Approximate three-quark Hamiltonian in heavy-flavor QCD

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K. Serafin, M. Gómez-Rocha, J. More, S.D. Głazek, arXiv: 1805.03436

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Many-body problem

$$H|\psi\rangle = P^-|\psi\rangle$$

$$|\psi\rangle = \begin{bmatrix} \dots \\ |4Q\,\bar{Q}\rangle \\ \dots \\ |3Q\,3G\rangle \\ |3Q\,2G\rangle \\ |3Q\,G\rangle \\ |3Q\rangle \end{bmatrix}$$

Many-body problem

$$H|\psi\rangle = P^-|\psi\rangle \qquad\qquad H_s|\psi_s\rangle = P^-|\psi_s\rangle$$

$$|\psi\rangle = \begin{bmatrix} \dots \\ |4Q\,\bar{Q}\rangle \\ \dots \\ |3Q\,3G\rangle \\ |3Q\,2G\rangle \\ |3Q\,G\rangle \\ |3Q\rangle \end{bmatrix} \longrightarrow |\psi_s\rangle = \begin{bmatrix} \dots \\ |4Q_s\,\bar{Q}_s\rangle \\ \dots \\ |3Q_s\,3G_s\rangle \\ |3Q_s\,2G_s\rangle \\ |3Q_s\,G_s\rangle \\ |3Q_s\rangle \end{bmatrix}$$

Many-body problem

$$H|\psi\rangle \quad = \quad P^-|\psi\rangle \qquad\qquad H_s|\psi_s\rangle \quad = \quad P^-|\psi_s\rangle$$

RGPEP

$$|\psi\rangle = \begin{bmatrix} \dots \\ |4Q\,\bar{Q}\rangle \\ \dots \\ |3Q\,3G\rangle \\ |3Q\,2G\rangle \\ |3Q\,G\rangle \\ |3Q\rangle \end{bmatrix} \longrightarrow |\psi_s\rangle = \begin{bmatrix} \dots \\ |4Q_s\,\bar{Q}_s\rangle \\ \dots \\ |3Q_s\,3G_s\rangle \\ |3Q_s\,2G_s\rangle \\ |3Q_s\,G_s\rangle \\ |3Q_s\rangle \end{bmatrix}$$

$$q_s = \mathcal{U}_s^\dagger q_0 \mathcal{U}_s$$
$$q_s^\dagger = \mathcal{U}_s^\dagger q_0^\dagger \mathcal{U}_s$$

Heavy quarks allow for several simplifications

- We choose s such that

$$0.9 \text{ fm} \approx \frac{1}{\Lambda_{\text{QCD}}} \gg \; s \; \gtrsim \frac{1}{m_Q} \approx 0.05 \text{ fm (bottom quark)}$$

- For $s \ll 1/\Lambda_{\text{QCD}}$ effective coupling constant is small in accordance with asymptotic freedom, we can expand in powers of g,

$$H_s \quad = \quad H_{s\,0} + gH_{s\,1} + g^2 H_{s\,2} + \ldots$$

- Choosing $s \gtrsim 1/m_Q$ we can neglect Fock sectors with extra $Q_s \bar{Q}_s$ pairs.
- Gluons, however, still pose a problem, bacause they are massless.

Gluon mass ansatz

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & H_{s\,0} + g^2 H_{s\,2} & g H_{s\,1} \\ \cdot & \cdot & g H_{s\,1} & H_{s\,0} + g^2 H_{s\,2} \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix} = P^- \begin{bmatrix} \cdot \\ \cdot \\ |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} H_{s\,0} + \mu_s^2 & g H_{s\,1} \\ g H_{s\,1} & H_{s\,0} + g^2 H_{s\,2} \end{bmatrix} \begin{bmatrix} |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix} = P^- \begin{bmatrix} |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix}$$

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$$\begin{bmatrix} H_{s\,0} + \mu_s^2 & g H_{s\,1} \\ g H_{s\,1} & H_{s\,0} + g^2 H_{s\,2} \end{bmatrix} \begin{bmatrix} |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix} = P^- \begin{bmatrix} |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix}$$

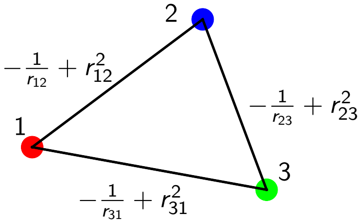
$$\downarrow$$

$$H_{\mathrm{eff}\ s} |3Q_s\rangle = \frac{M^2 + P^{\perp 2}}{P^+} |3Q_s\rangle \ ,$$

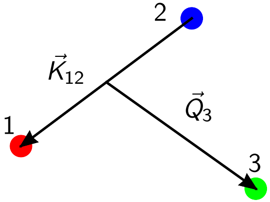
$$\langle l | H_{\mathrm{eff\ s}} | r \rangle = \langle l | \Bigg[H_{s\,0} + g^2 H_{s\,2} + \frac{1}{2} g H_{s\,1} \left(\frac{1}{E_l - H_{s\,0} - \mu_s^2} + \frac{1}{E_r - H_{s\,0} - \mu_s^2} \right) g H_{s\,1} \Bigg] | r \rangle \ .$$

The result

Three Coulomb potentials and three harmonic oscillator potentials.



In relative variables

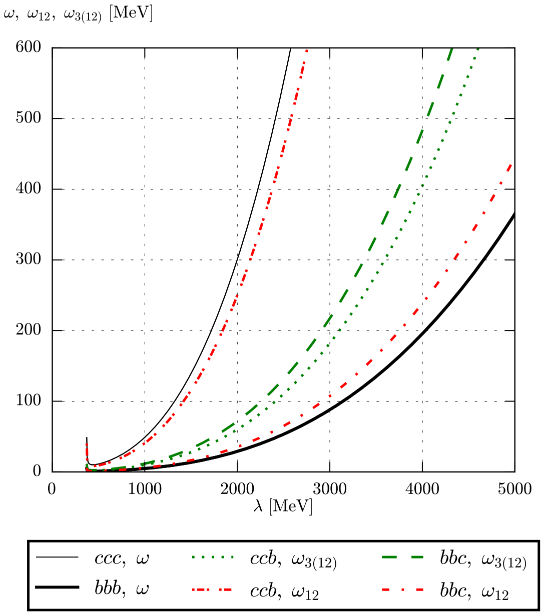


three Coulomb potentials and two collective harmonic oscillators with frequencies ω_{12} and $\omega_{3(12)}$.

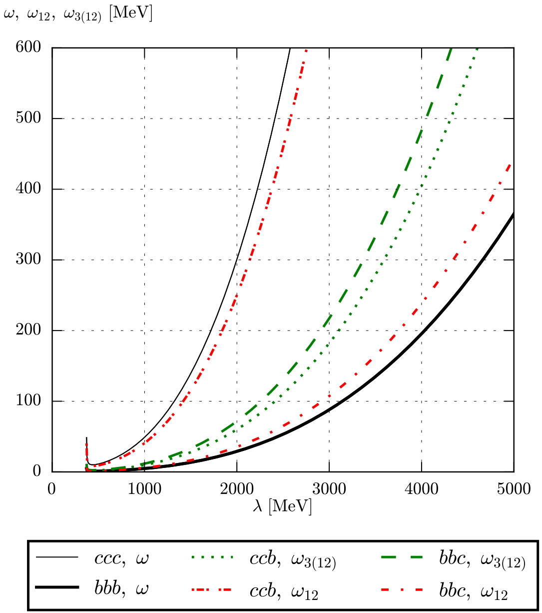
$$\omega_{12}^2 = \frac{1}{m_1}\frac{\alpha\lambda^3}{18\sqrt{\pi}}\left[\left(\frac{\lambda^2}{2m_1^2}\right)^{3/2} + \frac{1}{2}\left(\frac{\lambda^2}{m_1^2+m_3^2}\right)^{3/2}\right], \quad \lambda = 1/s$$

$$\omega_{3(12)}^2 = \frac{2m_1+m_3}{2m_1 m_3}\frac{\alpha\lambda^3}{18\sqrt{\pi}}\left(\frac{\lambda^2}{m_1^2+m_3^2}\right)^{3/2}.$$

λ dependence



λ dependence



We choose

$$\lambda_{Q\bar{Q}} = \sqrt{\alpha}\left(a\ \bar{m}_{Q\bar{Q}} + b\right)$$

$$\lambda_{3Q} = \sqrt{\alpha}\left(a\ \bar{m}_{3Q} + b\right)$$

Sketch of hadron spectra

Treating Coulomb as a perturbation to harmonic oscillator we obtain analytical formulas for masses of mesons and baryons.

$$\alpha(\lambda) = \left[\beta_0 \log(\lambda^2/\Lambda^2_{\rm QCD})\right]^{-1} , \quad \beta_0 = (33 - 2n_f)/(12\pi) , \quad n_f = 2$$
$$\alpha(M_Z) = 0.1181 .$$

Fit to $\Upsilon(1S)$, $\Upsilon(2S)$ and $\chi_{b1}(1P)$ $\rightarrow$ $m_b = 4698$ MeV, $\lambda_{b\bar{b}} = 4258$ MeV

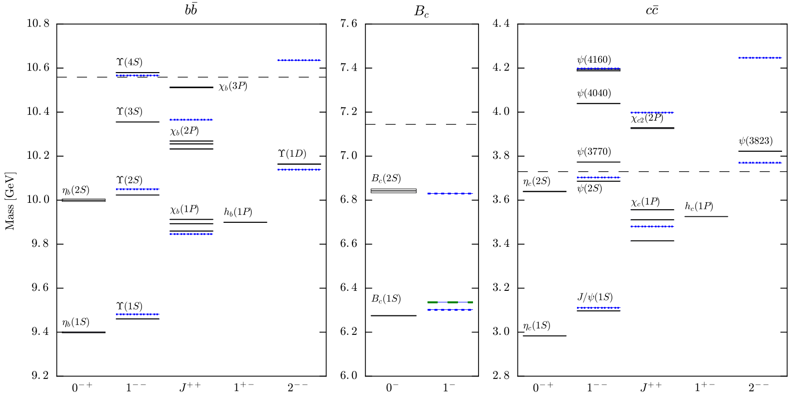
Fit to J/ψ, $\psi(2S)$ and $\chi_{c1}(1P)$ $\rightarrow$ $m_c = 1460$ MeV, $\lambda_{c\bar{c}} = 1944$ MeV

Values of $\lambda_{b\bar{b}}$, $\lambda_{c\bar{c}}$, and m_b, m_c allow us to fix

$$a = 1.589 ,$$
$$b = 783 \text{ MeV} .$$

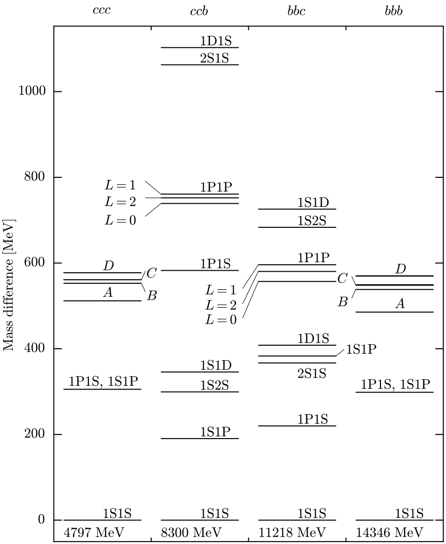
These coefficients imply λs and, hence, ωs for all other systems.

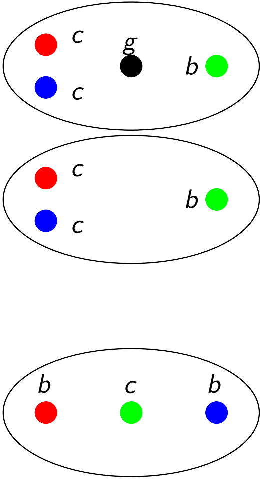
Meson spectra



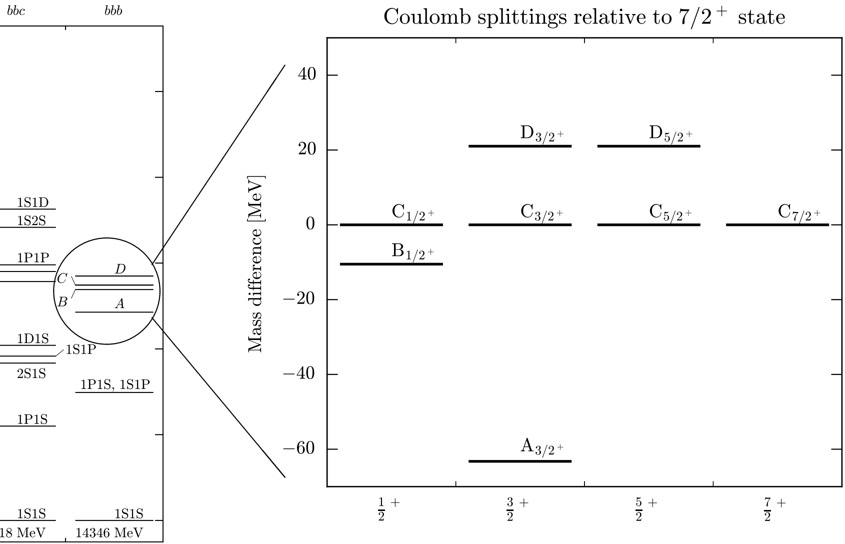
Dotted blue: our masses.
Solid black: recent PDG masses.
Dashed green: Gómez-Rocha, Hilger, Krassnigg, PRD93 074010 (2016)

Baryon spectra



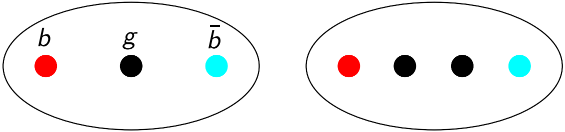


Splittings of the second band of harmonic oscillator

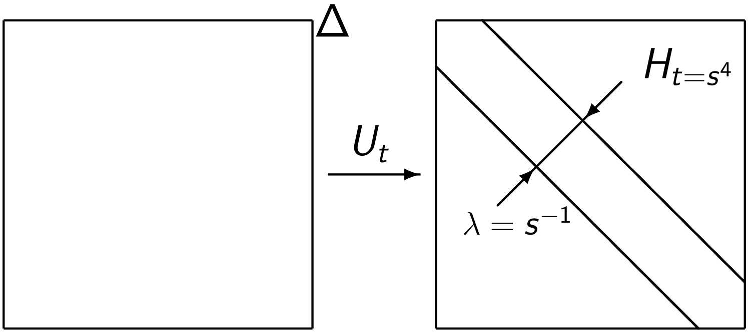


Outlook and perspectives

- 4^{th}-order RGPEP studies of H_s in QCD.
- Nonperturbative solutions of RGPEP equation.
- Gluon strings in a Hamiltonian approach?



- Hybrid mesons, glueballs, tetraquarks . . .
- Light quarks after gluons (gluon mass).



- The narrowing of the Hamiltonian is realized by the following differential equation,

$$\frac{d}{dt}\mathcal{H}_t = \left[[\mathcal{H}_{\text{free}}, \tilde{\mathcal{H}}_t], \mathcal{H}_t\right] \quad , \qquad \mathcal{H}_t = H_t(q_0) \ .$$

- It allows for nonperturbative calculations.

Effective-particle picture of nucleons

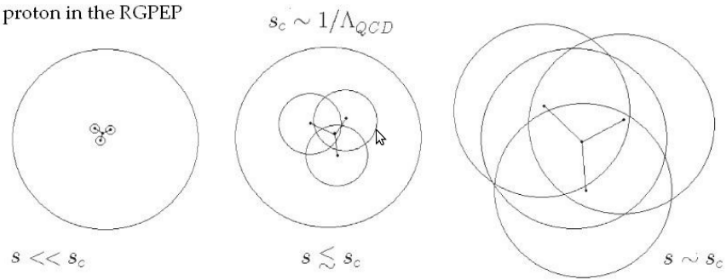


FIG. 6: The RGPEP scale-dependent proton picture

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Path to Schrödinger equation

- QCD Lagrangian $\rightarrow$ canonical Front Form Hamiltonian H_{can}.
- H_{can} is ill-defined $\rightarrow$ needs regularization and counterterms.
- The counterterms are determined using RGPEP.
- RGPEP gives us also a family of renormalized Hamiltonians parametrized by the size of effective particles.
- We reduce the Fock space to two sectors only at the prize of introducing the gluon mass function.
- Taking advantage of asymptotic freedom we reduce Fock space perturbatively to one sector only obtaining the effective Hamiltonian.
- Taking nonrelativistic limit we obtain Schrödinger equation for three quarks.

Baryon and meson Schrödinger equations I

The effective eigenvalue equation for heavy baryons in QCD with two heavy flavors, implied by our gluon mass hypothesis, is

$$
\begin{aligned}
& \left[\frac{K_{12}^2}{2\mu_{12}} + \frac{Q_3^2}{2\mu_{3(12)}} + \frac{\mu_{12}\omega_{12}^2\Delta_K^2}{2} + \frac{\mu_{3(12)}\omega_{3(12)}^2\Delta_Q^2}{2}\right]\psi_t(\vec{K}_{12},\vec{Q}_3) \\
+ & \int \frac{d^3q}{(2\pi)^3} V_C(q)\ \psi_t\left(\vec{K}_{12}-\vec{q},\vec{Q}_3\right) \\
+ & \int \frac{d^3q}{(2\pi)^3} V_C(q)\ \psi_t\left(\vec{K}_{12}+\frac{1}{2}\vec{q},\vec{Q}_3+\vec{q}\right) \\
+ & \int \frac{d^3q}{(2\pi)^3} V_C(q)\ \psi_t\left(\vec{K}_{12}+\frac{1}{2}\vec{q},\vec{Q}_3-\vec{q}\right) \\
= & \ E\ \psi_t(\vec{K}_{12},\vec{Q}_3)\ ,
\end{aligned}
$$

where Δ denotes Laplacian, reduced masses are $\mu_{12} = m_1/2$, $\mu_{3(12)} = 2m_1 m_3/(2m_1 + m_3)$ and

$$V_C(q) = -\frac{2}{3}\frac{g^2}{q^2}\ .$$

Baryon and meson Schrödinger equations II

The baryon mass eigenvalue is obtained from the eigenvalue E,

$$M = (2m_1 + m_3)\sqrt{1 + \frac{2E}{2m_1 + m_3}} \, .$$

We omitted the BF spin-dependent terms and the RGPEP form factors whose numerical inclusion requires the fourth-order RGPEP calculation. The associated quarkonium eigenvalue equation is

$$\left[\frac{K_{12}^2}{2\mu_{12}} + \frac{\mu_{12}\omega_{12}^2\Delta_K^2}{2}\right] \psi_t(\vec{K}_{12}) + 2\int \frac{d^3q}{(2\pi)^3} V_C(q)\, \psi_t(\vec{K}_{12} - \vec{q}) = E\, \psi_t(\vec{K}_{12}) \, ,$$

$$\omega_{12}^2 = \frac{\alpha(\lambda_{bc})}{18\sqrt{\pi}\,\mu_{bc}} \left(\frac{\lambda_{bc}^2}{\sqrt{m_b^2 + m_c^2}}\right)^3 ,$$

$$M = (m_1 + m_2)\sqrt{1 + \frac{2E}{m_1 + m_2}} \, ,$$

λs for baryons