Lorentz Invariance and QCD Equation of Motion Relations for Partonic Orbital Angular Momentum

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People Involved

- Simonetta Liuti, University of Virginia
- Michael Engelhardt, New Mexico State University
- Aurore Courtoy, UNAM Mexico

AR, Engelhardt and Liuti arxiv:1709.05770

AR, Courtoy, Engelhardt and Liuti PRD 94 (2016)

Outline

- Spin Crisis !
- Orbital Angular Momentum
  - GTMD definition
  - GPD definition Ji
- What's the connection? Lorentz Invariance Relations
- Equation of Motion
- Quark Gluon Structure of Twist Three GPDs
- Conclusions

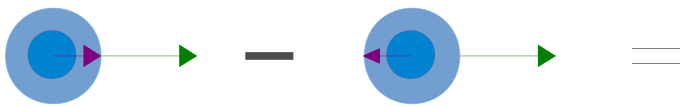
Proton Spin Crisis

$= g_1^P(x)$ Quark Spin Contribution

$$\frac{1}{2M}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\left\langle P,S\left|\bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right|P,S\right\rangle = \Lambda g_1(x)p_\mu + g_T(x)S_{\perp_\mu}$$

Measured by EMC experiment in 1980s to be only 33% of total !!

Proton Spin Crisis

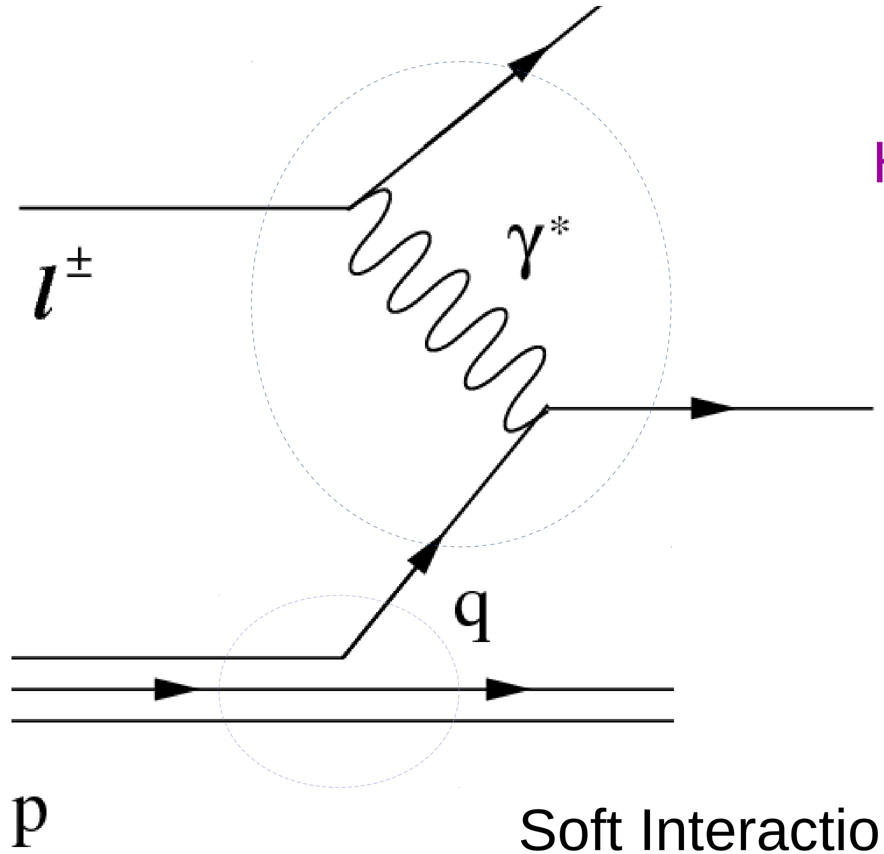
 $g_1^P(x)$ Quark Spin Contribution

$$\frac{1}{2M}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\left\langle P,S\left|\bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right|P,S\right\rangle = \Lambda g_1(x)p_\mu + g_T(x)S_{\perp_\mu}$$

Measured by EMC experiment in
1980s to be only 33% of total !!

What are other sources ?
Partonic Orbital Angular Momentum

Hard and Soft Parts



Hard Interaction

Deep Inelastic Scattering

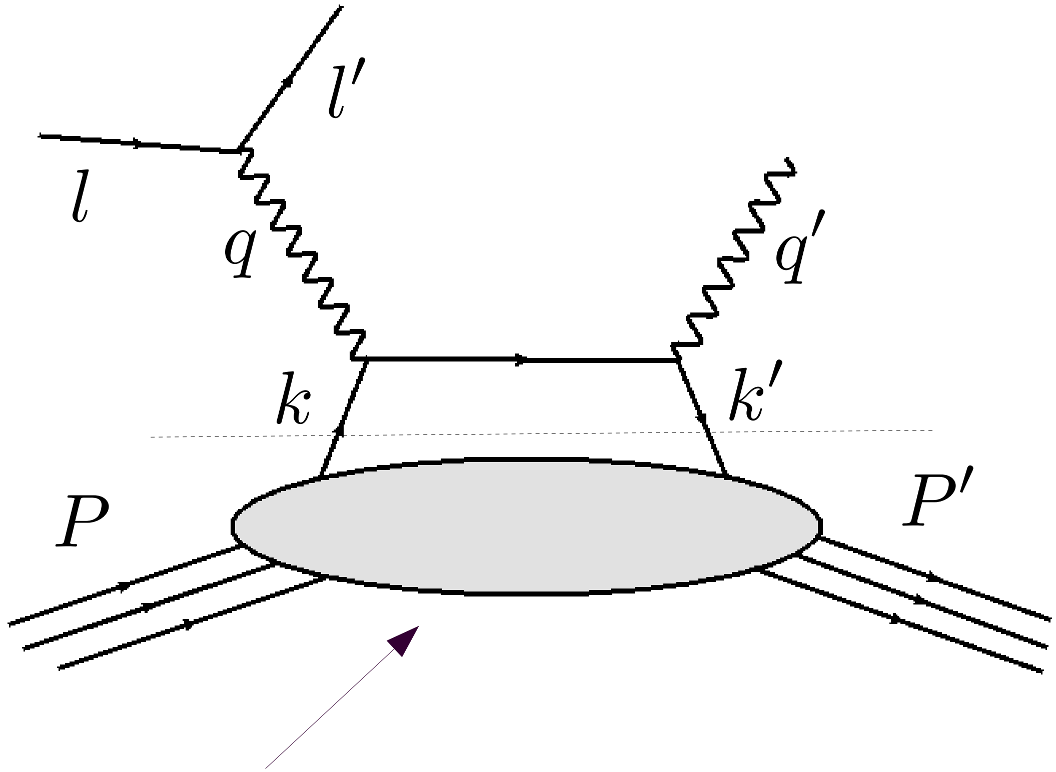
Soft Interaction

$$\int \frac{dz^-}{2\pi} e^{ik^+z^-} \langle p, S \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, S\rangle_{z^+=z_T=0} = f_1(x)$$

$$a^{\pm} = \frac{a^0 \pm a^3}{\sqrt{2}}$$

Where are the partons located?

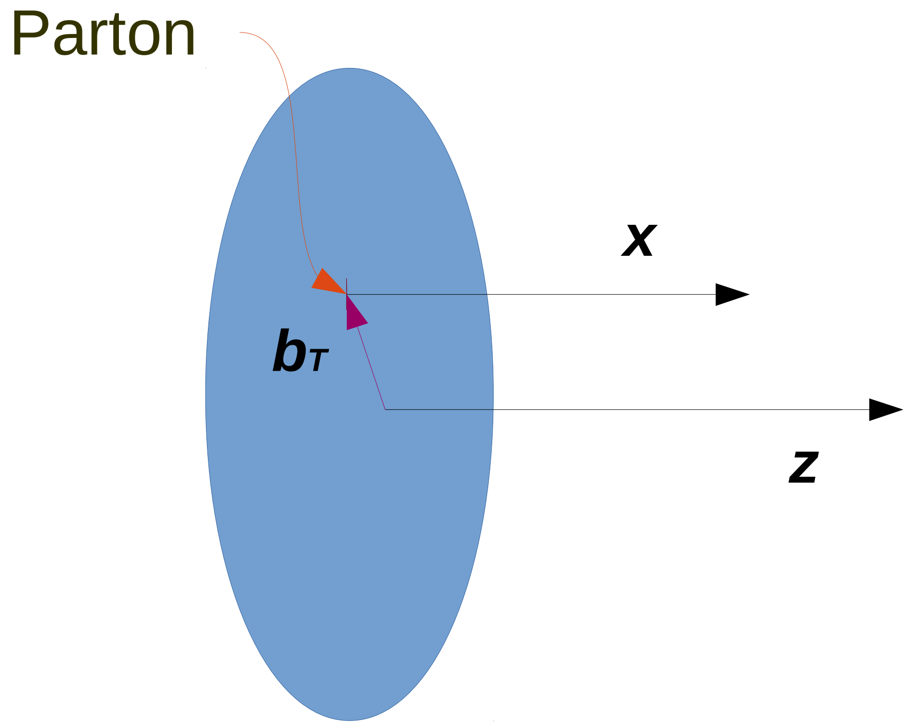
- Need a high energy photon to probe the partons
- The proton needs to remain intact to access spatial distribution
- **Deeply Virtual Compton Scattering**



Generalized Parton Distributions

Generalized Parton Distributions

- GPDs are the Fourier transform of the spatial distribution of partons in protons and neutrons.



GPD based definition of Angular Momentum

$$J_q = \frac{1}{2}\int_{-1}^{1} dx x (H_q(x,0,0) + E_q(x,0,0))$$

Xiangdong Ji, PRL 78.610,1997

To access OAM, we take the difference between total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

Direct description of OAM

$$\int dx\, x G_2 = \int dx\, x(H + E) - \int dx \tilde{H}$$

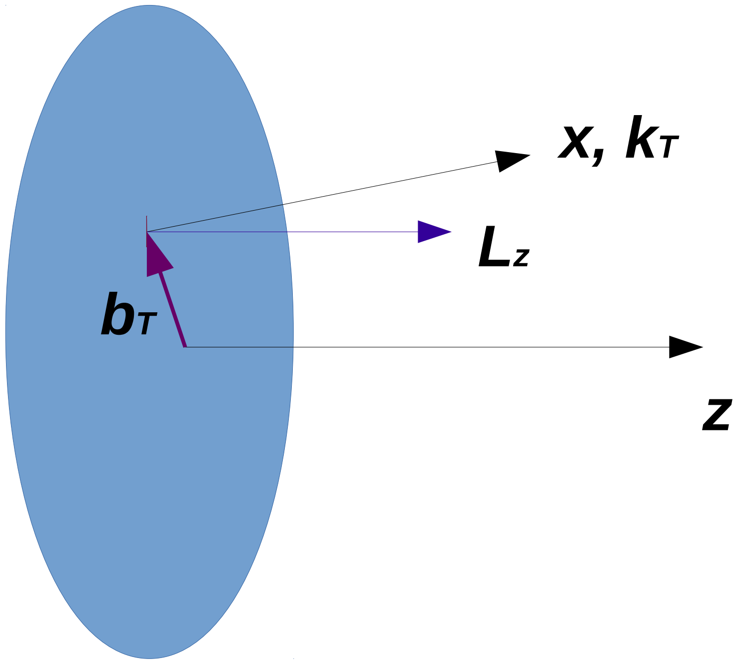
$$G_2 \equiv \tilde{E}_{2T} + H + E$$

Kiptily and Polyakov, Eur Phys J C 37 (2004)
Hatta and Yoshida, JHEP (1210), 2012

- The moment in x of the GPD G_2 shown to be OAM

Partonic Orbital Angular Momentum II

- Consider measuring both the intrinsic transverse momentum and the spatial distribution of partons
- $L_{q,z}$ = $\mathbf{b}_T \times \mathbf{k}_T$



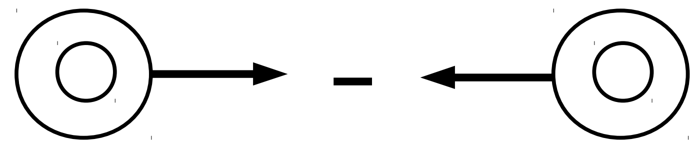
$$W^{[\gamma^+]}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k^i_T}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta^i_T}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k^i_T\Delta^j_T}{M^2}F_{14}]U(p,\Lambda)$$

Generalized Transverse Momentum Distributions (related by Fourier transform to Wigner Distributions)

Meissner Metz and Schlegel, JHEP 0908 (2009)

GTMDs that describe OAM

- How does F_{14} connect to OAM ?



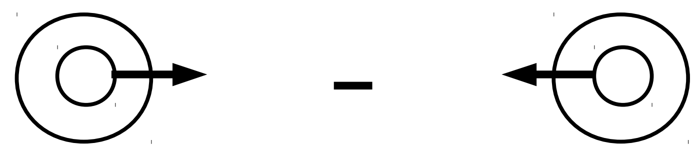
Unpolarized quark in a longitudinally polarized proton

$$\mathcal{W}(x,\mathbf{k}_T,\mathbf{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{ib.\Delta_T} \left[W_{++}^{\gamma^+} - W_{--}^{\gamma^+} \right]$$

$$L = \int dx \int d^2k_T \int d^2\mathbf{b} (\mathbf{b}\times\mathbf{k}_T) \mathcal{W}(x,\mathbf{k}_T,\mathbf{b}) = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce et al PRD84, (2011)

- Another GTMD relevant to OAM



G_{11} describes a longitudinally polarized quark in an unpolarized proton. Measures spin orbit correlation.

The Two Definitions

- Weighted average of b_T X k_T

$$L_z = -\int dx \int d^2 k_T \frac{k_T^2}{M^2} F_{14}$$

- Difference of total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x (H_q + E_q) \qquad\qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

The Two Definitions

- Weighted average of b_T X k_T

GTMD

$$L_z = -\int dx \int d^2 k_T \frac{k_T^2}{M^2} F_{14}$$

$\rightarrow F_{14}^{(1)}$

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x (H_q + E_q) \qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

Is there a connection ?

- We find that

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

- This is a form of Lorentz Invariant Relation (LIR)
- This is a distribution of OAM in x
- Derived for a straight gauge link

Quark Quark Correlator Function

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2z_T}{2\pi} e^{ixP^+z^- - k_T.z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations

- Parametrization of the quark quark correlator at different levels
- LIRs occur because the number of GPCFs is less than the number of GTMDs.

$$\begin{aligned}\mathcal{W}^{[\gamma^\mu]}_{\Lambda\Lambda'} &= \frac{\bar{U}U}{M}(P^\mu A_1^F + k^\mu A_2^F + \Delta^\mu A_3^F) + i\frac{\bar{U}\sigma^{\mu k}U}{M}A_5^F + i\frac{\bar{U}\sigma^{\mu\Delta}U}{M}A_6^F \\ &+ i\frac{\bar{U}\sigma^{k\Delta}U}{M^3}(P^\mu A_8^F + k^\mu A_9^F + \Delta^\mu A_{17}^F)\end{aligned}$$

Explicit k_T coefficient

$$W^{[\gamma^+]}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

$$F^{[\gamma^i]}_{\Lambda,\Lambda'} = \frac{1}{2(P^+)^2}\bar{U}\left[i\sigma^{+i}H_{2T} + \frac{\gamma^+\Delta_T^i}{2M}E_{2T} + \frac{P^+\Delta_T^i}{M^2}\tilde{H}_{2T} - \frac{P^+\gamma^i}{M}\tilde{E}_{2T}\right]U$$

Generalized Lorentz Invariance Relations

- The As are a function of the following scalar variables :

$$\sigma \equiv \frac{2k.P}{M^2}, \qquad \tau \equiv \frac{k^2}{M^2}, \qquad \sigma' \equiv \frac{k.\Delta}{\Delta^2} = \frac{k_T.\Delta_T}{\Delta_T^2}$$

For $\Delta^+ = 0$

$$\int dk^- A(k^2, k.P, k.\Delta \ldots) \;\rightarrow\; \frac{M^2}{2P^+}\int d\sigma A$$

$$\rightarrow\; \frac{M^2}{2P^+}\int d\sigma' d\sigma d\tau \delta\left(\frac{k_T^2}{M^2} - x\sigma + \tau + \frac{x^2 P^2}{M^2}\right)\delta\left(\sigma' - \frac{k_T.\Delta_T}{\Delta_T^2}\right) A(\sigma, \tau, \sigma')$$

Generalized Lorentz Invariance Relations

$$F_{14}^{(1)} = \int d\sigma d\sigma' d\tau \frac{M^3}{2} J \left[A_8^F + xA_9^F\right]$$

$$J = \sqrt{x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}}$$

$$\tilde{E}_{2T} = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \left[\left(x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}\right) A_9^F - \sigma' A_5^F - A_6^F\right]$$

$$H + E = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \sigma' A_5^F + A_6^F + \left(\frac{\sigma}{2} - \frac{xP^2}{M^2}\right)\left(A_8^F + xA_9^F\right)$$

$$-\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$

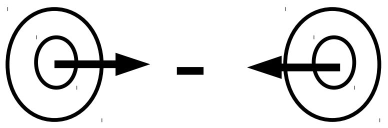
$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y)\right)$$

Distribution of OAM in x !

k_T^2 moment of a twist two function

Twist three function

Generalized Lorentz Invariance Relations



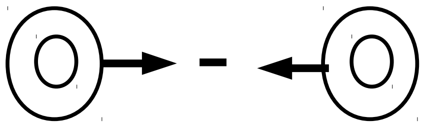
Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$

$$\frac{dG_{12}^{e(1)}}{dx} = H'_{2T} - \frac{\Delta_T^2}{4M^2} E'_{2T} - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$

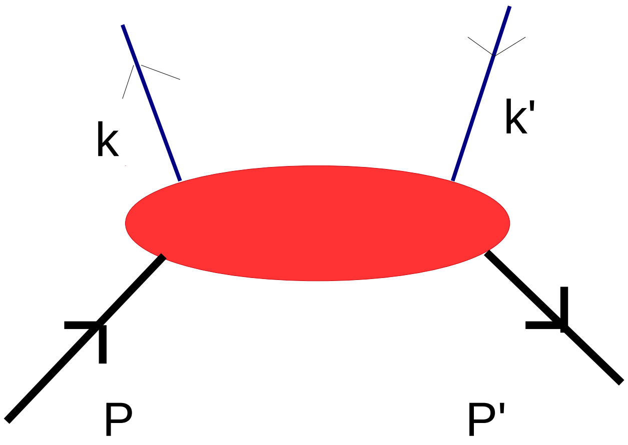


The GTMDs are complex in general.

$$X = X^e + iX^o$$

The imaginary part integrates to zero, on integration over k_T.

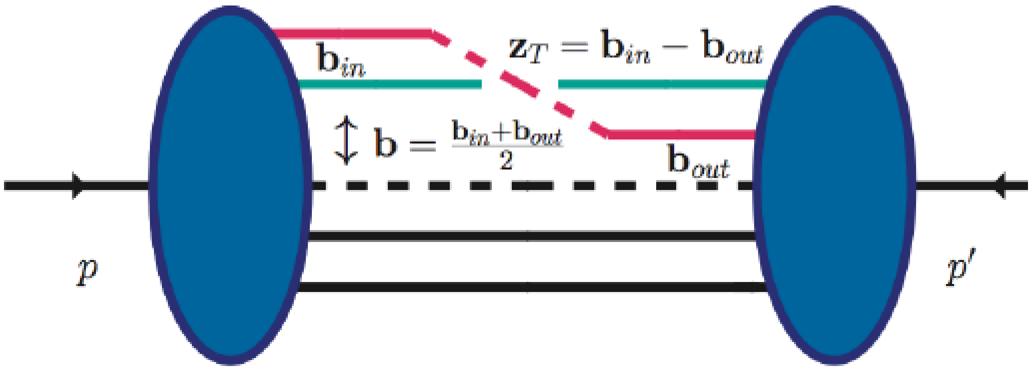
Intrinsic Momentum vs Momentum Transfer Δ

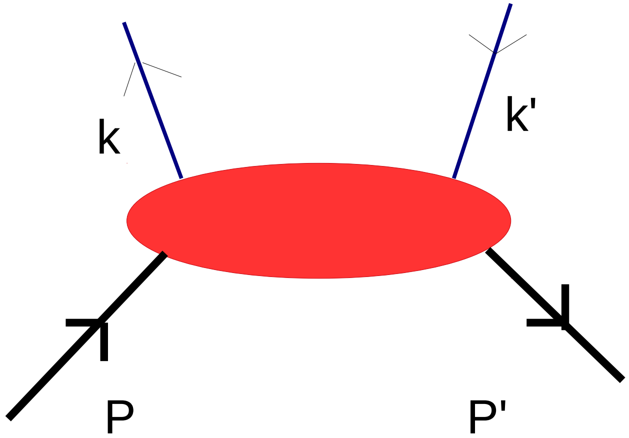


Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

Intrinsic Momentum vs Momentum Transfer Δ





$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

Equations of Motion Relations

How do we obtain these ?

$$
\begin{aligned}
(i\not{D} - m)\psi(z_{out}) &= (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) = 0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

$$\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) = \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$

$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0$$

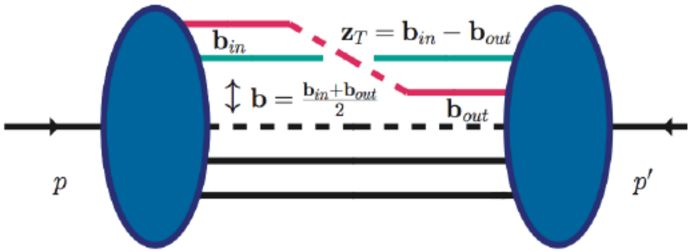
Equations of Motion Relations

How do we obtain these ?

$$\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) = \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$

$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2 b_T e^{-ib\cdot\Delta} \int dz^- d^2 z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5 (i\overrightarrow{\not{D}} - m) \right] \psi | p, \Lambda \rangle = 0$$

Equations of Motion R

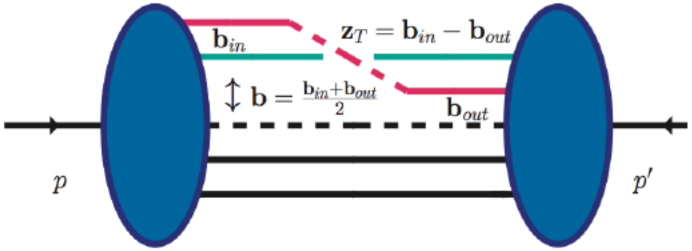
Crucial for understanding qgq contribution to GPDs!!

How do we obtain these ?

$$\mathcal{U} i\sigma^{i+}\gamma_5(i\not{D} - m)\psi(z_{out}) = \mathcal{U} i\sigma^{i+}\gamma_5(i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$

$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m)i\sigma^{i+}\gamma_5\mathcal{U} = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m)i\sigma^{i+}\gamma_5\mathcal{U} = 0$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2 b_T e^{-ib\cdot\Delta} \int dz^- d^2 z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5 (i\overrightarrow{\not{D}} - m) \right] \psi | p, \Lambda \rangle = 0$$

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + 2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T (\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

$$x\left(E'_{2T} + 2\tilde{H}'_{2T}\right) = -H + \frac{m}{M}(E_T + 2\tilde{H}_T) - 2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} G_{11} - \mathcal{M}_{G_{11}}$$

$$\mathcal{M}_{G_{11}} = \frac{2i\epsilon^{im}\Delta^m}{\Delta_T^2}(\mathcal{M}^{i,A}_{++} + \mathcal{M}^{i,A}_{--})$$

EoM relations for Transversely Polarized Proton

$$0 = \frac{\Delta_T^2}{4M^2}E + \frac{1}{2}G_{12}^{(1)} - \frac{\Delta_T^2}{4M^2}G_{11}^{(1)} - x\left(H'_{2T} + \frac{\Delta_T^2}{2M^2}\widetilde{H}'_{2T}\right) + \frac{m}{M}\left(H_T + \frac{\Delta_T^2}{2M^2}\widetilde{H}_T\right)$$
$$-\frac{i\epsilon^{ij}\Delta^j}{2M\Delta_T^2}\int d^2k_T \left((\Delta^1 + i\Delta^2)\mathcal{M}^{i,A}_{+-} + (-\Delta^1 + i\Delta^2)\mathcal{M}^{i,A}_{-+}\right)$$

Axial vector

$$- \quad x\left(F_{23} + \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{24}\right) + \frac{1}{2M^2}\left(\Delta_T^2 G_{13} + k_T\cdot\Delta_T G_{12}\right)$$
$$+ \quad \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{12} + \frac{\Delta^i}{2M\Delta_T^2}\left((\Delta^1 - i\Delta^2)\mathcal{M}^{i,S}_{-+} + (\Delta^1 + i\Delta^2)\mathcal{M}^{i,S}_{+-}\right) = 0.$$

Vector

EoM relations for Transversely Polarized Proton

Twist 3

$$0 = \frac{\Delta_T^2}{4M^2}E + \frac{1}{2}G_{12}^{(1)} - \frac{\Delta_T^2}{4M^2}G_{11}^{(1)} - x\left(H'_{2T} + \frac{\Delta_T^2}{2M^2}\widetilde{H}'_{2T}\right) + \frac{m}{M}\left(H_T + \frac{\Delta_T^2}{2M^2}\widetilde{H}_T\right)$$

$$-\frac{i\epsilon^{ij}\Delta^j}{2M\Delta_T^2}\int d^2k_T\left((\Delta^1 + i\Delta^2)\mathcal{M}_{+-}^{i,A} + (-\Delta^1 + i\Delta^2)\mathcal{M}_{-+}^{i,A}\right)$$

d_2 (in the forward limit)

Axial vector

H_{2T}

Twist 3

$$- \quad x\left(F_{23} + \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{24}\right) + \frac{1}{2M^2}\left(\Delta_T^2 G_{13} + k_T\cdot\Delta_T G_{12}\right)$$

$$+ \quad \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{12} + \frac{\Delta^i}{2M\Delta_T^2}\left((\Delta^1 - i\Delta^2)\mathcal{M}_{-+}^{i,S} + (\Delta^1 + i\Delta^2)\mathcal{M}_{+-}^{i,S}\right) = 0.$$

Vector

EoM relations for Transversely Polarized Proton

Twist 3

$$0 = \frac{\Delta_T^2}{4M^2}E + \frac{1}{2}G_{12}^{(1)} - \frac{\Delta_T^2}{4M^2}G_{11}^{(1)} - x\left(H'_{2T} + \frac{\Delta_T^2}{2M^2}\widetilde{H}'_{2T}\right) + \frac{m}{M}\left(H_T + \frac{\Delta_T^2}{2M^2}\widetilde{H}_T\right)$$
$$-\frac{i\epsilon^{ij}\Delta^j}{2M\Delta_T^2}\int d^2k_T\left((\Delta^1 + i\Delta^2)\mathcal{M}_{+-}^{i,A} + (-\Delta^1 + i\Delta^2)\mathcal{M}_{-+}^{i,A}\right)$$

d_2 (in the forward limit)

Axial vector

H_{2T}

Twist 3

$$- \quad x\left(F_{23} + \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{24}\right) + \frac{1}{2M^2}\left(\Delta_T^2 G_{13} + k_T\cdot\Delta_T G_{12}\right)$$
$$+ \quad \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{12} + \frac{\Delta^i}{2M\Delta_T^2}\left((\Delta^1 - i\Delta^2)\mathcal{M}_{-+}^{i,S} + (\Delta^1 + i\Delta^2)\mathcal{M}_{+-}^{i,S}\right) = 0.$$

$$f_{1T}^{\perp(1)} = -F_{12}^{o(1)} = \mathcal{M}_{F_{12}}|_{\Delta_T=0}$$

Vector

Wandzura Wilczek Relations

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

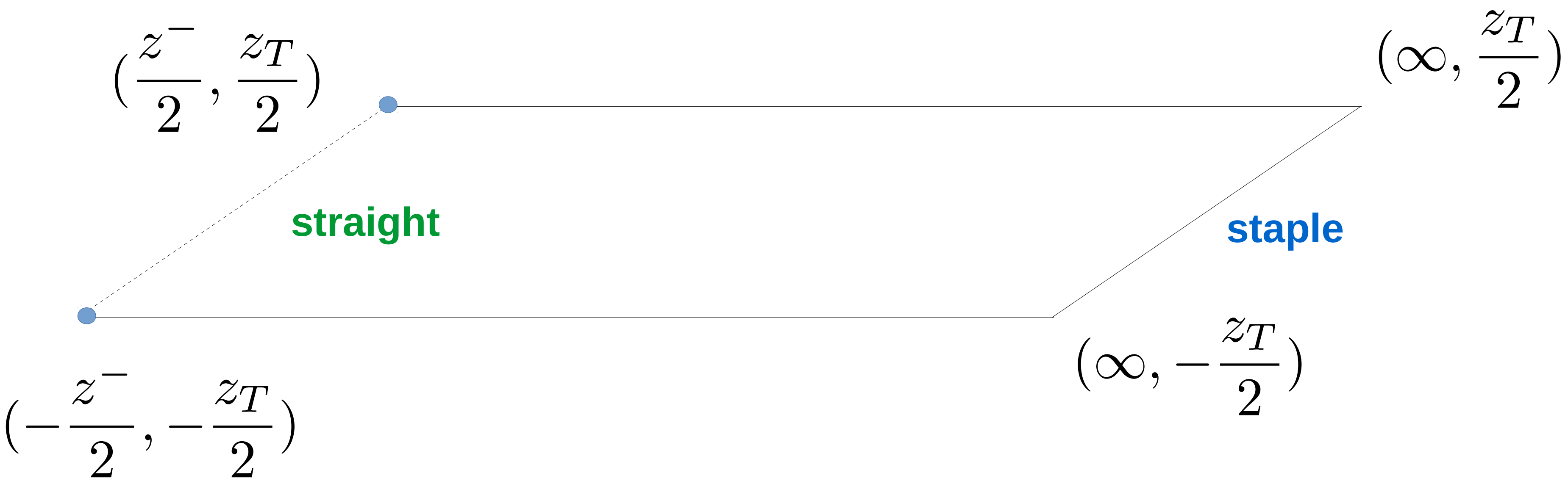
Twist three
vector GPD

Twist two

Axial vector GPD
contributes to a vector
GPD

Genuine Tw 3

Quark gluon quark contributions



$$(\overrightarrow{\partial} - igA)\mathcal{U}\Big|_{-z/2} =$$

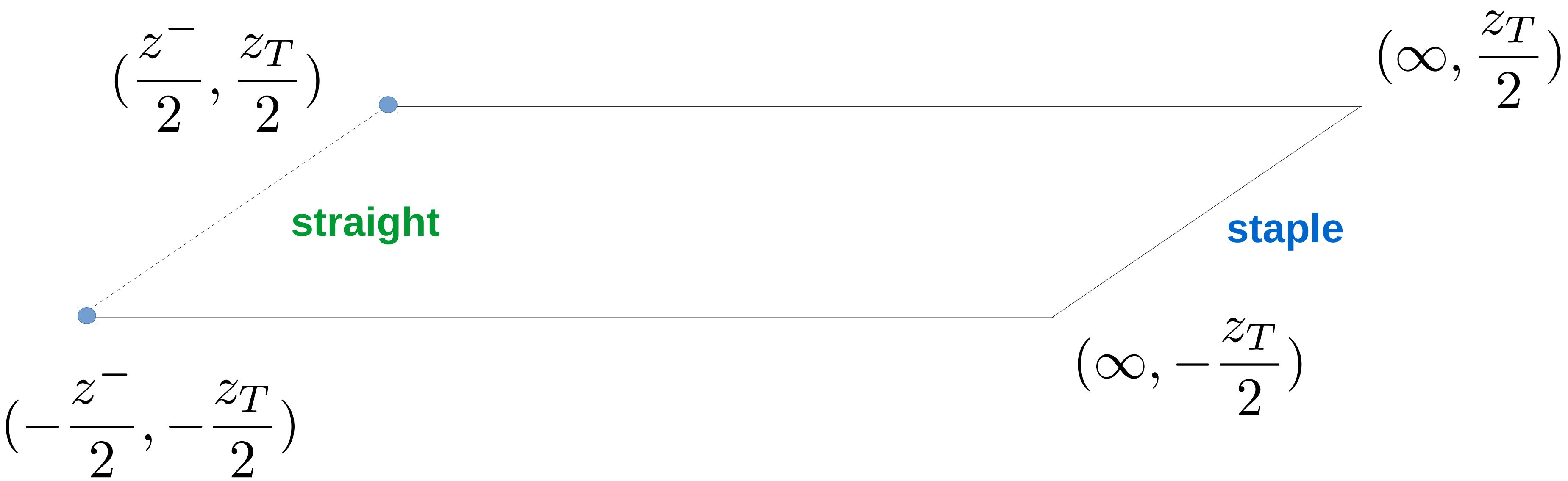
$$igz^- \int_0^1 ds\,(1-s)$$

$$\cdot\, U(-z/2, -z/2 + v + sz)\gamma_\mu F^{+\mu}(-z/2 + v + sz)U(-z/2 + v + sz, z/2)$$

$$+igv^- \int_0^1 ds\, U(-z/2, -z/2 + sv)\gamma_\mu F^{+\mu}(-z/2 + sv)U(-z/2 + sv, z/2)$$

staple arm

Quark gluon quark contributions



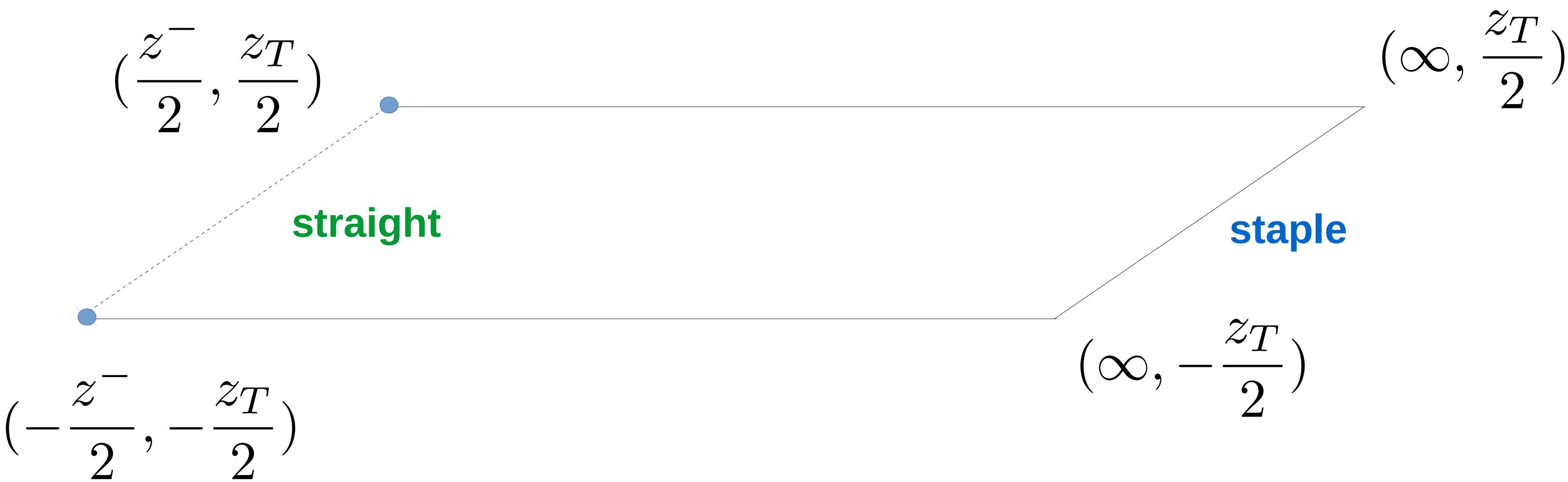
$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} \quad = $$

$$i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0) \psi(0) | p, \Lambda \rangle$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} \quad = $$

$$-g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ \gamma^5 U(0, sv) F^{+i}(sv) U(sv, 0) \psi(0) | p, \Lambda \rangle$$

Quark gluon quark contributions

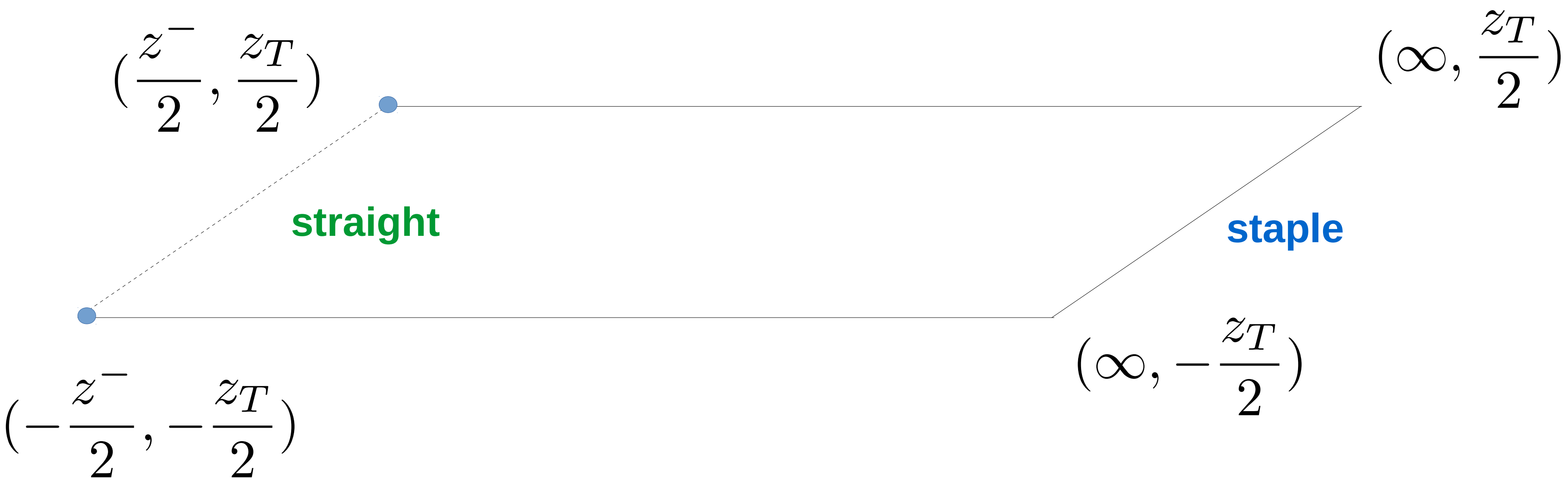


Zero for straight gauge link

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \\ i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0)\psi(0)|p, \Lambda\rangle$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \\ -g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 U(0, sv) F^{+i}(sv) U(sv, 0)\psi(0)|p, \Lambda\rangle$$

Quark gluon quark contributions



$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{ig}{4(P^+)^2} \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 F^{+i}(0)\psi(0)|p,\Lambda\rangle$$

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2} \epsilon^{ij} \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0)|p,\Lambda\rangle$$

Quark gluon structure of the moments of twist three GPDs

$$\int dx \widetilde{E}_{2T} = -\int dx (H+E) \qquad \Rightarrow \int dx \left(\widetilde{E}_{2T} + H + E\right) = 0$$

$$\int dx x \widetilde{E}_{2T} = -\frac{1}{2}\int dx x (H+E) - \frac{1}{2}\int dx \widetilde{H}$$

$$\int dx x^2 \widetilde{E}_{2T} = -\frac{1}{3}\int dx x^2 (H+E) - \frac{2}{3}\int dx x \widetilde{H} - \frac{2}{3}\int dx x \, \mathcal{M}_{F_{14}}\Big|_{v=0}$$

$$\int dx\, x \int d^2k_T \, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{ig}{4(P^+)^2} \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ \gamma^5 F^{+i}(0) \psi(0) | p, \Lambda \rangle$$

Quark gluon structure of the moments of twist three GPDs

$$\int dx \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\int dx \widetilde{H} \qquad \Rightarrow \int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} + \widetilde{H}\right) = 0$$

$$\int dx x \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{2}\int dx x \widetilde{H} - \frac{1}{2}\int dx H + \frac{m}{2M}\int dx (E_T + 2\widetilde{H}_T)$$

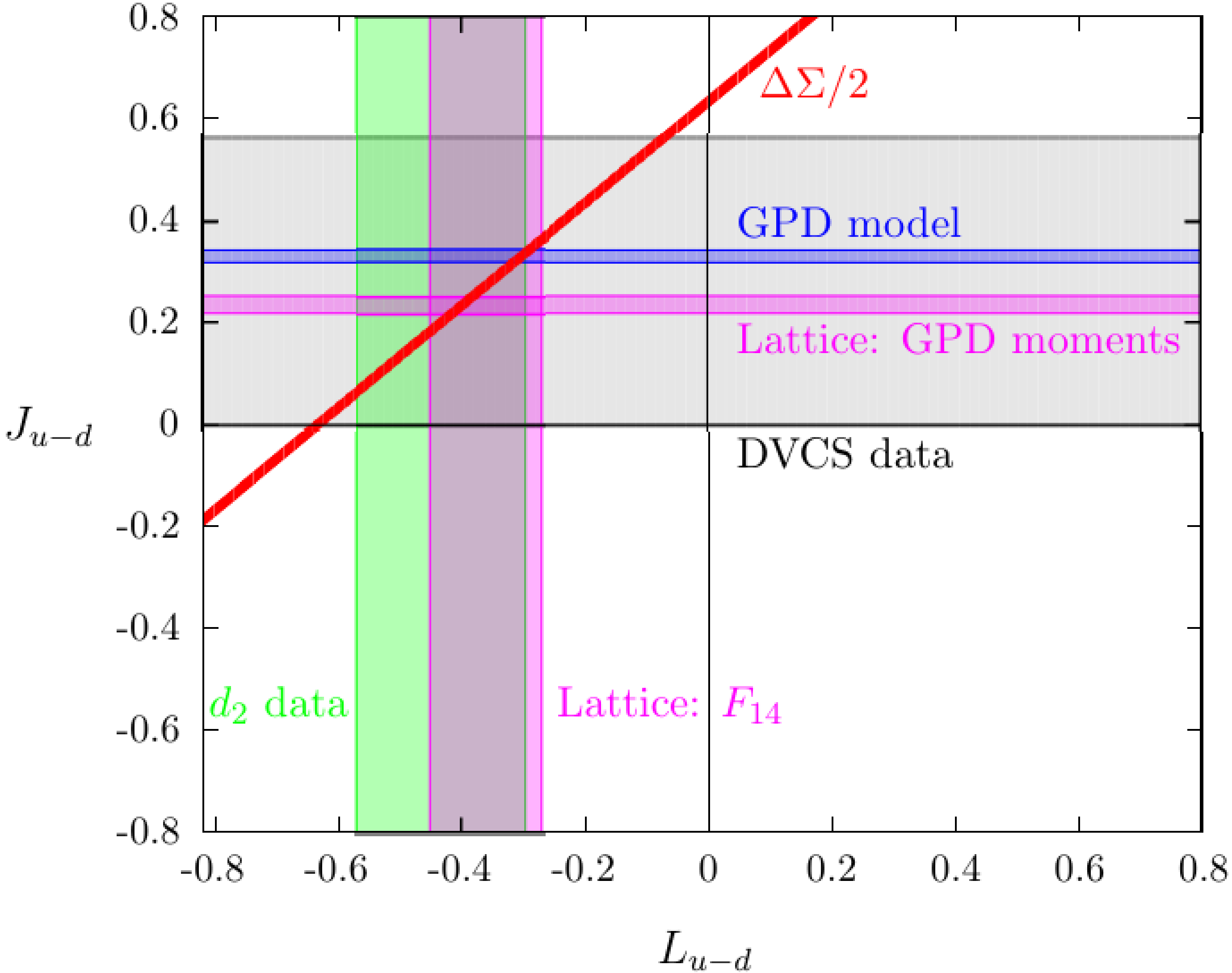
$$\int dx\, x^2 \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{3}\int dx x^2 \widetilde{H} - \frac{2}{3}\int dx x H + \frac{2m}{3M}\int dx x (E_T + 2\widetilde{H}_T) - \frac{2}{3}\int dx x \mathcal{M}_{G_{11}}\Big|_{v=0}$$

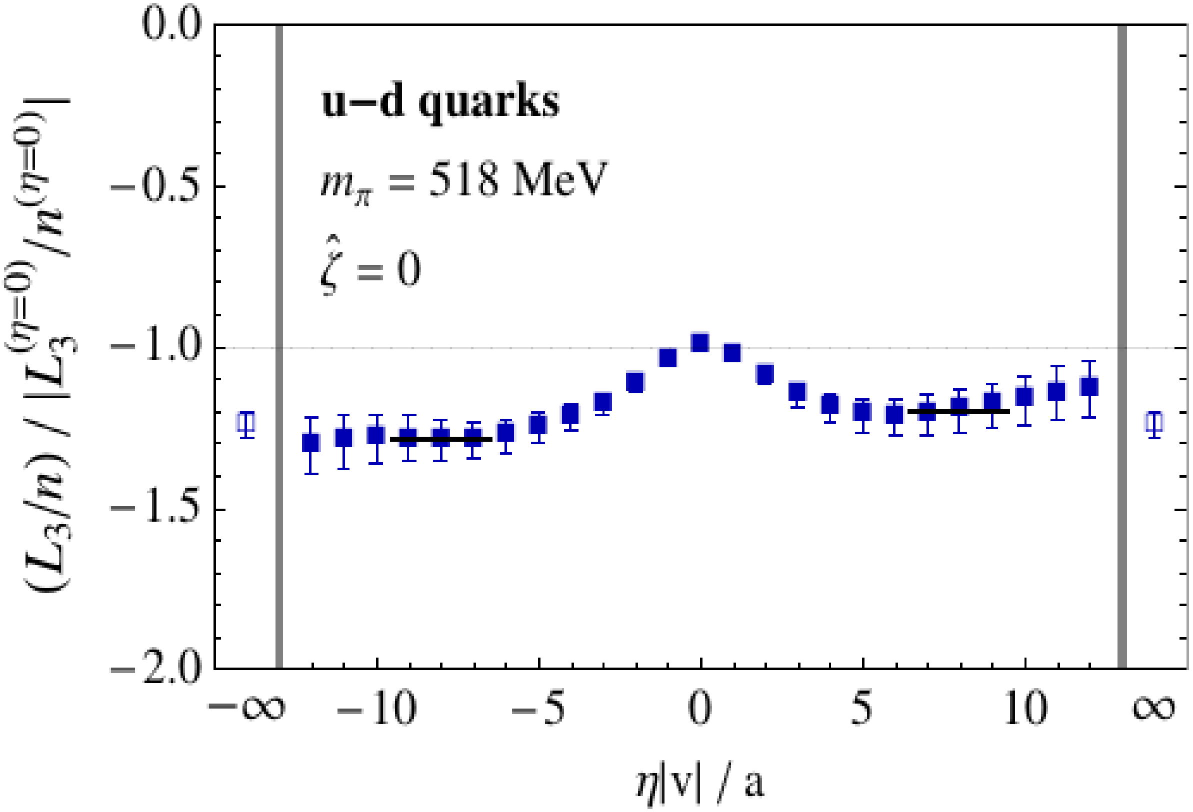
$$\int dx\, x \int d^2 k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2}\epsilon^{ij}\langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0)| p, \Lambda\rangle$$

Quark gluon structure of the moments of twist three GPDs

$$\begin{aligned}
\int dx \left(H'_{2T} - \frac{\Delta_T^2}{4M^2} E'_{2T} \right) &= \left(1 + \frac{\Delta_T^2}{4M^2} \right) \int dx \widetilde{H} \quad \overset{\Delta_T \to 0}{\Rightarrow} \int dx \left(H'_{2T} - \widetilde{H} \right) \\
&\equiv \int dx \, g_2 = 0 \\
\int dx \, x \left(H'_{2T} - \frac{\Delta_T^2}{4M^2} E'_{2T} \right) &= \frac{1}{2} \left(1 + \frac{\Delta_T^2}{4M^2} \right) \int dx x \widetilde{H} + \frac{\Delta_T^2}{8M^2} \int dx (H + E) \\
&+ \frac{m}{2M} \int dx \left(H_T - \frac{\Delta_T^2}{4M^2} E_T \right) \\
\int dx \, x^2 \left(H'_{2T} - \frac{\Delta_T^2}{4M^2} E'_{2T} \right) &= \frac{1}{3} \left(1 + \frac{\Delta_T^2}{4M^2} \right) \int dx x^2 \widetilde{H} + \frac{\Delta_T^2}{6M^2} \int dx x (H + E) \\
&+ \frac{2m}{3M} \int dx x \left(H_T - \frac{\Delta_T^2}{4M^2} E_T \right) + \frac{2}{3} \int dx \, x \, \mathcal{M}_{G_{12}} \Big|_{v=0} \quad .
\end{aligned}$$

Where do we stand experimentally?





Michael Engelhardt

Phys. Rev. D95 (2017)

Conclusions

- Our work probes the role played by the gauge link structure in the description of higher twist objects
- We find relations between GPDs and GTMDs and the respective genuine twist three contribution under different limits: kT unintegrated, x unintegrated and both integrated
- These relations are valid for both the forward and off forward case.
- This also provides a way to measure effects that were solely associated with GTMDs by measuring the associated GPD.
- Quark gluon quark interactions are at the heart of twist three effects.
- Alternate way of deriving the genuine twist twist three contributions and 'wandzura wilczek' terms. Allows us to write out precisely quark gluon contribution.

Thank you!

Equations of Motion Relations

Starting with the equation of motion and its conjugate we arrive at the following

$$-\frac{\Delta^+}{2} W_{\Lambda\Lambda'}^{[\gamma^i \gamma^5]} + i k^+ \epsilon^{ij} W_{\Lambda\Lambda'}^{[\gamma^j]} = -\frac{\Delta^i}{2} W_{\Lambda\Lambda'}^{[\gamma^+ \gamma^5]} + i \epsilon^{ij} k_T^j W_{\Lambda\Lambda'}^{[\gamma^+]} - \mathcal{M}_{\Lambda\Lambda'}^{i,S}$$

$$-k^+ W_{\Lambda\Lambda'}^{[\gamma^i \gamma^5]} + \frac{i\Delta^+}{2} \epsilon^{ij} W_{\Lambda\Lambda'}^{[\gamma^j]} + k^i W_{\Lambda\Lambda'}^{[\gamma^+ \gamma^5]} = i \epsilon^{ij} \frac{\Delta^j}{2} W_{\Lambda\Lambda'}^{[\gamma^+]} - m W_{\Lambda\Lambda'}^{[i\sigma^{i+} \gamma^5]} - i \mathcal{M}_{\Lambda\Lambda'}^{i,A}$$

- Each W is a correlator that can be parameterized using GTMDs / GPDs.

$$W_{\Lambda\Lambda'}^{\Gamma} = \int \frac{dz^- \, d^2\mathbf{z}_T}{(2\pi)^3} e^{ixP^+ z^- - i\bar{\mathbf{k}}_T \cdot \mathbf{z}_T} \langle p', \Lambda' \mid \bar{\psi}\left(-\frac{z}{2}\right) \mathcal{U} \Gamma \psi\left(\frac{z}{2}\right) \mid p, \Lambda \rangle \Big|_{z^+=0}$$

LIR violating term

$$\mathcal{A}_{F_{14}}(x) \equiv v^- \frac{(2P^+)^2}{M^2} \int d^2k_T \int dk^- \Bigg[\frac{k_T \cdot \Delta_T}{\Delta_T^2} (A_{11} + xA_{12}) + A_{14}$$

$$+ \frac{k_T^2 \Delta_T^2 - (k_T \cdot \Delta_T)^2}{\Delta_T^2} \left(\frac{\partial A_8}{\partial (k \cdot v)} + x \frac{\partial A_9}{\partial (k \cdot v)} \right) \Bigg]$$

$$= \frac{dF_{14}^{(1)}}{dx} - \left. \frac{dF_{14}^{(1)}}{dx} \right|_{v=0} \quad (1)$$

$$F_{14}^{(1)} - \left. F_{14}^{(1)} \right|_{v=0} = \mathcal{M}_{F_{14}} - \mathcal{M}_{F_{14}}|_{v=0} \, (1)$$

$$\mathcal{A}_{F_{14}}(x) = \frac{d}{dx} \left(\mathcal{M}_{F_{14}} - \mathcal{M}_{F_{14}}|_{v=0} \right) (1)$$

$$- \int dx \left(F_{14}^{(1)} - \left. F_{14}^{(1)} \right|_{v=0} \right) \Bigg|_{\Delta_T = 0} = \quad (1)$$

$$- \frac{\partial}{\partial \Delta^i} i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds \, \langle p', + | \bar{\psi}(0) \gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0) \psi(0) | p, + \rangle \Big|_{\Delta_T = 0},$$

Wandzura Wilczek Relations

$$2\tilde{H}'_{2T} + E'_{2T} = -\int_x^1 \frac{dy}{y}\tilde{H} + \left[\frac{H}{x} - \int_x^1 \frac{dy}{y^2} H\right]$$

$$+ \quad \frac{m}{M}\left[\frac{1}{x}(2\tilde{H}_T + E_T) - \int_x^1 \frac{dy}{y^2}(2\tilde{H}_T + E_T)\right] + \frac{\mathcal{M}_{G_{11}}}{x} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{G_{11}}$$

Wandzura Wilczek Relations

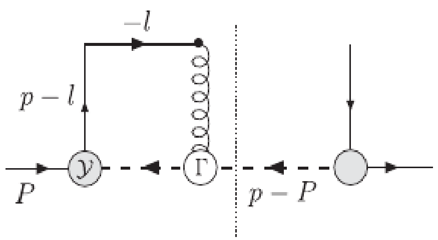
$$
\begin{aligned}
H'_{2T} - \frac{\Delta_T^2}{4M^2} E'_{2T} \quad &= \quad \left(1 + \frac{\Delta_T^2}{4M^2}\right) \int_x^1 \frac{dy}{y} \widetilde{H} + \frac{m}{M} \left[\frac{1}{x} \left(H_T - \frac{\Delta_T^2}{4M^2} E_T \right) \right. \\
&\left. - \int_x^1 \frac{dy}{y^2} \left(H_T - \frac{\Delta_T^2}{4M^2} E_T \right) \right] + \frac{\Delta_T^2}{4M^2} \left[\frac{1}{x} (H + E) - \int_x^1 \frac{dy}{y^2} (H + E) \right] \\
&+ \left[\frac{\mathcal{M}_{G_{12}}}{x} - \int_x^1 \frac{dy}{y^2} \mathcal{M}_{G_{12}} \right] - \int_x^1 \frac{dy}{y} \mathcal{A}_{G_{12}}. \qquad (1)
\end{aligned}
$$

Including Final State Interactions

Brandon Kriesten

- Ji → Straight Gauge link

- Jaffe Manohar → Staple Link



- The difference is the torque

$$\mathcal{L}_q^{JM} - L_q^{Ji} = \int \frac{d^2 z_T dz^-}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}(z) \gamma^+ (-g) \int_{z^-}^{\infty} dy^- U \left[z_1 G^{+1}(y^-) - z_2 G^{+2}(y^-) \right] U \psi(z) | P, \Lambda \rangle \Big|_{z^+=0}$$

Burkardt (2013)