

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

PATRICK BARRY

(IN COLLABORATION WITH: NOBUO SATO, WALLY MELNITCHOUK, AND CHUENG-RYONG JI)

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pcbarry@ncsu.edu

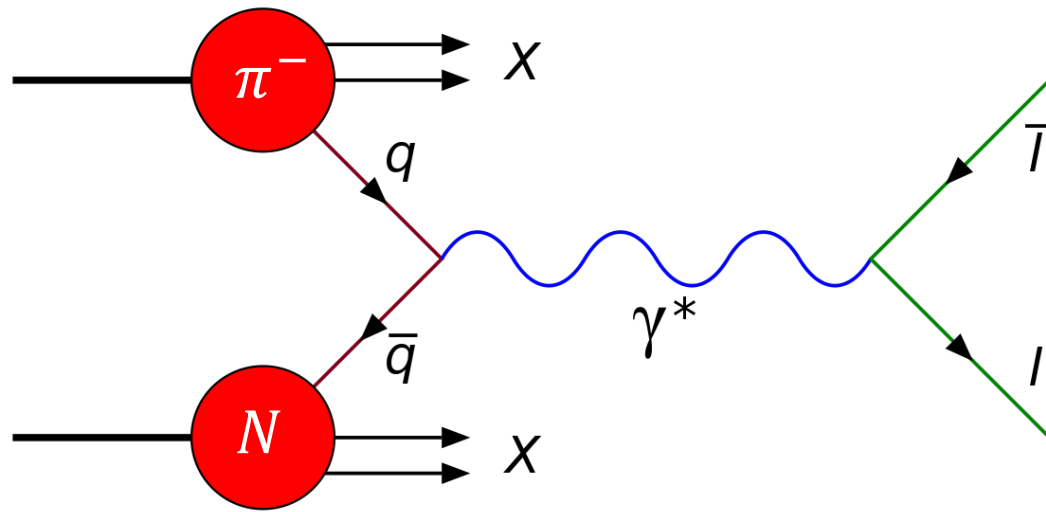
Pion Parton Distribution Functions (PDF)

Pions have been associated with the lightest bound state as well as the Goldstone boson of chiral effective theory

Understanding more about the pion's structure can tell us more about chiral symmetry

Since proton can be modeled as a core valence structure surrounded by a pion cloud, the pion PDF can lead us, *e.g.* to an understanding of the $\bar{d} - \bar{u}$ asymmetry in the proton

Drell-Yan (DY) Process



Two hadrons collide

- One donates a quark, the other donates an antiquark

Quark/antiquark annihilate forming a virtual photon, which creates a lepton/antilepton

Cross section of dilepton measured

Focuses on high- x_π region of PDF

$$\frac{d^2\sigma}{dQ^2 dY} \propto \sum_{i,j} \int_{x_\pi}^1 \frac{d\hat{x}_\pi}{\hat{x}_\pi} \int_{x_A}^1 \frac{d\hat{x}_A}{\hat{x}_A} C_{ij}(\hat{x}_\pi, \hat{x}_A, x_\pi, x_A, Q/\mu) f_i^\pi(\hat{x}_\pi, \mu) f_j^A(\hat{x}_A, \mu)$$

Pion PDF

DY Datasets

In this analysis, we use Drell-Yan data of a π^- incident on a Tungsten (W) target from:

- E615 done at Fermilab ($E_{\pi^-} = 252$ GeV). J. S. Conway et al., Phys. Rev. D 39, 1 (1989).
- NA10 ($E_{\pi^-} = 194$ GeV, and $E_{\pi^-} = 286$ GeV) done at CERN. B. Betev et al., Z. Phys. C 28, 9 (1985).

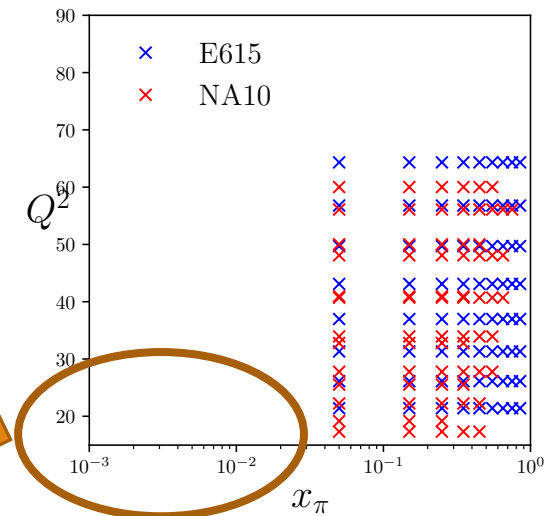
We use Tungsten PDFs as in K. Eskola, P. Paakkinen, H. Paukkunen and C. Salgado, Eur. Phys. J. C 77, 163 (2017).

Previous analyses have fit the pion PDF to DY data:

- P. J. Sutton, A. D. Martin, W. J. Stirling, R. G. Roberts, Phys. Rev. D 45, 2349 (1992).
- M. Glück, E. Reya and A. Vogt, Z. Phys. C 53, 651 (1992).
- M. Glück, E. Reya and I. Schienbein, Eur. Phys. J. C 10, 313 (1999).
- M. Aicher, A. Schäfer and W. Vogelsang, Phys. Rev. Lett. 105, 252003 (2010).

No data at low x_π !

Need to use data at low x_π region to gather the whole story.



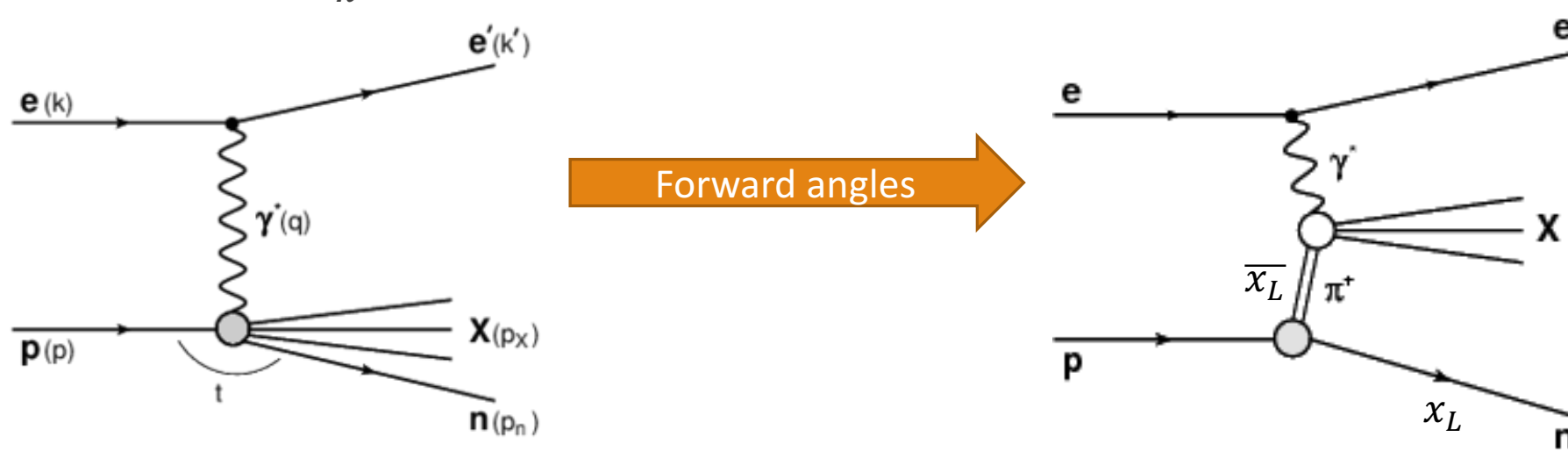
Leading Neutron (LN)

Semi-inclusive Deep Inelastic Scattering (SIDIS) electroproduction

Electron barely strikes the surface of the proton, knocking out and shattering the proton's periphery.

The exchange of a charged π^+ and leaves behind a neutron

Focuses on low- x_π of the PDF



LN Observables

The LN differential cross-section

$$\frac{d^3\sigma^{LN}}{dx dQ^2 dx_L}$$

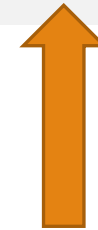
is parametrized by

$$F_2^{LN(3)}(x, Q^2, x_L)$$

where

$$F_2^{LN(3)}(x, Q^2, x_L) = 2f_{\pi N}(\bar{x}_L)F_2^\pi(x_\pi, Q^2)$$

x_L represents the longitudinal momentum fraction carried by the outgoing neutron.



Pion PDF lives in here

Splitting Function and Regulators

Amplitude for proton to dissociate into a π^+ and neutron:

$$f_{\pi N}(\bar{x}_L) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{\bar{x}_L [k_\perp^2 + \bar{x}_L^2 M^2]}{x_L^2 D_{\pi N}^2} |\mathcal{F}|^2,$$

$$D_{\pi N} \equiv t - m_\pi^2 = -\frac{1}{1-y} [k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]$$

$$\mathcal{F} = \begin{cases} \text{(i)} & \exp((M^2 - s)/\Lambda^2) & s\text{-dep. exponential} \\ \text{(ii)} & \exp(D_{\pi N}/\Lambda^2) & t\text{-dep. exponential} \\ \text{(iii)} & (\Lambda^2 - m_\pi^2)/(\Lambda^2 - t) & t\text{-dep. monopole} \\ \text{(iv)} & \bar{x}_L^{-\alpha_\pi(t)} \exp(D_{\pi N}/\Lambda^2) & \text{Regge} \\ \text{(v)} & [1 - D_{\pi N}^2/(\Lambda^2 - t)^2]^{1/2} & \text{Pauli-Villars} \end{cases}$$

We examine five regulators, and we fit Λ as a new parameter along with LN data

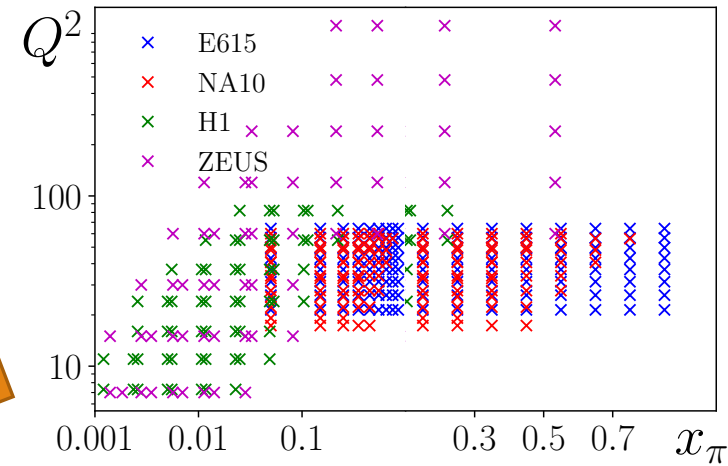
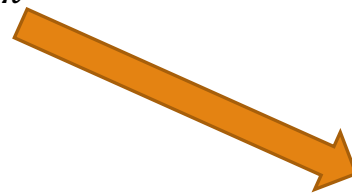
Where $\mathcal{F}$ is a UV regulator, which the data chooses

LN Datasets

We use data from HERA at DESY as:

- H1, which reports $F_2^{LN(3)}$. [F. D. Aaron et al., Eur. Phys. J. C 67, 381 \(2010\).](#)
- ZEUS, which reports the ratio $r(x, Q^2, \bar{x}_L) = 2f_{\pi N}(\bar{x}_L) \frac{F_2^\pi(x_\pi, Q^2)}{F_2^p(x, Q^2)} \Delta \bar{x}_L$ [S. Chekanov et al., Nucl. Phys. B637, 3 \(2002\).](#)
- Previous analysis has studied the pion structure function. [J. R. McKenney, N. Sato, W. Melnitchouk and C.-R. Ji, Phys. Rev. D 93, 054011 \(2016\).](#)

Now we have low x_π data



Nested Sampling

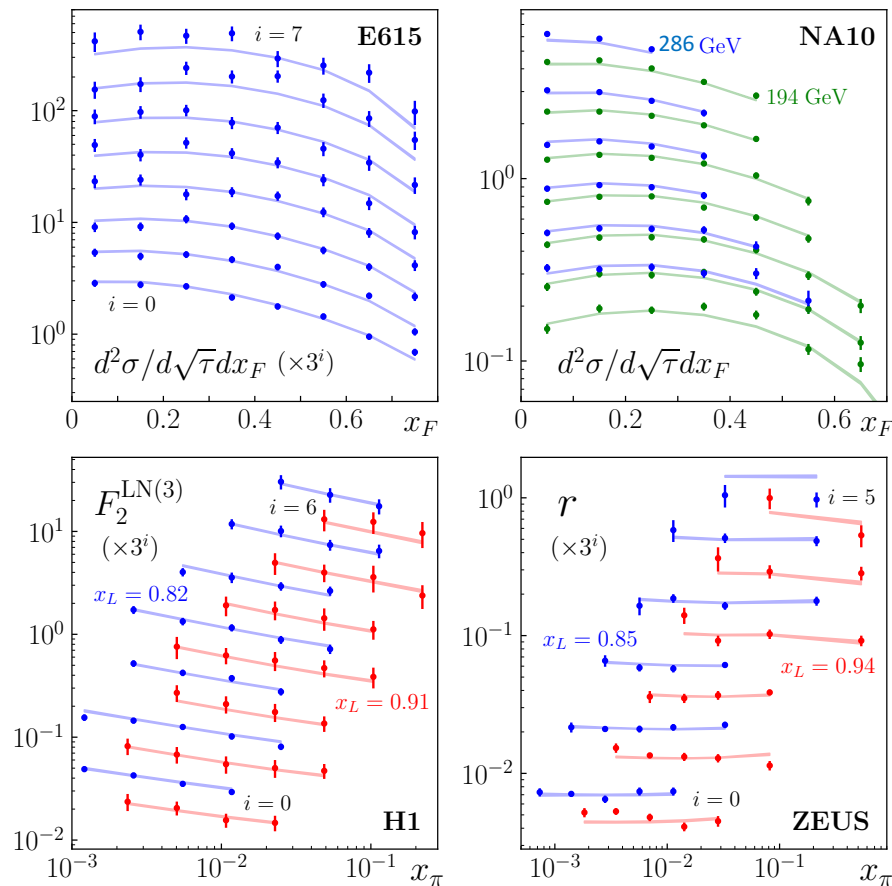
Use Monte Carlo (MC) fitting based on Nested Sampling

- Based on [J. Skilling, Bayesian Anal. 1, 4 \(2006\)](#).
- Also used in: [P. Mukherjee, D. Parkinson and A. Liddle, Astrophys. J. 638, L51 \(2006\)](#).
[R. Shaw, M. Bridges and M. P. Hobson, Mon. Not. Roy. Astron. Soc. 378, 1365 \(2007\)](#).
- Allows to robustly calculate the uncertainties on PDFs and other observables
- Recent publications have used MC method
 - In calculating nucleon PDFs: [N. Sato, W. Melnitchouk, S. E. Kuhn, J. J. Ethier and A. Accardi, Phys. Rev. D 93, 074005 \(2016\)](#)
[J. J. Ethier, N. Sato and W. Melnitchouk, Phys. Rev. Lett. 119, 132001 \(2017\)](#).
 - Fragmentation functions: [N. Sato, J. J. Ethier, W. Melnitchouk, M. Hirai, S. Kumano and A. Accardi, Phys. Rev. D 94, 114004 \(2016\)](#).
 - Transverse momentum dependent transversity distributions: [H. W. Lin, W. Melnitchouk, A. Prokudin, N. Sato and H. Shows, arXiv:1710.09858 \[hep-ph\], to appear in Phys. Rev. Lett.](#)
- Calculates directly the likelihood function (minimum χ^2)
- Gathers sample parameter vectors into a sample set, with weights

$$E[\mathcal{O}] = \sum_k w_k \mathcal{O}(\mathbf{a}_k)$$

$$V[\mathcal{O}] = \sum_k w_k (\mathcal{O}(\mathbf{a}_k) - E[\mathcal{O}])^2$$

Analysis of Data



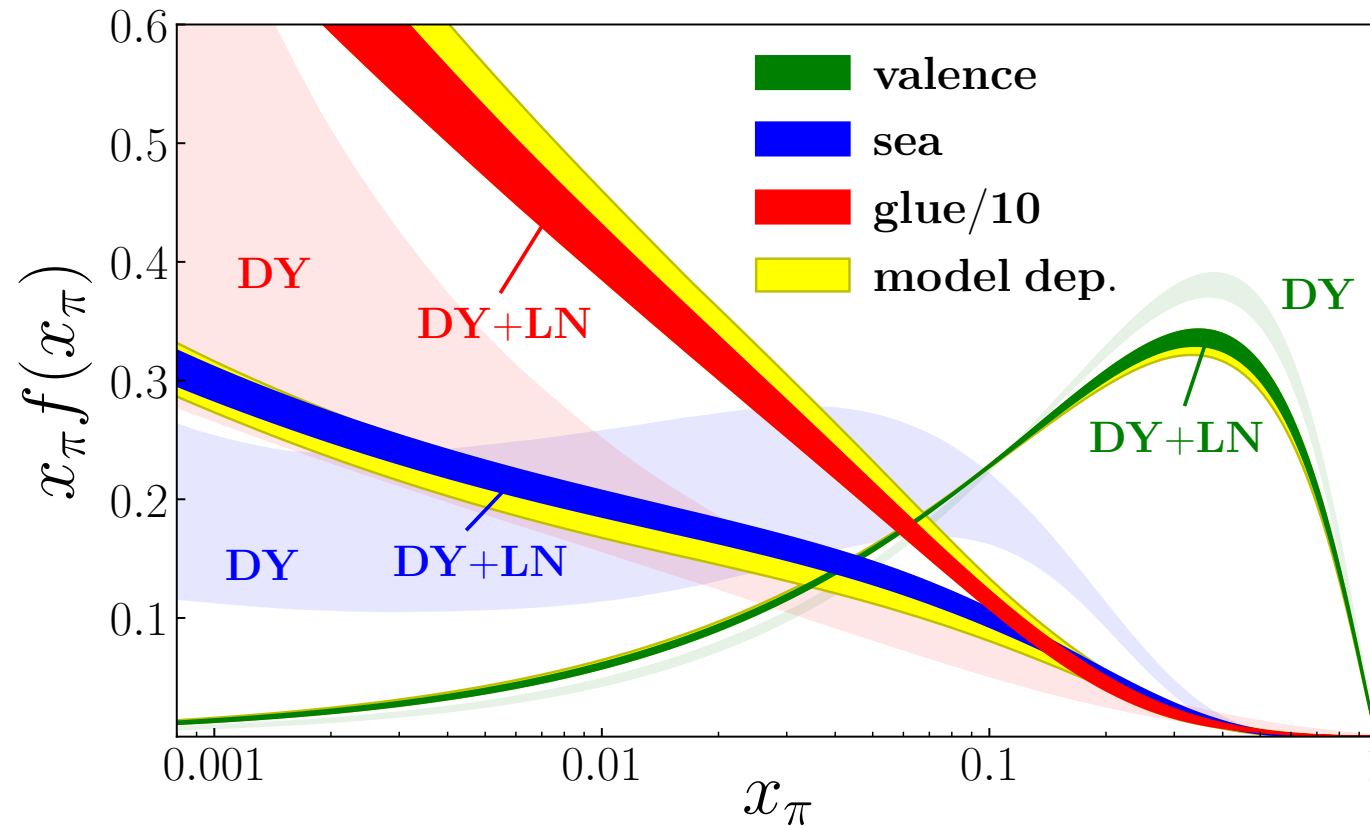
Showing fits to (almost) all data points using the s -dep. exponential model ($\chi^2 = 0.979$).

Each data set has good χ^2

Data set	# data points	χ^2
E615	72	81.82
NA10 (194 GeV)	44	41.52
NA10 (286 GeV)	26	28.85
H1	58	21.98
ZEUS	50	70.62

- Avoid the J/Ψ and Υ resonances, so limit the DY data to $4.16 < Q < 8.34$ GeV
- To ensure one pion exchange in LN electroproduction, limit our range of the longitudinal momentum fraction to $x_L > 0.8$

The Parton Distribution Functions (PDFs)



The lightly shaded bands are fits to only DY data

Uncertainties dramatically decrease with inclusion of LN data

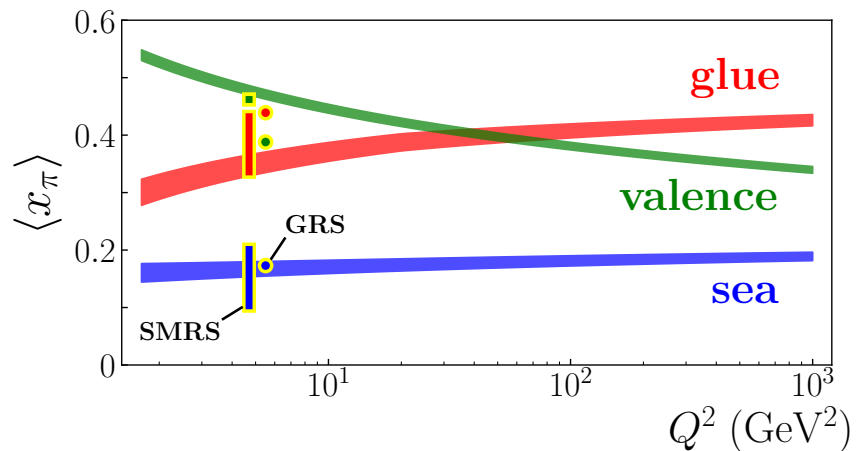
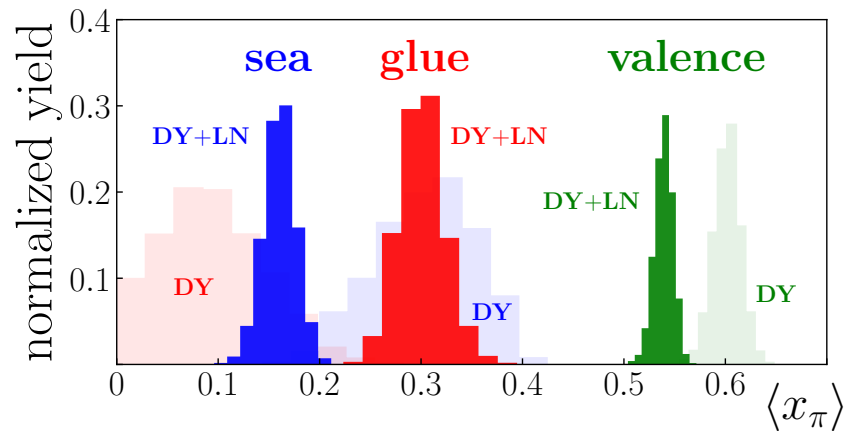
Yellow bands show the range of the regulators

Valence PDF shows $q_v \sim (1 - x)$ as $x \rightarrow 1$, as can be related with

- R.M. Davidson, E. Ruiz Arriola. Oct 1994. 10 pp. Phys.Lett. B348 (1995)
- T. Shigetani, K. Suzuki and H. Toki, Phys. Lett. B 308, 383 (1993).
- A. Szczepaniak, C.-R. Ji and S. R. Cotanch, Phys. Rev. D 49, 3466 (1994).
- W. Melnitchouk, Eur. Phys. J. A 17, 223 (2003); Phys. Rev. D 67, 077502 (2003).

Momentum Sum Rule

$$\int_0^1 dx_\pi x_\pi (2q_v^\pi + 6q_s^\pi + g^\pi) = 1$$

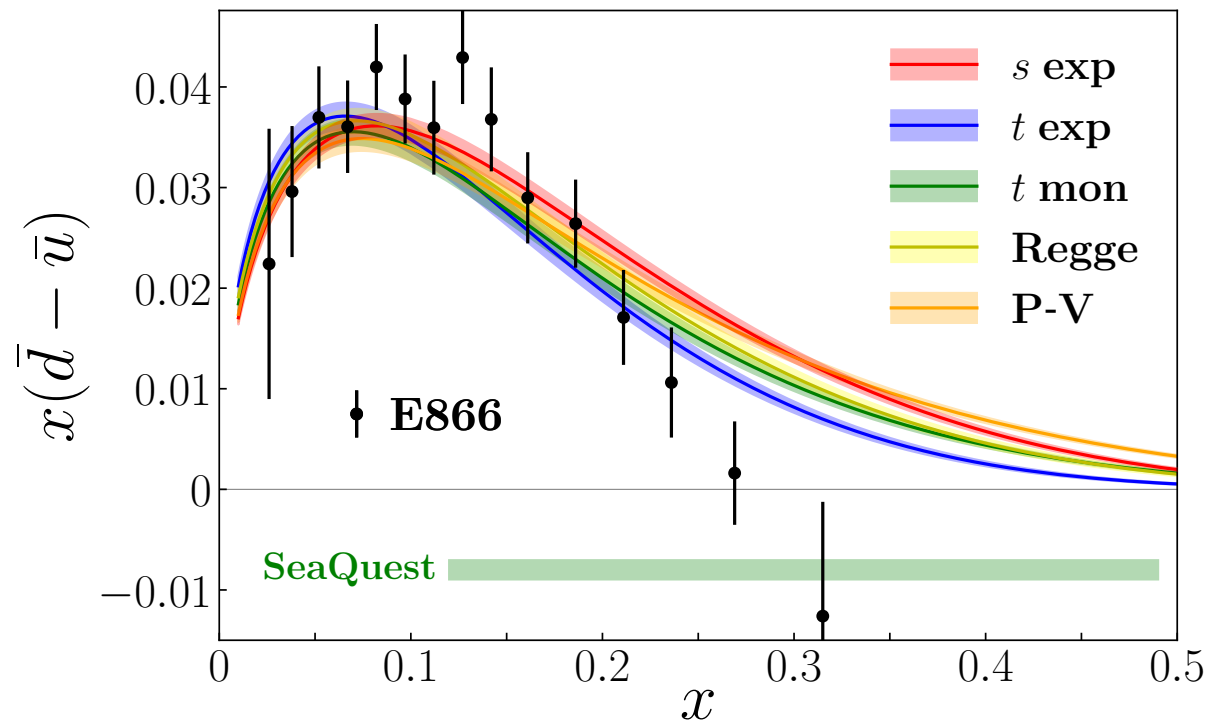


flavor	JAM	GRS	MRS (10% sea)	MRS (15% sea)	MRS (20% sea)
$\langle x_\pi \rangle_v$	0.445	0.365	0.438	0.438	0.438
$\langle x_\pi \rangle_s$	0.171	0.178	0.111	0.156	0.202
$\langle x_\pi \rangle_g$	0.371	0.456	0.441	0.398	0.355

(Upper panel): distribution of expectation value for momentum of parton at $Q_0^2 = m_c^2$

We see that closest results are for the SMRS parameterization at the same scale

$\bar{d} - \bar{u}$ Asymmetry in the Proton



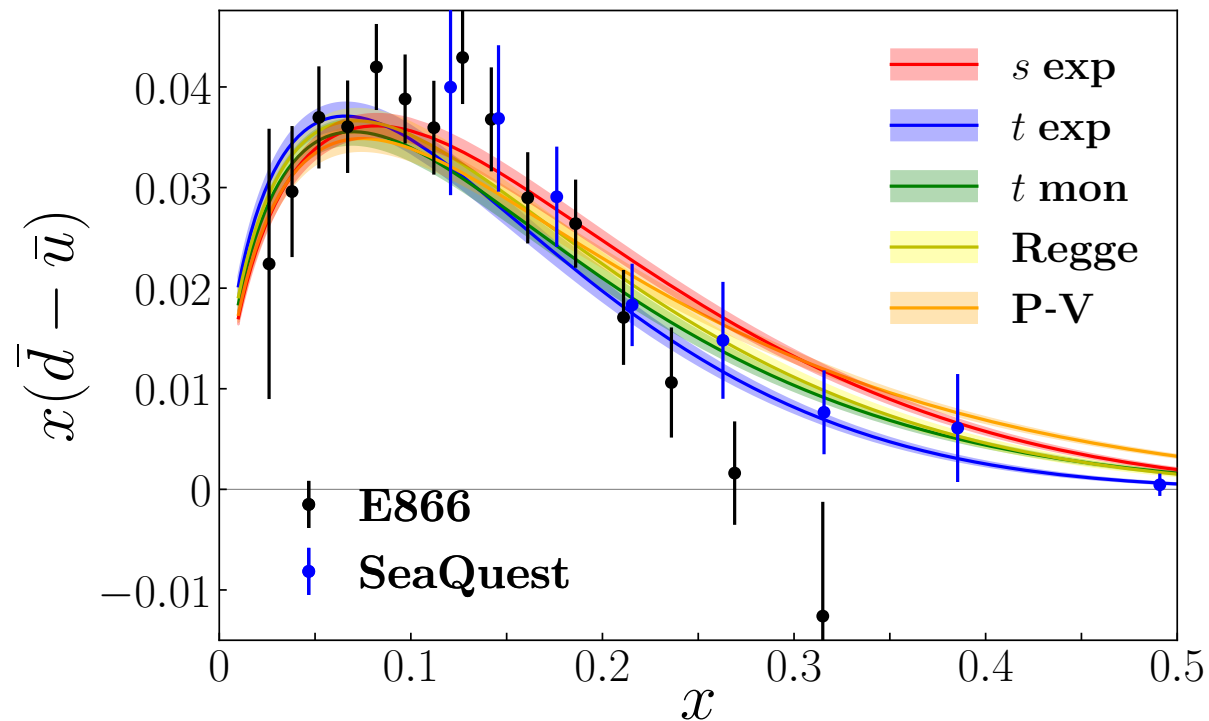
Model all of the asymmetry in the sea of the proton as totally from the pions

Perform a fit to all DY, LN, and $\bar{d} - \bar{u}$ asymmetry

E866 experiment done at Fermilab was the first "smoking gun" of the asymmetry

Again, s -dep. exponential has the best fit (lowest $\chi^2 = 1.03$)

$\bar{d} - \bar{u}$ Asymmetry in the Proton



SeaQuest (E906) is not yet published (nor fitted here), but our models predict well the data

Summary and Outlook

Presented the first Monte Carlo Global QCD determination of the pion PDF

First time LN data has been used to calculate PDFs

Learned that at our input scale ($Q_0^2 = m_c^2$) the gluon contributes higher to the total momentum than predicted by only DY data, and the sea quark contributes less

We will look to include threshold resummation, which can potentially change the large x_π shape of pion PDFs

- Implementation is not unique, but it requires a dedicated systematic study using various possible implementations from Mellin space method or SCET.

We look forward to results from the future tagged DIS (TDIS) experiment at JLab, as well as πA DY data from COMPASS

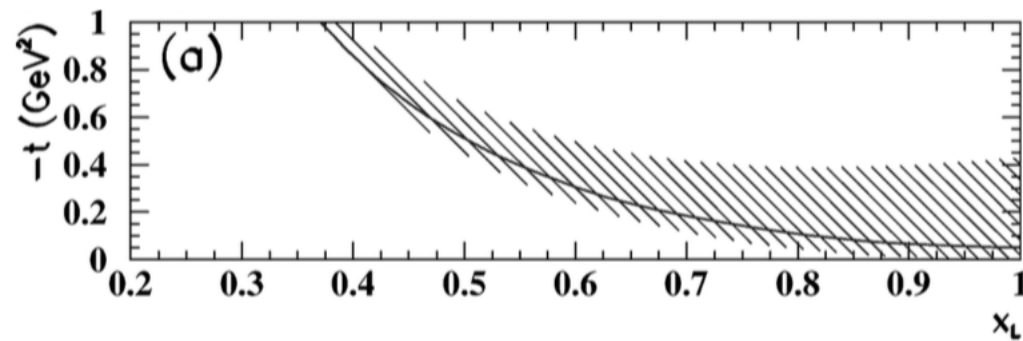
Thank you!

Backup Slides

High x_L

At low t , the neutron carries the majority of the longitudinal momentum of the proton, x_L

In this region, we can guarantee that a proton has split into a pion and a neutron (as opposed to another particle)



Parameterization of the PDF (in terms of π^-)

By isospin symmetry, we can say the valence PDF looks like

$$v_\pi \equiv \bar{u}_v^{\pi^-} = d_v^{\pi^-} = u_v^{\pi^+} = \bar{d}_v^{\pi^+}$$

with the quark sum rule:

$$\int_0^1 dx_\pi v_\pi(x_\pi, Q^2) = 1$$

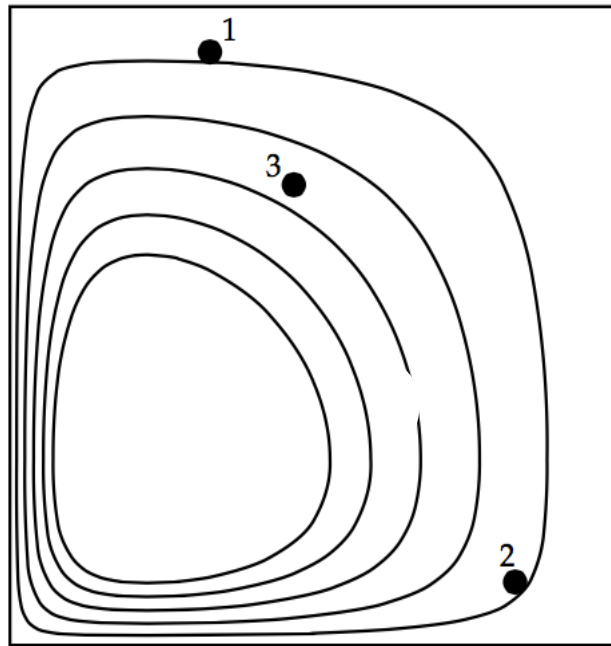
The sea of the pion is given by the sum of all sea quarks, parameterized by

$$u = \bar{u}_s = d_s = \bar{d} = s = \bar{s}$$

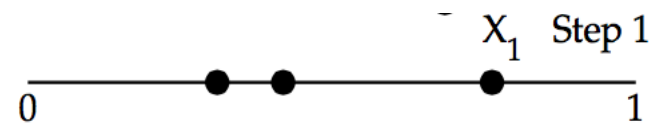
$$f(x_\pi, Q_0^2; \mathbf{a}) = \frac{N}{B(2 + \alpha, \beta)} x_\pi^\alpha (1 - x_\pi)^\beta$$

$$Q_0^2 = m_c^2$$

Nested Sampling



Parameter space

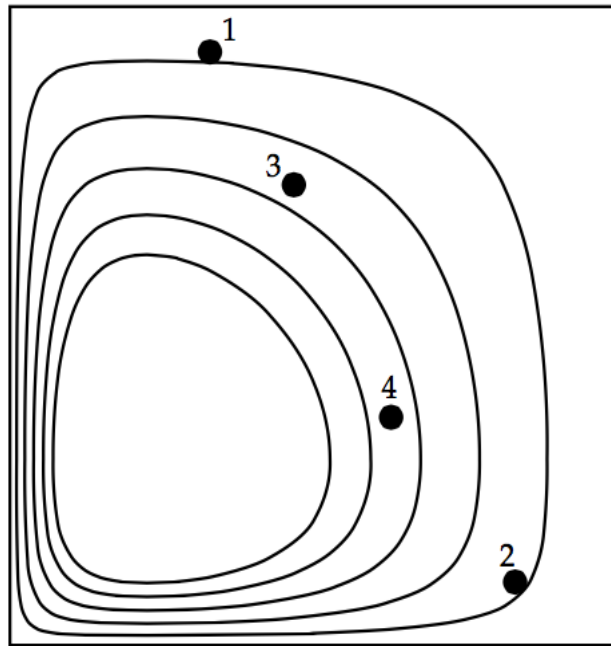


Enclosed prior mass X

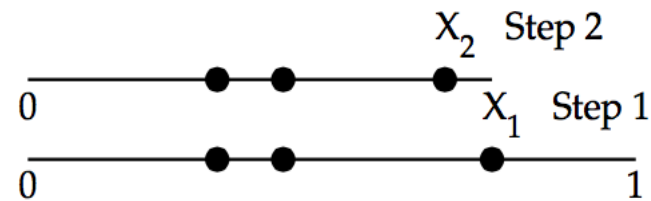
Start with random points on line of $0 < X < 1$.

$X = 0$ is the point of highest likelihood.

Nested Sampling



Parameter space



Enclosed prior mass X

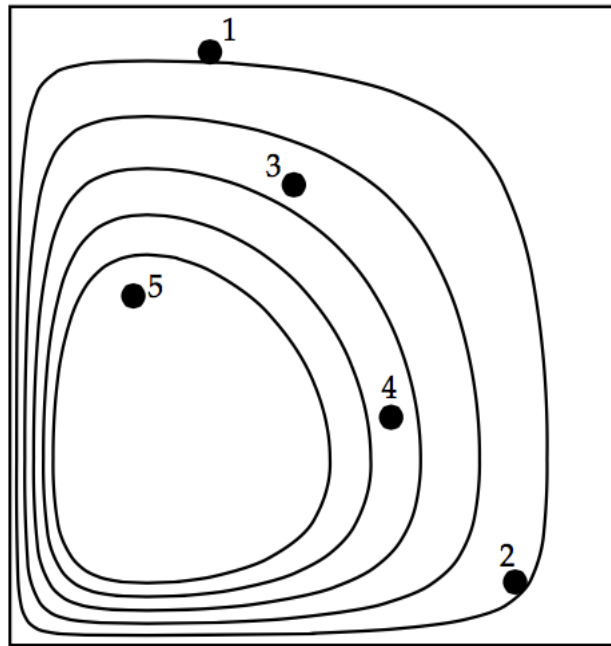
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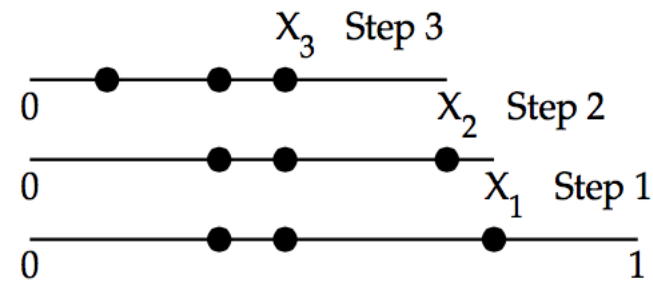
Move point of lowest likelihood to sample set

Make that point the upper-bound on new sampling boundary

Nested Sampling



Parameter space



Enclosed prior mass X

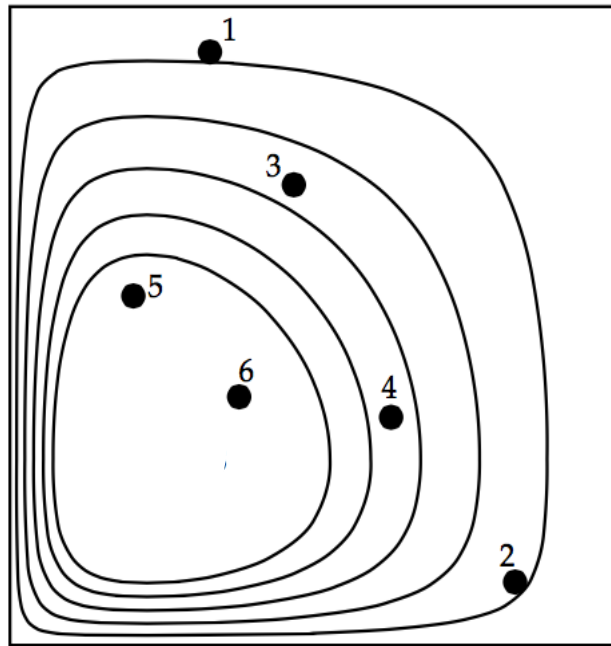
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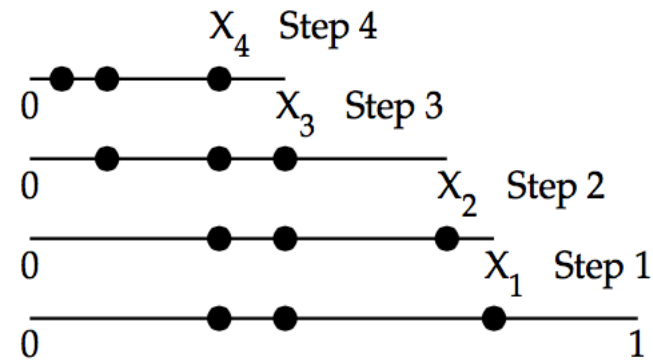
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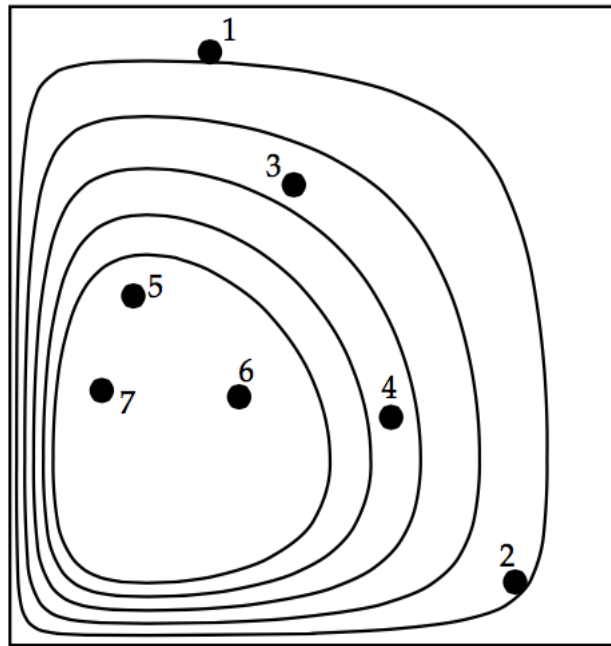
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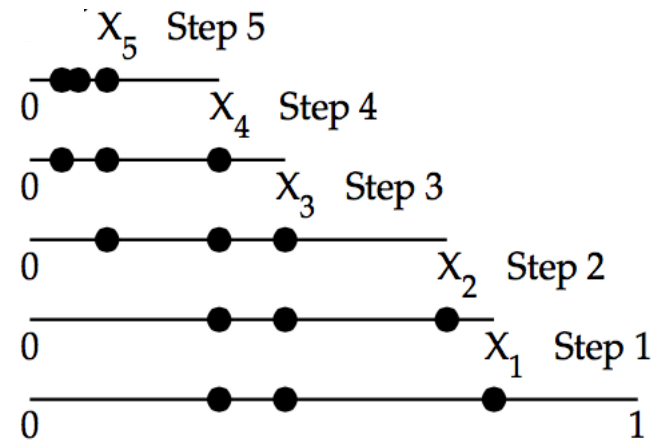
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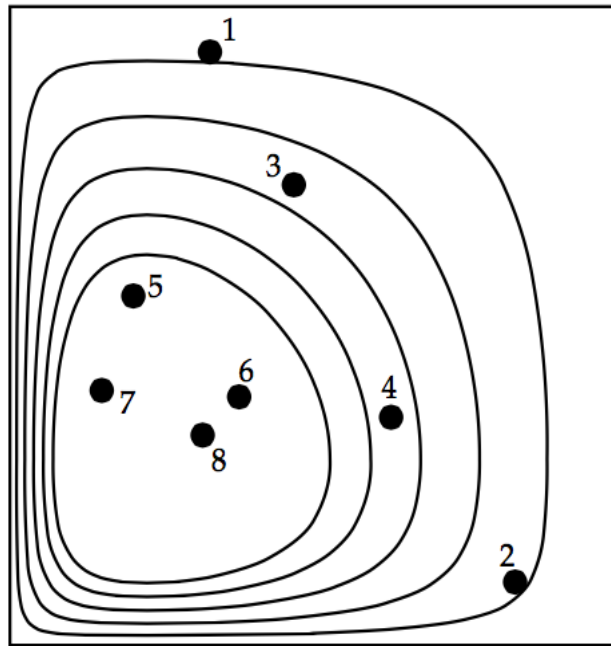
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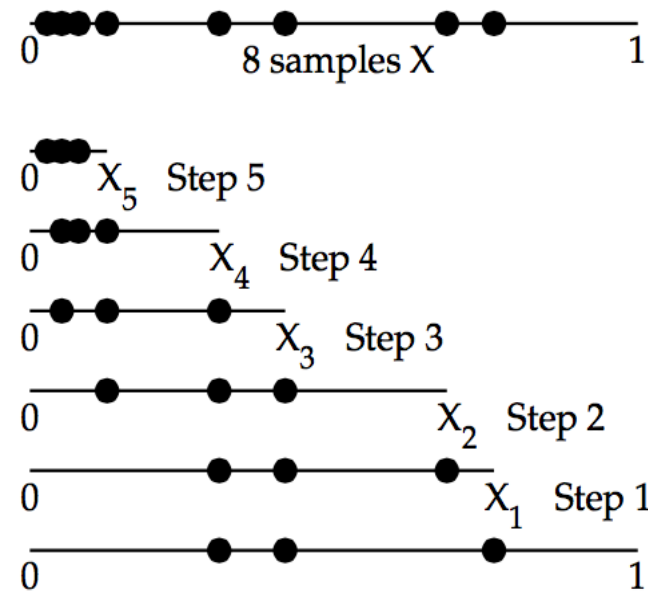
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