Pion Model inspired by Lattice QCD

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Collaborators

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Light-Cone 2018, Jefferson Lab May 14 – 18, 2018.

Motivation

- Develop methods in continuous nonperturbative QCD within a given dynamical simple framework in Minkowski space
- Compare to/ incorporate results Lattice in the space-like region (e. g. self-energies)
- Solve the Bethe-Salpeter bound state equation with dressed quantities
- Observables: spectrum, SL/TL momentum region
- Relation BSA to LF Fock-space expansion of the hadron wf

Problems to be addressed

Observables associated with the hadron structure in Minkowski Space obtainable from BSA

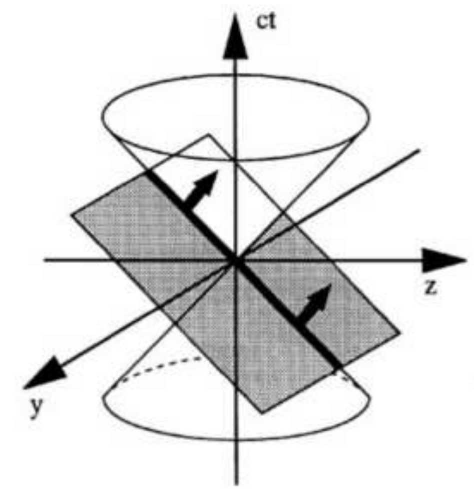
- parton distributions (pdfs)
- generalized parton distributions
- transverse momentum distributions (TMDs)
- Fragmentation functions
- SL and TL form factors ….

Light-Front WF (LFWF)

basic ingredient in PDFs, GPDs and TMDs

$$\tilde{\Phi}(x,p) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\Phi(k,p)$$

$$p^\mu = p_1^\mu + p_2^\mu \qquad k^\mu = \frac{p_1^\mu - p_2^\mu}{2}$$



$$\tilde{\Phi}(x,p) = \langle 0|T\{\varphi_H(x^\mu/2)\varphi_H(-x^\mu/2)\}|p\rangle$$
$$= \theta(x^+)\langle 0|\varphi(\tilde{x}/2)e^{-iP^-x^+/2}\varphi(-\tilde{x}/2)|p\rangle e^{ip^-x^+/4} + \cdot\cdot\cdot$$
$$= \theta(x^+)\sum_{n,n'} e^{ip^-x^+/4}\langle 0|\varphi(\tilde{x}/2)|n'\rangle\langle n'|e^{-iP^-x^+/2}|n\rangle\langle n|\varphi(-\tilde{x}/2)|p\rangle + \cdot\cdot\cdot$$

$x^+ = 0$ only valence state remains! How to rebuilt the full BS amplitude?

Iterated Resolvents: Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998)

- **From the valence→ full Fock Space w-f:** Sales, et al. PRC61, 044003 (2000)

Main Tool: Nakanishi Integral Representation (NIR)

"Parametric representation for any Feynman diagram for interacting bosons, with a denominator carrying the overall analytical behavior in Minkowski space" (Nakanishi 1962)

Bethe-Salpeter amplitude

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g(\gamma',z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

$$\kappa^2 = m^2 - \frac{M^2}{4}$$

BSE in Minkowski space with NIR for bosons

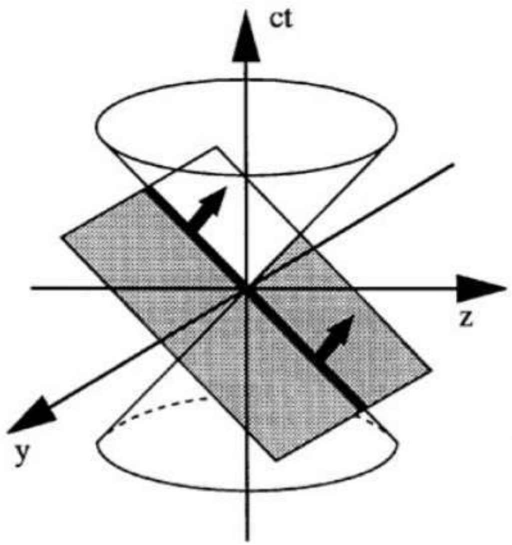
Kusaka and Williams, PRD 51 (1995) 7026;

Light-front projection: integration in k^-

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

TF, Salme, Viviani PRD89(2014) 016010,…

LF wave function



$$\boxed{\psi_{LF}(\gamma, z) = \frac{1}{4}(1 - z^2)\int_0^\infty \frac{g(\gamma', z)d\gamma'}{\left[\gamma' + \gamma + z^2 m^2 + \kappa^2(1 - z^2)\right]^2}}$$

$$\gamma = k_\perp^2 \qquad\qquad z = 2x - 1$$

Equivalent to Generalized Stietjes transform

Carbonell,Frederico, Karmanov PLB769 (2017) 418

1. *Bethe–Salpeter equation* & ***the pion***

The pion is the Goldston Boson of the dynamical chiral symmetry breaking
→Quark self energy and pseudo-scalar bound state equation has to be consistent in order that axial vector Ward Identity is fulfilled.

I.C. Cloët, C.D. Roberts, Prog. Part. Nucl. Phys. 77 (2014) 1.

$$\Phi(k,p) = S(\eta_1 p + k) S(\eta_2 p - k) \int \frac{d^4k'}{(2\pi)^4} iK(k,k',p)\Phi(k',p)$$

Källen–Lehmann spectral representation $\quad S(p') = \int_0^\infty d\gamma \frac{\rho(\gamma)}{p'^2 - \gamma + i\epsilon}$

$$\boxed{\Phi(k,p) = \int_0^\infty d\gamma \frac{\rho(\gamma)}{(\eta_1 p + k)^2 - \gamma + i\epsilon} \int_0^\infty d\gamma' \frac{\rho(\gamma')}{(\eta_2 p - k)^2 - \gamma' + i\epsilon} \int \frac{d^4k'}{(2\pi)^4} iK(k,k',p)\Phi(k',p)}$$

Non planar diagrams (cross-ladder) are unimportant with color degrees of freedom!

J.H. Alvarenga Nogueira, Chueng-Ryong Ji, E. Ydrefors, T. Frederico, *Phys. Lett. B 777 (2018) 207*

2. Quark model propagator

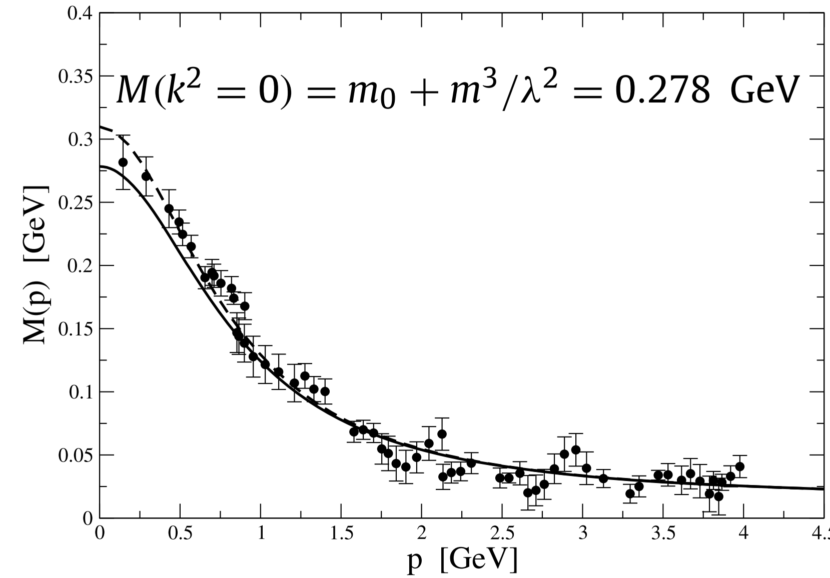


Fig. 1. The running quark mass, Eq. (3), as a function of the Euclidean momentum $p = \sqrt{-p^\mu p_\mu}$, with parameters from (4), is given by the solid line and compared to lattice QCD calculations from [37]. The dashed line shows the parametrization used in reference [51].

$$S_F(k) = \imath\, Z(k^2)\left[\not{k} - M(k^2) + \imath\epsilon\right]^{-1}$$

simplification $\quad Z(k^2) = 1$

$$M(k^2) = m_0 - m^3\left[k^2 - \lambda^2 + i\epsilon\right]^{-1}$$

$$m_0 = 0.014\ \text{GeV},\ m = 0.574\ \text{GeV and}\ \lambda = 0.846\ \text{GeV}$$

quark propagator poles
$$\begin{cases} m_i^2 = M^2(m_i^2) \\ m_i\left(m_i^2 - \lambda^2\right) = \pm\left[m_0\left(m_i^2 - \lambda^2\right) - m^3\right] \end{cases}$$

$$\boxed{S_F(k) = \imath\, \frac{(k^2-\lambda^2)^2\,(\not{k} + m_0) - \left(k^2-\lambda^2\right) m^3}{\prod_{i=1,3}(k^2 - m_i^2 + \imath\epsilon)}}$$

$$m_1 = 0.327\ \text{GeV},\ m_2 = 0.644\ \text{GeV and}\ m_3 = 0.954\ \text{GeV}$$

Källén–Lehmann spectral decomposition

$$S_F(k) = \imath \left[A(k^2)\, \not{k} + B(k^2) \right]$$

$$A(k^2) = \int_0^\infty d\mu^2 \frac{\rho_A(\mu^2)}{k^2 - \mu^2 + \imath\varepsilon} \quad \text{and} \quad B(k^2) = \int_0^\infty d\mu^2 \frac{\rho_B(\mu^2)}{k^2 - \mu^2 + \imath\varepsilon}$$

$$\rho_A(\mu^2) = -\frac{1}{\pi}\,\mathrm{Im}\,[A(\mu^2)] \quad \text{and} \quad \rho_B(\mu^2) = -\frac{1}{\pi}\,\mathrm{Im}\,[B(\mu^2)]$$

positivity constraints

$$\boxed{\mathcal{P}_a = \rho_A(\mu^2) \geq 0 \quad \text{and} \quad \mathcal{P}_b = \mu\,\rho_A(\mu^2) - \rho_B(\mu^2) \geq 0}$$

$$A(k^2) = \frac{H_2(m_1,m_2,m_3)}{k^2 - m_1^2 + \imath\epsilon} + \frac{H_2(m_2,m_1,m_3)}{k^2 - m_2^2 + \imath\epsilon} + \frac{H_2(m_3,m_2,m_1)}{k^2 - m_3^2 + \imath\epsilon}$$

$$B(k^2) = \frac{H_1(m_1,m_2,m_3)}{k^2 - m_1^2 + \imath\epsilon} + \frac{H_1(m_2,m_1,m_3)}{k^2 - m_2^2 + \imath\epsilon} + \frac{H_1(m_3,m_2,m_1)}{k^2 - m_3^2 + \imath\epsilon} + m_0\, A(k^2)$$

$$H_n(m_1,m_2,m_3) = \frac{(-m^3)^{2-n}(m_1^2 - \lambda^2)^n}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)}$$

$$\boxed{\begin{aligned}\rho_A(\mu^2) = {} & H_2(m_1,m_2,m_3)\;\delta\left(\mu^2 - m_1^2\right)\\ & + H_2(m_2,m_1,m_3)\;\delta\left(\mu^2 - m_2^2\right)\\ & + H_2(m_3,m_2,m_1)\;\delta\left(\mu^2 - m_3^2\right)\end{aligned}}$$

$$\boxed{\begin{aligned}\rho_B(\mu^2) = {} & H_1(m_1,m_2,m_3)\;\delta\left(\mu^2 - m_1^2\right)\\ & + H_1(m_2,m_1,m_3)\;\delta\left(\mu^2 - m_2^2\right)\\ & + H_1(m_3,m_2,m_1)\;\delta\left(\mu^2 - m_3^2\right) + m_0\,\rho_A(\mu^2)\end{aligned}}$$

Positivity Violation!

$$\mathcal{P}_a = \rho_A(\mu^2) \geq 0 \;\text{ and }\; \mathcal{P}_b = \mu\,\rho_A(\mu^2) - \rho_B(\mu^2) \geq 0$$

$$\mathcal{P}_a^\delta(m_1, m_2, m_3) = H_2(m_1, m_2, m_3) \;\text{ and}$$

$$\mathcal{P}_b^\delta(m_1, m_2, m_3) = H_1(m_1, m_2, m_3) + m_0\, H_2(m_1, m_2, m_3)$$

$m_1 = 0.327$ GeV	$\mathcal{P}_a^\delta(m_1, m_2, m_3) = 1.49$	$\mathcal{P}_b^\delta(m_1, m_2, m_3) \sim -0.001$GeV
$m_2 = 0.644$ GeV	$\mathcal{P}_a^\delta(m_2, m_1, m_3) = -0.580$	$\mathcal{P}_b^\delta(m_2, m_1, m_3) \sim 0.001$ GeV
$m_3 = 0.954$ GeV	$\mathcal{P}_a^\delta(m_3, m_2, m_1) = 0.095$	$\mathcal{P}_b^\delta(m_3, m_2, m_1) = 0.183$ GeV

3. Pion Bethe–Salpeter amplitude model

pion-quark-antiquark vertex

$$\Gamma_\pi(k;P) = \gamma_5[\imath E_\pi(k;P) + \not{P} F_\pi(k;P) + k^\mu P_\mu \not{k} G_\pi(k;P) + \sigma_{\mu\nu} k^\mu P^\nu H_\pi(k;P)]$$

chiral limit $m_\pi = 0$ axial-vector Ward–Takahashi identity

I.C. Cloët, C.D. Roberts, Prog. Part. Nucl. Phys. 77 (2014) 1.

$$\boxed{f_\pi E_\pi(k,P) = M(k^2)/\sqrt{Z(k^2)}}$$

OUR MODEL:

$$\boxed{\Gamma_\pi(k;P) = \imath \mathcal{N} \gamma_5 M(k^2)|_{m_0=0} \quad Z(k^2) = 1}$$

$$\boxed{\psi_\pi(k;P) = S_F(k+P/2)\,\Gamma_\pi(k;P)\,S_F(k-P/2)}$$

Other examples of analytic vertex models with fixed constituent quark masses

$$\Lambda_\pi(k,P) = \mathcal{N}\left[\frac{1}{(k+\frac{P}{2})^2 - m_R^2 + \imath\epsilon} + \frac{1}{(k-\frac{P}{2})^2 - m_R^2 + \imath\epsilon}\right]$$

J.P.B.C. de Melo, T. Frederico, E. Pace, G. Salmé, Nucl. Phys. A 707 (2002) 399; Braz. J. Phys. 33 (2003) 301.

$$\Lambda_\pi(k,P) = \mathcal{N}\frac{1}{\left[(k+\frac{P}{2})^2 - m_R^2 + \imath\epsilon\right]}\frac{1}{\left[(k-\frac{P}{2})^2 - m_R^2 + \imath\epsilon\right]}$$

T. Frederico, E. Pace, B. Pasquini, G. Salmè, Phys. Rev. D 80 (2009) 054021.
C. Fanelli, E. Pace, G. Romanelli, G. Salmè, M. Salmistraro, Eur. Phys. J. C 76 (2016) 253.

4. Integral representation of the BSA

$$\psi_\pi(k;P) = -\left[A(k_q^2)\,\not{k}_q + B(k_q^2)\right]\frac{\mathcal{N}\gamma_5}{k^2-\lambda^2+\imath\epsilon}\left[A(k_{\bar{q}}^2)\,\not{k}_{\bar{q}} + B(k_{\bar{q}}^2)\right]$$

$k_q = k + P/2$ and $k_{\bar{q}} = k - P/2$

$$\frac{1}{((k+\frac{p}{2})^2-\mu'^2+\imath\epsilon)(k^2-\lambda^2+\imath\epsilon)(k-\frac{p}{2})^2-\mu^2+\imath\epsilon)} = \int_{-\infty}^{+\infty} d\gamma \int_{-1}^{1} dz \frac{g(\gamma,z;\mu',\mu,p)}{\left(k^2+zk\cdot p+\gamma+\imath\epsilon\right)^3}$$

$$g(\gamma,z;\mu',\mu,p) = \\ = \frac{\theta(\alpha)\theta(1-\alpha)}{\frac{1}{2}-\alpha}\left[\theta(1-2\alpha-z)\theta(z)-\theta(z-1+2\alpha)\theta(-z)\right]$$

$$\alpha = \frac{\frac{p^2}{4}+\lambda^2-\mu^2-z^{-1}(\lambda^2+\gamma)}{\mu^2-\mu'^2+2z^{-1}(\lambda^2+\gamma)}$$

Nakanishi integral representation

$$\boxed{\psi_\pi(k;P) = \gamma_5\,\chi_1(k,P) + \not{k}_q\,\gamma_5\,\chi_2(k,P) + \gamma_5\,\not{k}_{\bar{q}}\,\chi_3(k,P) + \not{k}_q\,\gamma_5\,\not{k}_{\bar{q}}\,\chi_4(k,P)}$$

$$\chi_i(k,P) = \int_{-\infty}^{+\infty} d\gamma \int_{-1}^{1} dz \frac{g_i(\gamma,z;p)}{\left(k^2 + zk\cdot p + \gamma + \imath\epsilon\right)^3}$$

J. Carbonell, V.A. Karmanov, Eur. Phys. J. A 46 (2010) 387

W. de Paula, T. Frederico, G. Salmè , M. Viviani Phys.Rev. D94 (2016) R071901

W. de Paula, T.Frederico, G. Salmè, M.Viviani and R.Pimentel, Eur. Phys. J.C 77 (2017) 764

The Nakanishi weight functions given by:

$$g_i(\gamma,z;p) = -\mathcal{N}\int_0^\infty d\mu^2 \int_0^\infty d\mu'^2 \rho_{C_i'}(\mu'^2)\rho_{C_i}(\mu^2)\, g(\gamma,z;\mu',\mu,p)$$

$(C_1',C_1) = (B,B)$, $(C_2',C_2) = (A,B)$, $(C_3',C_3) = (B,A)$ and $(C_4',C_4) = (A,A)$

5. Electroweak decay constant

$$P^{\mu} f_{\pi} = N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\gamma^{\mu}\gamma^{5}\psi_{\pi}(k;\,P)\right]$$

6. Electromagnetic form factor *Space-like*

$$\boxed{-\imath\,\Gamma^{\mu}_{\pi}(P,P';q) \equiv \left\langle \pi(P') \right| J^{\mu} \left| \pi(P) \right\rangle = (P+P')^{\mu} F_{\pi}(Q^2) \qquad Q^2 = -q^2}$$

$$\Gamma^{\mu}_{\pi^+}(P,P';q) = \hat{Q}_u \Gamma^{\mu}_{\pi^+,u}(P,P';q) + \hat{Q}_{\bar{d}} \Gamma^{\mu}_{\pi^+,\bar{d}}(P,P';q)$$

$$\begin{aligned}
&\Gamma^{\mu}_{\pi^+,u}(P,P';q) \\
&\quad = N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[S_F(k'-P'/2)\bar{\Gamma}_{\pi^+}(k';\,P')S_F(k'+P'/2)\right. \\
&\qquad \times \left.\underbrace{\Gamma^{\mu}_{u}(k'+P'/2,k+P/2;\,P)}_{\text{Dressed quark photon vertex}} S_F(k+P/2)\Gamma_{\pi^+}(k;\,P)\right], \quad k' = k+q/2
\end{aligned}$$

7. Dressed quark current operator

$$q_\mu \Gamma_\pi^\mu(P, P'; q) = 0$$

Ward–Takahashi identity (WTI)

$$q_\mu \Gamma_q^\mu(p', p; q) = S_F^{-1}(p') - S_F^{-1}(p)$$

$$\boxed{-\imath\, \Gamma_q^\mu(p', p; q) = \gamma^\mu - \frac{m^3 (p' + p)^\mu}{\mathcal{D}(p'^2)\mathcal{D}(p^2)}}$$

$$\mathcal{D}(p^2) = \left(p^2 - \lambda^2 + i\epsilon\right) \qquad q = p' - p$$

Electromagnetic form-factor

8. Results

$$r_\pi = 0.672 \ \text{fm}, \ f_\pi = 90 \ \text{MeV}$$

$$r_\pi^{exp} = 0.672 \pm 0.008 \ \text{fm} \qquad f_\pi^{exp} = 92.42 \pm 0.021 \ \text{MeV}$$

K.A. Olive, et al., Particle Data Group, Chin. Phys. C 38 (2014) 090001

C.S. Mello et al. / Physics Letters B 766 (2017) 86–93 91

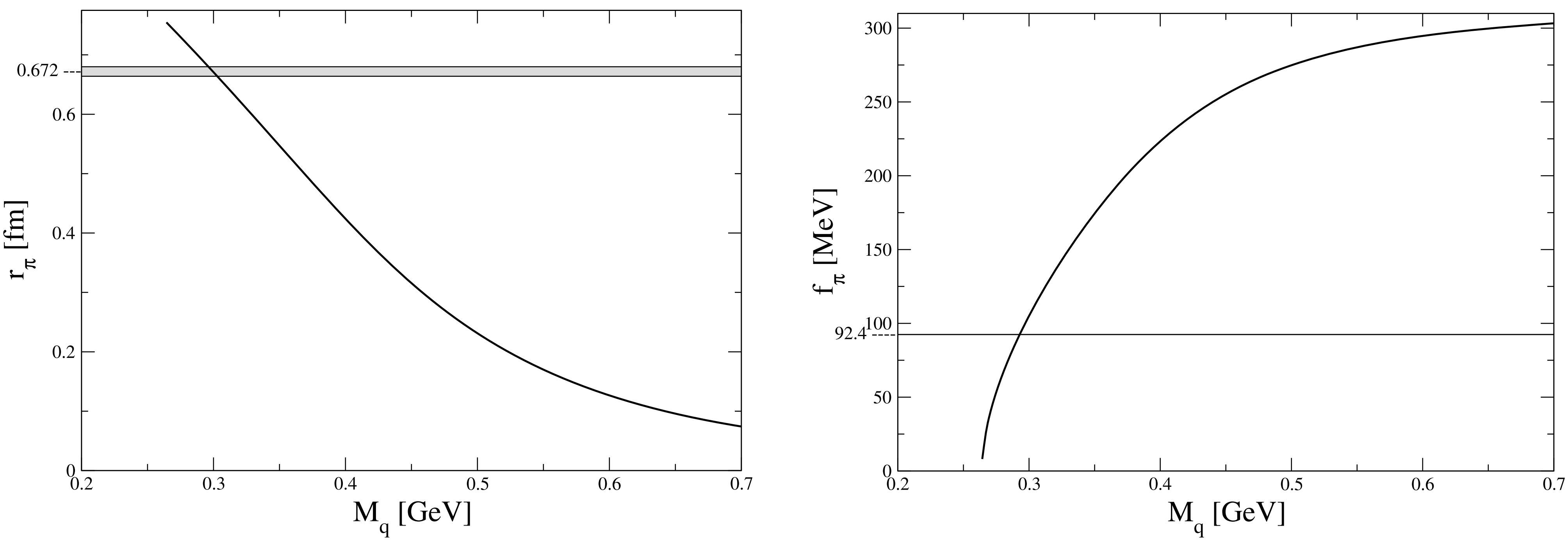
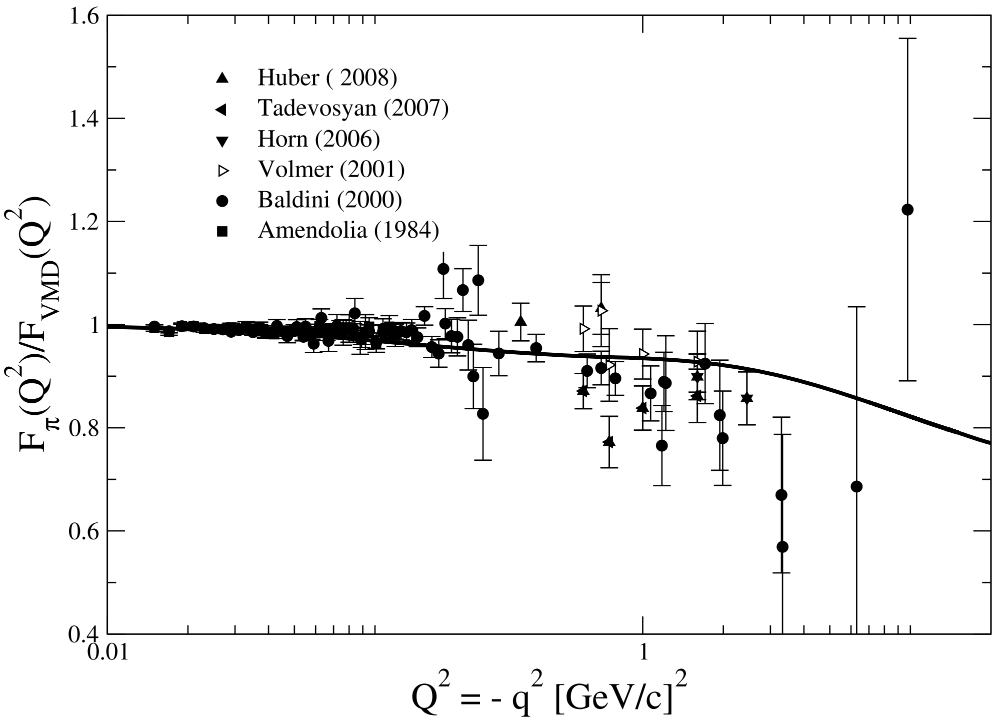


Fig. 2. Pion charge radius (left-frame) and decay constant (right-frame) as a function of the quark mass given for $M_q \equiv M(k^2 = 0)$. The experimental data for the charge radius and decay constant [67] are given by the horizontal lines.

$$\boxed{r_\pi \ f_\pi = \sqrt{N_c}/2\pi}$$

R. Tarrach, Z. Phys. C 2 (1979) 221.
S.B. Gerasimov, Sov. J. Nucl. Phys. 29 (1979) 259;
S.B. Gerasimov, Sov. J. Nucl. Phys. 32 (1980) 156 (Erratum)



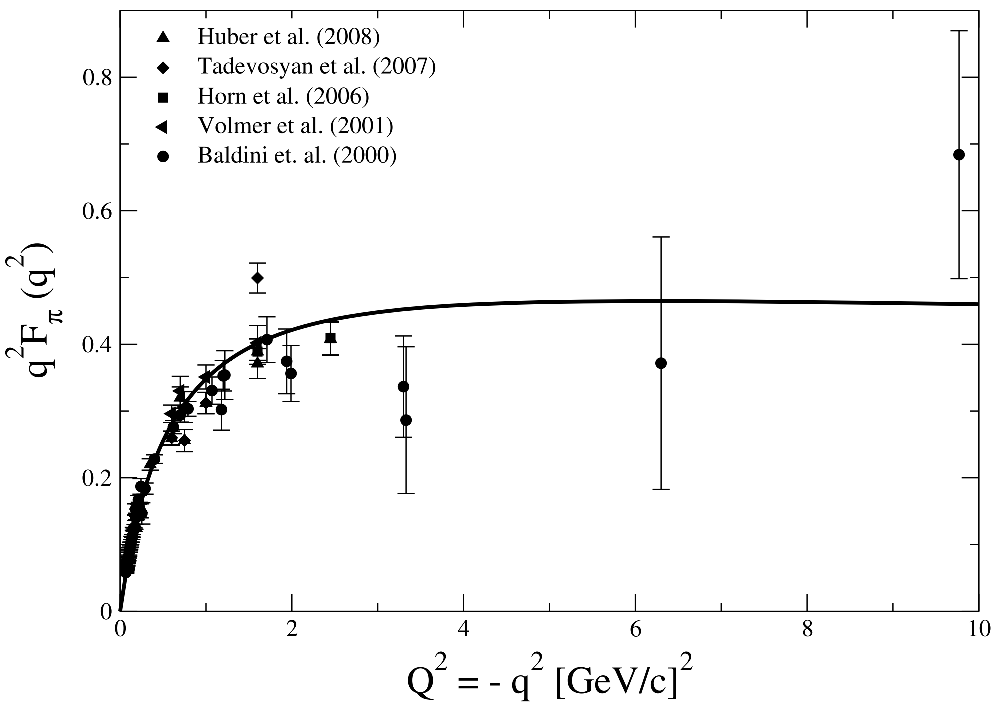


Fig. 3. Pion model electromagnetic form factor as a function of the space-like momentum transfer, $Q^2 = -q^2$, compared to the experimental values: Amendolia et al. [70], Baldini et al. [71], Volmer et al. [72], Horn et al. [73], Tadevosyan et al. [74] Huber et al. [75]. In the left frame it is presented the results normalized to the dipole form factor, $F_\pi(Q^2)(1 + Q^2/(0.77\ \text{GeV})^2)$, and in the right frame $Q^2 F_\pi(Q^2)$.

Conclusions and Perspectives

- **The pion structure in Minkowski space is described in terms of an analytic model of the Bethe–Salpeter amplitude combined with Euclidean Lattice QCD results;**
- **the model includes the running quark mass fitted to Lattice QCD data;**
- **the pion pseudoscalar vertex & quark mass function (dynamical chiral symmetry breaking requirements in the limit of vanishing current quark mass);**
- **the quark propagator violates the positivity constraints of the KL spectral repres.;**
- **the integral representation of the pion Bethe–Salpeter amplitude is sketched;**
- **a quark electromagnetic current built consistent with the WTI (current conservation);**
- **form factor and weak decay constant are consistent with the experimental data;**
- **Beyond the pion: the kaon**
- **Form-Factors, PDFs, TMDs, Fragmentation Functions...**
- **Incorporate in a dynamical model the running quark/gluon masses, quark-gluon vertex from Lattice calculations…**

THANK YOU!