Direct Calculation of Slope of Form Factors from Lattice QCD

David Richards (JLab)

Chia Cheng Chang (LBNL), Chris Bouchard (Glasgow)
Kostas Orginos (JLab/WM)

arXiv:1610.02354

Proton EM form factors

- Nucleon Pauli and Dirac Form Factors described in terms of matrix element of vector current

$$\langle N \mid V_\mu \mid N\rangle(\vec{q}) = \bar{u}(\vec{p}_f)\left[F_q(q^2)\gamma_\mu + \sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2m_N}\right]u(\vec{p}_i)$$

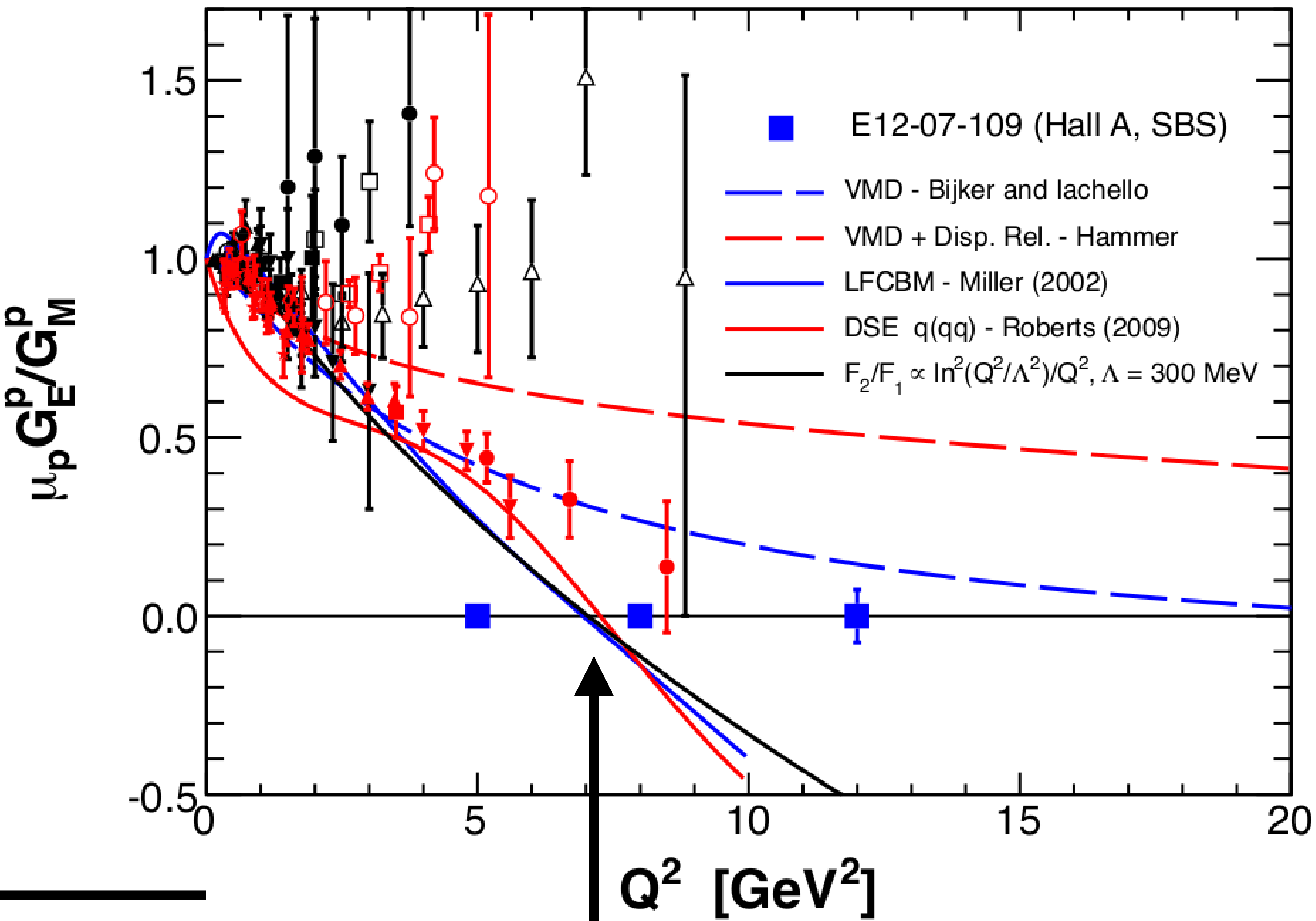
- Alternatively, Sach's form factors determined in experiment

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$

Charge radius is slope at $Q^2 = 0$

$$\left.\frac{\partial G_E(Q^2)}{\partial Q^2}\right|_{Q^2=0} = -\frac{1}{6}\langle r^2\rangle = \left.\frac{\partial F_1(Q^2)}{\partial Q^2}\right|_{Q^2=0} - \frac{F_2(0)}{4M^2}$$

EM Form factors - Expt

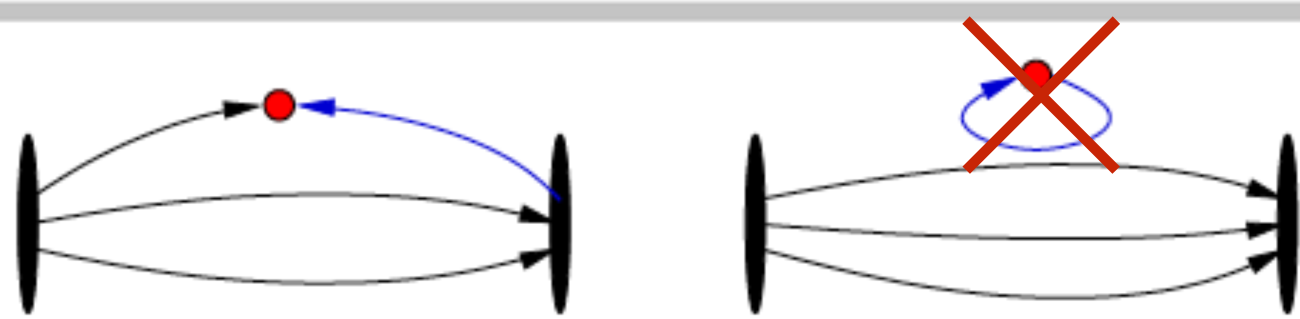


Nucleon Charge Radius

Quark and Gluon DOF

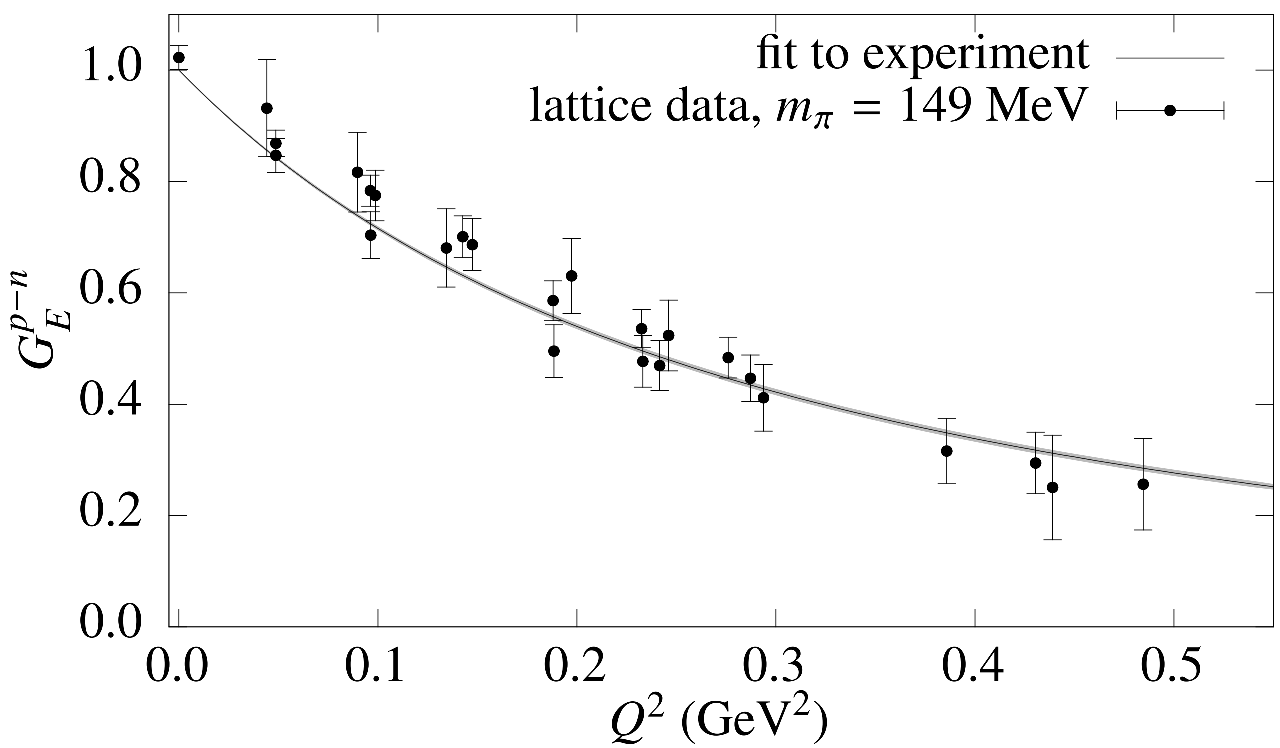
Cross zero

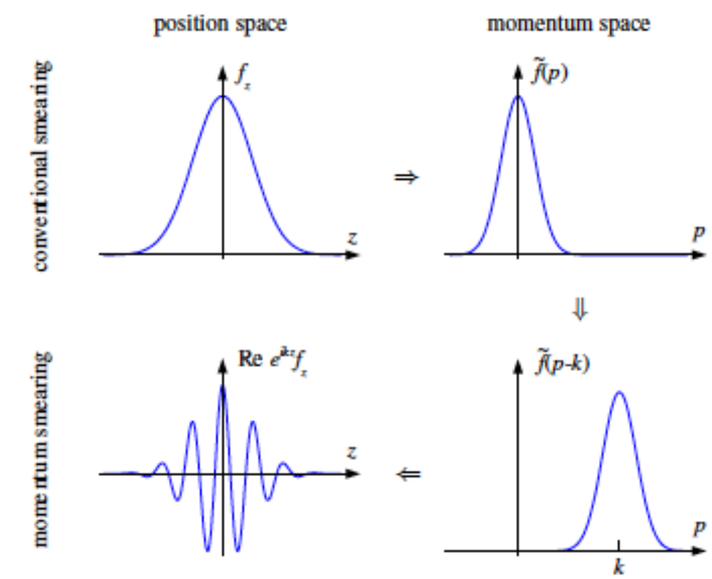
Form Factors in LQCD



$$C_{3\text{pt}}(t_{sep}, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0 \mid N(\vec{x}, t_{\text{sep}}) V_\mu(\vec{y}, t) \bar{N}(\vec{0}, 0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

Green et al (LHPC), Phys. Rev. D 90, 074507 (2014)





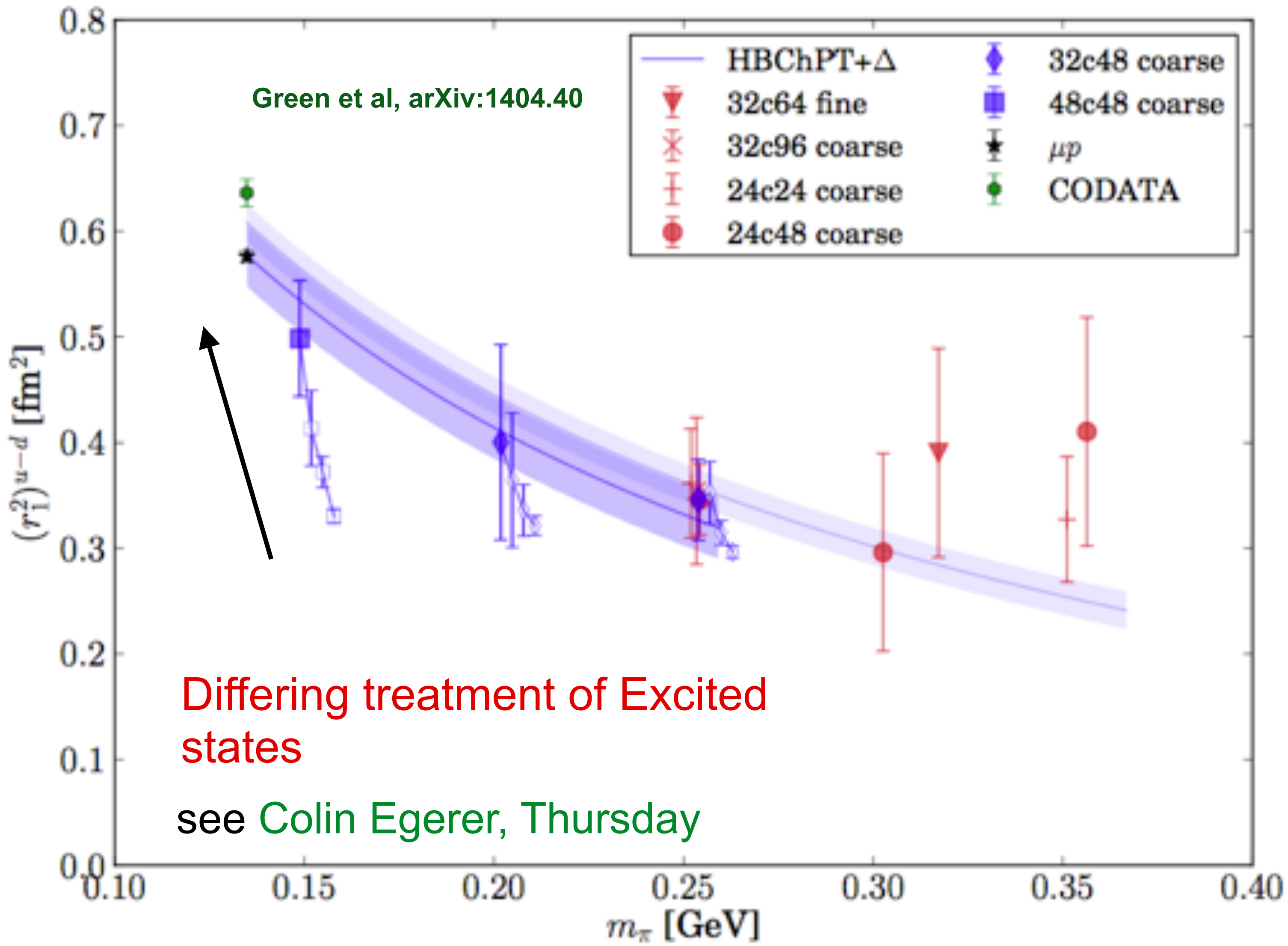
Boosted interpolating operators

Bali *et al.*, Phys. Rev. D 93, 094515 (2016)

Smallest non-zero Q^2 determined by spatial volume
⇒Calculate slope of form factor directly.

Highest momentum controlled by lattice spacing:
➡ Signal-to-noise Ratio
➡Discretization uncertainties
Key for pseudo/quasi/LCS distributions

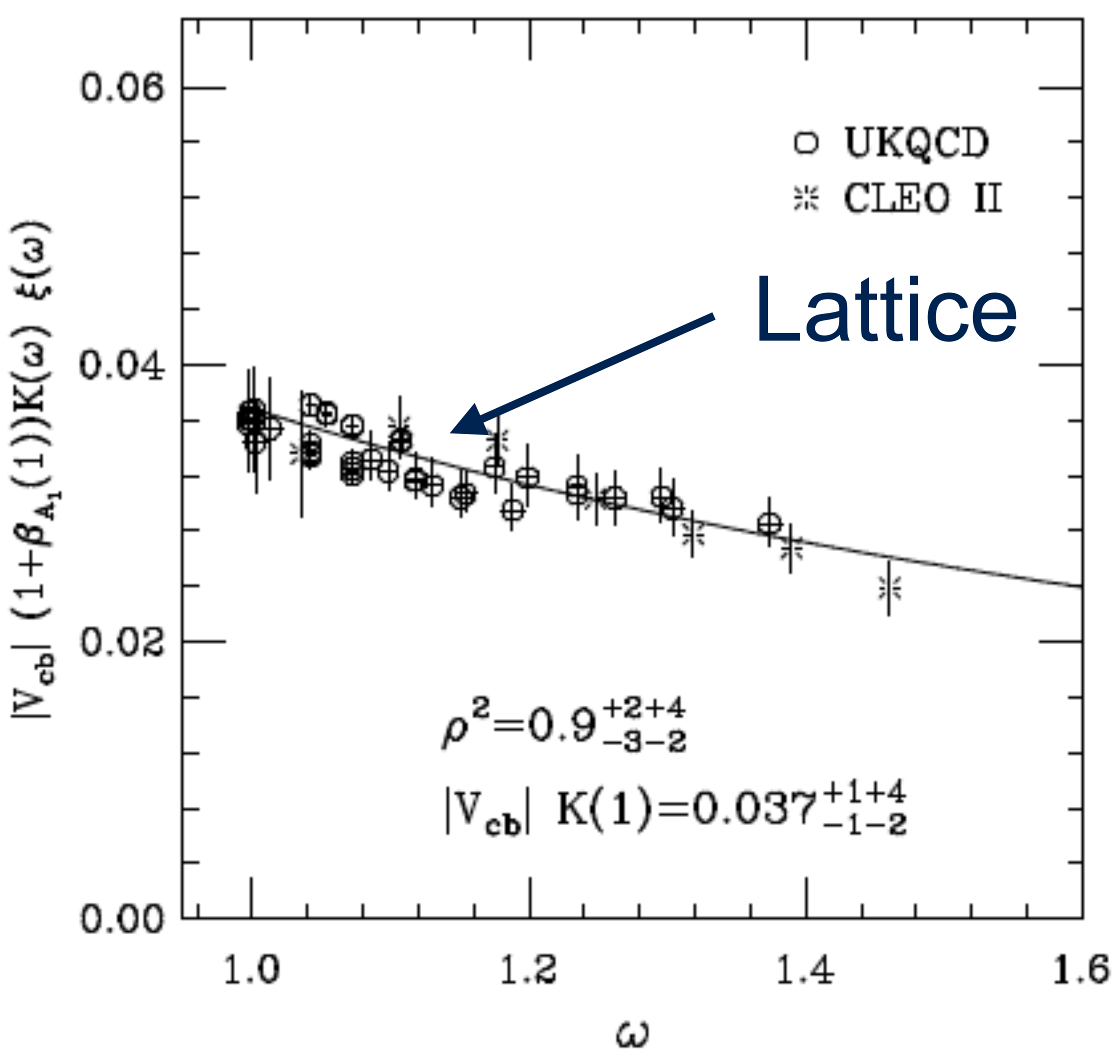
Charge Radius - Excited States



I will concentrate on direct calculation of *slope*

see also Green et al., Phys. Rev. D 97, 034504 (2018)
Twisted boundary conditions

Isgur-Wise Function and CKM matrix



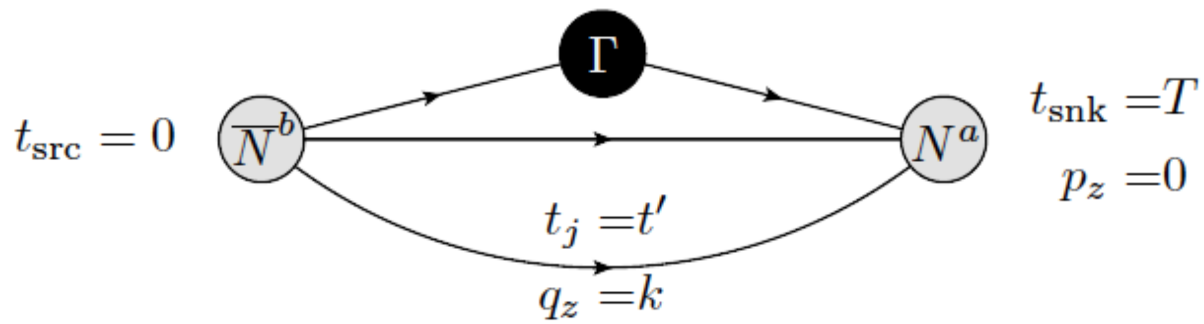
Extract V_{cb} if know intercept at zero recoil

↓

Calculate slope at zero recoil..

UKQCD, L. Lellouch et al., Nucl. Phys. B444, 401 (1995), hep-lat/9410013

Moment Methods



- Introduce three-momentum projected three-point function

$$C^{3\text{pt}}(t,t') = \sum_{\vec{x},\vec{x}'} \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle e^{-ikx'_z}$$

- Now take derivative w.r.t. k^2

whence

$$C'_{3\text{pt}}(t,t') = \sum_{\vec{x},\vec{x}'} \frac{-x'_z}{2k} \sin(kx'_z) \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle$$

$$\lim_{k^2 \to 0} C'_{3\text{pt}}(t,t') = \sum_{\vec{x},\vec{x}'} \frac{-x'^2_z}{2} \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle .$$

Odd moments vanish by symmetry

Moment Methods - II

- Analogous expressions for two-point functions:

$$C_{2\text{pt}}(t) = \sum_{\vec{x}} \left\langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \right\rangle e^{-ikx_z}$$

$$\Rightarrow$$

$$C'_{2\text{pt}}(t) = \sum_{\vec{x}} \frac{-x_z}{2k} \sin(kx_z) \left\langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \right\rangle$$

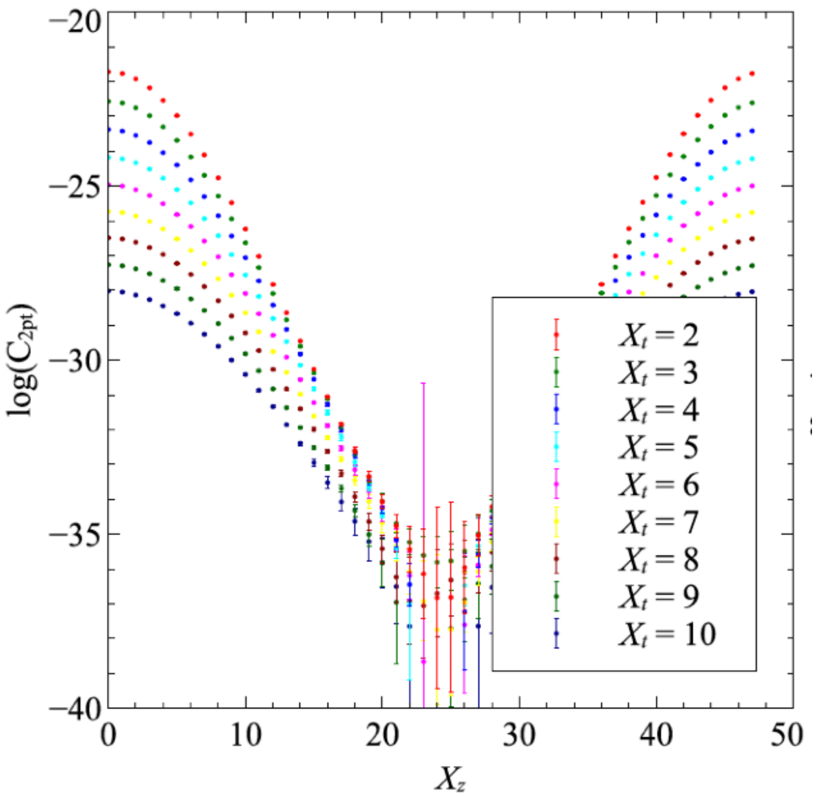
$$\Rightarrow$$

$$\lim_{k^2 \to 0} C'_{2\text{pt}}(t) = \sum_{\vec{x}} \frac{-x_z^2}{2} \left\langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \right\rangle .$$

Lowest coordinate-space moment ⇔ slope at zero momentum

Two-point correlator

Test Example



$\ln\left[C_{2\mathrm{pt}}(t, x_z)\right]$

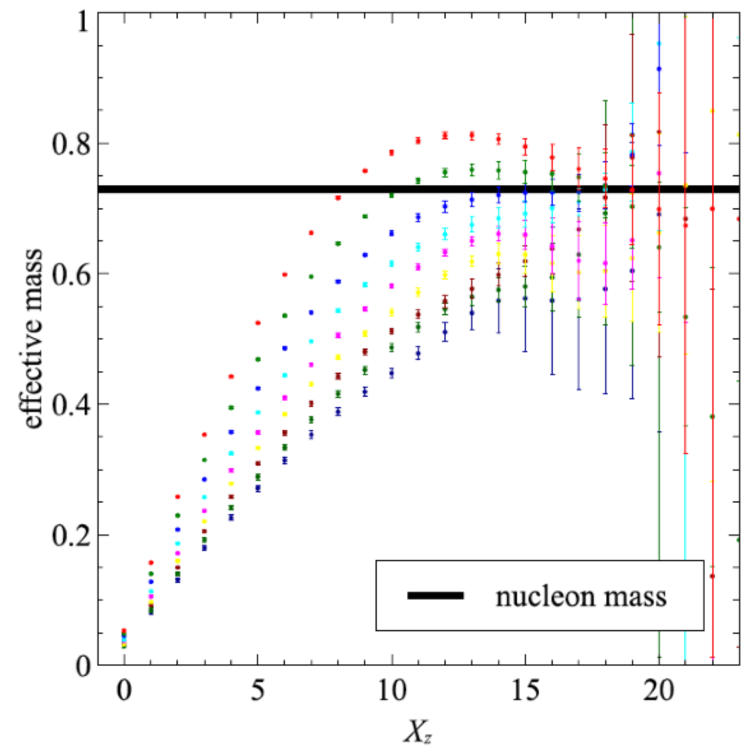
$$a \simeq 0.12 \text{ fm}$$
$$m_\pi \simeq 400 \text{ MeV}$$
$$\text{Lattice Size} : 24^3 \times 64$$

$$24^2 \times 48 \times 64$$

Any polynomial moment in x_z converges

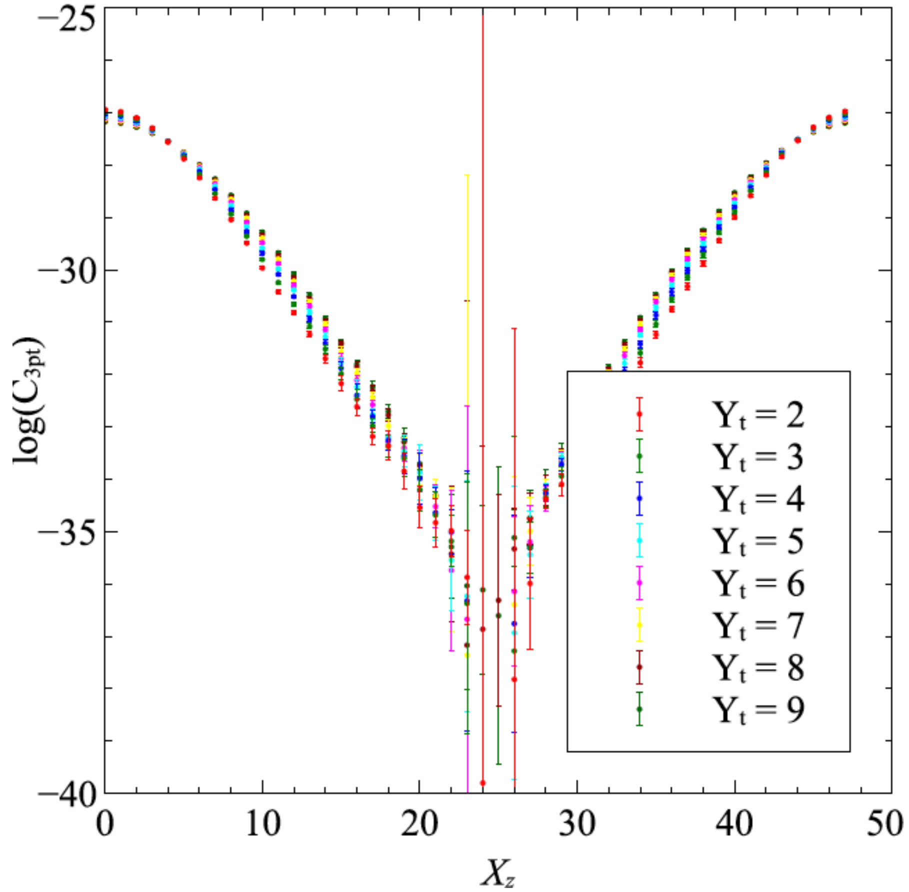
"Effective mass"

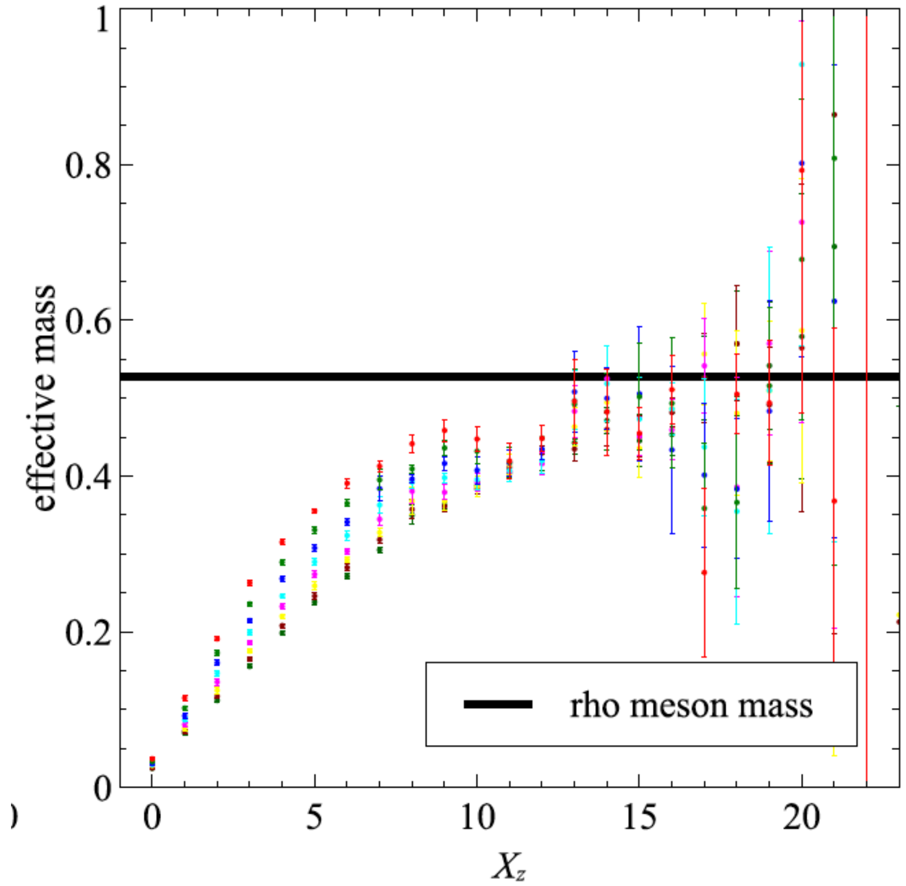
$\ln C_{2\mathrm{pt}}(t, x_z)/C_{2\mathrm{pt}}(t, x_z + 1)$



Three-point correlator

"Effective mass"





$\ln\left[C_{3\mathrm{pt}}(t', x'_z)\right]$

- Spatial moments push the peak of the correlator away from origin
- Larger finite volume corrections compared to regular correlators

Fitting the data…

$$C^{3\text{pt}}(t,t') = \sum_{n,m} \frac{Z_n^{\dagger a}(0)\Gamma_{nm}(k^2)Z_m^b(k^2)}{4M_n(0)E_m(k^2)} e^{-M_n(0)(t-t')} e^{-E_m(k^2)t'}$$

$$C_{2\text{pt}}(t) = \sum_m \frac{Z_m^{b\dagger}(k^2)Z_m^b(k^2)}{2E_m(k^2)} e^{-E_m(k^2)t}$$

Allow for multi-state contributions in the fit

where

$$Z_n^{\dagger a}(0) \equiv \langle \Omega | N^a | n, p_i = (0,0,0)\rangle$$

$$Z_m^b(k^2) \equiv \langle m, p_i = (0,0,k) | \overline{N}^b | \Omega \rangle$$

$$\Gamma_{nm}(k^2) \equiv \langle n, p_i = (0,0,0) | \Gamma | m, p_i = (0,0,k)\rangle$$

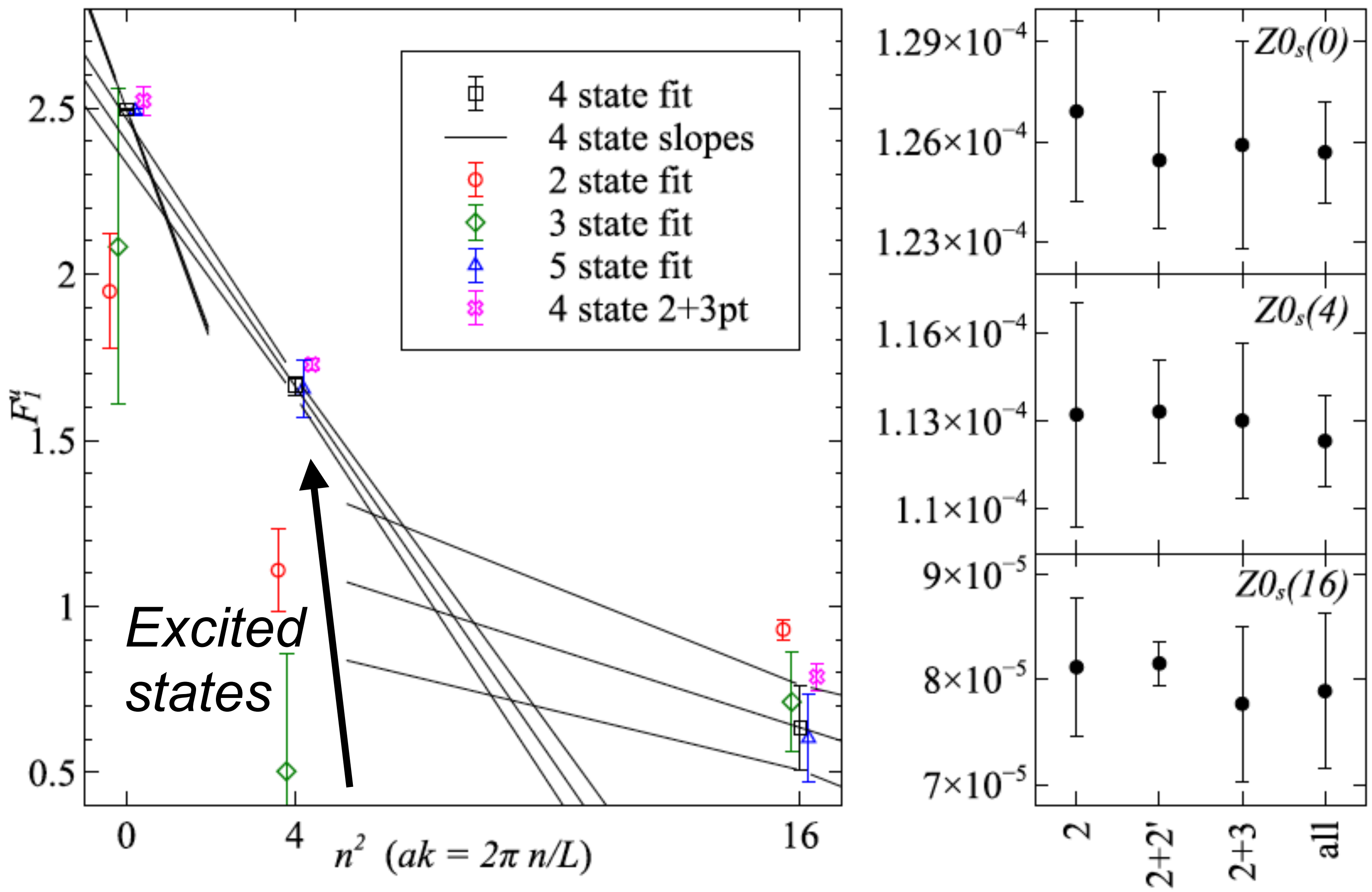
- Now look at the functional form of derivatives:

$$C'_{2\text{pt}}(t) = \sum_m C_m^{2\text{pt}}(t) \left(\frac{2Z_m^{b\prime}(k^2)}{Z_m^b(k^2)} - \frac{1}{2[E_m(k^2)]^2} - \frac{t}{2E_m(k^2)} \right)$$

Spatially extended sources - ***second distance scale***

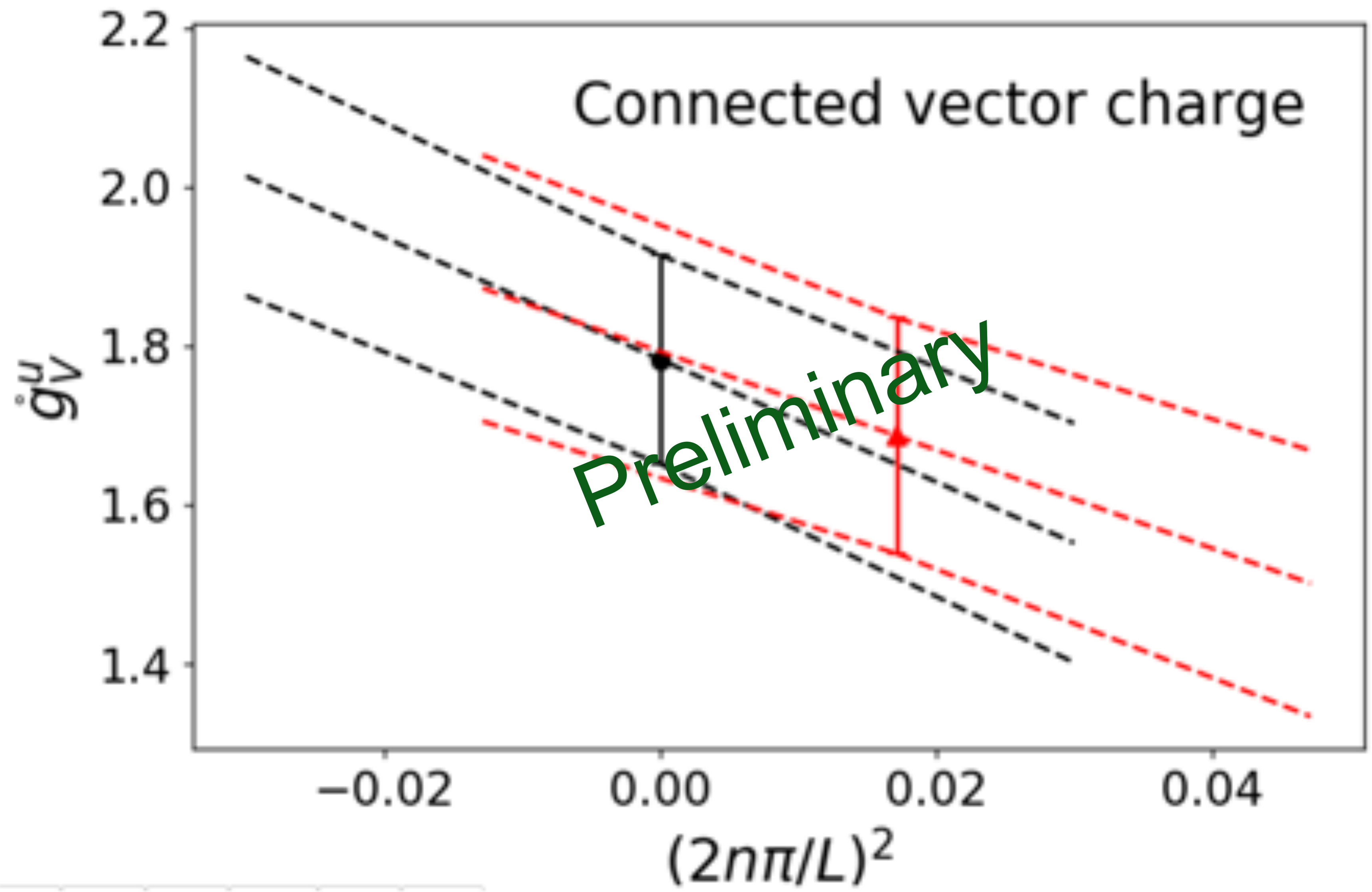
$$C'_{3\text{pt}}(t,t') = \sum_{n,m} C_{nm}^{3\text{pt}}(t,t') \left\{ \frac{\Gamma'_{nm}(k^2)}{\Gamma_{nm}(k^2)} + \frac{Z_m^{b\prime}(k^2)}{Z_m^b(k^2)} - \frac{1}{2[E_m(k^2)]^2} - \frac{t'}{2E_m(k^2)} \right\}$$

u-quark contribution to F_1 Form Factor



Multi-exponentials essential to control fit

Currently extending to $48^3 \times 96$



Conclusions

- Precise calculations of charge radius require fine control over systematic uncertainties.
- Moment methods allow direct calculations of slopes of form factors at *momenta allowed on lattice*
- Lowest (even) moment gives the slope at $Q^2 = 0$.
- Larger finite-volume effects than regular correlators (expected - no free lunch).
- Method illustrated here for u-quark contribution to EM form factor.
- Analysis on larger-volume lattices for complete isovector form factor near completion. *Can we understand volume dependence?*