Positronium on the Light-front

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With

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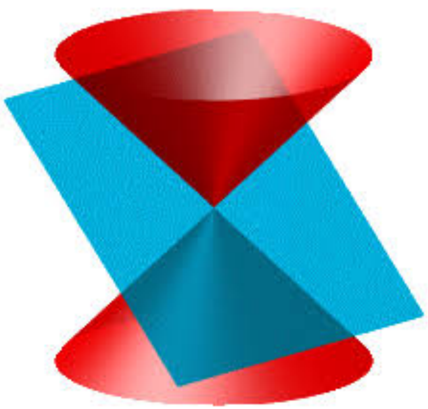

Institute of Modern Physics

Chinese Academy of Sciences

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Outline

- Motivation – Why Positronium?
- Methodology - Basis Light-front Quantization
- Numerical Results
  - Energy spectrum
  - Wave functions
- Summary and Outlook

Motivation

Positronium is a test bed for

- Relativistic bound state structure beyond leading Fock-sector

- Basis Light-front Quantization on first-principles, esp., nonperturbative renormalization procedure

- Connection with effective theory

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^{-}|\beta\rangle = P_{\beta}^{-}|\beta\rangle$$

  - P^{-} : light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P_{β}^{-} : eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

See Chandan Monal's talk
Shaoyang Jia's talk
Meijian Li's talk

Light-front QED Hamiltonian

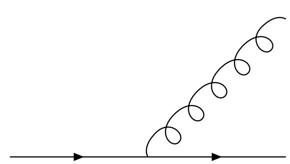
- QED Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m_e)\Psi$
- QED Light-front Hamiltonian

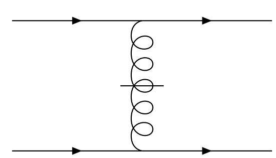
$$\left(A^{+} = 0\right)$$

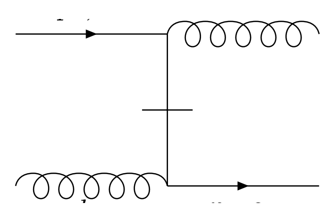
$$P^{-} = \int \mathrm{d}^2x^{\perp}\mathrm{d}x^{-}\ F^{\mu +}\partial_{+}A_{\mu} + i\bar{\Psi}\gamma^{+}\partial_{+}\Psi - \mathcal{L}$$

$$= \int \mathrm{d}^2x^{\perp}\mathrm{d}x^{-}\ \underbrace{\frac{1}{2}\bar{\Psi}\gamma^{+}\frac{m_e^2 + (i\partial^{\perp})^2}{i\partial^{+}}\Psi + \frac{1}{2}A^{j}(i\partial^{\perp})^2 A^{j}}_{\text{kinetic energy terms}}$$

$$+ \underbrace{e j^{\mu}A_{\mu}}_{\text{vertex interaction}} + \underbrace{\frac{e^2}{2}j^{+}\frac{1}{(i\partial^{+})^2}j^{+}}_{\text{instantaneous photon interaction}} + \underbrace{\frac{e^2}{2}\bar{\Psi}\gamma^{\mu}A_{\mu}\frac{\gamma^{+}}{i\partial^{+}}\gamma^{\nu}A_{\nu}\Psi}_{\text{instantaneous fermion interaction}}$$







Light-front QED Hamiltonian

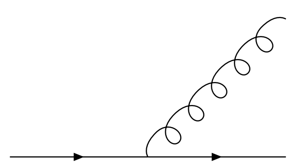
- QED Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$
- QED Light-front Hamiltonian

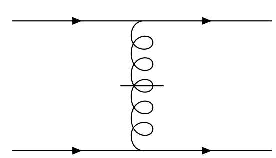
$$\left(A^+ = 0\right)$$

$$P^- = \int \mathrm{d}^2x^\perp \mathrm{d}x^- \;\; F^{\mu +}\partial_+ A_\mu + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L}$$

$$= \int \mathrm{d}^2x^\perp \mathrm{d}x^- \;\; \underbrace{\frac{1}{2}\bar{\Psi}\gamma^+\frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2}A^j(i\partial^\perp)^2 A^j}_{\text{kinetic energy terms}}$$

$$+ \underbrace{e j^\mu A_\mu}_{\text{vertex interaction}} + \underbrace{\frac{e^2}{2} j^+ \frac{1}{(i\partial^+)^2} j^+}_{\text{instantaneous photon interaction}}$$





Basis Construction

1. Fock-space expansion:

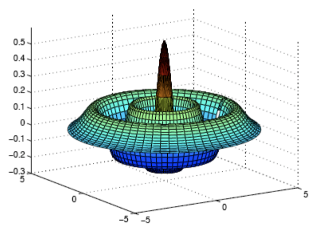
e.g. $|\boldsymbol{e_p}\rangle \quad = a|e\rangle + b|e\gamma\rangle + \mathrm{c}|ee\bar{e}\rangle \; + d|ee\bar{e}\gamma\rangle + \ldots.$

$|\mathbf{Ps}\rangle \quad = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + c|\gamma\rangle + d|e\bar{e}\bar{e}e\bar{e}\rangle \; + \ldots.$

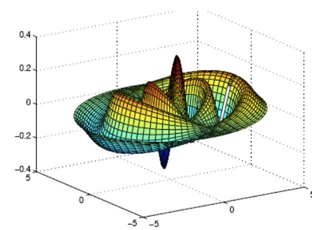
2. For each Fock particle:

- Transverse: 2D harmonic oscillator basis: $\Phi^{b}_{n,m}(\vec{p}_\perp)$
  labeled by radial (angular) quantum number n (m); scale parameter b

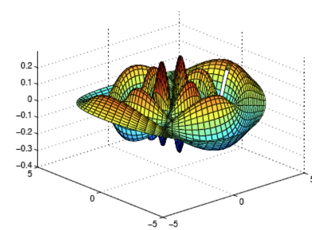
e.g., n=4



m=0



m=1



m=2

- Longitudinal: plane-wave basis, labeled by k
- Helicity: labeled by λ

e.g. $|e\gamma\rangle = |e\rangle \otimes |\gamma\rangle$ with $e = \{n^e, m^e, k^e, \lambda^e\}$ and $\gamma = \{n^\gamma, m^\gamma, k^\gamma, \lambda^\gamma\}$

Basis Truncation Scheme

- Symmetries of Hamiltonian:

  - Net fermion number: $\sum_i n_i^f = N^f$

  - Total angular momentum projection: $\sum_i (m_i + s_i) = J_z$

  - Longitudinal momentum: $\sum_i k_i = K$

- Further truncation:

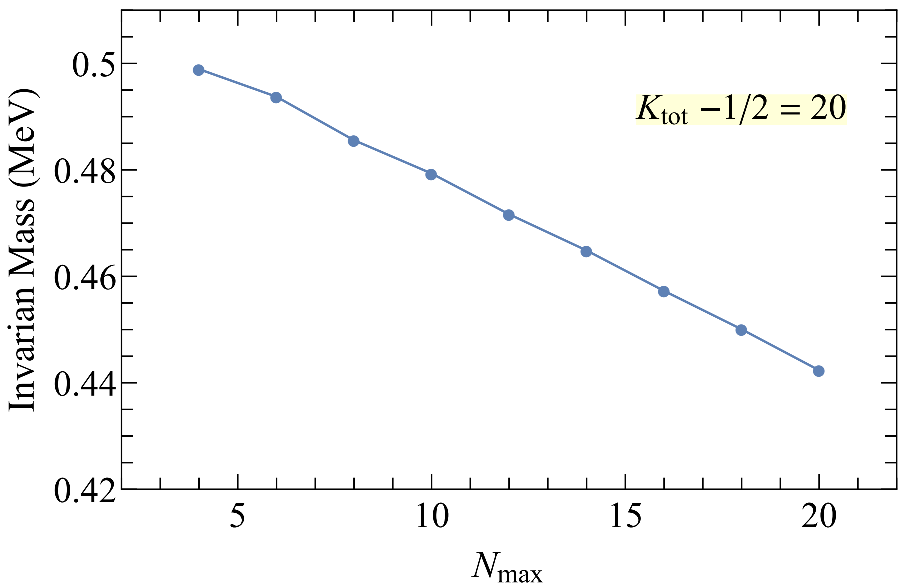
  - Fock sector truncation

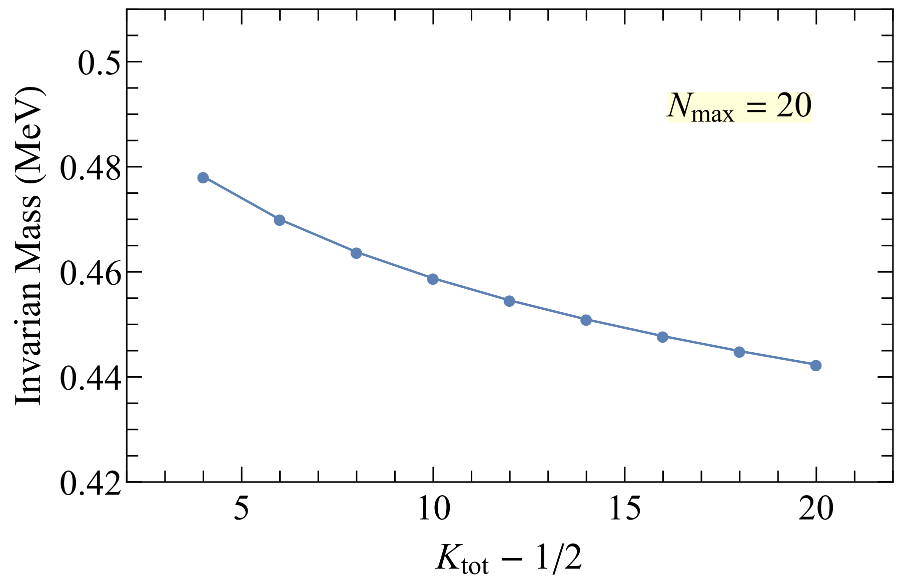
  - Discretization in longitudinal direction $k_i = \begin{cases} 1, 2, 3.... & \text{bosons} \\ 0.5, 1.5, 2.5 \ldots & \text{fermions} \end{cases}$

  - "N_{max}" truncation in transverse directions $\sum_i [2n_i + |m_i| + 1] \leq N_{\max}$

  UV cutoff $\Lambda \sim b\sqrt{N_{\max}}$; IR cutoff $\lambda \sim b/\sqrt{N_{\max}}$

Naïve Diagonalization

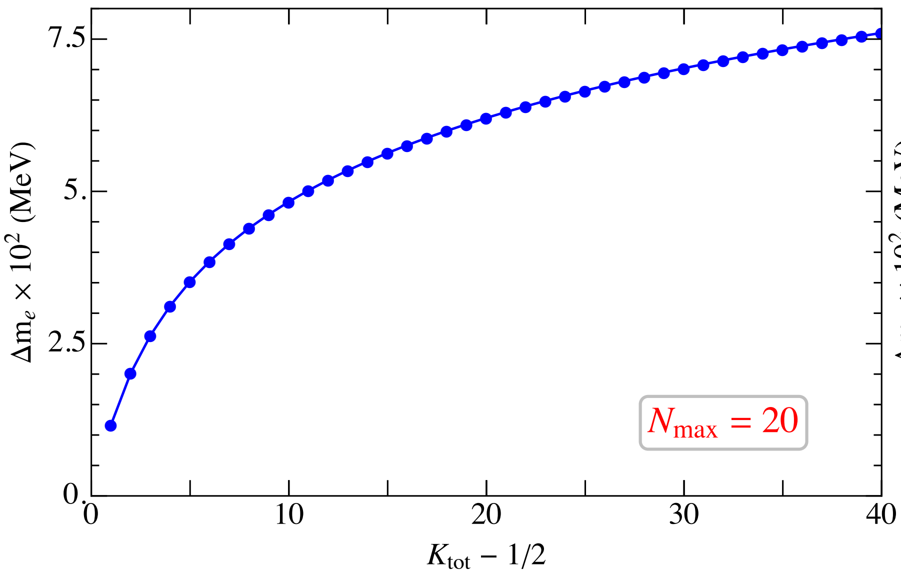


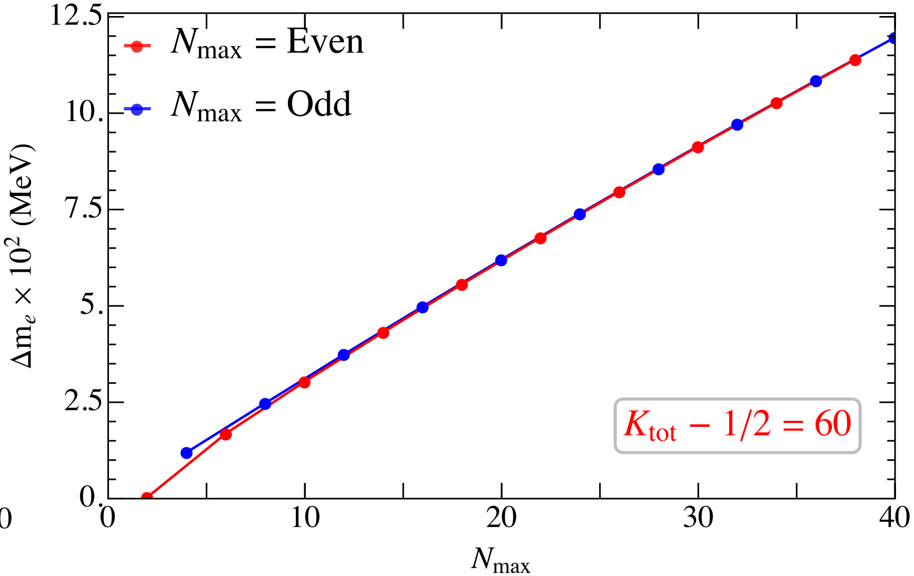


- Ground state (physical electron) mass drops as $\Lambda \to \infty$ and $\lambda \to 0$
- Mass counterterm is introduced $m_e \to m_e + \Delta m_e$ to match the ground state mass to that of the physical electron

[Shuo Tang et al, in preparation]

Mass Counterterm



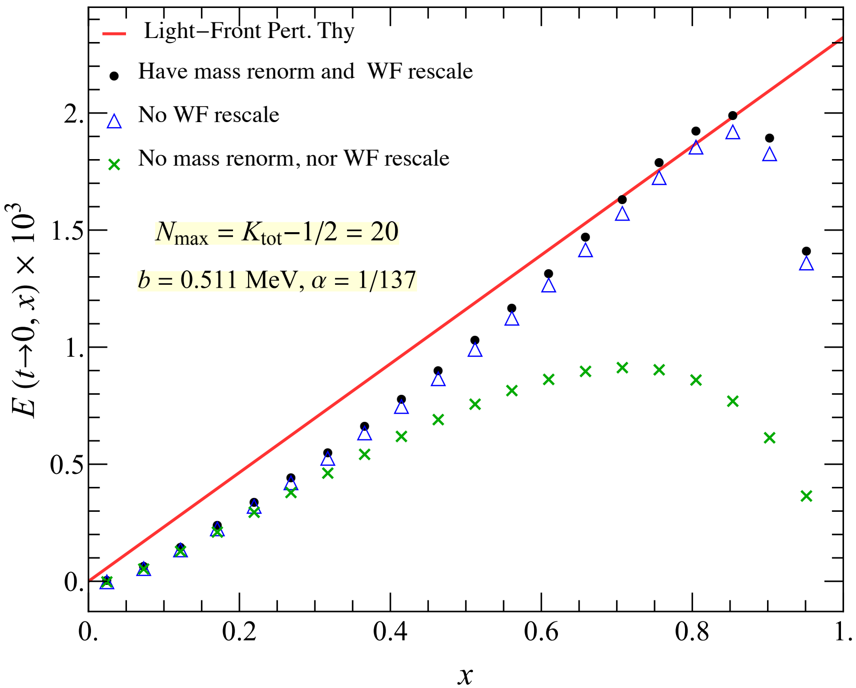


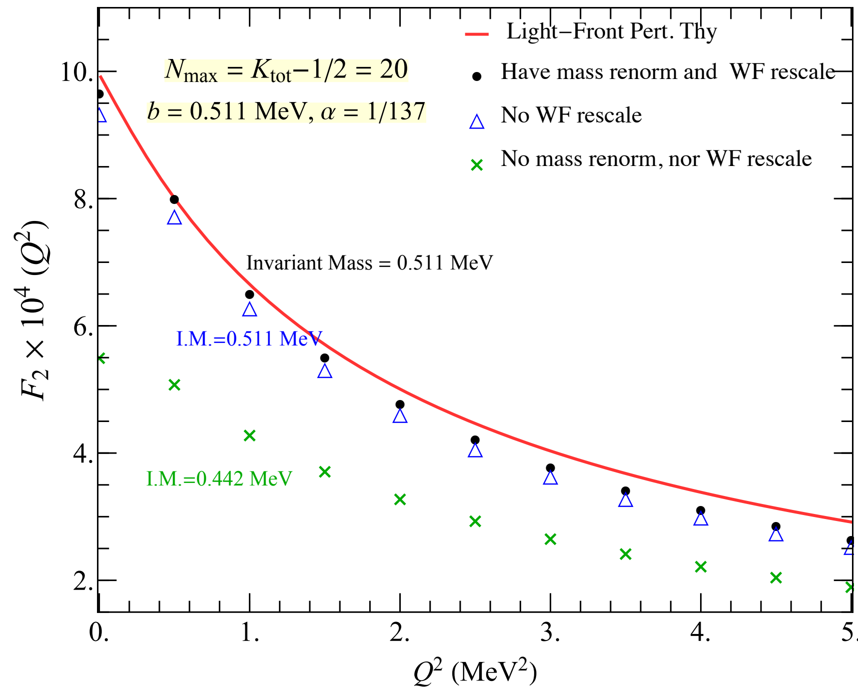
[Karmanov et al, 2008, 2012]

- Sector dependent renormalization ⟹ Δm_e is only applied to $|e\rangle$ sector
- Δm_e are applied iteratively so that the physical electron mass at 0.511 MeV
- Δm_e increase as the truncation parameters (regulators) increase
- Δm_e are divergent

[Shuo Tang et al, in preparation]

Electron GPDs and Form Factors

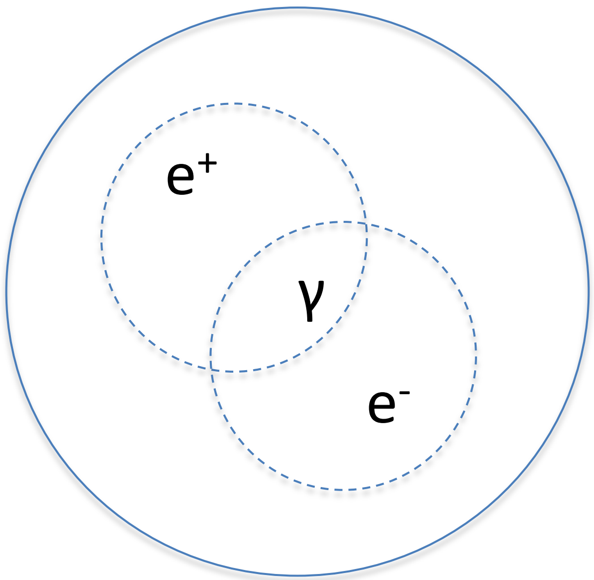




- $$\frac{q_1 - iq_2}{2M_e} E(x,0,\vec{q}) = \left\langle e^{\downarrow}{}_{phys}(\vec{q}) \right| \int dy^- e^{ixP^+y^-/2} \overline{\psi}(0)\gamma^+\psi(y) \left| e^{\uparrow}{}_{phys}(0) \right\rangle \Big|_{y^+=0,\, y_\perp=0}$$
- With mass renormalization, agreement with perturbation theory is reached

[Shuo Tang et al, in preparation]

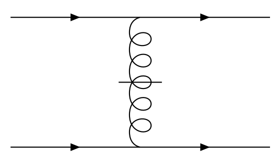
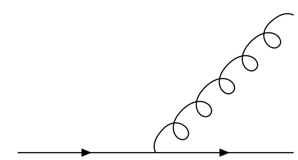
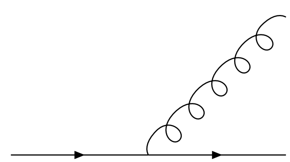
Application to Positronium



Structure of Hamiltonian

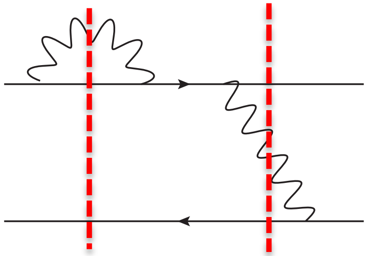
$|\mathbf{Ps}\rangle \quad = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + \mathrm{c}|\gamma\rangle + \mathrm{d}|e\bar{e}e\bar{e}\rangle \;+ . . . .$

m_e=1.0MeV $\alpha = 0.3$

H_{int}	$\|e\bar{e}\rangle$	$\|e\bar{e}\gamma\rangle$
$\langle e\bar{e}\|$		
$\langle e\bar{e}\gamma\|$		0

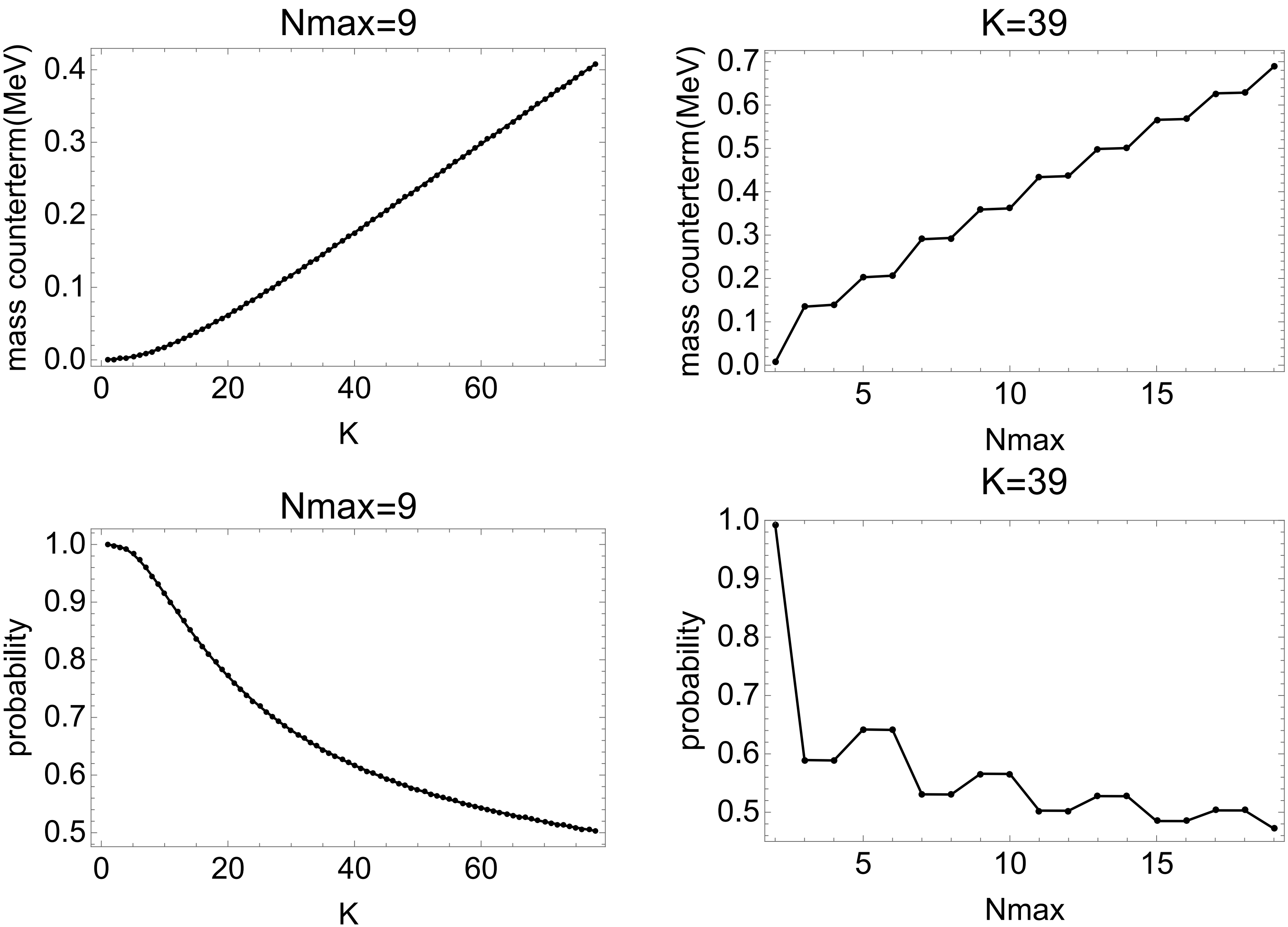
Challenge I

- Self-energy interaction



  - Photon generates both self-energy correction and binding
  - Mass renormalization needed
  - Each basis state has distinct phase space for self-energy interaction due to truncation
  - Need to solve a series of single electron problems to obtain $\Delta m_e(N_{\max}, K)$ for each basis state

Mass Counterterms and Z factors

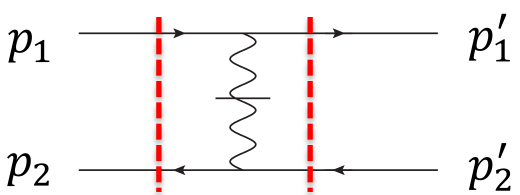


All of them are needed in the positronium problem with N_{max}=20, K=69

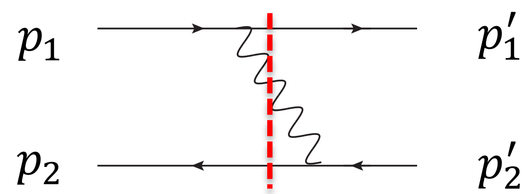
Challenge II

- Mismatch between explicit and instantaneous photon interactions:

for instantaneous photon:
$p_{\text{rel}} = p_1 - p_2$ not limited



for explicit photon:
$p_{\text{rel}} = p_1 - p_2$ subject to $\mathrm{N}_{\max}$ truncation

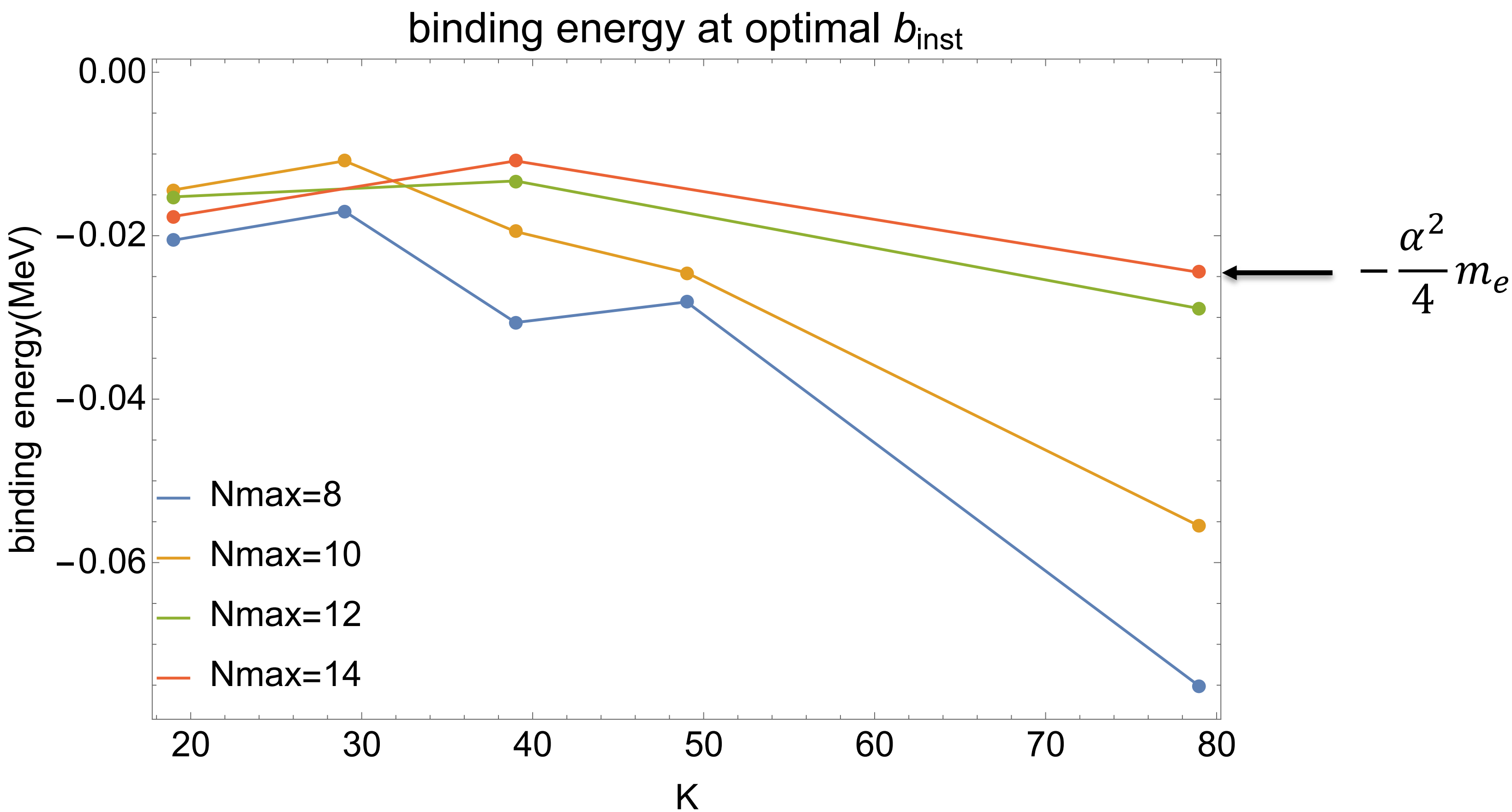


- Introduce cutoff parameter b_{inst} for instantaneous photon interaction:

$$V_{inst} \equiv \int \mathrm{d}^2 x^{\perp} \mathrm{d}x^{-} \, j^{+} \frac{1}{(i\partial^{+})^2} j^{+} \Longrightarrow V_{inst} \times \exp\left(-\frac{p_{\perp}^2}{b_{\text{inst}}^2}\right)$$

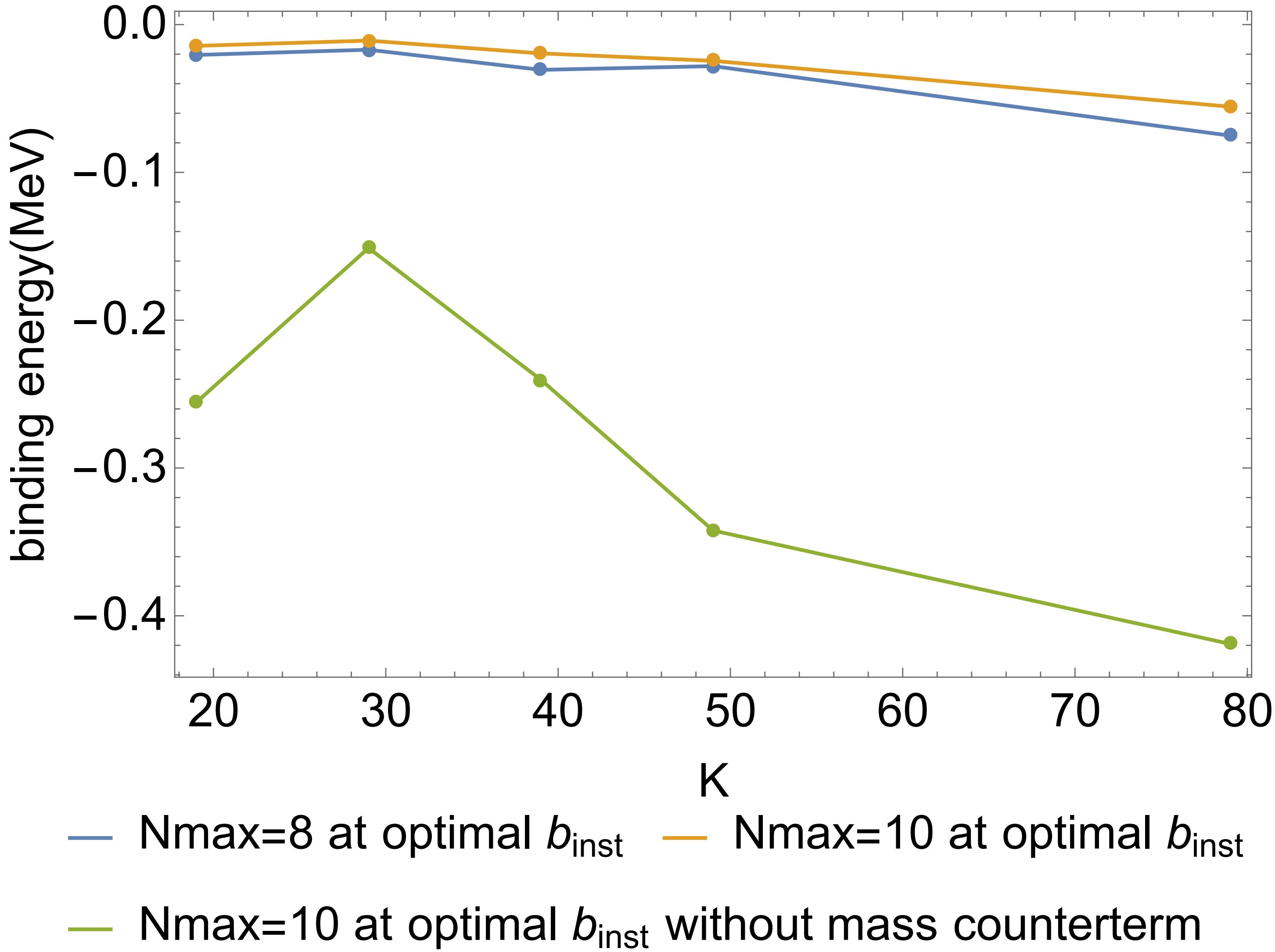
- b_{inst} is chosen by minimizing $|E_b(m_j = 0) - E_b(m_j = 1)|$ for $^3S_1$ state.

Ground State Binding Energy



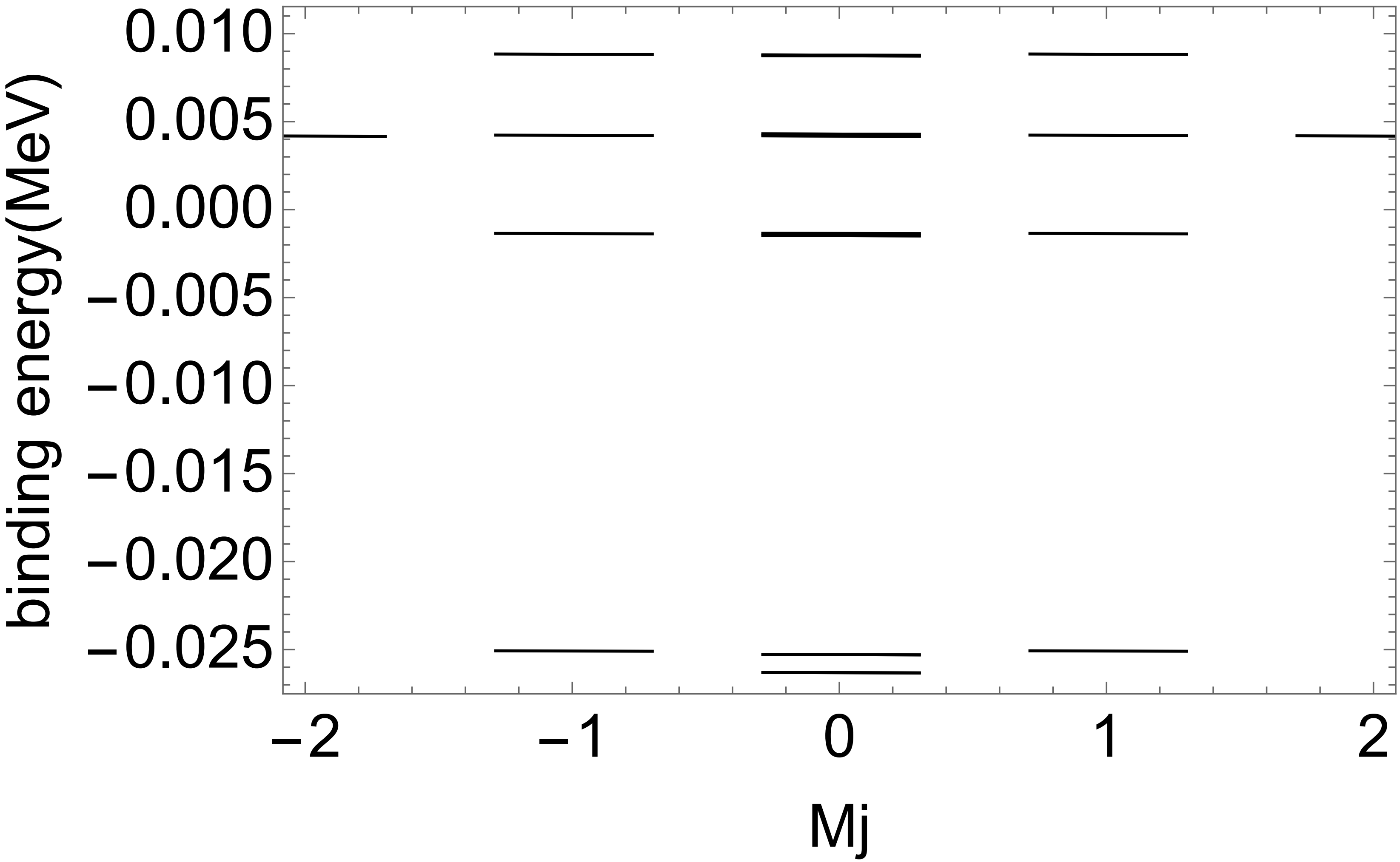
Convergence is better at smaller K

Ground State Binding Energy (Without Mass Renormalization)



Positronium Mass Spectrum (Shifted)

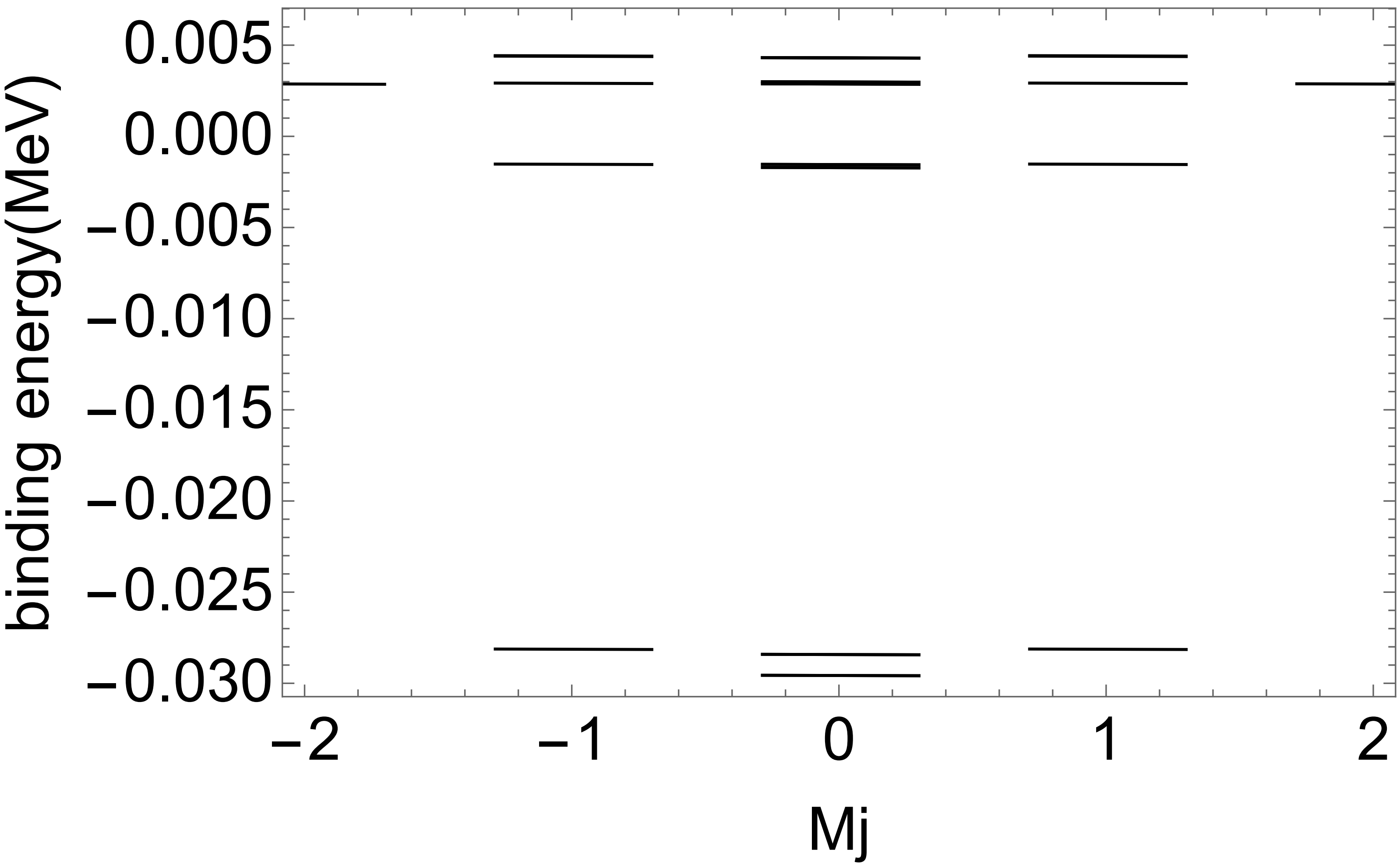
Nmax=12, K=19, $\alpha = 0.3$, M_e=1MeV



Mj=0 states(from down to up): 1^1S_0, 1^3S_1, 2^3S_1, 2^1S_0, 2^3P_2, 2^1P_1 , 2^3P_1, 2^3P_0

Positronium Mass Spectrum (Shifted)

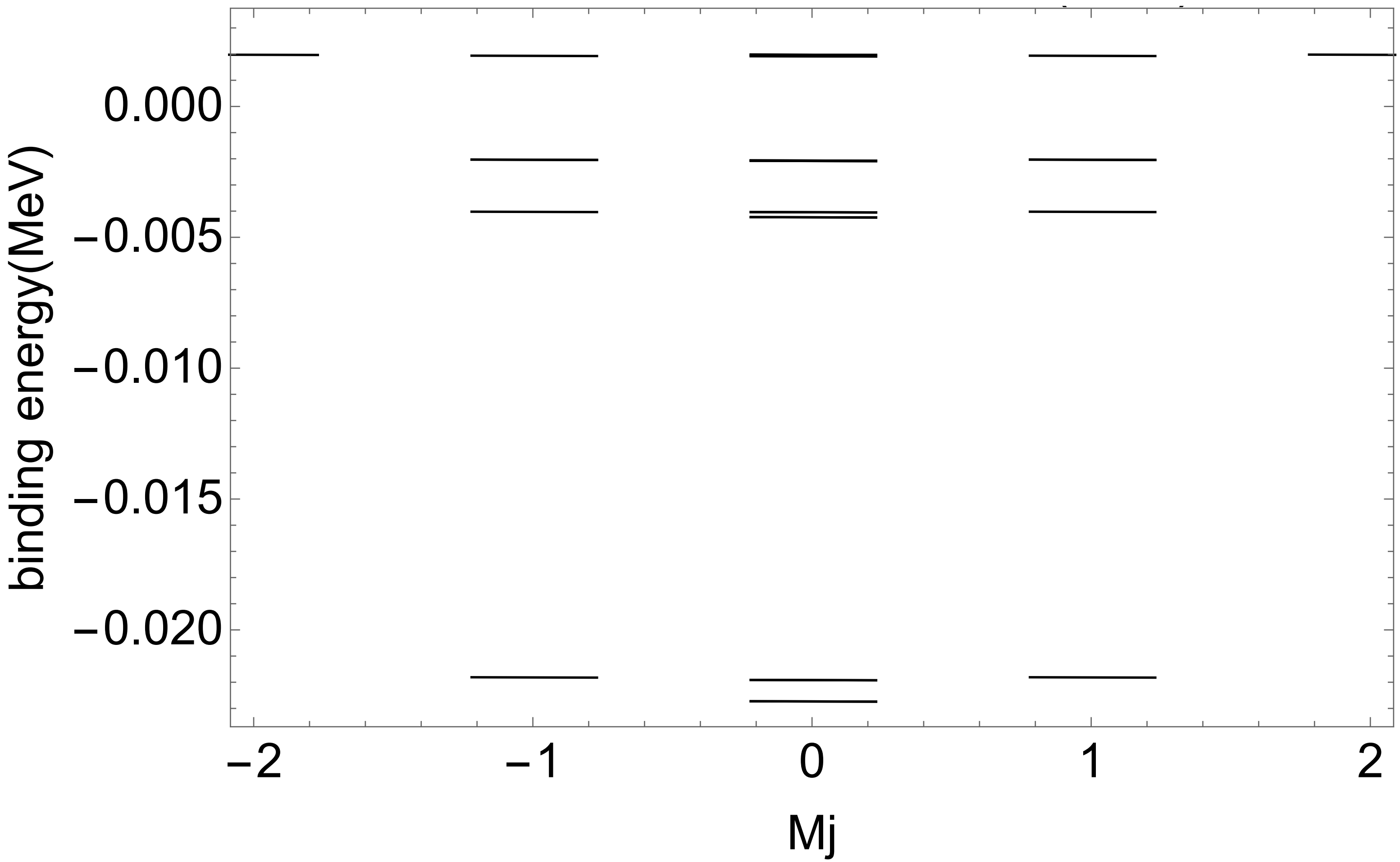
Nmax=14, K=19, $\alpha = 0.3$, M_e=1MeV



Mj=0 states(from down to up): 1^1S_0, 1^3S_1, 2^3S_1, 2^1S_0, 2^3P_2, 2^1P_1 , 2^3P_1, 2^3P_0

Positronium Mass Spectrum (Shifted)

Nmax=16, K=19, $\alpha = 0.3$, M_e=1MeV



Mj=0 states(from down to up): 1^1S_0, 1^3S_1, 2^3S_1, 2^1S_0, 2^3P_0, 2^3P_1, 2^3P_2, 2^1P_1

Mj=1 states(from down to up): 1^3S_1, 2^3S_1, 2^1P_1, 2^3P_1, 2^3P_2

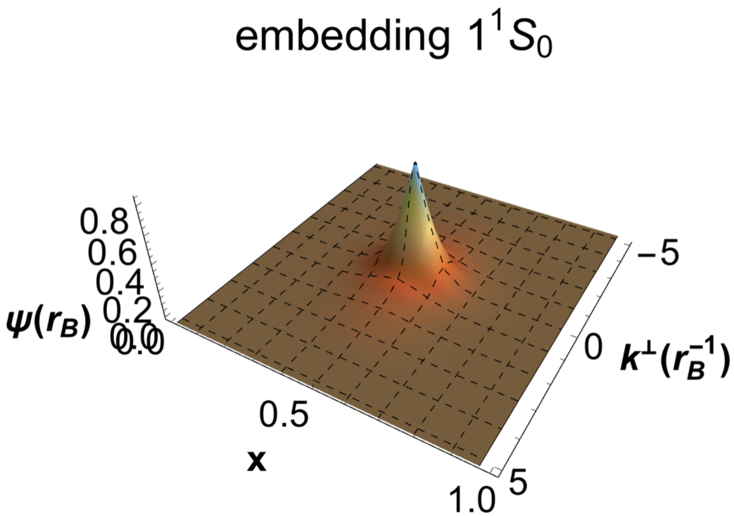
Mj=2 states(from down to up): 2^3P_2

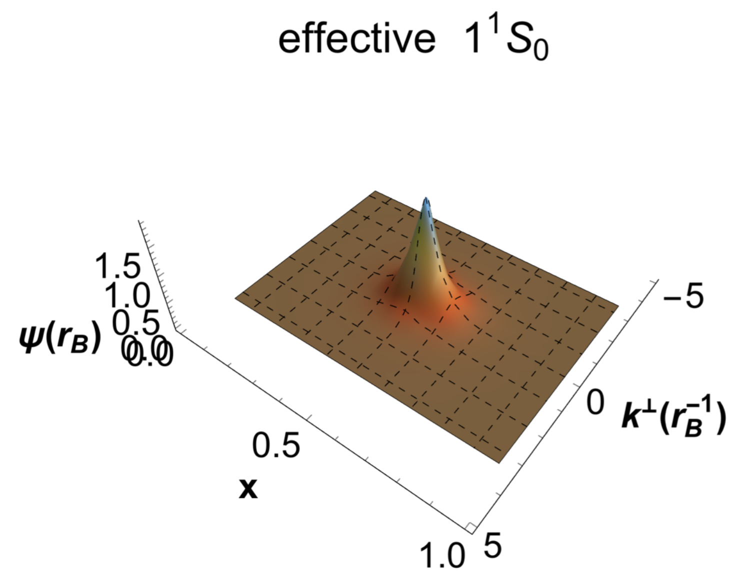
Wave Function Comparison

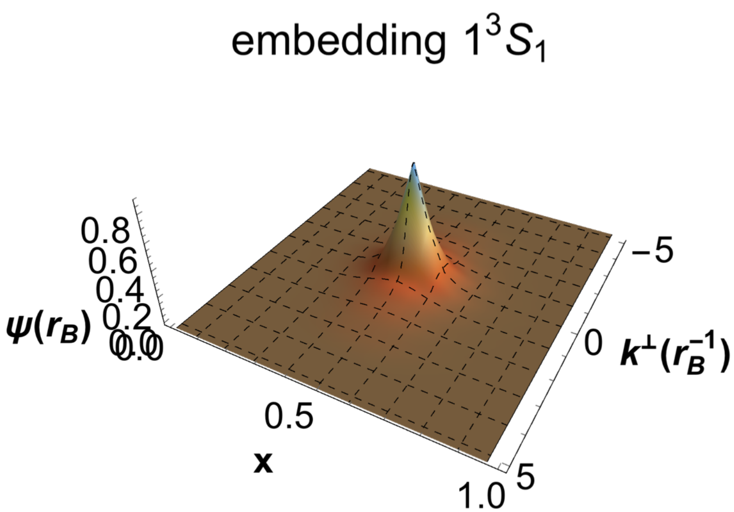
[Wiecki, et al, 2015]

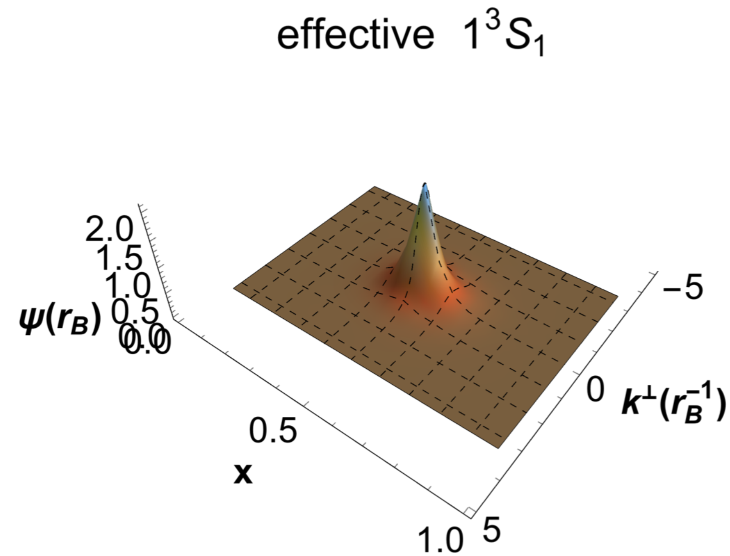
embedding wavefunction at Nmax=16, K=19, optimal b_{inst} and b=0.312MeV

effective wavefunction at Nmax=16, K=19, b=0.3MeV

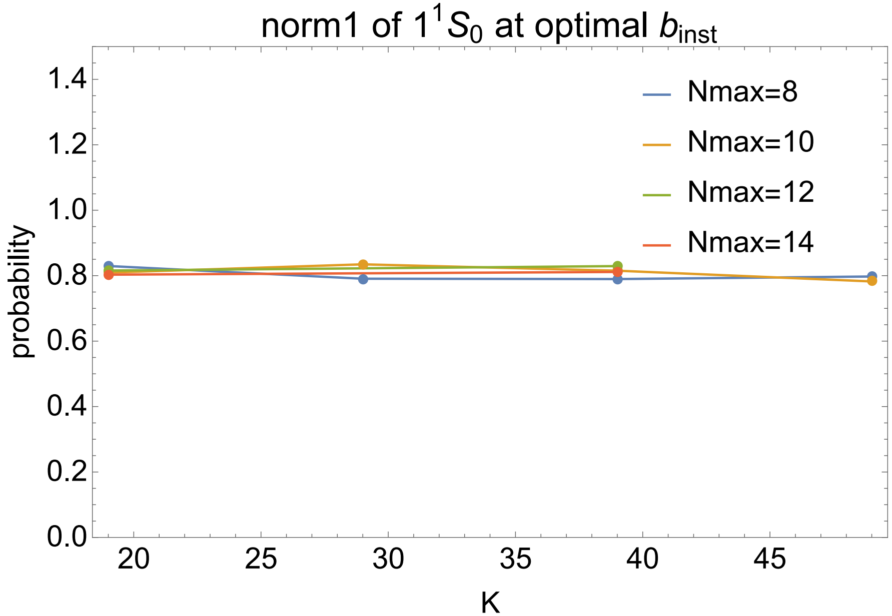


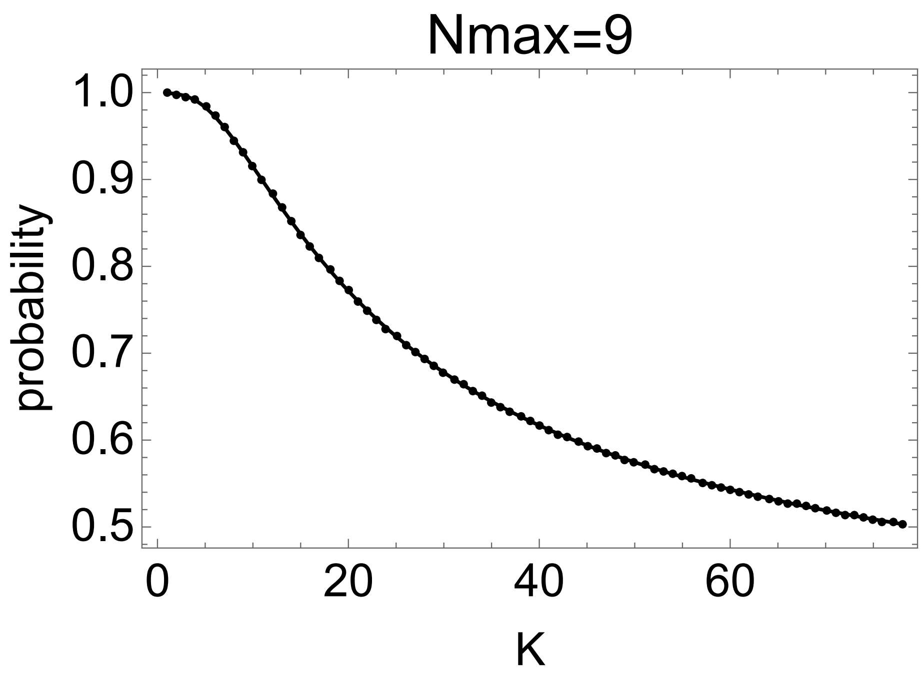






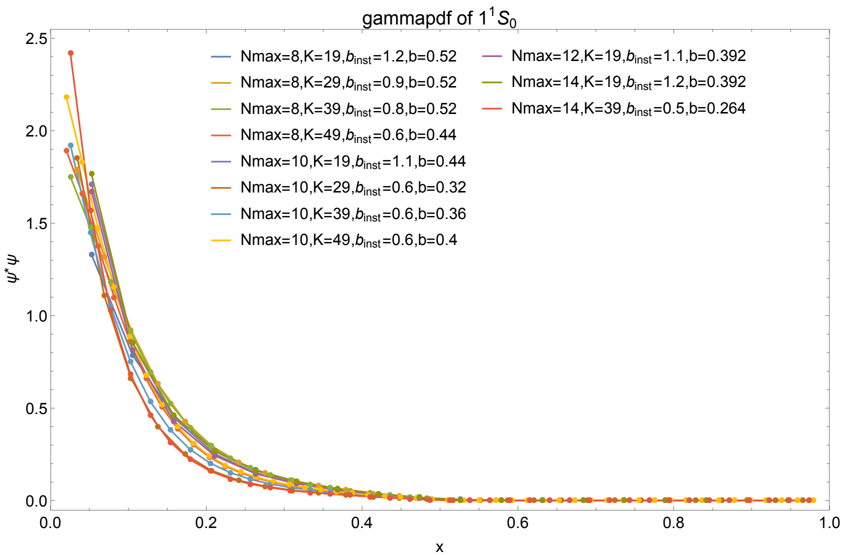
Probability of $|e\bar{e}\rangle$



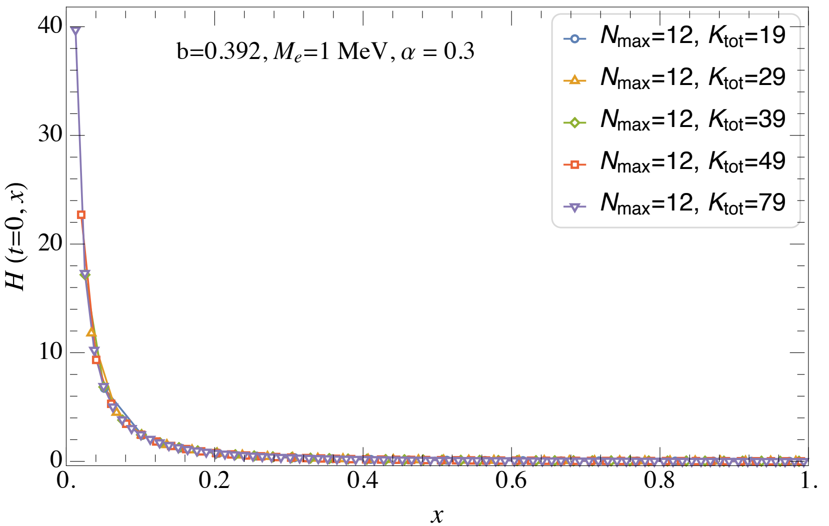


Probability of $|e\bar{e}\rangle$ seems converging with respect to Nmax and K in comparison with the single electron case

Compare with Photon distribution in Single Electron



Positronium $^1S_0$



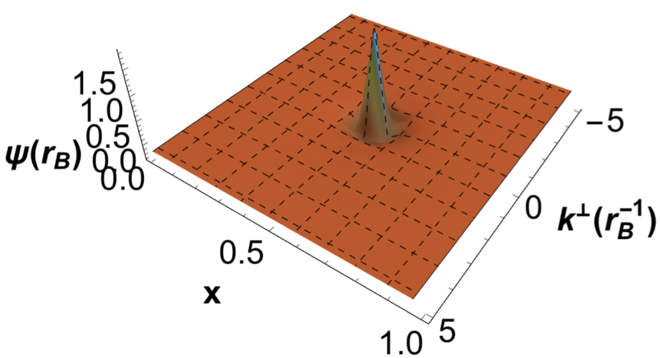
Single electron

[Shuo, et al, in preparation]

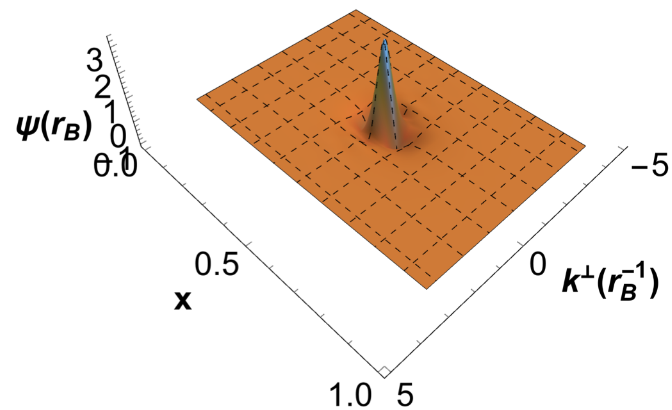
Small-x photons are largely suppressed
In consistency with the “smaller size” of positrtonium

Wave Function Comparison

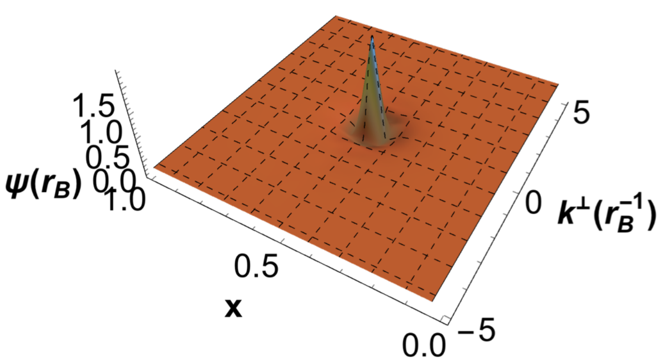
embedding 2^1S_0



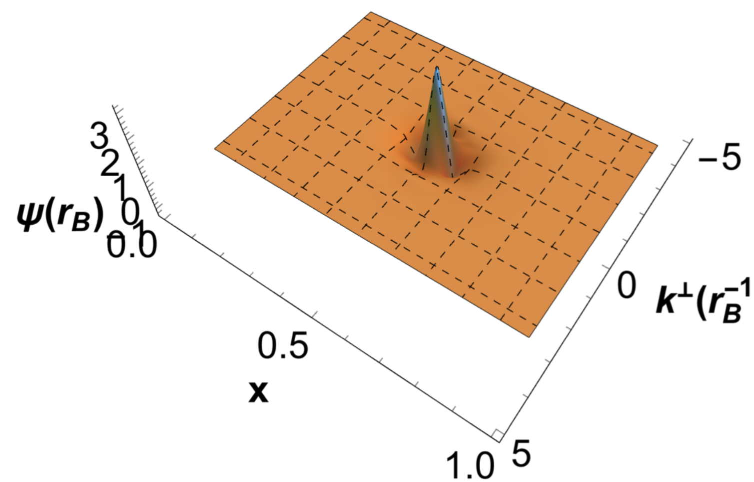
effective 2^1S_0



embedding 2^3S_1

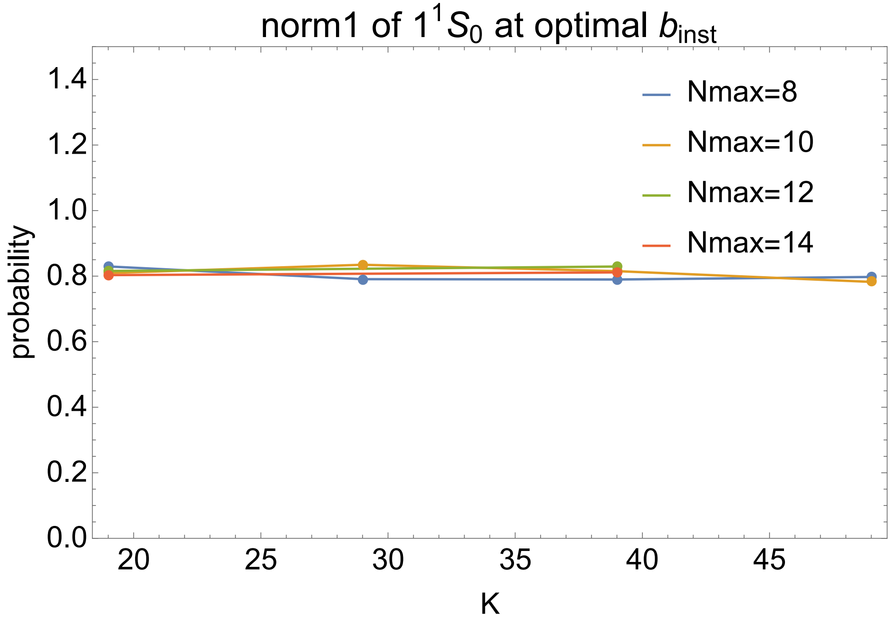


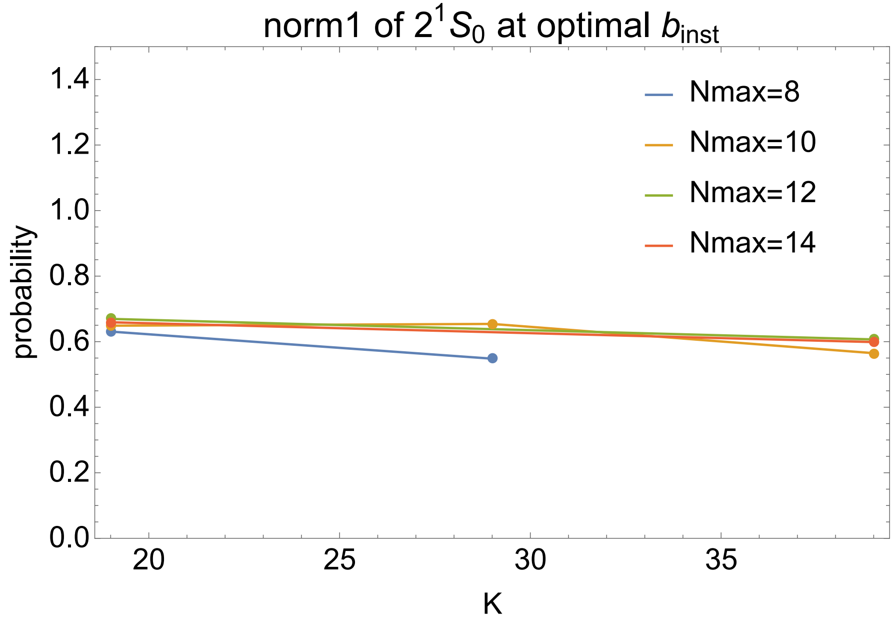
effective 2^3S_1



Nodal structure in radial direction

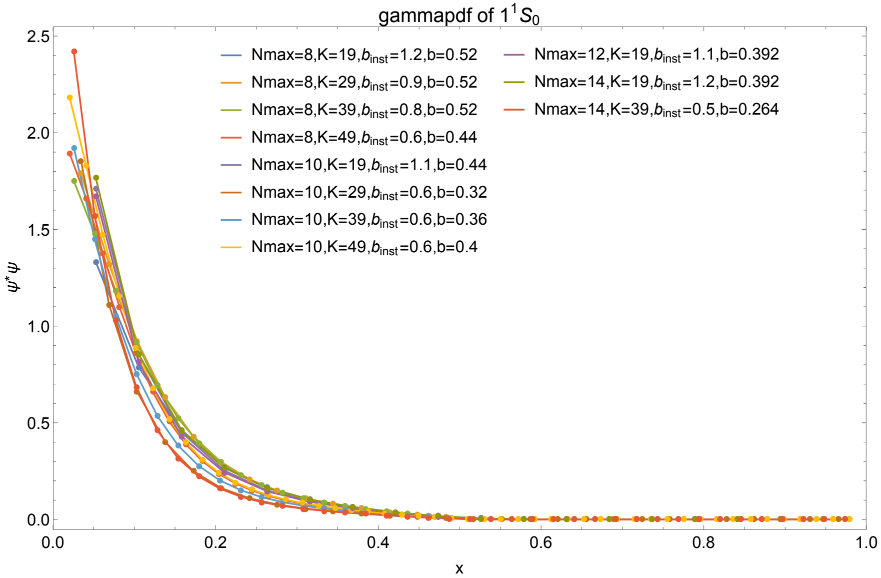
Probability of $|e\bar{e}\rangle$

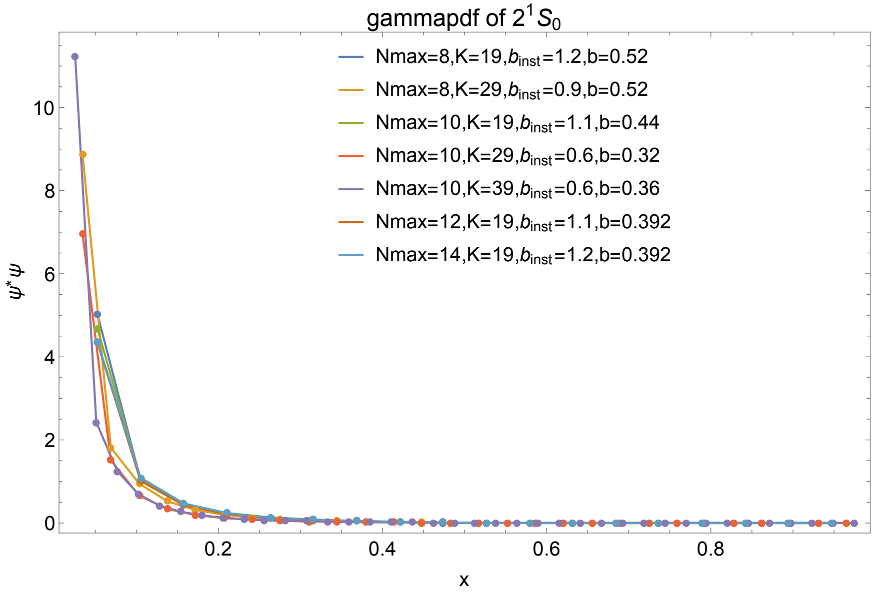




Excited states have larger $|e\bar{e}\gamma\rangle$ component

Photon Distribution in Positronium

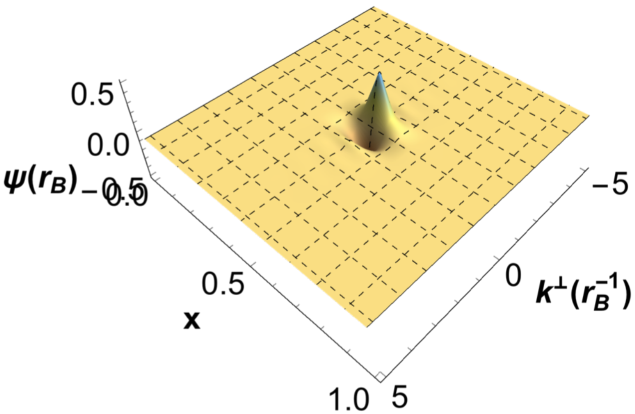




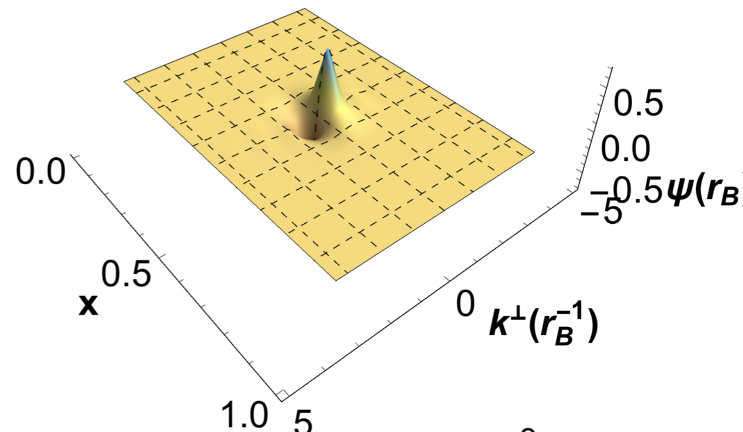
In excited states photons have larger probability at small-x region

Wave Function Comparison

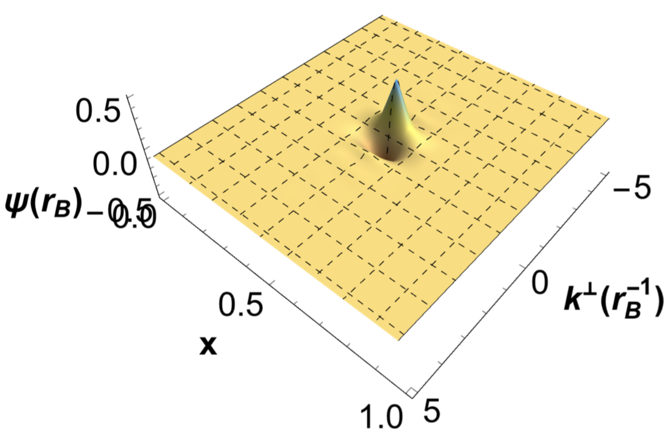
embedding 2^3P_0



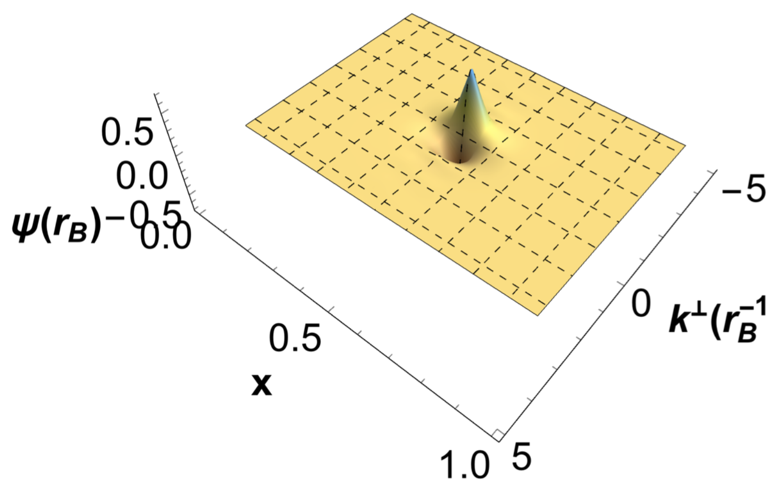
effective 2^3P_0



embedding 2^3P_1

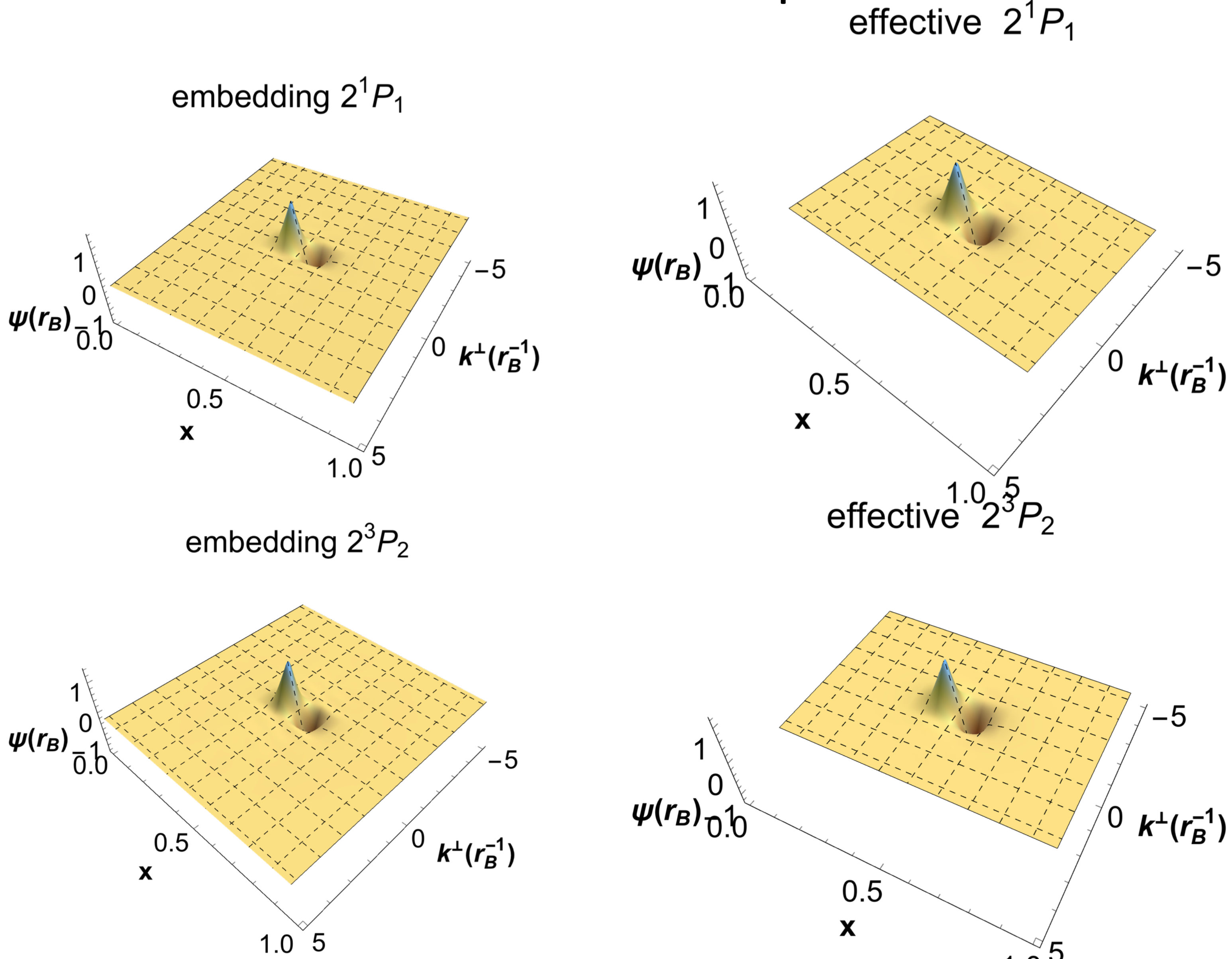


effective 2^3P_1



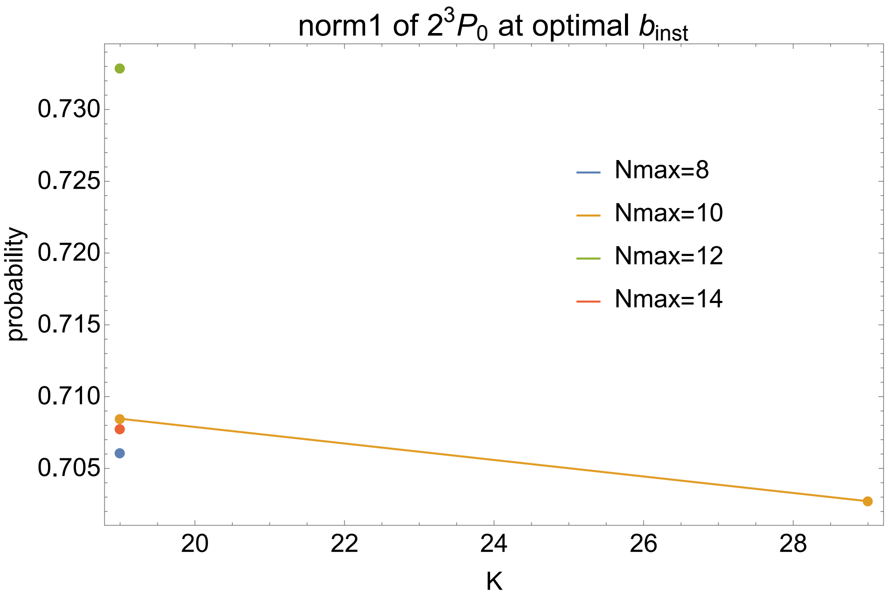
Nodal structure in angular direction

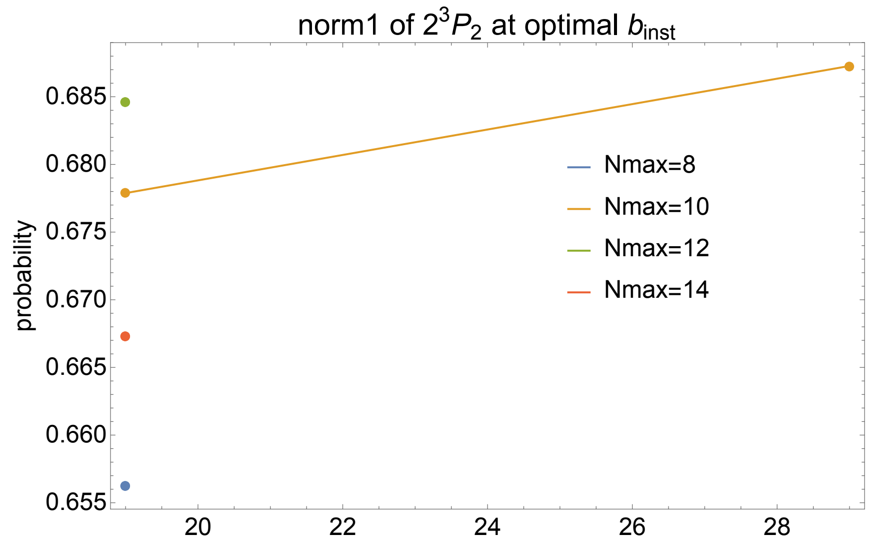
Wave Function Comparison



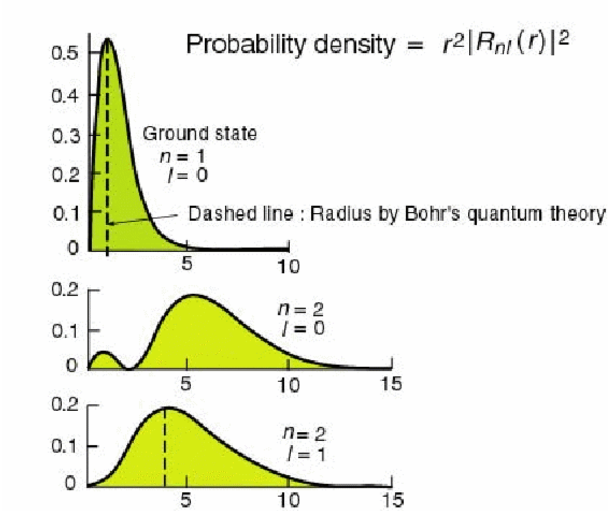
Nodal structure in angular direction

Photon Distribution in Positronium



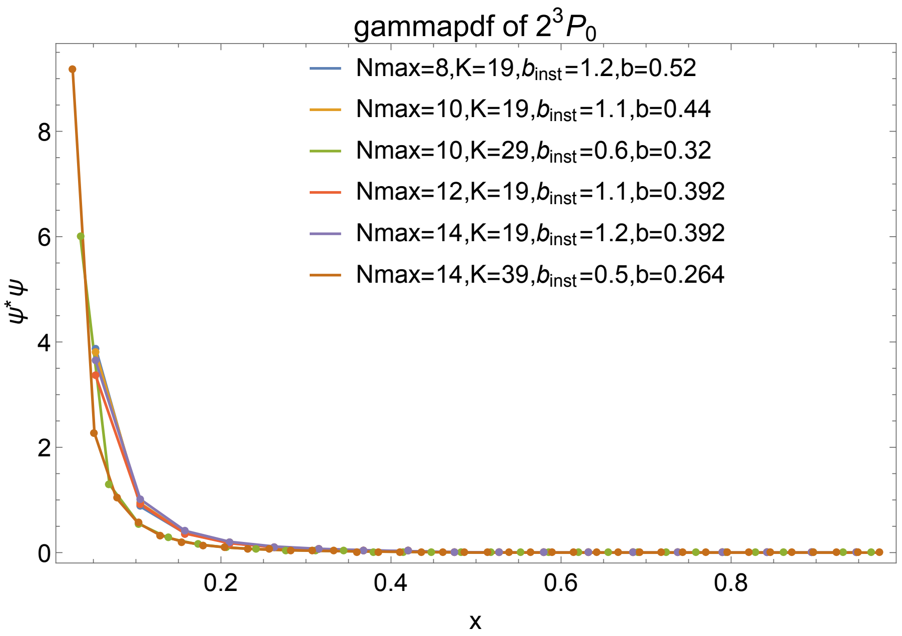


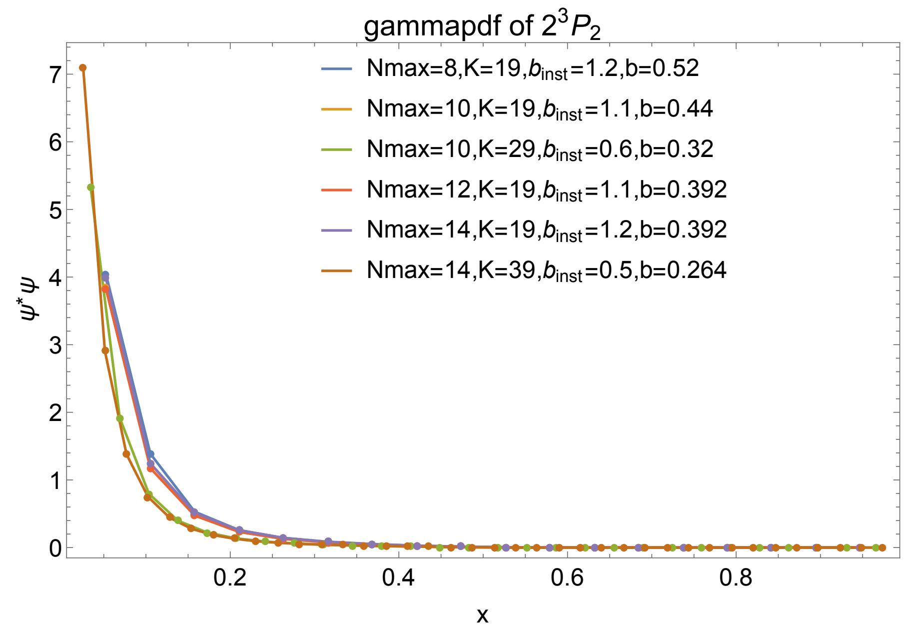
Probability of $|e\bar{e}\rangle$ for P-wave states seem to be between 1S and 2S states



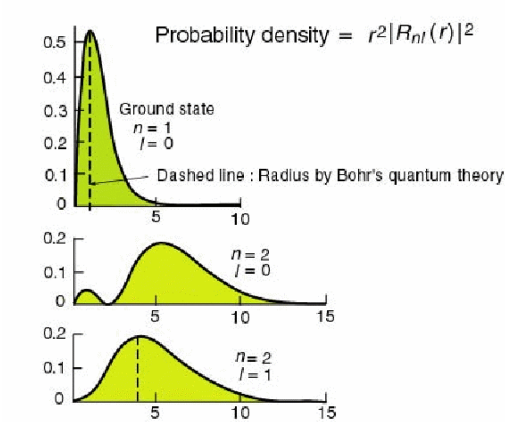
https://astarmathsandphysics.com/university-physics-notes/quantum-mechanics/1644-table-of-the-radial-parts-of-the-wavefunctions-for-the-hydrogen-atom.html

Photon Distribution in Positronium





Small-x photon probability for P-wave states seem to be between 1S and 2S states



Conclusions

- Solve positronium system based on first-principles
- Hamiltonian framework provides wave functions beyond leading Fock-sector
- Mass renormalization is needed
- Wave function and energy spectrum for low lying states reasonably agree with those from potential approach
- Small-x photons seem less divergent compared to those in single electron
- Indicateing less divergent infrared behavior for positronium compared to single electron

Outlook

- Convergence study
- Observables
- Connection with effective potential approach
- Heavy quarkonium systems
- Baryon system beyond leading Fock-sector

Thank you!

Hyperfine Splitting ($^3S_1$-$^1S_0$)