Deuteron GPDs from a covariant contact model

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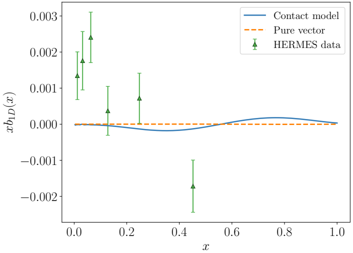
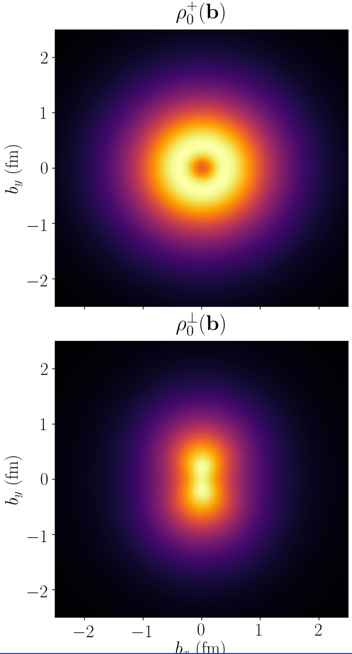
Why the deuteron?

- The deuteron is a spin-1 system: has **more structure** than the proton or neutron.
- The deuteron has an electric quadrupole moment—and a huge one, 0.286 fm^2.
- The deuteron also has a tensor polarization, which is sensitive to exotic components like hidden color.
- JLab experiment **E12-13-011** will measure tensor-polarized DIS of the deuteron.



Left: Tensor-polarized deuteron DIS

Right: Light cone transverse charge density of the deuteron. (top) longitudinally polarized (bottom) transversely polarized.



Deuteron polarization relevant to EIC design!

What are GPDs?

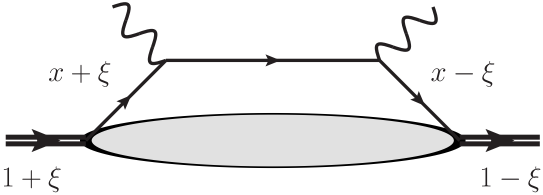
Generalized parton distributions (GPDs) are defined using the same operators (light cone correlators) as PDFs.

- A familiar example: vector quark correlator for the nucleon.

$$\frac{1}{2}\int \frac{dz}{2\pi} e^{-iP\cdot nzx} \langle p'|\bar{q}\left(\frac{nz}{2}\right) \not{n} q \left(-\frac{nz}{2}\right)|p\rangle = \bar{u}(p')\left[\not{n} H_N(x,\xi,t) + \frac{i\sigma^{n\Delta}}{2m_N} E_N(x,\xi,t)\right] u(p)$$

...in the light cone gauge. Other gauges require a Wilson line.

- GPDs are defined using *different* momenta in the initial and final states.
- The limit $p' \to p$ gives us traditional PDFs.



- x is the *average* light cone momentum fraction between initial and final states.
- 2ξ is the light cone momentum fraction *lost* by the target.
- t is the invariant momentum transfer.

Going up in spin

The deuteron (as a spin-1 system) has **more GPDs** than the proton.

- A spin-0 system (π, ^{4}He) has **one** vector GPD.
- A spin-$\frac{1}{2}$ system (p, n, ^{3}H, ^{3}He) has **two** vector GPDs.
- A spin-1 system (**deuteron**, ρ) has **five** vector GPDs.

This increase in the number of GPDs is analogous to the increasing number of form factors, or of DIS structure functions, as spin increases.

$$\langle \not{n} \rangle = -(\varepsilon \cdot \varepsilon'^*) H_1 + \frac{(n \cdot \varepsilon'^*)(\Delta \cdot \varepsilon) - (n \cdot \varepsilon)(\Delta \cdot \varepsilon'^*)}{2P \cdot n} H_2 + \frac{(\varepsilon \cdot \Delta)(\varepsilon'^* \cdot \Delta)}{2M_D^2} H_3$$
$$- \frac{(n \cdot \varepsilon)(\Delta \cdot \varepsilon'^*) + (n \cdot \varepsilon'^*)(\Delta \cdot \varepsilon)}{2P \cdot n} H_4 + \left[\frac{(n \cdot \varepsilon)(n \cdot \varepsilon'^*) M_D^2}{(P \cdot n)^2} + \frac{1}{3}(\varepsilon \cdot \varepsilon'^*) \right] H_5$$

This big equation tells us how the five vector GPDs are defined.

- Helpful mnemonic: H_1-H_3 are defined by same Lorentz structures as EM form factors F_1-F_3.

Polynomiality rules for the nucleon

Nucleon GPDs are known to obey **polynomiality sum rules** [X. Ji, J.Phys. G24 (1998) 1181]:

$$\int_{-1}^{1} x^s H_N(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} A_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)C_N(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s E_N(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} B_{s+1,l}(t)(2\xi)^l - \mathrm{mod}(s,2)C_N(t)(2\xi)^{s+1}$$

- A, B, and C are called **generalized form factors**.
- These rules are a **result of Lorentz covariance**.
- They are **violated for models that break covariance** (*e.g.*, models with Fock space truncations or which use non-relativistic nuclear wave functions).

Spin-1 systems will have polynomiality rules too (due to Lorentz symmetry).

Polynomiality sum rules for the deuteron

I have derived the following sum rules for spin-1 systems (with $x \in [-1,1]$ convention):

$$\int_{-1}^{1} x^s H_1(x,\xi,t)dx = \sum_{\substack{l=0\\ 2|l}}^{s} \mathcal{A}_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)\mathcal{F}_{s+1}(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s H_2(x,\xi,t)dx = \sum_{\substack{l=0\\ 2|l}}^{s} \mathcal{B}_{s+1,l}(t)(2\xi)^l$$

$$\int_{-1}^{1} x^s H_3(x,\xi,t)dx = \sum_{\substack{l=0\\ 2|l}}^{s} \mathcal{C}_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)\mathcal{G}_{s+1}(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s H_4(x,\xi,t)dx = \sum_{\substack{l=1\\ 2\nmid l}}^{s} \mathcal{D}_{s+1,l}(t)(2\xi)^l$$

$$\int_{-1}^{1} x^s H_5(x,\xi,t)dx = \sum_{\substack{l=0\\ 2|l}}^{s-1} \mathcal{E}_{s+1,l+1}(t)(2\xi)^l$$

Only H_1 and H_3 (related to **electric distribution**, not magnetic) have the $(2\xi)^{s+1}$ term. **Two D-terms???**

Special cases of generalized form factors

The first Mellin moments ($s = 0$) give **electromagnetic form factors**:

$$\int_{-1}^{1} H_1(x,\xi,t)dx = F_1(t) \qquad \int_{-1}^{1} H_2(x,\xi,t)dx = F_2(t)$$

$$\int_{-1}^{1} H_3(x,\xi,t)dx = F_3(t) \qquad \int_{-1}^{1} H_4(x,\xi,t)dx = \int_{-1}^{1} H_5(x,\xi,t)dx = 0$$

The second Mellin moments ($s = 1$) give **gravitational form factors**:

$$\int_{-1}^{1} xH_1(x,\xi,t)dx = \mathcal{G}_1(t)+(2\xi)^2\mathcal{G}_3(t) \qquad \int_{-1}^{1} xH_2(x,\xi,t)dx = \mathcal{G}_5(t)$$

$$\int_{-1}^{1} xH_3(x,\xi,t)dx = \mathcal{G}_2(t)+(2\xi)^2\mathcal{G}_4(t)$$

$$\int_{-1}^{1} xH_4(x,\xi,t)dx = (2\xi)\mathcal{G}_6(t) \qquad \int_{-1}^{1} xH_5(x,\xi,t)dx = \mathcal{G}_7(t)$$

Information contained in GFFs

The GFFs contain extra information that electromagnetic FFs don't.

- Can construct a **Newtonian form factor** (monopole gravitational) and define a **gravitational radius**:

$$\mathcal{G}_N(t) = \left(1 + \frac{2}{3}\tau\right)\mathcal{G}_1(t) - \frac{2}{3}\tau\mathcal{G}_5(t) + \frac{2}{3}\tau(1+\tau)\mathcal{G}_2(t)$$

  where $\tau = -t/(4M_D^2)$.

$$\langle r_G^2 \rangle = 6\frac{d}{dt}\left[\mathcal{G}_N(t)\right]$$

- Taneja *et al.* (Phys.Rev. **D86** (2012) 036008) tell us that

$$J(t) = \frac{1}{2}\mathcal{G}_5(t)$$

- To unambiguously extract this information requires GPD calculations to obey polynomiality. **Lorentz covariance in GPD calculations is a necessity.**

Convolution formalism

First step (baseline) in nuclear GPDs: establish an impulse approximation convolution formalism. This is ostensibly straightforward:

- Get a model for the nucleon GPDs H_N and E_N.
- Compute the matrix element

$$\langle p', \lambda' | \left[\not{n} H_N + \frac{i\sigma^{n\Delta}}{2m_N} E_N \right] | p, \lambda \rangle$$

  assuming pointlike, on-shell nucleons.
  (The factors H_N and E_N fold in the non-pointlike structure.)
- An **ambiguity** arises: identities like Gordon decomposition that are true for on-shell nucleons **will lead to different results for kinematically off-shell nucleons**.
- This turns out to matter for the nucleon D-terms.

The D-term and Gordon decomposition

In models such as [Goeke *et al.*, Prog. Part. Nucl. Phys. 47 (2001)], the nucleon GPD is broken into a **double distribution** and a **D-term**:

$$H_N(x,\xi,t) = H_{DD}(x,\xi,t) + D\left(\frac{x}{\xi},t\right) \qquad E_N(x,\xi,t) = E_{DD}(x,\xi,t) - D\left(\frac{x}{\xi},t\right)$$

- The D-term here contributes to the $(2\xi)^{s+1}$ GFF in the polynomiality sum rules.
- The same D-term enters both H_N and E_N with opposite sign.
- **This is due to Lorentz invariance.** [X. Ji, J.Phys. G24 (1998) 1181]

Using **Gordon decomposition**, we can write:

$$\bar{u}(\mathbf{p}',\sigma')\left[\not{n}H_N + \frac{i\sigma^{n\Delta}}{2m_N}E_N\right]u(\mathbf{p},\sigma) = \bar{u}(\mathbf{p}',\sigma')\left[\not{n}H_{DD} + \frac{i\sigma^{n\Delta}}{2m_N}E_{DD} + \frac{p\cdot n}{m_N}D_N\right]u(\mathbf{p},\sigma)$$

for *on-shell* spinors.
We must decide between the LHS and RHS for the "unmodified" deuteron GPD.
(I've chosen the RHS since it gives us polynomiality.)

The master convolution formula

Evaluating the matrix element

$$\langle p', \lambda' | \left[\not{n} H_{DD} + \frac{i\sigma^{n\Delta}}{2m_N} E_{DD} + \frac{p \cdot n}{m_N} D_N \right] | p, \lambda \rangle$$

gives a **master convolution formula**:

$$H_i(x,\xi,t) = \int_{-1}^{1} \frac{dy}{y} \left[h_i(y,\xi,t) H_{DD}\left(\frac{x}{y}, \frac{\xi}{y}, t\right) + e_i(y,\xi,t) E_{DD}\left(\frac{x}{y}, \frac{\xi}{y}, t\right) + y d_i\left(\frac{y}{\xi}, t\right) D_N\left(\frac{x}{\xi}, t\right)\right]$$

- h_i, e_i, and d_i describe how the nucleons are distributed in the nucleus, using GPD language. Call them **generalized nucleon distributions** (GNDs).
- By construction, H_{DD}, E_{DD}, and D_N already obey polynomiality.
- We can prove that when the GNDs obey polynomiality sum rules, so do the deuteron GPDs.
- **The only ingredient needed to ensure the GNDs observe polynomiality is a Lorentz-covariant model of nuclear structure.**
- Taking Mellin moments of the master convolution formula will give **discrete convolution relations** for the GFFs. Don't have time to discuss these.

Motivation for a contact model

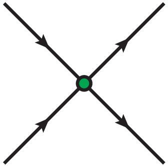
For computing the GPDs themselves, covariance is of the utmost importance.

- Can be difficult to maintain covariance while solving a bound state equation.
- Covariantly solving a four-Fermi contact interaction is tractable.
- Success of the Nambu-Jona-Lasinio (NJL) model suggests this approach has promise.
- The skeptic may ask: what about the deuteron's D-wave?
  What about the deuteron's huge quadrupole moment?
- The magic of relativity will produce these things, even in a contact interaction model.

Lagrangian

Construct most general possible NN Lagrangian that:

- Has four-fermi contact interactions.
- Has no derivatives in interaction terms.
- Obeys $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_A$ isospin symmetry.
- Satisfies Pauli exclusion principle (enforced by ψ being Grassmann-number-valued!).

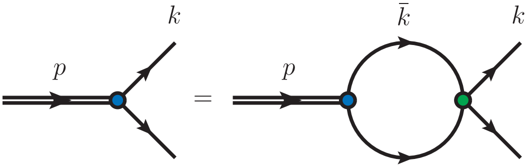


$$\begin{aligned}\mathcal{L}_{NN} &= \bar{\psi}(i\partial\!\!\!/ - m)\psi \\ &- G_S\left[\left(\bar{\psi}\tau_j C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\tau_j\psi\right) - \left(\bar{\psi}\tau_j\gamma^5 C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma^5\tau_j\psi\right)\right] \\ &- G_V\left[\left(\bar{\psi}\tau_j\gamma^5\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma^5\gamma_\mu\tau_j\psi\right) + \left(\bar{\psi}\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_\mu\psi\right)\right] \\ &- \frac{1}{2}G_T\left[\left(\bar{\psi}i\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2 i\sigma_{\mu\nu}\psi\right)\right]\end{aligned}$$

Neglect charge-symmetry violation (assume $m_p = m_n \equiv m_N$).
Interactions decouple into separate isoscalar and isovector sectors.

Bethe-Salpeter vertex

Bethe-Salpeter equation in the covariant contact model:



Solution is the Bethe-Salpeter vertex:

$$\Gamma_D(p,\lambda) = \left[\alpha_V \not{\varepsilon}(p,\lambda) + i\alpha_T \frac{\sigma^{\varepsilon p}}{M_D}\right] C\tau_2$$

We can solve for α_V and α_T in terms of couplings G_V and G_T . . . and a UV regulator Λ (from proper time regularization).

Solution and static observables

Solution has parameters: G_V, G_T, and Λ.
These must be chosen somehow.
Fit to static observables:

- Deuteron binding energy
- Deuteron electromagnetic moments
- ${}^3\mathrm{S}_1$-${}^3\mathrm{D}_1$ scattering parameters.

	Contact model	Empirical
ϵ_D (MeV)	2.18	2.22
r_E (fm)	2.09	2.14
μ_D	0.879	0.857
$\mathcal{Q}_D$ (fm^2)	0.285	0.286
${}^3a_1$ (fm)	5.26	5.42
${}^3r_1$ (fm)	1.78	1.76
Λ (MeV)	139	—
G_V (GeV^{-2})	-683	—
G_T (GeV^{-2})	-715	—

- $\Lambda = 139$ MeV is a result of a fit—is not chosen by us.
- Suggests the model "knows" it breaks down when pion exchange becomes relevant.
- Note we have a non-zero, almost correct quadrupole moment.
- We do actually have a D-wave!

Origin of the D-wave

Whence the D-wave?
Bethe-Salpeter wave function takes the form

$$\psi_D(p,k,\lambda) = S(k)\Gamma_D(p,\lambda)S^T(p-k)$$

The numerator of the top-right 2×2 corner (where both nucleons have **positive energy**):

$$\psi_D^{(++)}(p,k,\lambda) \propto m_N(M_D+m_N)(\alpha_V+\alpha_T)(\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}) + 2(\alpha_V-\alpha_T)(\mathbf{k}\cdot\boldsymbol{\varepsilon})(\mathbf{k}\cdot\boldsymbol{\sigma})$$

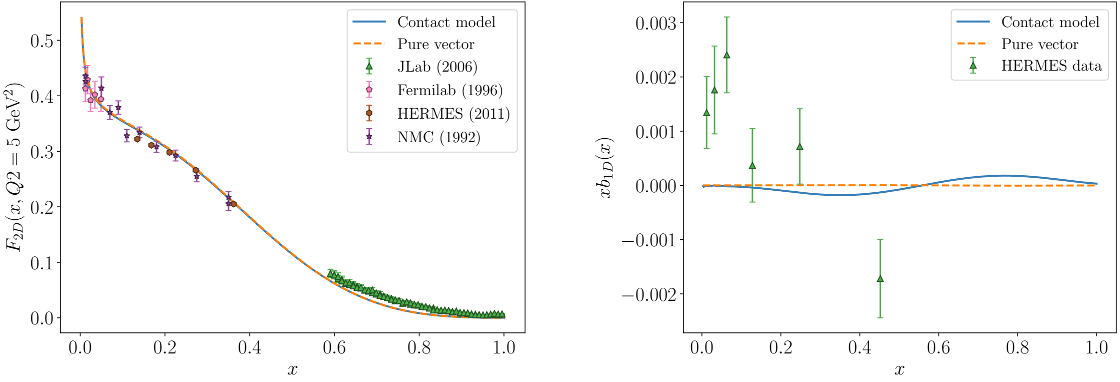
D-wave comes from second part of structure.
Ensures that even non-relativistic reductions, with:

$$\psi_{\mathrm{NR}}(p,k,\lambda) \propto \bar{u}(k,s_1)\Gamma_D(p,\lambda)\bar{u}^T(p-k,s_2)$$

have D-wave—that is, $(\mathbf{k}\cdot\boldsymbol{\varepsilon})(\mathbf{k}\cdot\boldsymbol{\sigma})$ terms—in them.
Answer to **whence**: the **lower components** of u! **This is a relativistic effect.**

DIS structure functions

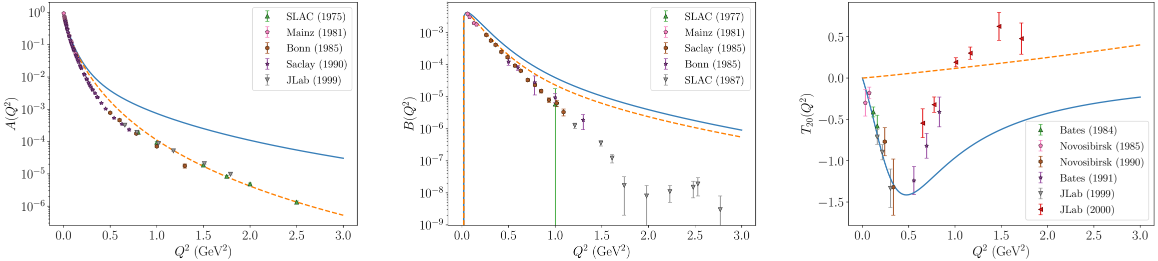
How well can this model describe DIS structure functions?
(Use CJ15 for nucleon PDFs.)



- Not bad for $F_2(x, Q^2)$ (underestimate at high x due to lack of short range correlations).
- Doesn't describe HERMES data for $b_1(x, Q^2)$, but that's expected.

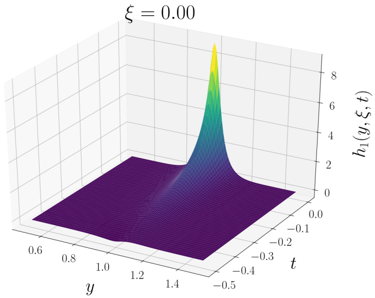
Electromagnetic form factors

What about electromagnetic form factors?
(Use Kelly-Riordan nucleon form factors ... Blue is full model, orange is G_V only.)

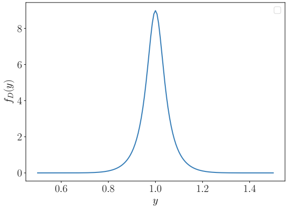


- Absolute size is too big at moderate-to-large Q^2.
- Agreement is OK for $Q^2 \lesssim 0.5$ GeV^2.
- Suggests our GPDs will be applicable to only low $-t$.

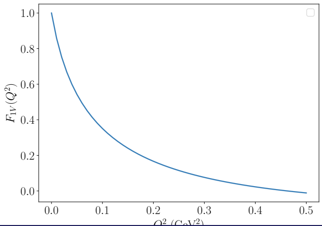
Generalized nucleon distributions



$\xrightarrow{t=0}$



$$\downarrow \sum_q e_q \int_{-1}^{1} dx$$



h_1 is the "typical" GND.

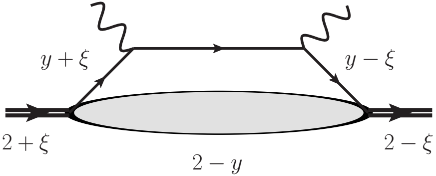
- Reduces to light cone density in forward limit.
- Gives F_{1V} "body" form factor (for pointlike nucleons) when integrated over x.

GNDs are nucleon densities in the deuteron using GPD language.

Skewed GNDs

What does skewness do to a GND?

- It breaks $y \to (2-y)$ symmetry.
- $y = 1 \pm \frac{\xi}{2}$ means both nucleons carry equal fractions in final/initial state.
- Harder to keep deuteron together with momentum transfer.



Click here for GND animation

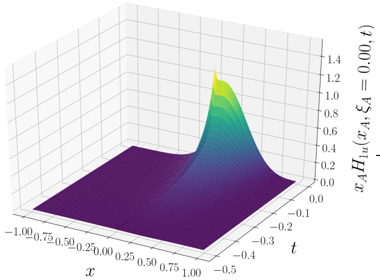
To obtain deuteron GPDs...

- We use the master convolution formula.
- Generalized nucleon distributions (GNDs) are computed in the covariant contact model.
- For the nucleon GPDs, we use the model of [Goeke *et al.*, Prog. Part. Nucl. Phys. 47 (2001)], since it
  - obeys polynomiality
  - has a non-trivial t-dependence
  - contains a D-term
- For Q^2 dependence, we use our own GPD evolution code, with splitting functions from [X. Ji, PRD55 (1997), 7114].

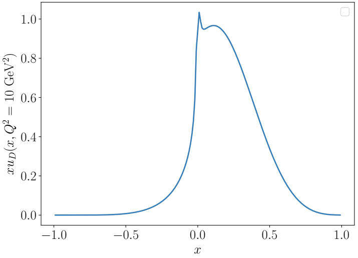
A reminder that there are five (vector/helicity-independent) GPDs:

$$\langle \not{n} \rangle = -(\varepsilon \cdot \varepsilon'^*) H_1 + \frac{(n \cdot \varepsilon'^*)(\Delta \cdot \varepsilon) - (n \cdot \varepsilon)(\Delta \cdot \varepsilon'^*)}{2P \cdot n} H_2 + \frac{(\varepsilon \cdot \Delta)(\varepsilon'^* \cdot \Delta)}{2M_D^2} H_3$$
$$- \frac{(n \cdot \varepsilon)(\Delta \cdot \varepsilon'^*) + (n \cdot \varepsilon'^*)(\Delta \cdot \varepsilon)}{2P \cdot n} H_4 + \left[\frac{(n \cdot \varepsilon)(n \cdot \varepsilon'^*) M_D^2}{(P \cdot n)^2} + \frac{1}{3}(\varepsilon \cdot \varepsilon'^*) \right] H_5$$

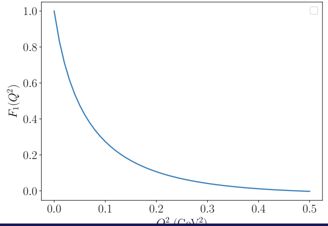
GPD results



$\xrightarrow{t=0}$



$$\downarrow \sum_q e_q \int_{-1}^{1} dx$$



H_1 is the "typical" GPD. (Dominated by monopole.)

- Reduces to unpolarized PDF in the forward limit.
- Gives F_1 form factor (for real nucleons) when integrated over x.

Skewed GPD results

What does skewness do to a GPD?

- There are well-known ridges at $x = \pm\xi$.
- These ridges are where **deeply virtual Compton scattering** (DVCS) occurs!
- $x \pm \xi = 0$ is the zero-fraction limit for the initial (final) quark—limit of the sea.
- $|x| < |\xi|$ means pulling quark and antiquark from target while leaving it in tact—*suppressed*.

Click here for GPD animation

Conclusions and outlook

In conclusion:

- We have calculated deuteron GPDs in a manifestly covariant contact model.
- Our GPDs obey polynomiality sum rules, and allow an unambiguous extraction of generalized form factors.

Future work to be done:

- We will use these GPDs to make predictions for cross sections and asymmetries in DVCS, for both JLab and the EIC.
- The model will be extended to other light nuclei (triton and helium).
- The NJL model can be used to compute covariant *nucleon* GPDs.

Thanks for your time and attention!