Light-Front Hadronic Models

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Introduction

- A class of phenominological models of hadronic systems that have dual represenations as models of mesons and nucleons or constituent quarks.
- Quarks, antiquarks and gluons in confined color singlets interact by string breaking interaction.
- The degrees of freedom are gauge invariant sub systems of quarks, antiquarks and gluons
- Locally and globally color invarant, hence no color indices
- Model using mass operator with kinetic and potential term
- Interaction between confined color singelts is by string breaking
- Fully relativistic model with light front kinematic symmetry
- Assymptotically linear confinement
- Vertices can be calculated analytically
- The role of sea quarks in spectral calculations, lifetime calculations, scattering and electromagnetic calculations

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Kinematic considerations

Vectors in the irreducible representation space $\mathcal{H}_{mj}$ are square integrable functions of the light-front momentum components $\tilde{\mathbf{p}} = (p^+, \mathbf{p}_\perp)$ and the eigenvalues μ of the $\hat{\mathbf{n}}$ component of the light-front spin,$\mathbf{j}_f \cdot \hat{\mathbf{n}}$,

$$\langle (m,j)\tilde{\mathbf{p}}, \tilde{\mu} | \psi \rangle . \tag{1}$$

Poincaré transformation on the basis states can be expressed as

$$U(\Lambda, a)|(m,j), \tilde{\mathbf{p}}, \mu\rangle = \sum_{\mu'=-j}^{j} |(m,j), \tilde{\mathbf{p}}', \mu'\rangle e^{-ia\cdot p'} \sqrt{\frac{p'^+}{p^+}} D^j_{\mu'\mu}[B_f^{-1}(p'/m)\Lambda B_f(p/m)] \tag{2}$$

For a non-interacting quark-antiquark pair

$$|P = p_q + p_{\bar{q}} \qquad M_0 = \sqrt{-P^2} \qquad Q = P/M_0 .$$

The momentum of the quark in the rest frame of the non-interacting quark-anti-quark pair is defined by transforming to the rest frame with a light-front preserving boost

$$\mathrm{k} = k_f = k_q = B_f^{-1}(Q)p_q \qquad k_{f\bar{q}} = B_f^{-1}(Q)p_{\bar{q}} .$$

The light-front momentum fractions are defined by

$$\xi = \frac{k^+}{M_0} = p_q^+/P^+ \qquad 1 - \xi = p_{\bar{q}}^+/P^+ \tag{3}$$

which can be used to express

$$\mathbf{k}_\perp = \mathbf{p}_{q\perp} - \xi \mathbf{P}_\perp \qquad k^+ = M_0 \xi \tag{4}$$

Kinematic considerations

The tensor product and two particle Poincaré irreducible light-front bases are related by

$$|(|M_0|, j)\tilde{\mathbf{P}}, \tilde{\mu}(l, s)\rangle =$$

$$\sum_{\tilde{\nu}_q, \tilde{\nu}_{\bar{q}}, \mu_q, \mu_{\bar{q}}, m, \mu_s} |(m_q, j_q)\tilde{\mathbf{p}}_q, \tilde{\nu}_q\rangle \otimes |(m_{\bar{q}}, j_{\bar{q}})\tilde{\mathbf{p}}_{\bar{q}}, \tilde{\nu}_{\bar{q}}\rangle D^{j_q}_{\tilde{\nu}_q \mu_q}[R_{fc}(\mathbf{k}_q/m_q)] D^{j_{\bar{q}}}_{\tilde{\nu}_{\bar{q}} \mu_{\bar{q}}}[R_{fc}(\mathbf{k}_{\bar{q}}/m_{\bar{q}})] \times$$

$$\langle j_q, \mu_q, j_{\bar{q}}, \mu_{\bar{q}} | s, \mu_s\rangle Y^m_l(\hat{\mathbf{k}}) \langle s, \mu_s, l, m | j, \tilde{\mu}\rangle \times$$

$$\sqrt{\frac{p^+_q p^+_{\bar{q}} (\omega_q(\mathbf{k}_q) + \omega_{\bar{q}}(\mathbf{k}_{\bar{q}}))}{\omega_q(\mathbf{k}_q)\omega_{\bar{q}}(\mathbf{k}_{\bar{q}})(p^+_q + p^+_{\bar{q}})}} \tag{5}$$

With the interaction

$$M = M_0 + V \tag{6}$$

Poincaré transformation

$$U(\Lambda, a)|(|M|, j)\tilde{\mathbf{P}}, \tilde{\mu}(l, s)\rangle = \sum_{\mu'=-j}^{j} |(M, j), \tilde{\mathbf{P}}', \mu'\rangle e^{-ia\cdot P'} \sqrt{\frac{P'^+}{P^+}} D^j_{\mu'\mu}[B_f^{-1}(P'/m)\Lambda B_f(P/m)] \tag{7}$$

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Confined Quarks

The confined singlets contain quark-antiquark pairs. The Mass operator with the quadratic confining interaction is,

$$M_c = \sqrt{k^2 + V_c + m_q^2} + \sqrt{k^2 + V_c + m_{\bar{q}}^2}. \tag{8}$$

$$V_c = -\frac{\lambda^2}{4}\nabla_k^2 + V_0 \tag{9}$$

The resulting bare meson mass eigenvalues are

$$M_{nl} \to \sqrt{m_q^2 + \lambda(2n + l + \frac{3}{2})} + \sqrt{m_{\bar{q}}^2 + \lambda(2n + l + \frac{3}{2})} \tag{10}$$

Adding a spin term

$$M_{nls} \to \sqrt{m_q^2 + \lambda(2n + l + \frac{3}{2})} + \sqrt{m_{\bar{q}}^2 + \lambda(2n + l + \frac{3}{2})} + \delta_{l0}(a + \frac{b}{2}(s(s+1) - 3/2))) \tag{11}$$

We can solve for a and b to get the correct π and ρ masses

$$a = \frac{1}{4}(3m_\rho + m_\pi) - \sqrt{m_q^2 + \lambda\frac{3}{2}} - \sqrt{m_{\bar{q}}^2 + \lambda\frac{3}{2}}) \tag{12}$$

$$b = m_\rho - m_\pi \tag{13}$$

Confined Quarks

The harmonic oscillator wave functions in the coordinate and momentum-space have the form

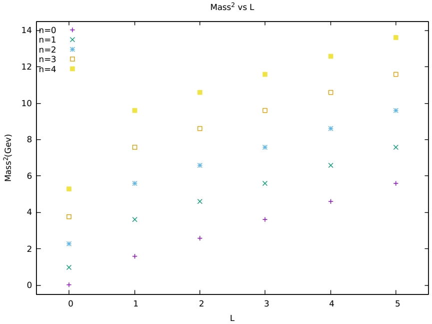
$$\langle \mathbf{r}|n, l, m\rangle = R_{nl}(r)Y_{lm}(\hat{\mathbf{r}}). \quad (14)$$

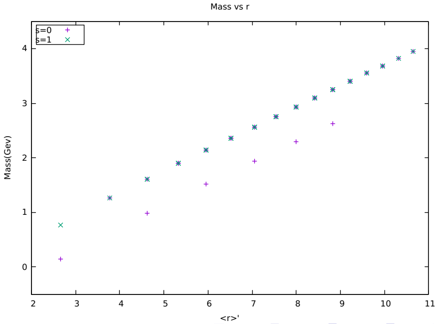
$$\langle \mathbf{k}|n, l, m\rangle = \tilde{R}_{nl}(k)Y_{lm}(\hat{\mathbf{k}}). \quad (15)$$

The RMS relative momentum and displacement of the pair is

$$\langle r^2\rangle^{1/2} = \sqrt{\frac{2}{\lambda}(2n + l + \frac{3}{2})} \qquad \langle k^2\rangle^{1/2} = \sqrt{\frac{\lambda}{2}(2n + l + \frac{3}{2})} \quad (16)$$

In the limit $2n + l$ gets large, $M \approx \sqrt{2}\lambda\langle r^2\rangle^{1/2}$





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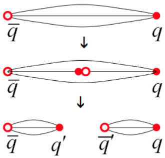
String Breaking Vertex

This allowes the confined singlets to interact. Its a string breaking interaction that assumes a quark-antiquark pair is created with equal probability at any point on the path between the quark and antiquark causing it to break up into a pir of confined singlets. The string breaking interaction in r space have the form

$$v(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12}; \mathbf{r}) := \gamma\delta(\mathbf{r} - 2\mathbf{r}_{12}) \int_0^1 d\eta \delta_{\sqrt{\frac{\lambda}{2}}}(\mathbf{r}_1 - \eta\mathbf{r})\delta_{\sqrt{\frac{\lambda}{2}}}(\mathbf{r}_2 - (1-\eta)\mathbf{r}) \tag{17}$$



The expression for kernel of the vertex as an expansion in harmonic oscillator states is

$$v(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12}; \mathbf{r}) :=$$

$$\frac{g}{\lambda}\delta(\mathbf{r} - 2\mathbf{r}_{12})(\frac{\lambda}{4\pi})^3 4\pi^3 (\frac{2}{\lambda})^{3/2} \sum \int_0^1 d\eta e^{-\frac{\lambda}{2} r_{12}^2(\eta^2+(1-\eta)^2)} \eta^{l_1+2n_1}(1-\eta)^{l_2+2n_2} \times$$

$$(\sqrt{\frac{\lambda}{2}} r_{12})^{2n_1+l_1+2n_2+l_2} \times \frac{\psi_{n_1 l_1 m_1}(\mathbf{r}_1) Y^*_{l_1 m_1}(\hat{\mathbf{r}}_{12}) \psi_{n_2 l_2 m_2}(\mathbf{r}_2) Y^*_{l_2 m_2}(\hat{\mathbf{r}}_{12})}{\sqrt{2n_1!\Gamma(n_1 + l_1 + \frac{3}{2})}\sqrt{2n_2!\Gamma(n_2 + l_2 + \frac{3}{2})}} \tag{18}$$

Matrix Elements of the Vertex

Integrating the vertex against three oscillator states

$$\langle n_1, l_1, m_1, n_2, l_2, m_2, \mathbf{r}_{12}|v|n, l, m\rangle = \int \psi^*_{n_1 l_1 m_1}(\mathbf{r}_1)\psi^*_{n_2 l_2 m_2}(\mathbf{r}_2)v(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12}; \mathbf{r})\psi_{nlm}(\mathbf{x})d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r} =$$

$$\frac{g}{\lambda} R_{nl}(2r_{12})(2\lambda)^{3/2} \times \frac{(\sqrt{\frac{\lambda}{2}} r_{12})^{2n_1+l_1+2n_2+l_2}}{\sqrt{2n_1!\Gamma(n_1 + l_1 + \frac{3}{2})}\sqrt{2n_2!\Gamma(n_2 + l_2 + \frac{3}{2})}} \times$$

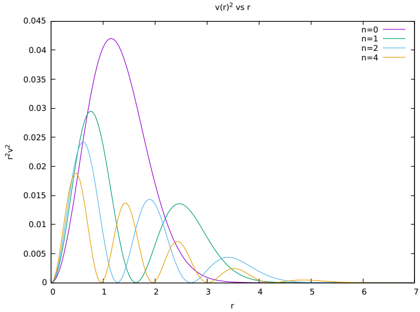
$$e^{-\frac{\lambda}{4} r_{12}^2} \sum_{k_1+k_2=2r} \frac{(l_1 + 2n_1)!(l_2 + 2n_2)!}{k_1!k_2!(l_1 + 2n_1 - k_1)!(l_2 + 2n_2 - k_2)!}(-)^{k_2}(\frac{1}{2})^{l_1+2n_1+l_2+2n_2} \times$$

$$\frac{1}{2r + 1} M(\frac{1}{2} + r, \frac{3}{2} + r, -\frac{\lambda r_{12}^2}{4}) Y_{lm}(\hat{\mathbf{r}}_{12}) Y^*_{l_1 m_1}(\hat{\mathbf{r}}_{12}) Y^*_{l_2 m_2}(\hat{\mathbf{r}}_{12}) \qquad (19)$$

The following is added to include the spin dependence

$$\langle 0, 0|1, m_l, 1, m_s\rangle\langle 1, m_s|\frac{1}{2}, \mu_3, \frac{1}{2}, \mu_4\rangle Y^{*1}_{m_1}(\hat{\mathbf{r}}_{12}) \qquad (20)$$

Vertex strength



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Dynamics

The model Hilbert space is the orthogonal direct sum of the one-singlet Hilbert space with the tensor product of two copies of the one-singlet Hilbert space

$$\mathcal{H} = \mathcal{H}_Q \oplus (\mathcal{H}_Q \otimes \mathcal{H}_Q) \tag{21}$$

The mass operator on $\mathcal{H}$ is given by

$$M = M_{c0} + V \tag{22}$$

and the mass eigenvalue problem has the form

$$\begin{pmatrix} M_c & v_{1:2} \\ v_{2:1} & \sqrt{M_{c1}^2 + \mathbf{q}_{12}^2} + \sqrt{M_{c2}^2 + \mathbf{q}_{12}^2} \end{pmatrix} \begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \end{pmatrix} = \eta \begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \end{pmatrix} \tag{23}$$

$$\left(I - \frac{1}{\eta - M_c} v_{1:2} \frac{1}{\eta - \sqrt{M_{c1}^2 + \mathbf{q}_{12}^2} + \sqrt{M_{c2}^2 + \mathbf{q}_{12}^2}} v_{2:1} \right) |\Psi_1\rangle = 0 \tag{24}$$

to label the one and two singlet channels, and

$$M_n := M_{nls} \qquad M_{n_1,n_2,q_{12}} := \sqrt{q_{12}^2 + M_{n_1,l_1,s_1}^2} + \sqrt{q_{12}^2 + M_{n_2,l_2,s_2}^2} \tag{25}$$

to label the mass eigenvalues of the bare meson systems. In this notation the mass eigenvalue problem has the form

$$\sum_m \left(\delta_{nm} - \sum_{n_1,n_2} \int_0^\infty \frac{1}{\eta - M_n} \langle n|v_{1:2}|n_1, n_2, q\rangle \frac{q^2 dq}{\eta - M_{n_1 n_2 q}} \langle n_1, n_2, q|v_{2:1}|m\rangle \right) \langle m|\Psi_1\rangle = 0. \tag{26}$$

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Summary

- A simple relativistic quark model with a string breaking enteraction between confined color singlets
- Study the effect of sea quarks in the spectral calculation, lifetime calculations, scattering and Electromagnetic phenomena.
- See if it can be extended for baryon studies with diquark approximation

THANK YOU