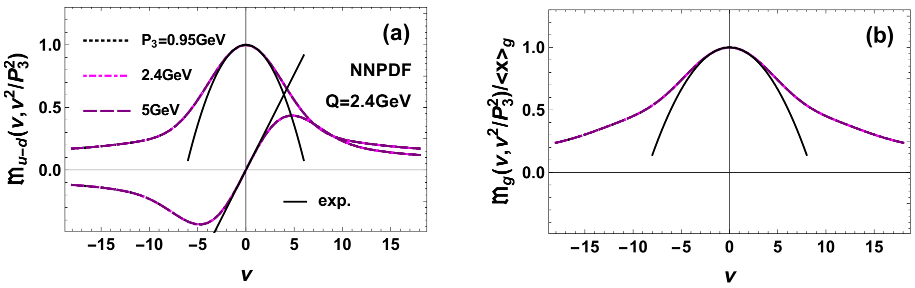
(in the factorization model independent of P_3)

In general

$$\frac{d}{d\nu} \mathfrak{M}(\nu, \nu^2/P_3^2)\Big|_{\nu=0} = i\langle x \rangle_q = i\langle y \rangle_q,$$

$$\frac{d^2}{d\nu^2} \mathfrak{M}(\nu, \nu^2/P_3^2)\Big|_{\nu=0} = -\langle x^2 \rangle_q = -\langle y^2 \rangle_q(P_3) + \frac{1}{P_3^2} \int dx \langle k_T^2(x) \rangle q(x)$$

Sum rules from reduced IDFs (nucleon)

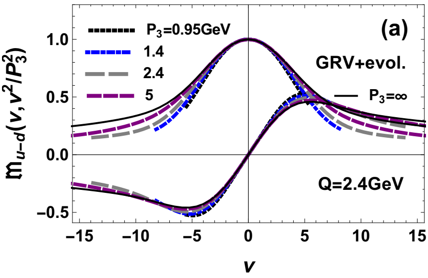


Lowest moments approximate well at low $|\nu|$

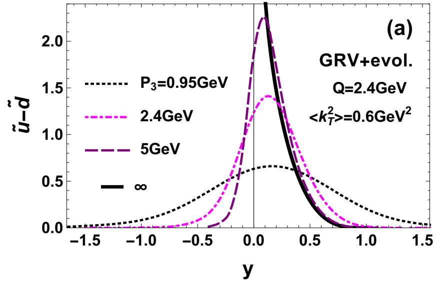
Long ν tails result from the low-x singularity in PDF: $\sim x^{-\alpha} \to \sim \nu^{-1+\alpha}$

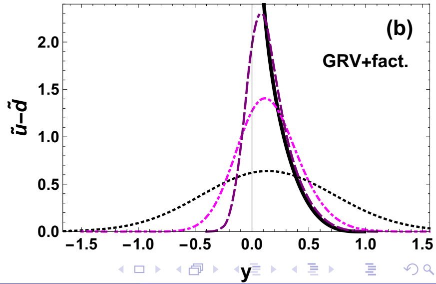
Factorization breaking from QCD evolution (nucleon)

seen

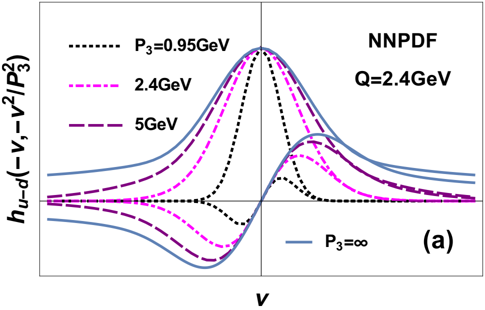


not seen



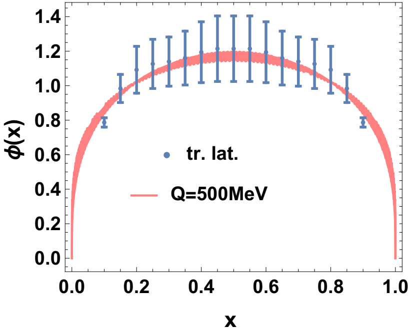


IDFs from TMDs (nucleon)



real (symmetric) and imaginary (antisymmetric) parts
solid lines: $h(-\nu, 0)$

Pion DA, QM vs transverse lattice



points: transverse lattice data [Dalley, van de Sande 2003]
band: QM + ERBL

GPD [WB, ERA, Golec-Biernat 2008], TDA [WB, ERA 2007], equal-time (ET) pion wave functions [WB, ERA, S. Prelovsek, L. Šantelj 2009] . . .