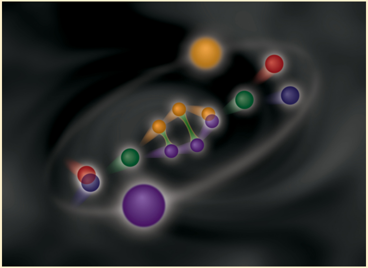
Light-Front Simulations of True Muonium



Hank Lamm

with Rich Lebed

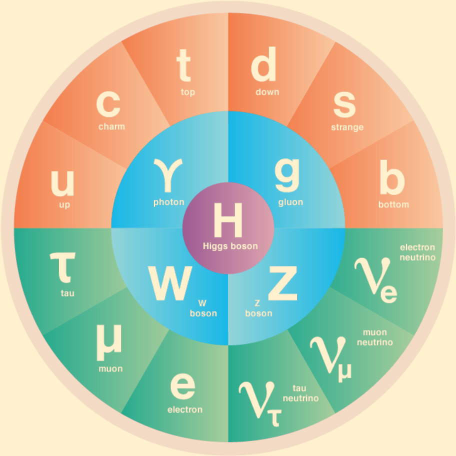
17 May 2018

LIGHTCONE 2018

UNIVERSITY OF MARYLAND

Who ordered that?

Within the Standard Model, *lepton universality* is broken only by the Higgs interaction



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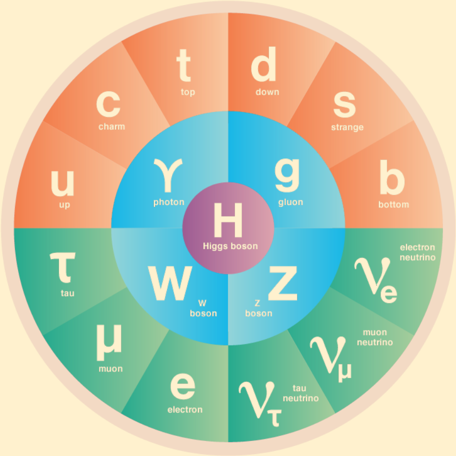
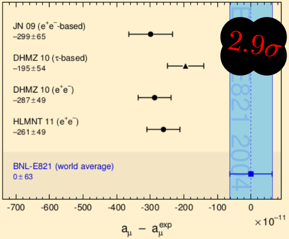
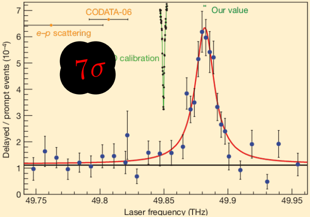


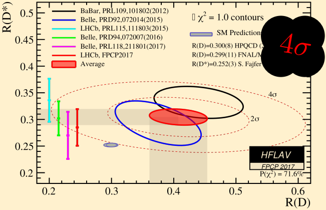
...but m_ν implies this isn't the end of the story

What's the deal with muon physics?[1,2,3]

mu·on prob·lem: (n) the curious observation that discrepancies exist in muon sector



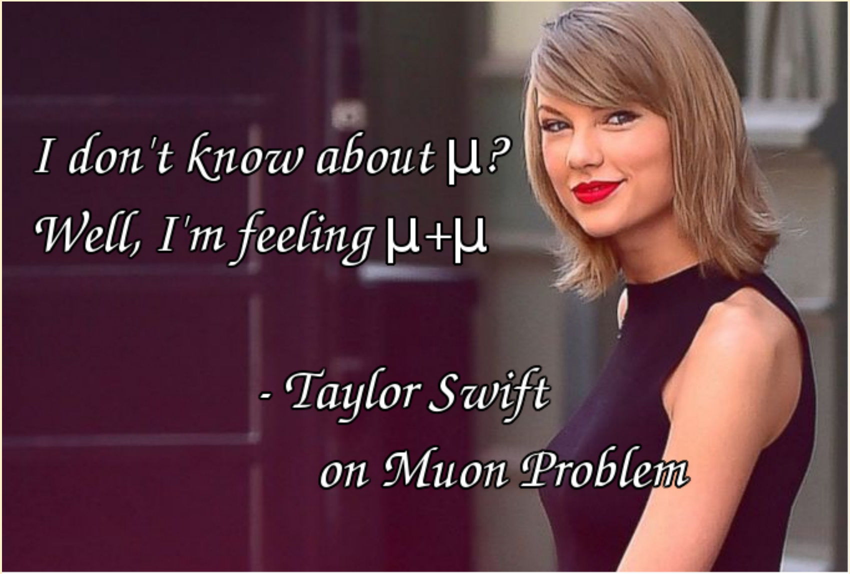




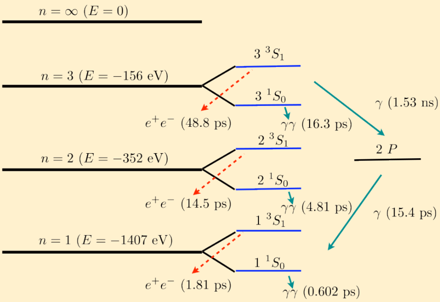
[1] G. Bennett et al. "Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL". In: *Phys. Rev.* D73 (2006), p. 072003. arXiv: hep-ex/0602035 [hep-ex].

[2] A. Antognini et al. "Proton Structure from the Measurement of $2S-2P$ Transition Frequencies of Muonic Hydrogen". In: *Science* 339 (2013), pp. 417–420.

[3] R. Pohl et al. "Laser spectroscopy of muonic deuterium". In: *Science* 353.6300 (2016), pp. 669–673.


I don't know about μ?
Well, I'm feeling μ+μ
- Taylor Swift
on Muon Problem

Wait? I can bind muons?



- Proposed in 1961, still undetected

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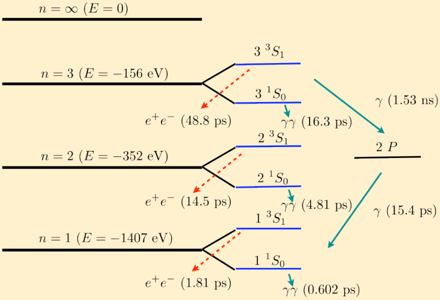

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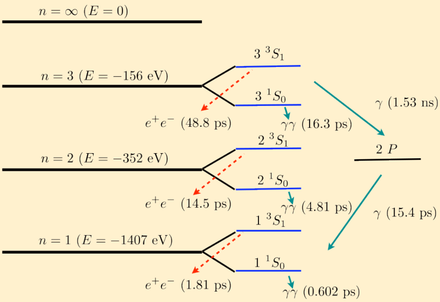

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- **...but many channels suggested**[4,5,6]

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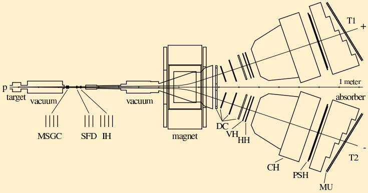
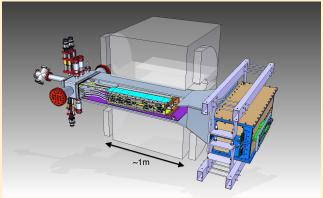
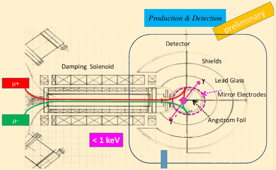
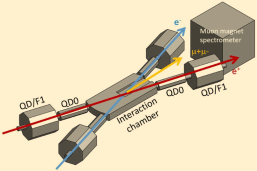
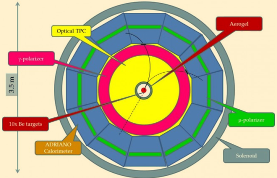
Difficult, but it's what we pay experimentalists for











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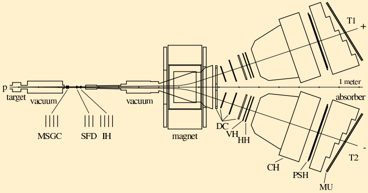
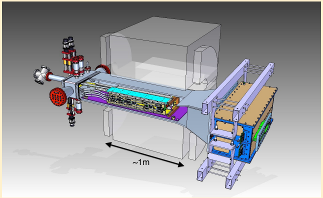
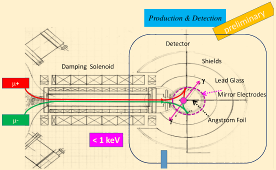
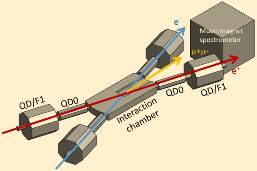
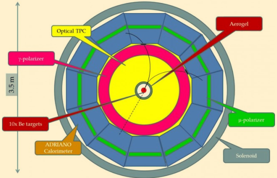




- DIRAC, HPS are existing fixed targets







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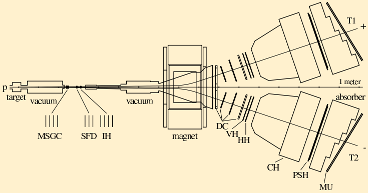

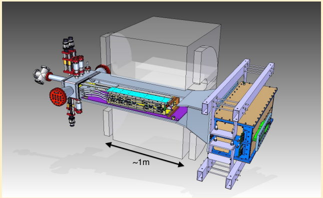

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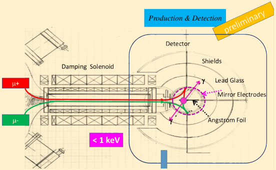

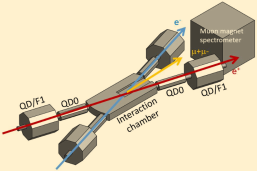

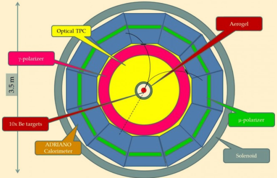

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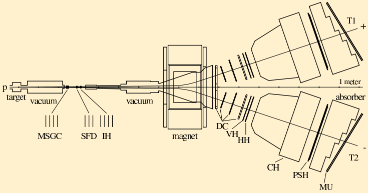

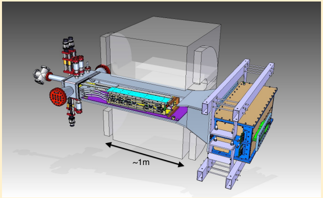

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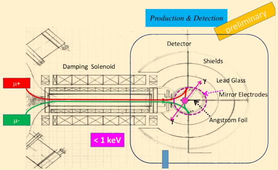

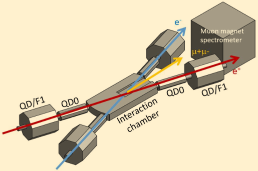

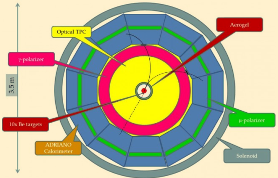

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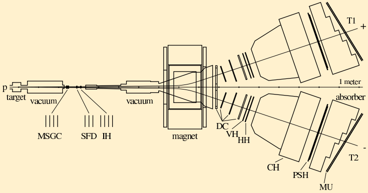

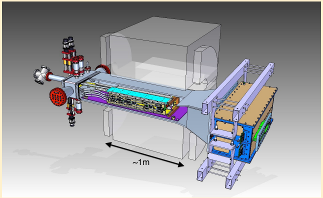

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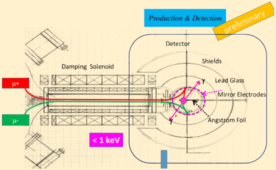
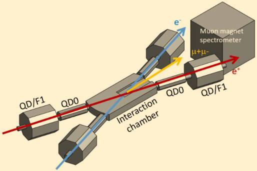
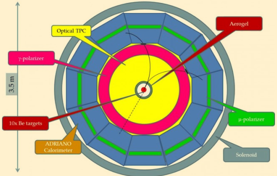
- REDTOP is a proposed η/η' factory







How can we construct a LF model for true muonium?

- Fock state expansion with partons

$$
\begin{aligned}
|\Psi\rangle &\equiv \sum_n \int [\mathrm{d}\mu_n]\, |\mu_n\rangle \langle \mu_n | \Psi; M, P^+, \boldsymbol{P}_\perp, S^2, S_z; h\rangle \\
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- $H_{LC} = P^- P^+ = M^2$
- Acting on a state with this gives

$$\sum_{\lambda'_j} \int_D \mathrm{d}x'_j \mathrm{d}^2\boldsymbol{k}'_{\perp,j} \langle x_i, \boldsymbol{k}_{\perp,i}; \lambda_i \, | H_{LC} | \, x_j, \boldsymbol{k}_{\perp,j}; \lambda'_j, \rangle \psi(x'_j, \boldsymbol{k}'_{\perp,j}; \lambda'_j) = M^2 \psi(x_i, \boldsymbol{k}_{\perp,i}; \lambda_i) \quad (2)$$

Hamiltonian operator have sparse structure[7]

Gauge fixing required for defining H_{LC}. Standard choice is $A^+ = 0$

$$H_{LC} = T + V + S + C + F \tag{3}$$

Sector	n	0	1	2	3	4
$\lvert\gamma\rangle$	0	$\bullet$	V	V	F	F
$\lvert e\bar{e}\rangle$	1	V	$\bullet$	S	V	$\cdot$
$\lvert\mu\bar{\mu}\rangle$	2	V	S	$\bullet$	$\cdot$	V
$\lvert e\bar{e}\,\gamma\rangle$	3	F	V	$\cdot$	$\bullet$	S
$\lvert\mu\bar{\mu}\,\gamma\rangle$	4	F	$\cdot$	V	S	$\bullet$

The Hamiltonian matrix for two-flavor QED, where n labels Fock states. The vertex, seagull and fork interactions are denoted by V, S, F respectively.

[7] U. Trittmann and H.-C. Pauli. "Quantum electrodynamics at strong couplings". In: (1997). arXiv: hep-th/9704215 [hep-th].

Use effective interactions, or you'll have a bad time[8]

- **IF** we truncate Fock space, we have **model wavefunctions (Yay)** but breaks **gauge invariance (Boo)**

[8] See Xingbo Zhao's talk on positronium earlier today!

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$$H_{LC} = \begin{pmatrix} H_{PP} & H_{PQ} \\ H_{QP} & H_{QQ} \end{pmatrix} \tag{4}$$

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$$\hat{Q}|\Psi\rangle = \hat{Q}(\omega - H_{LC})^{-1}\hat{Q}H_{LC}\hat{P}|\Psi\rangle \quad (5)$$

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- With this, we write an effective Hamiltonian **without** Q states

$$H^{eff}(\omega)_{LC} = \hat{P}H_{LC}\hat{P} + \hat{P}H_{LC}\hat{Q}(\omega - H_{LC})^{-1}\hat{Q}H_{LC}\hat{P} \tag{6}$$

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- Replace ω by $f(x', \boldsymbol{k}'_\perp, x, \boldsymbol{k}_\perp)$ to remove divergences

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TMSWIFT: Two-particle Hamiltonian Solver

```
TMSWIFT
True Muonium Solver with Iterative Front-Form Techniques

---TMSWIFT Parameters---
-Output File: OUT/truem_7p-*Ei_PWz>3,_CwFtn0_o.=A50
-Number of Flavors: 3
-J_z: 0
-Alpha: 0.3
-N_tot: 27
-N_mu Discretization Flag: clenshaw_curtis
-N_theta Discretization Flag: clenshaw_curtis
-Annihilation Flag: 0
-Flavor Mixing Flag: 0
-Asymptotic G2 Flag: 0
-MUMPS Flag: 1
------
-Mass_0: 1
-P_Bohr_0: 0.15
-N_mu_0: 3
-N_theta_0: 3
-Lambda_0: 12
------
-Mass_1: 0.5
-P_Bohr_1: 0.075
-N_mu_1: 3
-N_theta_1: 3
-Lambda_1: 10.1
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-Mass_2: 2
-P_Bohr_2: 0.3
-N_mu_2: 3
-N_theta_2: 3
-Lambda_2: 90
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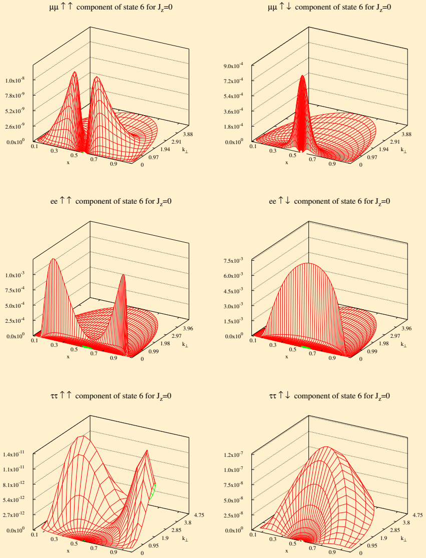

Regularization improves singlet state Λ dependence

- Remove remaining uncancelled divergence in $\langle\uparrow\downarrow|H_{\rm eff}|\downarrow\uparrow\rangle$

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- Remove remaining uncancelled divergence in $\langle\uparrow\downarrow|H_{\text{eff}}|\downarrow\uparrow\rangle$

$$\lim_{k'_\perp\to\infty}\delta G_2(x,k_\perp;x',k'_\perp)=\frac{1}{(k'_\perp)^2}\to\delta(\boldsymbol{r}) \tag{7}$$

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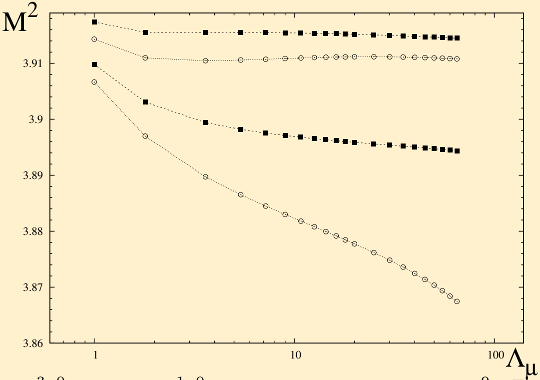

M^2 for (top) $1\,{}^3S_1^0$, (bottom) $1\,{}^1S_0^0$ as function of Λ_μ. (○) are from[9], (■) remove δG_2.

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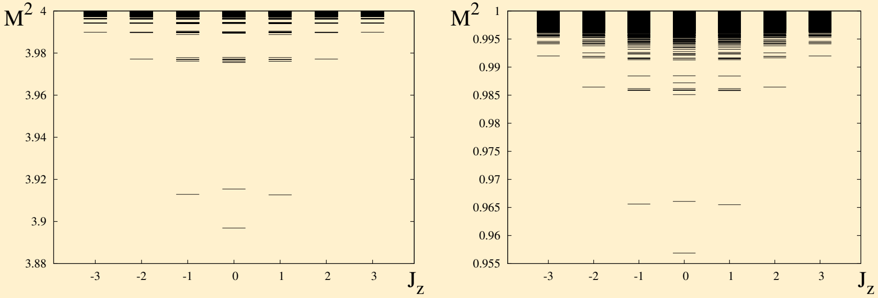
All the bound states!

Units of m_μ for $m_e = 1/2 m_\mu$, $\alpha = 0.3$, $\Lambda_e = 1/2\Lambda_\mu$, $\Lambda_\mu = m_\mu \alpha/2$



- Spectroscopic notation $^{2S+1}L_J^{J_z}$

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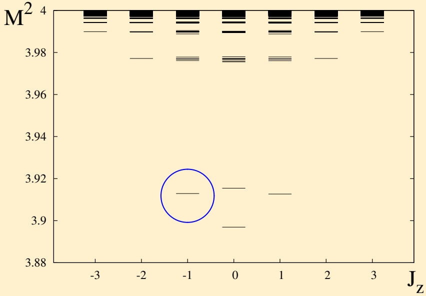

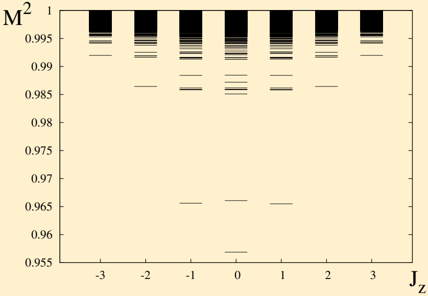

- Spectroscopic notation $^{2S+1}L_J^{J_z}$
- Rotational invariance is made **worse** for regularization

All the bound states!

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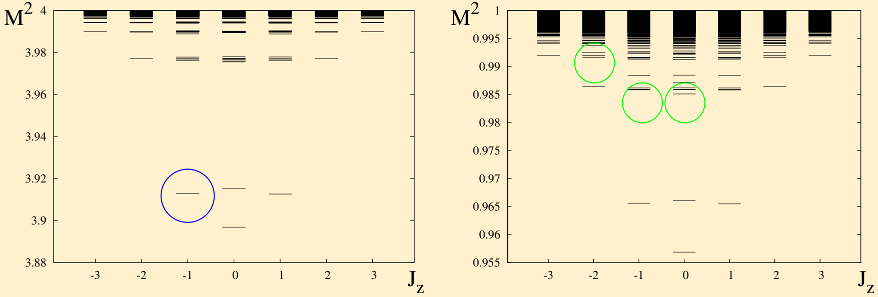

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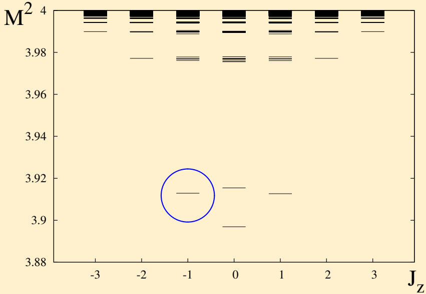
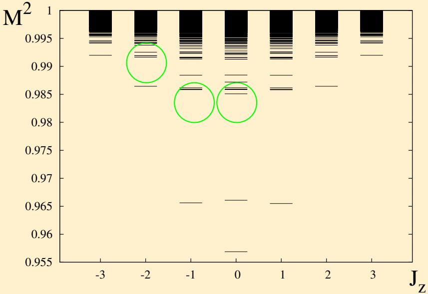
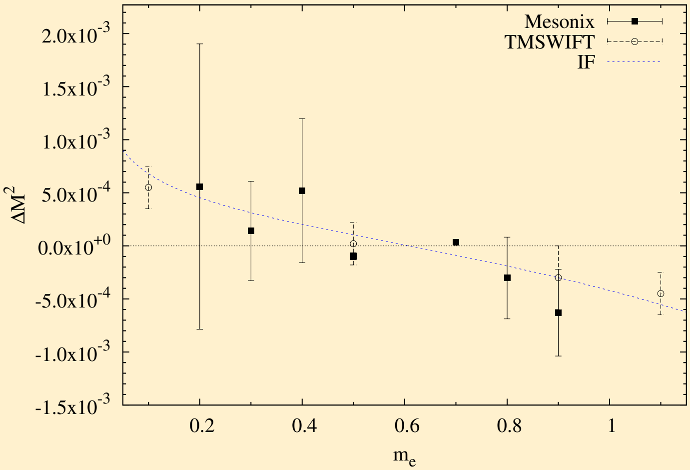




- Spectroscopic notation $^{2S+1}L_J^{J_z}$
- Rotational invariance is made **worse** for regularization
- **Renormalization effects** seen in positronium
- Discrepancy comes from excluding high momentum electron states

Effects of lepton loop consistent with instant form

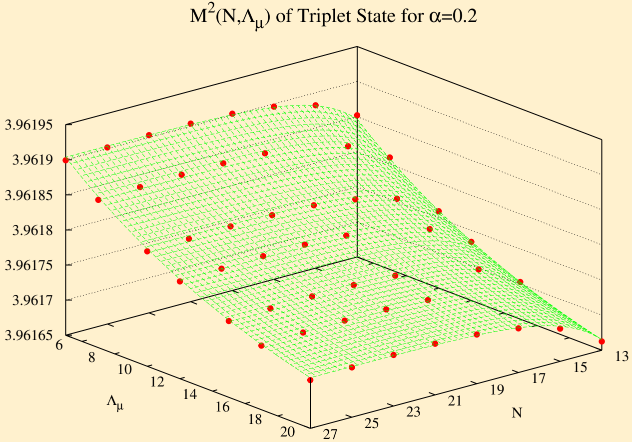


Eigenvalue shifts $\Delta M^2 \equiv M_{\mu\mu}^2 - M_0^2$ (in units of m_μ^2) for $1\,{}^3S_1^0$. The dashed line IF is the instant-form prediction, using the non-relativistic wave function, while the light-front (LF) points are obtained by taking $\alpha = 0.3$

N, Λ limits appear regulated

- Extract M^2 and f_i from finite-size and finite-cutoff through **Padé expansion**

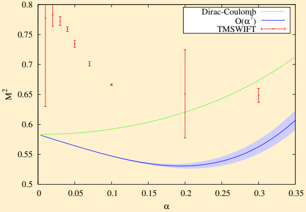
$$M^2(N,\Lambda) = \frac{M_\Lambda^2 + \frac{b}{N} + \frac{c}{N^2}}{1 + \frac{d}{N} + \frac{e}{N^2}}$$



Results for the $M^2(\alpha)$ and $f_{V,P}(\alpha)$[10]

α	$M^2(1^1S_0)$	$f_V(1^1S_0)$	$M^2(1^3S_1)$	$f_P(1^3S_1)$	$C_{\text{HFS,LF}}$	$C_{\text{HFS,ET}}$
0.01	3.99989993(3)	$4.18(10) \times 10^{-5}$	3.99989996(3)	$3.893(6) \times 10^{-5}$	0.76(77)	0.5834
0.02	3.9995997(2)	$1.1(4) \times 10^{-4}$	3.9996002(2)	$1.088(7) \times 10^{-4}$	0.79(42)	0.5837
0.03	3.9990987(4)	$2.05(9) \times 10^{-4}$	3.999101(2)	$1.93(6) \times 10^{-4}$	0.74(34)	0.5841
0.04	3.998397(4)	$3.15(5) \times 10^{-4}$	3.998404(5)	$3.07(7) \times 10^{-4}$	0.76(56)	0.5847
0.05	3.9974914(4)	$4.466(2) \times 10^{-4}$	3.9975098(3)	$3.95(2) \times 10^{-4}$	0.74(2)	0.5855
0.07	3.995068(3)	$7.404(7) \times 10^{-4}$	3.9951351(8)	$5.908(5) \times 10^{-4}$	0.7(4)	0.5877
0.1	3.98987(6)	$1.273(2) \times 10^{-3}$	3.990137(3)	$9.16(3) \times 10^{-4}$	0.67(2)	0.5922
0.2	3.9576(6)	$3.9(2) \times 10^{-3}$	3.9614(5)	$1.9(2) \times 10^{-3}$	0.6(2)	0.6204
0.3	3.8996(6)	$1.02(3) \times 10^{-2}$	3.91538(4)	$2.39(2) \times 10^{-3}$	0.49(2)	0.6735

$$f_{V(P)} = \int \frac{\mathrm{d}x}{\sqrt{x(1-x)}} \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^3} \left[\psi^J_{J_z=0}(\boldsymbol{k}_\perp, x, \uparrow\downarrow) \mp \psi^J_{J_z=0}(\boldsymbol{k}_\perp, x, \downarrow\uparrow)\right]$$



[10] H. Lamm and R. Lebed. "High Resolution Nonperturbative Light-Front Simulations of the True Muonium Atom". In: *Phys. Rev.* D94.1 (2016), p. 016004. arXiv: 1606.06358 [hep-ph].

Only minor issues with α-dependence

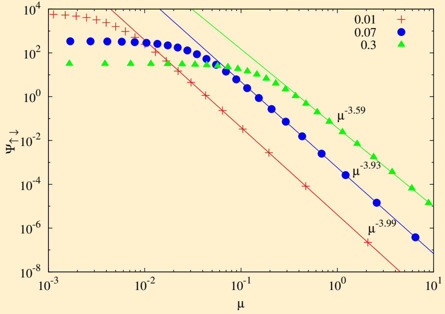
Fit parameters of $M^2(\alpha) = \left(\sum_\beta N_\beta \alpha^\beta\right)^2$. The perturbative predictions are $N_2 = -\frac{1}{4}, N_{4s} \approx -0.328, N_{4t} \approx 0.255$

E_n	α	N_0	N_2	N_4	N_5
1^1S_0	[0.01,0.3]	1.99999998(2)	-0.2500(2)	-0.37(5)	-0.04(21)
	[0.01,0.1]	1.999999990(2)	-0.25004(2)	-0.35(2)	0.08(10)
1^3S_1	[0.01,0.3]	1.99999998(2)	-0.24990(8)	0.39(3)	-0.78(8)
	[0.01,0.1]	1.999999979(6)	-0.24993(5)	0.38(3)	-0.60(26)

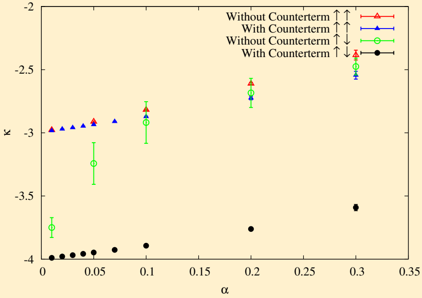
Fit parameters of $f_i(\alpha) = N\alpha^\beta$. The perturbative prediction is $\beta = 3/2$.

f_i	α	N	β
f_V	[0.01,0.3]	0.0412(9)	1.510(7)
	[0.01,0.1]	0.0411(3)	1.509(3)
f_P	[0.01,0.3]	0.022(3)	1.37(4)
	[0.01,0.1]	0.0240(8)	1.394(10)

Studying $\Psi(\mu) = a\mu^{-\kappa}$ can guide phenomenology



Use to build **models** of mesons



Strongly-coupled QED seems to indicate that $\kappa \to 3.5$ for dominant term, and $\kappa \to 2.5$ for subdominant term.

True muonium has pushed LF an inch further up the hill

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- TMSWIFT allows for renormalization implementations and new Fock states