Controlling excited-state contributions to nucleon charges g_A, g_T and g_S in Lattice QCD Calculations

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William and Mary

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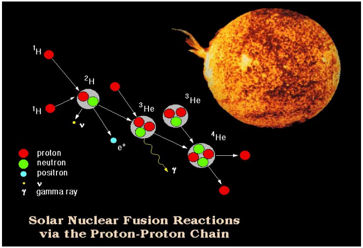
Outline

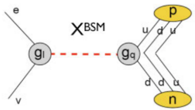
- Preliminaries - Isovector Nucleon Charges
- Ab initio QCD calculations using the Lattice
- Systematics & excited-state contamination
- Distillation
- Calculational Details
- g_A, g_S, g_T results and discussion

Motivation - Nucleon Isovector Scalar/Axial/Tensor Charges

- g_A and neutron decay
- Goldberger-Treiman relation

  ↳ $M_N g_A = f_\pi g_{\pi NN}$
- proton-proton fusion rate
- precision measurements of neutron decay
  - novel BSM interactions at TeV scale





[LANL: T-2 Nuclear Physics]

Axial Charge of the Nucleon

- Axial form-factors of the nucleon

$$\langle p(p',s')|\gamma^\mu\gamma^5\frac{\tau^3}{2}(k)|n(p,s)\rangle = \overline{q}_p(p',s')\left[\gamma^\mu\gamma^5F_1^5(k^2)+\frac{i\sigma^{\mu\nu}k_\nu}{2m}\gamma^5F_2^5(k^2)+\gamma^5\frac{k^\mu}{2m}F_3^5(k^2)\right]u_n(p,s)$$

- $F_2^5(k^2) = F_3^5(k^2) = 0$
  assuming isospin symmetry

- $F_1^5(0) \equiv g_A^{u-d}$

- Benchmark calculation in LQCD

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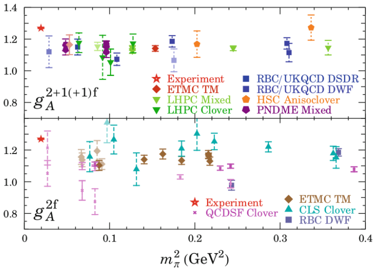

Figure 1: [H-W. Lin, H. Meyer, (2014) *Lattice QCD for Nuclear Physics*]

A Controlled Approximation

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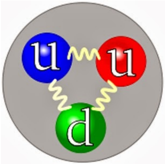
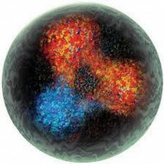
Smearing

$$C\left(t',t\right) = \langle \mathcal{O}\left(t'\right)\mathcal{O}^{\dagger}\left(t\right)\rangle = \sum_n \left|\langle \mathcal{O}|n\rangle\right|^2 e^{-E_n(t'-t)}$$

- Hadrons as extended objects
  - expect poor overlap of pt-like operators
- smearing $\Longrightarrow$ better overlap to low lying part of spectrum





Jacobi Smearing

- start with discretized Laplacian

$$-\nabla^2_{xy}(t) = 6\delta_{xy} - \sum_{j=1}^{3}\left[\tilde{U}_j(x,t)\,\delta_{x+\hat{\jmath},y} + \tilde{U}_j^{\dagger}(x-\hat{\jmath},t)\,\delta_{x-\hat{\jmath},y}\right]$$

- smearing operator:
  $J_{\sigma,n_\sigma}(t) = \left(1 + \frac{\sigma\nabla^2(t)}{n_\sigma}\right)^{n_\sigma} \rightarrow \lim_{n_\sigma\to\infty} J_{\sigma,n_\sigma}(t) = \mathrm{Exp}\left(\sigma\nabla^2(t)\right)$
- exp. suppression of high eigenmodes of Laplacian
  - lowest modes contribute appreciably to J_{σ,n_σ}
- when applied to quark fields $\tilde{\psi}(\vec{x},t) = J_{\sigma,n_\sigma}(t)\,\psi(\vec{x},t)$
  - reduces excited-state contributions to hadron correlators

Distillation

Consider 3D gauge-covariant Laplacian

$$-\nabla^2_{ab}\left(\vec{x},\vec{y};t\right) = 6\delta_{xy}\delta_{ab} - \sum_{j=1}^{3}\left[\tilde{U}_j\left(\vec{x},t\right)_{ab}\delta_{x+\hat{j},y} + \tilde{U}_j^{\dagger}\left(\vec{x}-\hat{j},t\right)_{ab}\delta_{x-\hat{j},y}\right]$$

- solutions to $-\nabla^2 v^{(k)} = \lambda^{(k)} v^{(k)}$ - ordered by $\lambda^{(k)}$

$$\square(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{N} v_a^{(k)}(\vec{x},t)\, v_b^{(k)\dagger}(\vec{y},t)$$

- define *Distillation* op. of rank $N \ll M = N_c \times N_X \times N_Y \times N_Z$
- **Our Aim**: rather than precise determination of g_A, g_S, g_T, can Distillation demonstrate controlled excited-states?

[M. Peardon, et. al., (2009) arXiv: 0905.2160v1]

Distillation - Properties & Advantages

- interpolator construction separable from quark propagation

- can construct interpolators to probe angular structure of states w/o recalculating $M^{-1}(t,t')$

- momentum projection at source & sink

$$C\left(t',t\right)=\sum_{\vec{x},\vec{y}} e^{-i\vec{p}\cdot\vec{x}}e^{i\vec{p}\cdot\vec{y}}\langle\mathcal{O}\left(\vec{x},t'\right)\mathcal{O}^{\dagger}\left(\vec{y},t\right)\rangle$$

- number of eigenvectors scales with V_3

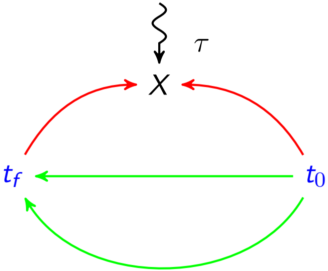
Components of a Calculation using Distillation

- **Solution Vectors**

$$M_f^{-1}(t,0)\,v(0)$$

- **Perambulators**

$$\tau_{\alpha\beta}(t,0) = v^{\dagger}(t)\,M_{\alpha\beta}^{-1}(t,0)\,v(0)$$



- **Elementals** - operator insertions in Distillation space

$$\Phi^{(i,j,k)}_{\alpha_1,\alpha_2,\alpha_3}(t) = \epsilon^{abc}\left(\mathcal{D}_1 v^{(i)}\right)^a \left(\mathcal{D}_2 v^{(j)}\right)^b \left(\mathcal{D}_3 v^{(k)}\right)^c (t)\, S_{\alpha_1,\alpha_2,\alpha_3}$$

- Correlators comprised of perambulators and elementals

Lattice & Operator Specifics

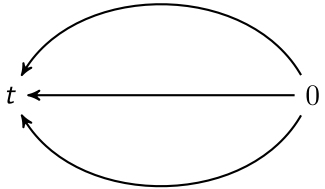
- $32^3 \times 64$ isotropic clover lattices
- $u/d + s$ flavor QCD
- $m_\pi = 356$ MeV
- $a = 0.098$ fm
- Distilled operators
  - 64 eigenvectors

2pt Correlation Function

- 'Local' interpolator

$$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[0]}_{L=0,S}\right)^{J^P=\frac{1}{2}^+}$$

- 'Projected' interpolator
  - interpolator basis $\mathcal{B} = \{\mathcal{O}_1, \ldots, \mathcal{O}_N\}$



[R. Edwards, et. al., (2011) arXiv: 1104.5152v2]

[J. Dudek, R. Edwards, (2012) arXiv: 1201.2349v1]

- $\mathcal{O}_i(t) \propto \epsilon^{abc}\mathcal{S}_i^{\alpha\beta\gamma}(\mathcal{D}_1\Box d)_a^\alpha(\mathcal{D}_2\Box u)_b^\beta(\mathcal{D}_3\Box d)_c^\gamma(t)$

$$C_{ij}^{\text{2pt}}(t,\vec{p}) = \sum_{\vec{w}} e^{-i\vec{p}\cdot\vec{w}} \sum_{\vec{y}} e^{i\vec{p}\cdot\vec{y}} \langle \mathcal{O}_i(t,\vec{w})\overline{\mathcal{O}}_j(0,\vec{y})\rangle$$

$$= V_3 \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_i(t,\vec{x})\overline{\mathcal{O}}_j(0)\rangle$$

Variational Analysis

- $\mathcal{B} = \{\mathcal{O}_1, \ldots, \mathcal{O}_N\}$
- optimal linear combination of interpolators to project onto $|n\rangle$, solve

$$C(t)\, u_n(t, t_0) = \lambda_n(t, t_0)\, C(t_0)\, u_n(t, t_0)$$

- solved for fixed reference time t_0 and all later times t
- solutions organized by magnitude of eigenvalues $\lambda_n(t, t_0)$ - principle correlators
- components of eigenvectors $u_n(t, t_0)$ yield weight of each $\mathcal{O}_i \in \mathcal{B}$ to interpolate $|n\rangle$ from vacuum
- projected operator $\quad \equiv \quad \mathcal{O}_{\text{proj}} = \sum_i u_n^i \mathcal{O}_i^\dagger$

2pt Decomposition

- With either 'local' or 'projected' interpolator

$$C_{2\text{pt}}\left(t,\vec{p}\right)=\sum_{\vec{x}}e^{-i\vec{p}\cdot\vec{x}}\sum_{\vec{y}}e^{i\vec{p}\cdot\vec{y}}\langle\mathcal{O}\left(\vec{x},t\right)\mathcal{O}^{\dagger}\left(\vec{y},0\right)\rangle$$

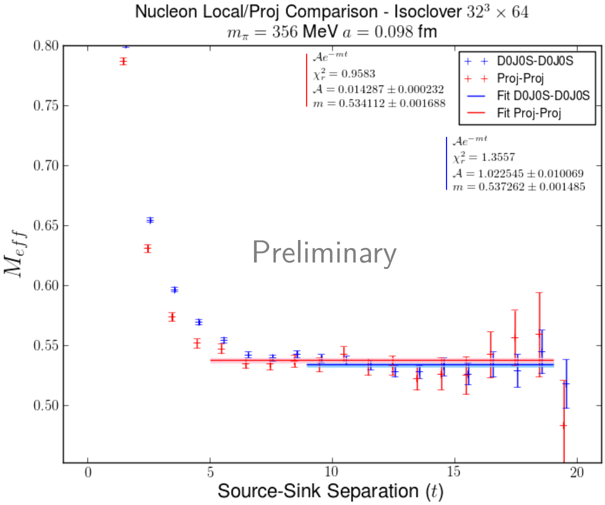
- Spectral decomp. at zero-momentum, keeping ground and first-excited states:

$$C_{2\text{pt}}\left(t,\vec{p}\right)=V_3\left(\frac{Z_0Z_0^{\dagger}}{2m_0}e^{-m_0t}+\frac{Z_1Z_1^{\dagger}}{2m_1}e^{-m_1t}\right)$$

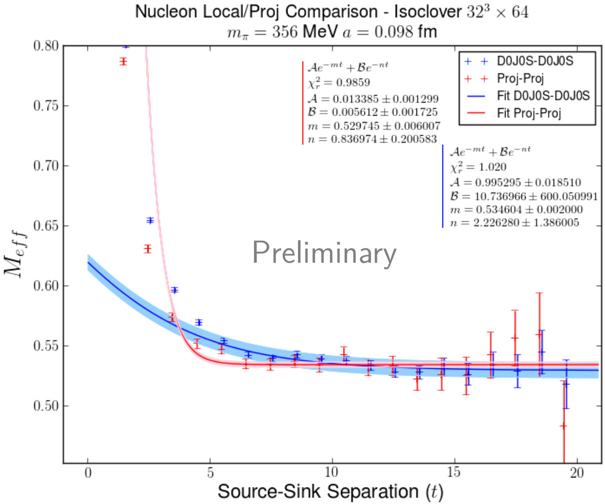
- A simple 2-state fit to extract masses

$$C_{2\text{pt}}\left(t\right)=Ae^{-m_0t}+Be^{-m_1t}$$

Effective Mass Comparison - Single State



Effective Mass Comparison - Two State



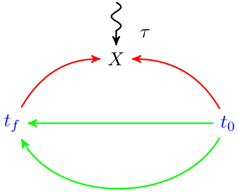
3pt Decomposition

- 2-state decomposition of 3pt correlator

$$C_{3\text{pt}}\left(t_{\text{sep}},\tau\right)=V_3\left(\frac{Z_0Z_0^\dagger}{4m_0^2}J_{00}e^{-m_0t_{\text{sep}}}+\frac{Z_0Z_1^\dagger}{4m_0m_1}e^{-m_0(t_{\text{sep}}-\tau)}e^{-m_1\tau}J_{01}+\frac{Z_1Z_0^\dagger}{4m_0m_1}e^{-m_1(t_{\text{sep}}-\tau)}e^{-m_0\tau}J_{10}+\frac{Z_1Z_1^\dagger}{4m_1^2}e^{-m_1t_{\text{sep}}}J_{11}\right)$$

- For zero-momentum states at fixed t_{sep}

$$C_{3\text{pt}}\left(\tau\right)=\left(\frac{V_3\left|Z_0\right|^2}{4m_0^2}J_{00}e^{-m_0t_{\text{sep}}}+\frac{V_3\left|Z_1\right|^2}{4m_1^2}J_{11}e^{-m_1t_{\text{sep}}}\right)$$



$$+\frac{V_3Z_0Z_1^\dagger}{4m_0m_1}J_{01}e^{-m_0t_{\text{sep}}}e^{-(m_1-m_0)\tau}+\frac{V_3Z_1Z_0^\dagger}{4m_0m_1}J_{10}e^{-m_1t_{\text{sep}}}e^{(m_1-m_0)\tau}$$

- Fit form: $C_{3\text{pt}}\left(\tau\right)=A+B\cosh\left[\left(m_1-m_0\right)\left(\tau-\frac{t_{\text{sep}}}{2}\right)\right]$

Extraction of Ground-state Matrix Element

$$A = \frac{V_3 \left|Z_0\right|^2}{4m_0^2} J_{00} e^{-m_0 t_{\text{sep}}} + \frac{V_3 \left|Z_1\right|^2}{4m_1^2} J_{11} e^{-m_1 t_{\text{sep}}}$$

- Fit t_{sep}-dependence of A to extract J_{00}

$$A\left(t_{\text{sep}}\right) = Xe^{-m_0 t_{\text{sep}}} + Ye^{-m_1 t_{\text{sep}}}$$
$$\Rightarrow J_{00} = \frac{4m_0^2}{V_3 \left|Z_0\right|^2} X$$

Method for renormalized g_Γ Extraction

- Relation between continuum and lattice charges

$$g_\Gamma = Z_\Gamma g_\Gamma^{\text{lat}}$$

- Z_V set based on knowledge of continuum vector charge
  - i.e. $1 + \mathcal{O}\left(a^2\right) = Z_V g_V^{\text{lat}} \quad \Longrightarrow \quad Z_V = 1/g_V^{\text{lat}} + \mathcal{O}\left(a^2\right)$
- Utilized definition of arbitrary renormalized charge

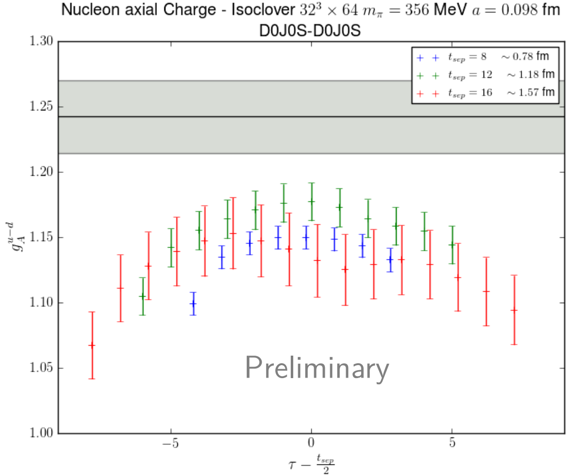
$$g_\Gamma = \frac{Z_\Gamma}{Z_V}\frac{g_\Gamma^{\text{lat}}}{g_V^{\text{lat}}}$$

- Z_Γ/Z_V on isoclover lattices with $a = 0.094$ fm and $m_\pi = 280$ MeV
  - n.b. Z_Γ appears to be weakly dependent on pion mass, but depends on lattice spacing

[B. Yoon, et. al., (2017) arXiv: 1611.07452v3]

Preliminary Estimate of g_A - Local Operator

$$Z_A/Z_V = 1.065(14)$$



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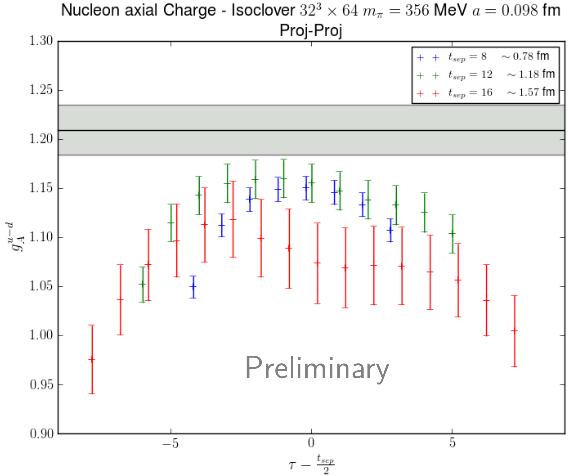
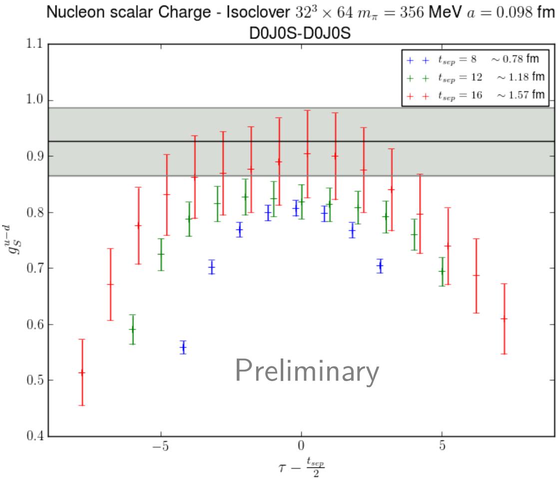


Preliminary Estimate of g_S - Local Operator

$$Z_S/Z_V = 0.959(27)$$



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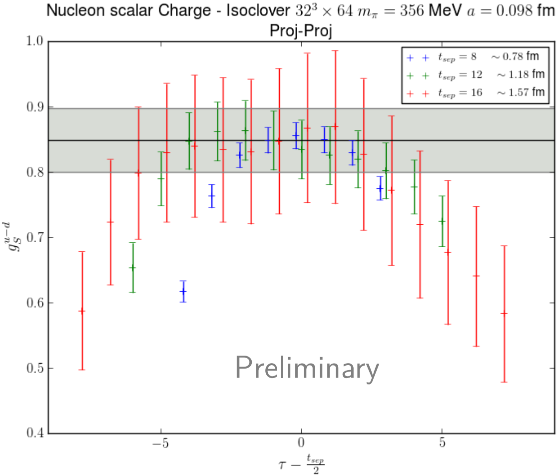
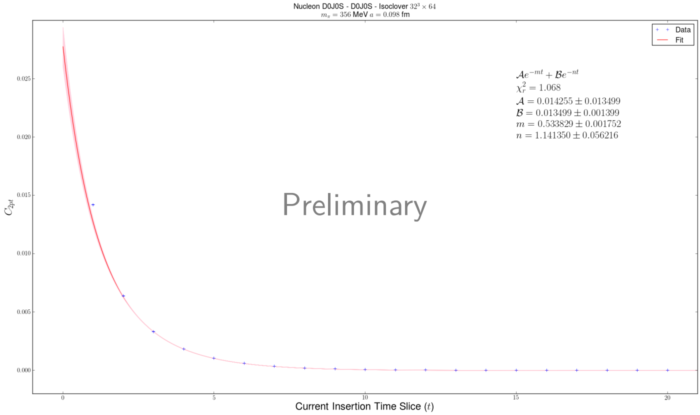


Closing Thoughts

- Distillation is a form of smearing
- Variational method applied to an extended basis of operators
  - effective mass plateaus for much smaller source-sink separations
  - better statistics
  - clear separation of ground & excited states - c.f. projected 2-state fit
- 3pt-2pt ratios demonstrate clear plateau
  - little to no src-snk dependence when using variationally optimized nucleon interpolators
- Considered forward-scattering of at rest nucleons
  - worse excited-state contamination for states in motion
- $\mathcal{Q}^2$ axial/vector form factors? Other structure calculations?

BACKUPS

Local Correlator - 2-state Fit



Operator Construction

- Non-hybrid Operators
  - $$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[0]}_{L=0,S}\right)^{J^P=\frac{1}{2}^+}$$
  - $$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=0,M}\right)^{J^P=\frac{1}{2}^+}$$
  - $$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=0,S}\right)^{J^P=\frac{1}{2}^+}$$
  - $$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=1,A}\right)^{J^P=\frac{1}{2}^+}$$
  - $$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=1,M}\right)^{J^P=\frac{1}{2}^+}$$
- Hydrid Operators
  - $$\left(N_M \otimes \left(\tfrac{3}{2}^+\right)^1_S \otimes D^{[2]}_{L=1,M}\right)^{J^P=\frac{1}{2}^+}$$
  - $$\left(N_M \otimes \left(\tfrac{3}{2}^+\right)^1_S \otimes D^{[2]}_{L=2,M}\right)^{J^P=\frac{1}{2}^+}$$

Principle Correlators - $t_0 = 2, 3$

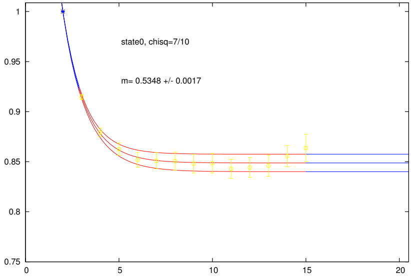


Figure 2: Ground-state principle correlator for $t_0 = 2$ and $t_Z = 5$.

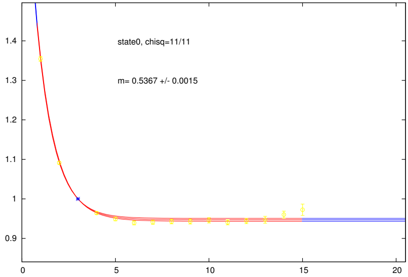


Figure 3: Ground-state principle correlator for $t_0 = 3$ and $t_Z = 5$.

Principle Correlators - $t_0 = 4, 5$

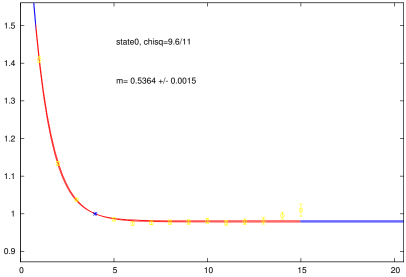


Figure 4: Ground-state principle correlator for $t_0 = 4$ and $t_Z = 5$.

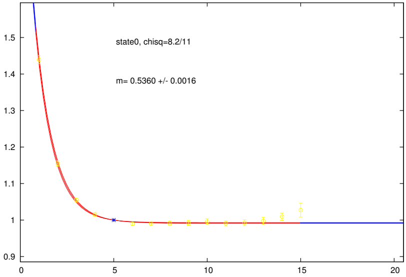


Figure 5: Ground-state principle correlator for $t_0 = 5$ and $t_Z = 7$.