Radiative transitions between 0^{-+} and 1^{--} heavy quarkonia on the light front

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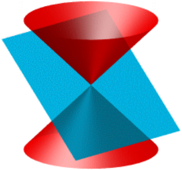

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Newport News, VA, May 17th, 2018

Introduction

Basis Light-Front Quantization (BLFQ):
A non-perturbative approach to solve QFT. [J.P.Vary, et al.,PRC81,2010]

- Light-front (LF) Hamiltonian formalism is a natural framework for tackling relativistic bound-state problems in QCD.
  - The quantum field is quantized on light-front time $x^+ = x^0 + x^3$.
  - LF energy P^-, LF 3-momentum (P^+, P^x, P^y), where $P^\pm = P^0 \pm P^3$.
  - Dispersion relation $P^- = (m^2 + \vec{P}_\perp^2)/P^+$
- By solving the eigenvalue equation, it directly produces the invariant masses and the boost invariant wavefunctions:

$$(P^+ \hat{P}^- - \vec{P}_\perp^2) \left|\psi_h(P, j, m_j)\right\rangle = M_h^2 \left|\psi_h(P, j, m_j)\right\rangle$$

- Basis representation
  - Basis can encode the analytical approximation to the solution.
  - Optimal basis is the key to numerical efficiency.

Heavy Quarkonia [Y.Li, et al.,PLB758,2016; PRD96,2017]

- The effective Hamiltonian at the $|q\bar{q}\rangle$ Fock sector:

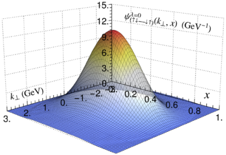
$$H_{\text{eff}} = \overbrace{\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\frac{\partial}{\partial x}\Big(x(1-x)\frac{\partial}{\partial x}\Big)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$

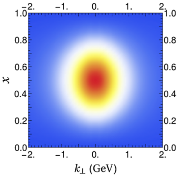
where $x = p_q^+/P^+$, $\vec{k}_\perp = \vec{p}_{q\perp} - x\vec{P}_\perp$, $\vec{r}_\perp = \vec{r}_{q\perp} - \vec{r}_{\bar{q}\perp}$.

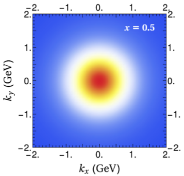
  - Confinement
    transverse holographic confinement [S.J.Brodsky, et al.,PR584,2015]
    longitudinal confinement [Y.Li, et al.,PLB758,2016]
  - One-gluon exchange with running coupling
    $V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$
- Basis representation
  - valence Fock sector: $|q\bar{q}\rangle$
  - basis functions: eigenfunctions of H_0
    $\phi_{nm}(\vec{k}_\perp/\sqrt{x(1-x)})$, $\chi_l(x)$,
    with basis truncation: $2n + |m| + 1 \leq N_{\max}$, $0 \leq l \leq L_{\max}$.

Heavy Quarkonia [Y.Li,et al.,PLB758,2016; PRD96,2017]

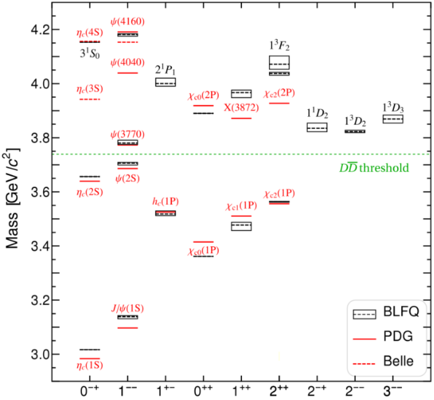
Light-front wavefunctions (LFWFs):
e.g. $\eta_c(1S)$

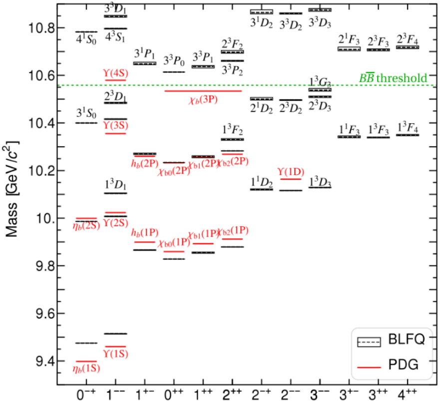






Mass spectra:





Radiative transitions

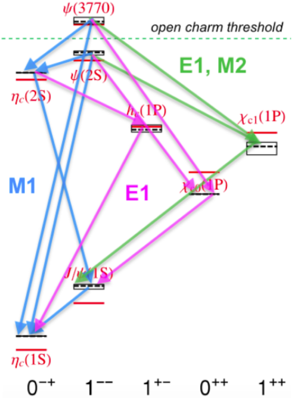
The electromagnetic transition between quarkonium states via emission of a photon, offers an insight into the internal structure of quark-antiquark bound states.

$$\psi_i \to \psi_f + \gamma$$

Each process is governed by its hadron matrix,

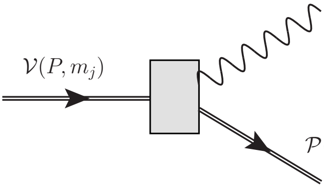
$$\langle \psi_f | \, J^\mu(0) \, | \psi_i \rangle \ ,$$

$J^\mu(x)$ is the current operator.



Radiative transitions characterized by multipoles. (Figure by Y.Li.)

The M1 transition



vector(1^{--}) $\leftrightarrow$ pseudoscalar (0^{-+})

spin $J: 1 \leftrightarrow 0$
parity $\mathrm{P}: -1 \leftrightarrow -1$
charge conjugation $\mathrm{C}: -1 \leftrightarrow +1$

$\mathcal{V} \to \mathcal{P} + \gamma$ or $\mathcal{P} \to \mathcal{V} + \gamma$

Transition form factor:

$$I^{\mu}_{m_j}(P,P') \equiv \langle \mathcal{P}(P') | J^{\mu} | \mathcal{V}(P,m_j) \rangle = \frac{2V(Q^2)}{m_{\mathcal{P}} + m_{\mathcal{V}}} \epsilon^{\mu\alpha\beta\sigma} P'_{\alpha} P_{\beta} e_{\sigma}(P,m_j)$$

Decay width:

$$\Gamma(\mathcal{V} \to \mathcal{P} + \gamma) = \frac{(m^2_{\mathcal{V}} - m^2_{\mathcal{P}})^3}{(2m_{\mathcal{V}})^3 (m_{\mathcal{P}} + m_{\mathcal{V}})^2} \frac{|V(0)|^2}{(2J_{\mathcal{V}} + 1)\pi}$$

where $q = P' - P$, $Q^2 \equiv -q^2$ and $e_{\sigma}(P,m_j)$ is the polarization vector of the vector meson. $J_{\mathcal{V}} = 1$ is the spin of the initial vector meson.

Calculation on the light front

Current components: J^{μ} $(\mu = +, -, x, y)$

J^{+}: *"good current"* (suppress contributions from pair production/annihilation in vacuum)

J^{-}: *"bad current"* (associate with the zero-mode contributions)

$J^{\perp}(J^{R} \equiv J^{x} + iJ^{y})$: good/bad?

[J.Carbonell, et al.,PR300,1998; S.J.Brodsky, et al., NPB543,1999; D.Melikhov, et al.,PRD65,2002]

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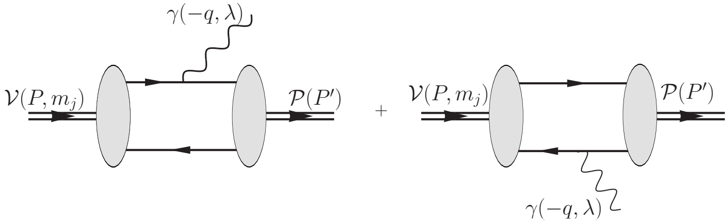
$$I^+_{m_j} = \frac{2V(Q^2)}{m_\mathcal{P} + m_\mathcal{V}} \begin{cases} 0, & m_j = 0 \\ \frac{i}{\sqrt{2}} P^+ \Delta^R, & m_j = 1 \\ -\frac{i}{\sqrt{2}} P^+ \Delta^L, & m_j = -1 \end{cases}$$

$$I^R_{m_j} = \frac{2V(Q^2)}{m_\mathcal{P} + m_\mathcal{V}} \begin{cases} -i m_\mathcal{V} \Delta^R, & m_j = 0 \\ \frac{i}{\sqrt{2}} P^R \Delta^R, & m_j = 1 \\ \frac{i}{\sqrt{2} z} (z^2 m^2_\mathcal{V} - m^2_\mathcal{P} - P'^R \Delta^L), & m_j = -1 \end{cases}$$

Boost invariants :
$z \equiv P'^+ / P^+$
$\vec{\Delta}_\perp \equiv \vec{P}'_\perp - z \vec{P}_\perp$.

Complex forms :
$k^{R/L} \equiv k^x \pm i k^y$.

Light-front wavefunction representation in $|q\bar{q}\rangle$



Impulse approximation: $V(Q^2) = 2e\mathcal{Q}_f \hat{V}(Q^2)$, $\mathcal{Q}_f$ is the quark charge.

The hadron matrix element in the Drell-Yan frame ($q^+ = 0$) is,

$$\begin{aligned}
&\langle \mathcal{P}(P')|J_q^\mu(0)|\mathcal{V}(P,m_j)\rangle \\
&= \sum_{s,\bar{s},s'} \int_0^1 \frac{\mathrm{d}x}{2x^2(1-x)} \int \frac{\mathrm{d}^2\vec{k}_\perp}{(2\pi)^3} \psi^{(m_j)}_{s\bar{s}/\mathcal{V}}(\vec{k}_\perp, x)\psi^*_{s'\bar{s}/\mathcal{P}}(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \\
&\bar{u}_{s'}(xP^+, \vec{k}_\perp + x\vec{P}_\perp + \vec{q}_\perp)\gamma^\mu u_s(xP^+, \vec{k}_\perp + x\vec{P}_\perp)\ .
\end{aligned}$$

Convolutions of LFWFs: $\psi^{(m_j)}_{s\bar{s}/\mathcal{V}}, \psi_{s'\bar{s}/\mathcal{P}}$.

Transition form factor at $|q\bar{q}\rangle$

$\hat{V}(Q^2)$ as convolutions of LFWFs: $\psi^{(m_j)}_{s\bar{s}/\mathcal{V}}(\vec{k}_\perp, x)\psi^*_{s'\bar{s}/\mathcal{P}}(\vec{k}_\perp + (1-x)\vec{q}_\perp, x)$.

- J^+ and $m_j = 1$
  $m_j = 0$ is not available, $m_j = -1$ is equivalent to $m_j = 1$.

$$\hat{V}_{m_j=1}(Q^2) = \frac{\sqrt{2}(m_\mathcal{P} + m_\mathcal{V})}{iq^R} \int_0^1 \frac{\mathrm{d}x}{2x(1-x)} \int \frac{\mathrm{d}^2\vec{k}_\perp}{(2\pi)^3}$$
$$[\psi^{(m_j=1)}_{\uparrow\uparrow/\mathcal{V}}\psi^*_{\uparrow\uparrow/\mathcal{P}} + \psi^{(m_j=1)}_{\uparrow\downarrow-\downarrow\uparrow/\mathcal{V}}\psi^*_{\uparrow\downarrow-\downarrow\uparrow/\mathcal{P}} + \psi^{(m_j=1)}_{\uparrow\downarrow+\downarrow\uparrow/\mathcal{V}}\psi^*_{\uparrow\downarrow+\downarrow\uparrow/\mathcal{P}} + \psi^{(m_j=1)}_{\downarrow\downarrow/\mathcal{V}}\psi^*_{\downarrow\downarrow/\mathcal{P}}]$$

- J^R and $m_j = 0$
  $m_j = \pm 1$ can be related to that in J^+ through a transverse Lorentz boost.

$$\hat{V}_{m_j=0}(Q^2) = \frac{i(m_\mathcal{P} + m_\mathcal{V})}{m_\mathcal{V}} \int_0^1 \frac{\mathrm{d}x}{2x^2(1-x)} \int \frac{\mathrm{d}^2\vec{k}_\perp}{(2\pi)^3}$$
$$[-\frac{1}{2}\psi^{(m_j=0)}_{\uparrow\downarrow+\downarrow\uparrow/\mathcal{V}}\psi^*_{\uparrow\downarrow-\downarrow\uparrow/\mathcal{P}} + \psi^{(m_j=0)}_{\downarrow\downarrow/\mathcal{V}}\psi^*_{\downarrow\downarrow/\mathcal{P}}]$$

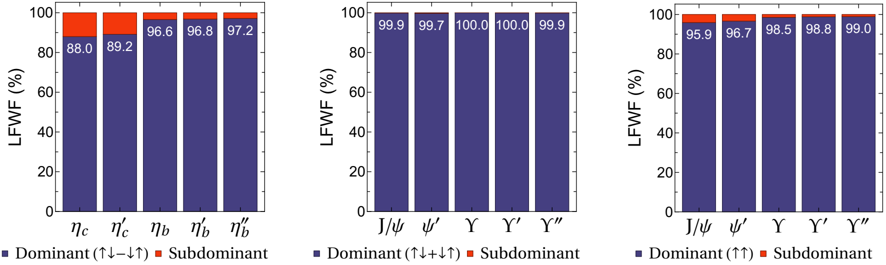
Dominant spin components: exist in the nonrelativistic limit
Subdominant spin components: relativistic origin

Transition form factor at $|q\bar{q}\rangle$

For the less relativistic heavy systems:

$\hat{V}_{m_j=0}(Q^2)$: depends on dominant components→ more robust! $(= \hat{V}(Q^2))$
$\hat{V}_{m_j=1}(Q^2)$: relies on subdominant components

dominant components and subdominant components of charmonium and bottomonium:



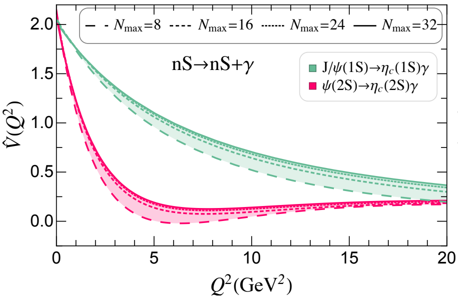
(a) $\mathcal{P}(nS)$ (b) $\mathcal{V}^{(m_j=0)}(nS)$ (c) $\mathcal{V}^{(m_j=1)}(nS)$

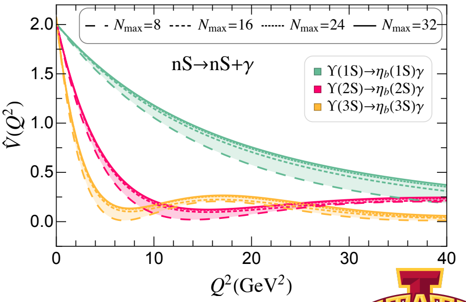
Transition form factor I

The *allowed* transition: initial and final states have the same radial or angular quantum numbers.

$$nS \to nS + \gamma$$

Nonrelativistic limit: $\hat{V}(0) \to 2$.





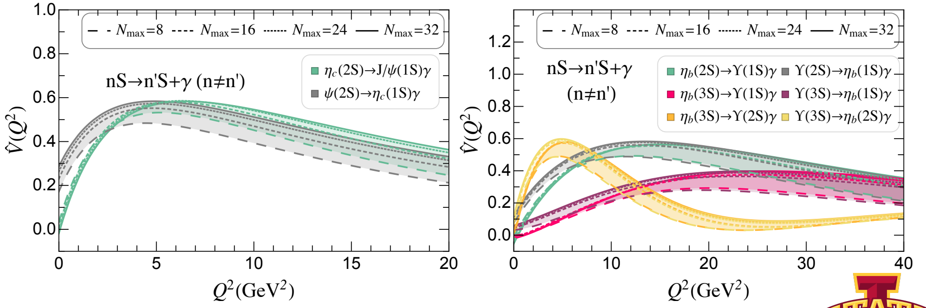
Shaded area: numerical uncertainty from basis truncation.

Transition form factor II

The *hindered* transition: initial and final states have different radial or angular quantum numbers.

$$nS \to n'S + \gamma (n \neq n'), \qquad nD \to n'S + \gamma$$

Nonrelativistic limit: $\hat{V}(0) \to 0$.



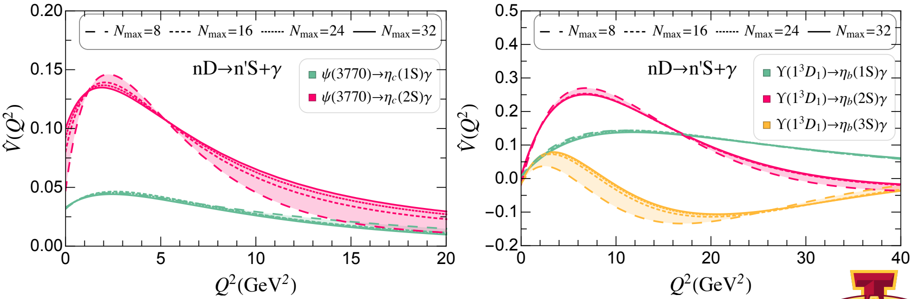
Shaded area: numerical uncertainty from basis truncation.

Transition form factor III

The *hindered* transition: initial and final states have different radial or angular quantum numbers.

$$nS \to n'S + \gamma (n \neq n'), \qquad nD \to n'S + \gamma$$

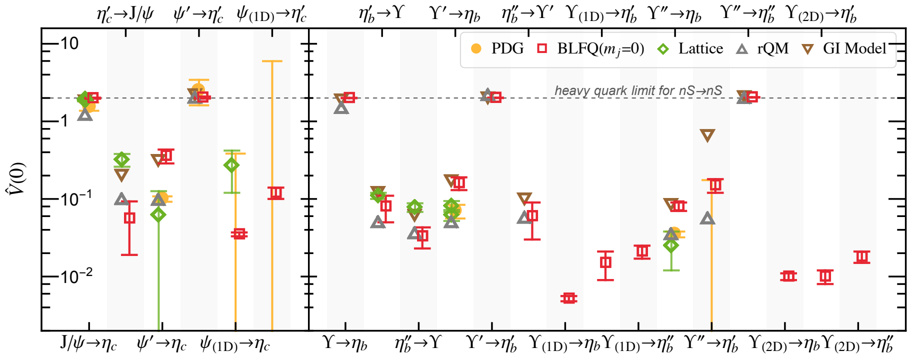
Nonrelativistic limit: $\hat{V}(0) \to 0$.



Shaded area: numerical uncertainty from basis truncation.

$\hat{V}(0)$

Transitions between vector and pseudoscalar mesons for charmonia and bottomonia below their open flavor threshold. (#21)



PDG: [C.Patrignani, et al.,CPC40,2016]

Lattice: [J.J.Dudek, et al.,PRD73,2006; PRD79,2009; D.Bečirević, et al.,JHEP01,2013; JHEP05,2015; C. Hughes, et al.,PRD92,2015; R.Lewis, et al.,PRD86,2012]

relativistic quark model (rQM): [D.Ebert, et al.,PRD67,2003]

Godfrey-Isgur model (GI model): [T.Barnes, et al.,PRD72,2005; S.Godfrey, et al.,PRD92,2015]

Decay constant of the vector meson $f_{\mathcal{V}}$

$$\langle 0| J^\mu(0) |\mathcal{V}(P, m_j)\rangle = e^\mu(P, m_j) M_{\mathcal{V}} f_{\mathcal{V}}$$

Integrals of LFWFs:

$f_{\mathcal{V}}|_{J^+/J^R, m_j=0} : \psi^{(m_j=0)}_{\uparrow\downarrow+\downarrow\uparrow/\mathcal{V}}$. $f_{\mathcal{V}}|_{J^R, m_j=1} : \psi^{(m_j=1)}_{\uparrow\uparrow/\mathcal{V}}, \psi^{(m_j=1)}_{\uparrow\downarrow+\downarrow\uparrow/\mathcal{V}}, \psi^{(m_j=1)}_{\uparrow\downarrow-\downarrow\uparrow/\mathcal{V}}$.

Dominant spin components for the S-wave states.

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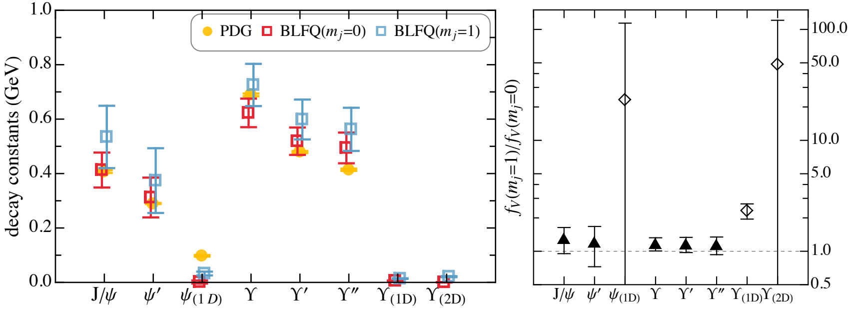
Dominant spin components for the S-wave states.



- Lorentz symmetry is reasonably preserved.
- $\hat{V}(Q^2)_{m_j=0}$ using the dominant components is reliable.

Summary and outlook

- The radiative transitions between 0^{-+} $(\mathcal{P})$ and 1^{--} $(\mathcal{V})$ heavy quarkonia is calculated from the LFWFs, providing predictions.
- Comparison of different components of current operator $(J^+$ and $J^\perp)$ provides insights on light-front dynamics.
  - $J^\perp$ is preferred to J^+ for the M1 transition in heavy systems
  - J^+ and $J^\perp$ agree on the decay constants of the vector mesons.
- Other radiative transitions: $\mathcal{V} \to \mathcal{S} + \gamma$, $\mathcal{V} \to \mathcal{A} + \gamma$
- EM Dalitz decay: $\mathcal{V} \to \mathcal{P} + \gamma* \to \mathcal{P} + e^+ + e^-$.

[M.Li, Y.Li, P.Maris and J.P.Vary, submitted to Phys.Rev.D, arXiv:1803.11519]

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- EM Dalitz decay: $\mathcal{V} \to \mathcal{P} + \gamma* \to \mathcal{P} + e^+ + e^-$.

[M.Li, Y.Li, P.Maris and J.P.Vary, submitted to Phys.Rev.D, arXiv:1803.11519]

Thank you very much!

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- Others
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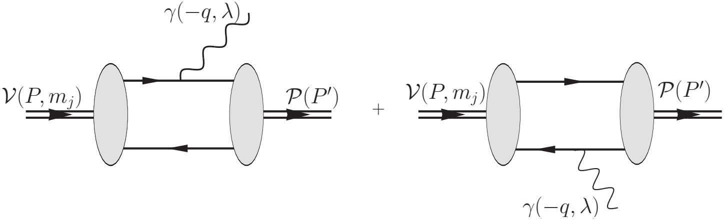
Backup: Impulse approximation



$$I^{\mu}_{m_j} = e\mathcal{Q}_f \langle \mathcal{P}(P')| J^{\mu}_{q}(0) |\mathcal{V}(P,m_j)\rangle - e\mathcal{Q}_f \langle \mathcal{P}(P')| J^{\mu}_{\bar{q}}(0) |\mathcal{V}(P,m_j)\rangle \ .$$

quark term $\overset{\text{charge conjugation}}{=\!=\!=\!=\!=}$ **antiquark term**

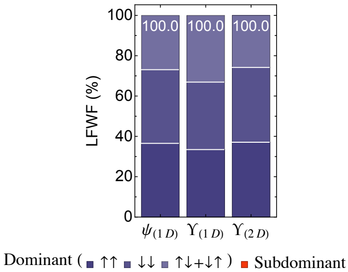
The transition form factor $\hat{V}(Q^2)$ extracted from the quark current is related to $V(Q^2)$ as,

$$V(Q^2) = 2e\mathcal{Q}_f \hat{V}(Q^2)$$

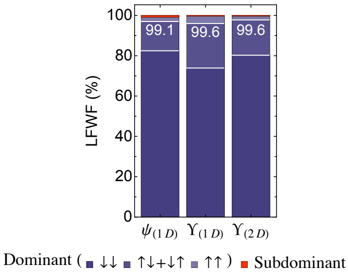
Quark charge: $\mathcal{Q}_f = \mathcal{Q}_c = +2/3$, $\mathcal{Q}_f = \mathcal{Q}_b = -1/3$.

Backup: More on the spin components

dominant components and subdominant components of D-wave states:



(a) $\mathcal{V}^{(m_j=0)}(n^3D_1)$



(b) $\mathcal{V}^{(m_j=1)}(n^3D_1)$

Dominant components: exist in the nonrelativistic limit

Subdominant components: relativistic origin

Backup: Decay constant of the vector meson

The decay constant $f_{\mathcal{V}}$ is defined from the vacuum-to-hadron matrix elements:

$$\langle 0| J^\mu(0) |\mathcal{V}(P, m_j)\rangle = e^\mu(P, m_j) M_{\mathcal{V}} f_{\mathcal{V}}$$

- Same current operator as for the transition form factor.
- Help check Lorentz symmetry.

$$f_{\mathcal{V}}(m_j = 0) : f^+_{\mathcal{V}}(m_j = 0) = f^\perp_{\mathcal{V}}(m_j = 0)$$

$$f_{\mathcal{V}}(m_j = \pm 1) : f^+_{\mathcal{V}}(m_j = \pm 1)(\text{not available}), f^\perp_{\mathcal{V}}(m_j = \pm 1)$$

The light-front wavefunctions representation,

$$f_{\mathcal{V}}(m_j = 0) = \sqrt{2N_c} \int_0^1 \frac{\mathrm{d}x}{\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2\vec{k}_\perp}{(2\pi)^3} \psi^{(m_j=0)}_{\uparrow\downarrow+\downarrow\uparrow/\mathcal{V}}(\vec{k}_\perp, x) \, .$$

$$f_{\mathcal{V}}(m_j = 1) = \frac{\sqrt{N_c}}{2m_{\mathcal{V}}} \int_0^1 \frac{\mathrm{d}x}{[x(1-x)]^{3/2}} \int \frac{\mathrm{d}^2\vec{k}_\perp}{(2\pi)^3} [k^L(1-2x)\psi^{(m_j=1)}_{\uparrow\downarrow+\downarrow\uparrow/\mathcal{V}}(\vec{k}_\perp, x) - k^L\psi^{(m_j=1)}_{\downarrow\uparrow-\downarrow\uparrow/\mathcal{V}}(\vec{k}_\perp, x) + \sqrt{2}m_q\psi^{(m_j=1)}_{\uparrow\uparrow/\mathcal{V}}(\vec{k}_\perp, x)] \, .$$

Dominant spin configurations for S-wave states.

Backup: the transverse boost

The two sets of hadron matrix elements $\langle \mathcal{P}(P')| J^+ |\mathcal{V}(P, m_j)\rangle$ and $\langle \mathcal{P}(P')| \vec{J}_\perp |\mathcal{V}(P, m_j)\rangle$ can be related through the transverse Lorentz boost specified by the velocity vector $\vec{\beta}_\perp$,

$$v^+ \to v^+, \quad \vec{v}_\perp \to \vec{v}_\perp + v^+ \vec{\beta}_\perp \ .$$

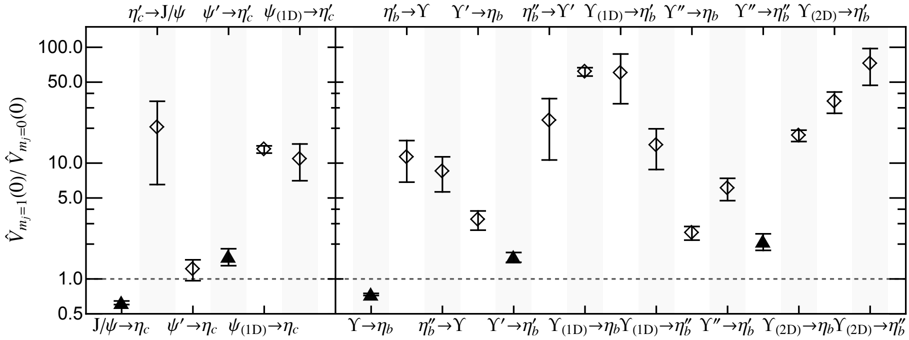
That is,

$$\begin{aligned} &\left\langle \mathcal{P}(P'^+, \vec{P}'_\perp + P'^+ \vec{\beta}_\perp) \middle| \vec{J}_\perp \middle| \mathcal{V}(P^+, \vec{P}_\perp + P^+ \vec{\beta}_\perp, m_j) \right\rangle \\ =& \left\langle \mathcal{P}(P'^+, \vec{P}'_\perp) \middle| \vec{J}_\perp \middle| \mathcal{V}(P^+, \vec{P}_\perp, m_j) \right\rangle + \vec{\beta}_\perp \left\langle \mathcal{P}(P'^+, \vec{P}'_\perp) \middle| J^+ \middle| \mathcal{V}(P^+, \vec{P}_\perp, m_j) \right\rangle \ . \end{aligned}$$

In consequence, $V(Q^2)$ obtained using J^+ and $\vec{J}_\perp$ with the same m_j should be the same.

Backup: $\hat{V}(0)$

Ratio of $\hat{V}_{m_j=1}(0)$ (using J^+) to $\hat{V}_{m_j=0}(0)$ (using J^R).



▲ Allowed transition: the transition with the same radial or angular quantum numbers (e.g. $nS \to nS + \gamma$)

◇ Hindered transition: the transition between states with different radial or angular excitations. (e.g. $nS \to n'S + \gamma$ and $nD \to n'S + \gamma$)