LFQED in LF gauge
Form of the gauge boson propagator
Equivalence of Covariant and LFQED
Summary & Conclusion

Equivalence of the covariant and LFQED and the form of the gauge boson propagator

Anuradha Misra

University of Mumbai

Jefferson Lab

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Based on

- Anuradha Misra and Swati Warwadekar, Equivalence of covariant and light-front QED at one loop level, Phys. Rev. D71, 125011(2005)

- Equivalence of covariant and light-front QED revisited, Deepesh Bhamre, Anuradha Misra and Vivek Kumar Singh (in preparation)

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Light-Front QED

Light front Hamiltonian P^-, is the operator conjugate to the "time" evolution variable x^+ and is given by,

$$P^- = H_0 + V_1 + V_2 + V_3$$

H_0 is the free hamiltonian, V_1 is the standard, order-e three-point interaction,

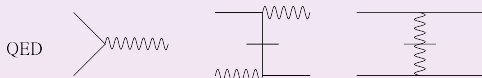$$V_1 = e \int d^2 x_\perp dx^- \bar{\xi} \gamma^\mu \xi a_\mu$$

V_2 is an order-e^2 non-local effective four-point vertex corresponding to an instantaneous fermion exchange,

$$V_2 = -\frac{i}{4} e^2 \int d^2 x_\perp dx^- dy^- \epsilon(x^- - y^-)(\bar{\xi} a_k \gamma^k)(x)\gamma^+ (a_j \gamma^j \xi)(y)$$

V_3 is an order-e^2 non-local effective four-point vertex corresponding to an instantaneous photon exchange,

$$V_3 = -\frac{e^2}{4} \int d^2 x_\perp dx^- dy^- (\bar{\xi} \gamma^+ \xi)(x)|x^- - y^-|(\bar{\xi} \gamma^+ \xi)(y)$$

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- Interaction vertices in LFQED

- This leads to many more one loop diagrams in LFQED as compared to covariant theory

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Light Front QED at one loop level

- Renormalization of light-front QED at one loop level addressed by
  D. Mustaki, S. Pinsky, J. Shigemitsu and K. Wilson (1991)
- In light-cone time-ordered Hamiltonian perturbation theory, amplitudes
  are evaluated by calculating the matrix elements of the transition matrix

$$T = V + V\frac{1}{p^- - H_0}V + \cdots$$

  between appropriate initial and final states up to the desired order

- Fermion self energy, vacuum polarization and vertex correction diagrams
  have been evaluated at one loop level using light-cone time-ordered
  Hamiltonian perturbation theory (Mustaki *et al*)

- Equivalence of one loop self energy, vacuum polarization and vertex
  correction diagrams was addressed
  (AM and S. Warwadekar, PR D 71, 125011 (2005), AM and S. Patel,
  Phys.Rev.D82:125024,(2010))

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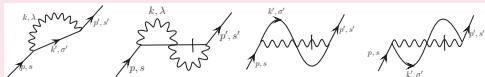Summary & Conclusion

Fermion Self Energy in LFQED

In light-cone time-ordered perturbation theory(LCTOPT), fermion self energy at $O(e^2)$ has three contributions given by

$$\bar{u}(p, s') \Sigma_1(p) u(p, s) = \langle p, s' | V_1 \frac{1}{p^- - H_0} V_1 | p, s \rangle$$

$$\bar{u}(p, s') \Sigma_2(p) u(p, s) = \langle p, s' | V_2 | p, s \rangle$$

and

$$\bar{u}(p, s') \Sigma_3(p) u(p, s) = \langle p, s' | V_3 | p, s \rangle$$

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The contributions of these diagrams are given by

$$\bar{u}(p,s')\Sigma_1(p)u(p,s) = \frac{e^2}{m} \int \frac{d^2k_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+(p^+-k^+)} \frac{\bar{u}(p,\sigma)\gamma^\mu(\slashed{k}'+m)\gamma^\nu u(p,s)d_{\mu\nu}}{p^- - k^- - k'^-}$$

$$\bar{u}(p,s')\Sigma_2(p)u(p,s) = \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2k_\perp}{(2\pi)^3} \int_0^{+\infty} \frac{dk^+}{k^+(p^+-k^+)}$$

$$\bar{u}(p,s')\Sigma_3(p)u(p,s) = \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2k_\perp}{(2\pi)^3} \left[\int_0^{+\infty} \frac{dk^+}{(p^+-k^+)^2} - \int_0^{+\infty} \frac{dk^+}{(p^++k^+)^2} \right]$$
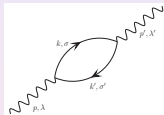
Note that all the momenta are on shell:

$$p = \left(p^+, \frac{p_\perp^2 + m^2}{2p^+}, p_\perp \right), k = \left(k^+, \frac{k_\perp^2}{2k^+}, k_\perp \right),$$
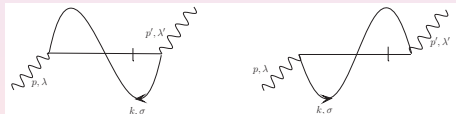
$$k' = \left(p^+ - k^+, \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+-k^+)}, p_\perp - k_\perp \right)$$

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Vacuum Polarization

- Vacuum polarization at one loop level

- Additional diagram in LFQED

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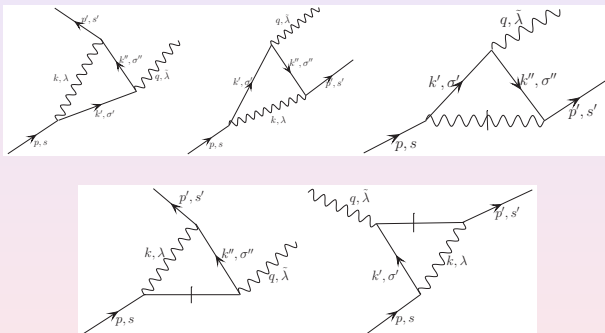
Vertex correction

- A total of 5 irreducible one loop diagrams in LFQED (Mustaki *etal* 1991)

- The last two diagrams do not contribute to Λ^+. The first three have been calculated by Mustaki *etal* using light-cone time-ordered Hamiltonian perturbation theory but for the $+$ component only.

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Equivalence of covariant and light-front field theory

- J. Eycke & F. Rohrlich (1974) : Equivalence of LFQED in LF gauge and conventional QED in Coulomb gauge formally at S-matrix level
- N. Ligterink & B. Bakker(1994) : N. Schoonderwoerd & Bakker (1998) : Equivalence of covariant and light-front perturbation theory at the level of Feynman diagrams
- AM & S. Warwadekar (2005, 2010) : Equivalence of conventional QED and LFQED

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Equivalence of Covariant and LFQED one loop diagrams : Fermion Self Energy

- One loop self energy correction in covariant QED
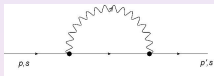
- To establish equivalence, one starts with covariant expression for electron self energy in the light- front gauge ,

$$\sum(p) = \frac{(ie)^2}{2mi} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\not{p} - \not{k} + m)\gamma^\nu d'_{\mu\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][k^2 + i\epsilon]}$$

and performs the integration over k^-, where $\frac{d'_{\mu\nu}}{k^2}$ is the photon propagator in light-cone gauge in covariant perturbation theory

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Gauge boson propagator in light-front gauge

- Two forms of gauge boson propagator in light-front gauge

- $d_{\mu\nu} = \left[- g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{k^+} \right]$

- $d'_{\mu\nu} = \left[- g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{k^+} - \frac{k^2 n_\mu n_\nu}{(k^+)^2} \right]$

  where $n_\mu = \delta^+_\mu$
  (S.J. Brodsky & P.P.Srivastava (2003), A. Suzuki & J. Sales (2003, 2004), C-R. Ji, Z. Li & A. Suzuki (2015))

- The third term is consistently dropped in the actual calculations arguing that such terms have no physical significance since they do not propagate any information

- In LF Hamiltonian QED calculations, the third term is not needed as all the particles are on shell

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Doubly transverse gauge boson propagator

- P.P.Srivastava and S.J. Brodsky, PRD64, 045006(2001)
  - LFQCD in LF gauge - free field satisfies both Lorentz condition as an operator condition as well as light-cone gauge condition
  - Propagator transverse to both its 4-momentum as well as the gauge direction
  - Interacting theory has additional instantaneous interactions

- A. Suzuki & J. Sales, NP A725 (2003)139, Mod. Phy.Lett. A19 (2004) 2831
  - Necessary and sufficient condition to uniquely define the light-cone gauge is $n \cdot A = \partial \cdot A = 0$ so the corresponding Lagrangian multiplier to be added to Lagrangian density is proportional to $(n \cdot A)(\partial \cdot A) = 0$
  - Doubly transverse gauge boson propogator can be derived by using the Lagrange multiplier method
  - The constraint $\partial \cdot A = 0$, together with the constraint $A^+ = 0$, substituted in the Lagrangian, leads to the 2-component formalism in LF with only physical degrees of freedom

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Proof of equivalence

- $\Sigma(p)$ can be rewritten as a sum of three terms

$$\Sigma(p) = \Sigma_1^{(a)}(p) + \Sigma_1^{(b)}(p)$$

- $\Sigma_1^{(a)}(p)$ is given by

$$\Sigma_1^{(a)}(p) = \frac{ie^2}{2m} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\not{p} - \not{k} + m)\gamma^\nu d_{\mu\nu}(k)}{[(p-k)^2 - m^2][k^2 + i\epsilon]}$$

  This is the contribution of two term photon propagator

- $\Sigma_1^{(b)}(p)$ given by

$$\Sigma_1^{(b)}(p) = -\frac{ie^2}{2m} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\not{p} - \not{k} + m)\gamma^\nu \delta_{\mu+}\delta_{\nu+}k^2}{[(p-k)^2 - m^2 + i\epsilon][k^2 + i\epsilon](k^+)^2}$$

  This arises from the third term in the photon propagator

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In $\Sigma_1^{(a)}(p)$, we substitute

$$\not{p} - \not{k} = \gamma^+ \left[\left(\frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)} \right) \right] + \gamma^-(p^+ - k^+) - \gamma_\perp(p_\perp - k_\perp)$$
$$+ \gamma^+ \left[p^- - k^- - \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)} \right]$$

and integrate over light-cone energy k^- to get

$$\sum_1^{(a)}(p) = \frac{e^2}{m} \int \frac{d^2 k_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \frac{\gamma^\mu (\not{k}' + m)\gamma^\nu d_{\mu\nu}(k)}{p^- - k^- - k'^-}$$
$$+ \frac{e^2}{2m} \int_0^\infty \frac{dk^+}{2k^+} \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{\gamma^\mu \gamma^+ \gamma^\nu d_{\mu\nu}(k)}{2(p^+ - k^+)}$$

where

$$\not{k}' = (\not{p} - \not{k})_{on} = \gamma^+ \left[\left(\frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)} \right) \right] + \gamma^-(p^+ - k^+) - \gamma_\perp(p_\perp - k_\perp)$$

The first term is the integral representing the standard propagating diagram and the second can be shown to be the expression for the diagram involving the four point vertex corresponding to instantaneous fermion exchange

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- The other two diagrams involving the 4-point instantaneous photon exchange vertex are obtained by performing the k^- integration in $\Sigma_1^{(b)}(p)$ given by

$$\Sigma_1^{(b)}(p) = -\frac{ie^2}{2m} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\not{p} - \not{k} + m)\gamma^\nu \delta_{\mu+}\delta_{\nu+}k^2}{[(p-k)^2 - m^2 + i\epsilon][k^2 + i\epsilon](k^+)^2}$$

- A.M. & S. Warwadekar (2005) : This contribution comes from the third term in the photon propagator and by carefully performing the k^--integration leads to the light-front expression for the self energy diagram corresponding to the instantaneous photon exchange

- Hence the equivalence cannot be proved if we neglect the third term in the doubly transverse photon propagator

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Issue of the form of photon propagator

- L. Mantovani, B. Pasquini, X. Xiong & A. Bacchetta, Phys. Rev. D 94, 116005 (2016) : using the same procedure as us, claimed that the equivalence can be proved without the third term in the photon propagator - "Revisiting the equivalence of light-front and covariant QED in the light-cone gauge"
- This work : (D. Bhamre, AM and V.K.Singh, in preparation) : "Re-Revisiting the ...... "
- Mantovani *et al* :
  (a) "Equivalence is actually not achieved by AM & SW, as $d_{\mu\nu}(k)$ in

$$\sum_{1}^{(a)}(p) = \frac{e^2}{m} \int \frac{d^2 k_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \frac{\gamma^\mu (\not{k}' + m)\gamma^\nu d_{\mu\nu}(k)}{p^- - k^- - k'^-}$$
$$+ \frac{e^2}{2m} \int_0^\infty \frac{dk^+}{2k^+} \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{\gamma^\mu \gamma^+ \gamma^\nu d_{\mu\nu}(k)}{2(p^+ - k^+)}$$

  is not on-shell" - not true as k^- integration has already been done
 (b) Proof of equivalence for vertex correction is only for the + component

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Vertex correction

- The standard covariant expression for vertex correction in the light-front gauge $\Lambda^\mu(p, p', q)$ is given by

$$e^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha(\slashed{p}' - \slashed{k} + m)\gamma^\mu(\slashed{p} - \slashed{k} + m)\gamma^\beta d'_{\alpha\beta}(k)}{[(p-k)^2 - m^2 + i\epsilon][(p'-k)^2 - m^2 + i\epsilon][k^2 + i\epsilon]}$$

- Performing the k^--integration, one obtains the light-front expressions for Λ^+ component ie the sum of following diagrams
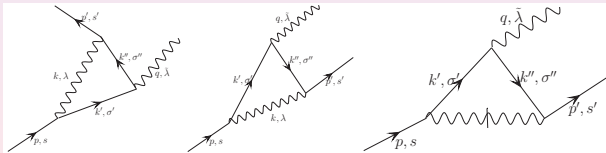
in case of $+$ component of one loop correction to Λ^μ
AM & S. Warwadekar(2005)
The remaining two diagrams do not contribute to Λ^+

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To prove equivalence, we rewrite the fermion momenta in terms of on-shell momenta

$(\not{p} - \not{k}) = \not{k}'_{on} + \frac{\gamma^+[(p-k)^2 - m^2]}{2(p^+ - k^+)}$

$(\not{p}' - \not{k}) = \not{k}''_{on} + \frac{\gamma^+[(p'-k)^2 - m^2]}{2(p'^+ - k^+)}$

Performing the k^- integration carefully, the one loop vertex correction can be written as

$$\Lambda^\mu(p, p', q) = \Lambda^{\mu(a)} + \Lambda^{\mu(b)} + \Lambda^{\mu(c)} + \Lambda^{\mu(d)} + \Lambda^{\mu(e)}$$

where

$\Lambda^{\mu(a)}(p, p', q) = e^3 \int \frac{d^2 k_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+ k'^+ k''^+} \frac{\gamma^\alpha (\not{k}''_{on} + m) \gamma^\mu (\not{k}'_{on} + m) \gamma^\beta d_{\alpha\beta}(k)}{(p^- - k_{on}^- - k_{on}'^-)(p^- - q^- - k_{on}^- - k_{on}''^-)}$
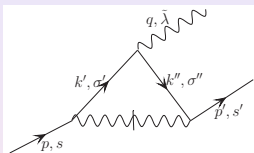
$\Lambda^{\mu(b)}(p, p', q) = -e^3 \int \frac{d^2 k_\perp}{(4\pi)^3} \int_{p'^+}^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \frac{\gamma^\alpha (\not{k}''_{on} + m) \gamma^\mu (\not{k}'_{on} + m) \gamma^\beta d_{\alpha\beta}(k)}{(p^- - k_{on}^- - k_{on}'^-)(p^- - p'^- - k_{on}^- + k_{on}''^-)}$

$\Lambda^{\mu(c)}(p, p', q) = 2e^3 \int \frac{d^2 k_\perp}{(4\pi)^3} \int_{p'^+}^{p^+} \frac{dk^+}{(k^+)^2 k'^+ k''^+} \frac{\gamma^+ (\not{k}''_{on} + m) \gamma^\mu (\not{k}'_{on} + m) \gamma^+}{(p^- - p'^- - k_{on}^- + k_{on}''^-)}$

$\Lambda^{\mu(d)}(p, p', q) = e^3 \int \frac{d^2 k_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+ k'^+ k''^+} \frac{\gamma^\alpha (\not{k}''_{on} + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k)}{(p'^- - k_{on}^- - k_{on}''^-)}$

$\Lambda^{\mu(e)}(p, p', q) = e^3 \int \frac{d^2 k_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \frac{\gamma^\alpha \gamma^+ \gamma^\mu (\not{k}'_{on} + m) \gamma^\beta d_{\alpha\beta}(k)}{(p^- - k_{on}^- - k_{on}'^-)}$

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- The third term in the photon propagator reproduces the expression for the the one loop vertex correction diagram containing the instantaneous interaction vertex

$\Lambda^{\mu(c)}(p, p', q) = 2e^3 \int \frac{d^2 k_{\perp}}{(4\pi)^3} \int_{p'^+}^{p^+} \frac{dk^+}{(k^+)^2 k'^+ k''^+} \frac{\gamma^+(k''_{on}+m)\gamma^\mu(k'_{on}+m)\gamma^+}{(p^- - p'^- - k'^-_{on} + k''^-_{on})}$
arises from the third term in the photon propagator

- $\Lambda^{\mu(d)}(p, p', q)$ and $\Lambda^{\mu(e)}(p, p', q)$ are zero when $\mu = +$ and hence do not contribute to Λ^+.

- D. Bhamre, AM and V.K.Singh (2018) : The two diagrams that do not contribute to Λ^+ (and hence were not calculated by Mustaki *etal*) have been calculated using light-cone time ordered Hamiltonian perturbation theory and found to be equal to $\Lambda^{\mu(d)}(p, p', q)$ and $\Lambda^{\mu(e)}(p, p', q)$ above.

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Issue of the form of photon propagator

- Mantovani *et al* : use two term photon propagator and split it into an on-shell pat and an off-shell part
- Mantovani *et al* : Equivalence can be proved with the two term photon propagator by rewriting it as
  $d_{\mu\nu}(k) = d_{\mu\nu}(k_{on}) + d_{\mu\nu}(k_{off-shell})$
  where $k^{\mu}_{on-shell} = (k^+, \frac{k_{\perp}^2 + m^2}{2k^+}, \mathbf{k}_{\perp}^2)$
  and
  $k^{\mu}_{off-shell} = (0, \frac{k^2 - m^2}{2k^+}, \mathbf{0}_{\perp}) = \frac{k^2 - m^2}{2k^+} n^{\mu}$
  Authors claim : second term reproduces the instantaneous diagram
- However, they use $d_{\mu\nu}(k) = d_{\mu\nu}(k_{on}) - \frac{n_{\mu}n_{\nu}k^2}{k^{+2}}$ which is nothing but the doubly transverse photon propagator

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- In the proof of equivalence for vertex correction diagrams, Mantovani *et al* start with the following diagrams and perform k^- integration along with the two term photon propagator to arrive at the LFQED result
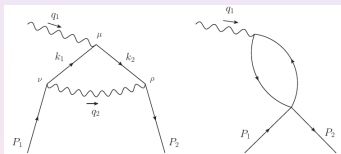
- The second diagram, however, involves an instantaneous 4-fermion vertex, which appears only after one uses the constraint equations to eliminate A^- from the Lagrangian - a procedure which does not leave the formalism covariant anymore.

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Summary

- The on-shell part of the fermion propagator, when considered with the two term photon propagator leads to the regular self energy and vertex correction diagrams

- The remaining part of the fermion propagator in covariant theory leads to the instantaneous fermion exchange diagram in LFTOPT

- The third term in the photon propagator is necessary to reproduce the instantaneous photon exchange diagrams and hence to establish equivalence of the two formalisms at this level

- The earlier proof of equivalence for Λ^+ is generalized and equivalence is established for Λ^μ by calculating the diagrams involving the instantaneous fermion exchange in LFTOPT and showing that they can be obtained by k^--integration from the terms containing $\not{k}' - \not{k}'_{on}$, which give zero result when $\mu = +$ and hence were not considered in the earlier proof (k' is fermion momentum).

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