Deeply virtual Compton scattering: the Compton form factors of the ^{4}He nucleus[1]

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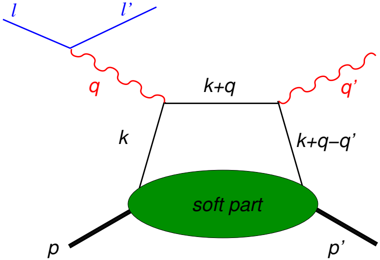
[1] This work was done in collaboration with Chueng-Ryong Ji

Outline

Motivation

Deeply-virtual Compton scattering (DVCS) has been proposed to determine the generalized-parton distributions (GPDs) of hadrons.



Handbag diagram for VCS, including the leptonic part

A hard photon, $q^2 = -Q^2$, with Q much larger than the characteristic hadronic scales, probes the quark content of the hadronic target. The detection of the outgoing, real photon provides information not contained in deep-inelastic scattering (DIS).

It is commonly assumed that to allow for the extraction of the GPDs, the experiments should be set-up in (approximately) collinear kinematics. Such kinematics may not always be possible to realize in concrete experiments.

We propose to first analyze the experimental data in terms of Lorentz-invariant amplitudes, Compton form factors (CFFs).

By definition, the CFFs can be determined in any suitable kinematics. Once they are measured, theorists may use them to extract the GPDs.

Here, we present our work on VCS off the ^{4}He nucleus, motivated by the work of M. Hattawy [2]

We shall in particular discuss the importance of including the three CFFs.

[2] M. Hattawy, *Thesis*, Université Paris-Sud XII (2015);M. Hattawy *et al.*, Phys. Rev. Lett. **119**, 202004 (2017)

Formalism

In Compton scattering the physical amplitudes is written as the contraction of a tensor operator with the photon polarization vectors.

In order not to introduce **unwarranted restrictions**, it is important to use the most general form of that tensor operator consistent with EM gauge invariance.

The quark-gluon structure of hadrons is supposed to manifest itself most transparently in processes where the hadrons are subjected to strongly virtual probes.

The amplitudes must scale with the virtuality Q to allow for a partonic interpretation.

To obtain the **complete** amplitudes, one must add the ones associated with the **Bethe-Heitler process**. These amplitudes can be written as the convolution of the leptonic (QED) amplitude and a hadronic amplitude, which involves the electro-magnetic form factor of the ^{4}He nucleus, which is well known.We shall use the parametrisation by Frosch *et al.*[3].

Otherwise, the Bethe-Heitler amplitudes are a straight forward exercise in QED. We neglect the electron mass.

Similarly, the VCS amplitude can be written as the convolution of another leptonic amplitude and the hadronic VCS amplitude, which is parameterised in terms of the CFFs.

[3] R.F. Frosch, J.S. McCarthy, R.E. Rand, and M.R. Yearian, Phys. Rev. **180**, 874 (1967)

Tarrach's tensor

We write the physical amplitudes as contractions of a tensor with the polarization vectors of the photons:

$$A(h', h) = \epsilon^*(q'; h')_\mu T^{\mu\nu} \epsilon(q; h)_\nu .$$

This tensor must be transverse, *i.e.*,

$$q'_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} q_\nu = 0.$$

It is written in terms of scalars (CFFs) and basis tensors. The number of independent tensor structures is known to be 5.[4]

The relevant momenta in the hadronic part are $p(p')$ and $q(q')$, the momenta of the incoming(outgoing) hadron and photon.

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[4]M. Perrottet, Lett. Nuovo Cim. **7**, 915 (1973) and R. Tarrach, Nuovo Cim. **28 A**, 409 (1975) and numerous more recent papers.

Using momentum conservation, one finds that there are 3 independent momenta. We keep q and q', to simplify a check of the transversity of the tensor. For the remaining one we choose the sum of the hadronic momenta, $\bar{P} = p' + p$. Our basis is then $k_1 = \bar{P}$, $k_2 = q'$, $k_3 = q$.

Following Tarrach, we find it useful to construct the tensor $T^{\mu\nu}$ by applying a two-sided projector $\tilde{g}^{\mu\nu}(q, q')$ to the most general second-rank tensor expressed in terms of our basis:

$$T^{\mu\nu} = \tilde{g}^{\mu m}\, t_{mn}\, \tilde{g}^{n\nu}, \quad t_{mn} = t_0\, g_{mn} + \sum_{i,j} t_{ij}\ k_{i\,m} k_{j\,n}.$$

The two-sided projector $\tilde{g}(q, q')$ is defined as follows:

$$\tilde{g}^{\mu\nu}(q, q') = g^{\mu\nu} - \frac{q^\mu q'^\nu}{q \cdot q'}.$$

This projector has the properties

$$\tilde{g}^{\mu m}\, g_{mn}\, \tilde{g}^{n\nu} = \tilde{g}^{\mu\nu}, \quad q'_\mu\, \tilde{g}^{\mu\nu} = 0, \quad \tilde{g}^{\mu\nu} q_\nu = 0.$$

We define the reduced momenta, $(k = \bar{P},\ q',\ q)$:

$$\tilde{k}_{\mathrm{L}}^{\mu} = \tilde{g}^{\mu m} k_m \quad \tilde{k}_{\mathrm{R}}^{\nu} = k_n \, \tilde{g}^{n\nu}$$

and find for unrestricted kinematics the following result for $T^{\mu\nu}$

$$T^{\mu\nu} = \mathcal{H}_0 \ \tilde{g}^{\mu\nu} + \mathcal{H}_1 \ \tilde{P}_{\mathrm{L}}^{\mu} \tilde{P}_{\mathrm{R}}^{\nu} + \mathcal{H}_2 \ \tilde{P}_{\mathrm{L}}^{\mu} \tilde{q}_{\mathrm{R}}^{\nu} + \mathcal{H}_3 \ \tilde{q}_{\mathrm{L}}^{\prime\mu} \tilde{P}_{\mathrm{R}}^{\nu} + \mathcal{H}_4 \ \tilde{q}_{\mathrm{L}}^{\prime\mu} \tilde{q}_{\mathrm{R}}^{\nu}.$$

Contracting the tensor with $\epsilon_{\mu}^{*}(q')$ and $\epsilon_{\nu}(q)$ we find that all five pieces of the tensor contribute to the Compton amplitude, if $q'^2 \neq 0$ and $q^2 \neq 0$.

Alternative tensors

The method proposed by Tarrach is by no means the only possible one. Other conventions were proposed by Andreas Metz [5] and Chueng Ji [6]

Defining the DNA tensor

$$d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\beta} g^{\nu\alpha},$$

novel projectors are defined by

$$G^{\mu\nu}(k_i, k_j) = k_{i\,\alpha} d^{\mu\nu\alpha\beta} k_{j\,\beta},$$

where the k's are just the basis vectors defined before.

[5] A. Metz, *Virtuelle Comptonstreuung und die Polarisierbarkeiten des Nukleons* (in German), PhD thesis, Universität Mainz

[6] B.L.G. Bakker and C.-R. Ji,Few-Body Syst., **58**, 1 (2017)

The complete tensor can now be written as

$$\tilde{T}^{\mu\nu}_{\text{DNA}} := \sum_{i=1}^{5} \mathcal{S}_i \, \tilde{T}^{(i)\,\mu\nu}_{\text{DNA}} =$$

$$\mathcal{S}_1 G^{\mu\nu}(q',q) + \mathcal{S}_2 G^{\mu\lambda}(q',q') G_\lambda^{\ \nu}(q,q) + \mathcal{S}_3 G^{\mu\lambda}(q',\bar{P}) G_\lambda^{\ \nu}(\bar{P},q)$$

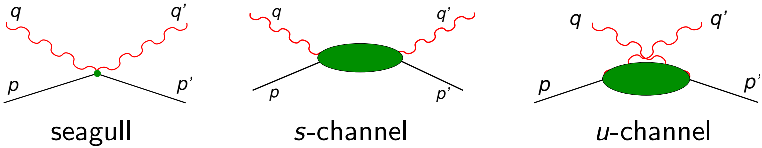
$$+\mathcal{S}_4 \left(G^{\mu\lambda}(q',\bar{P}) G_\lambda^{\ \nu}(q,q) + G^{\mu\lambda}(q',q') \right) G_\lambda^{\ \nu}(\bar{P},q) \big)$$

$$+\mathcal{S}_5 G^{\mu\lambda}(q',q') \bar{P}_\lambda \bar{P}_{\lambda'} G^{\lambda'\nu}(q,q).$$

Where the $\mathcal{S}_i$ are the CFFs in the DNA construction.

The CFFs $\mathcal{H}_i$ of the Tarrach basis can be written as linear combinations of the $\mathcal{S}_i$.

Illustration: Tree-level DVCS



As a **default model** one may consider the tree-level case, which of course describes completely structureless particles. Any deviation of the cross sections from the predictions of this model implies that the hadron has structure.

The tree-level DVCS amplitude corresponds to the CFFs

$$\mathcal{H}_0^{\text{tree}} = -2,\ \mathcal{H}_1^{\text{tree}} = \left(\frac{1}{s_{\text{had}} - M^2} + \frac{1}{u_{\text{had}} - M^2} \right),$$

$$s_{\text{had}} = (p+q)^2,\ u_{\text{had}} = (p-q')^2.$$

Thus, only 2 out of 5 CFFs contribute. We note that at large Q, $\mathcal{H}_1$ is of relative order $1/Q^2$ compared to $\mathcal{H}_0$.

Kinematics

We shall in general work in the hadronic CMF. The amplitudes can be expressed in terms of three invariants and two azimuthal angles. We start with the invariants.

For the VCS amplitude the relevant invariants are the mass M of the hadron and

$$\begin{aligned} Q^2 &= -q^2, \quad x_{\mathrm{Bj}} = \frac{Q^2}{(2p \cdot q)}, \\ s_{\mathrm{had}} &= (p+q)^2, \quad t_{\mathrm{had}} = (p-p')^2, \quad u_{\mathrm{had}} = (p-q')^2. \end{aligned} \tag{1}$$

For a given values of the Bjorken variable x_{Bj}, the squared invariant mass is given by

$$s_{\mathrm{had}} = M^2 + \frac{1-x_{\mathrm{Bj}}}{x_{\mathrm{Bj}}}\, Q^2.$$

Thus s_{had} is of order Q^2, Because in the CMF all non-vanishing momentum components can be expressed in terms of M, Q, s_{had}, and t_{had}, they are of order Q. The invariants t_{had} and u_{had} may become small in special points in the kinematic domain.

We calculate the Mandelstam variables t_{had} and u_{had} for large Q:

$$t_{\text{had}} \to -\frac{1-\cos\vartheta}{2x_{\text{Bj}}}\,Q^2 + \mathcal{O}(M^2), \quad u_{\text{had}} \to -\frac{1+\cos\vartheta}{2x_{\text{Bj}}}\,Q^2 + \mathcal{O}(M^2).$$

The quantity $\vartheta = \theta'_{\text{C}} - \theta_{\text{C}}$ is the scattering angle in the CMF.

If $\vartheta \to 0$, t_{had} goes to zero up to corrections of $\mathcal{O}(M^2)$, thus t_{had} does not strictly vanish in the forward limit. If the target mass M is not small compared to Q, which is the case in e.g. the VCS-on-^{4}He experiment done at Jlab, one must go to almost completely forward kinematics to make t_{had} small compared to Q^2,

If the experimental set-up limits the scattering angle to values greater than ϑ_{lim}, t remains of order Q^2.

For large Q and small ϑ_{lim} one finds $|t| > \frac{\vartheta^2_{\text{lim}}}{4x_{\text{Bj}}}\,Q^2$.

The Compton tensor

Looking at the Compton tensor of VCS

$$T^{\mu\nu} = \mathcal{H}_0 \; \tilde{g}^{\mu\nu} + \mathcal{H}_1 \; \tilde{P}^{\mu}_{\mathrm{L}} \tilde{P}^{\nu}_{\mathrm{R}} + \mathcal{H}_2 \; \tilde{P}^{\mu}_{\mathrm{L}} \tilde{q}^{\nu}_{\mathrm{R}},$$

it is clear from the fact that all three pieces must have the same mass dimension, that we must expect that the scaling of the CFFs must obey the rule

$$\mathcal{H}_0 : \mathcal{H}_1 : \mathcal{H}_2 = 1/Q^2 : 1/Q^2.$$

$T^{\mu\nu}$ in the forward kinematics and in the limit of large Q becomes

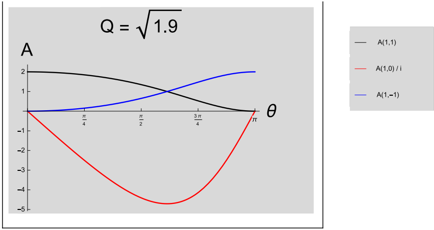
$$T^{\mu\nu} = Q^2 \begin{pmatrix} \frac{\mathcal{H}_1'}{4x_{\mathrm{Bj}}^2} & 0 & 0 & \frac{(1-2x_{\mathrm{Bj}})\mathcal{H}_1'}{4x_{\mathrm{Bj}}^2} \\ 0 & -\frac{\mathcal{H}_0}{Q^2} & 0 & 0 \\ 0 & 0 & -\frac{\mathcal{H}_0}{Q^2} & 0 \\ \frac{\mathcal{H}_1'}{4x_{\mathrm{Bj}}^2} & 0 & 0 & \frac{(1-2x_{\mathrm{Bj}})\mathcal{H}_1'}{4x_{\mathrm{Bj}}^2} \end{pmatrix}$$

where the compound CFF $\mathcal{H}_1'$ is defined by

$$\mathcal{H}_1' = 2x_{\mathrm{Bj}}^2 \frac{\mathcal{H}_0}{Q^2} + (2 - x_{\mathrm{Bj}})^2 \mathcal{H}_1 - x_{\mathrm{Bj}}(2 - x_{\mathrm{Bj}})\mathcal{H}_2.$$

Hadronic amplitudes

Starting from the Compton tensor with three CFFs, we can calculate the amplitudes for VCS on ^{4}He for any values of Q^2, $x_{\rm Bj}$, and scattering angle θ. We shall use the scattering angle in the hadronic CMF in our plots of the amplitudes. This angle is simply related to $t_{\rm had}$. At **tree level** we find the following results.



The value $Q^2 = 1.9$ GeV^2 is typical for the JLab experiment. At very small θ, the spin non-flip amplitude dominates, but in the backward hemisphere $A(1,-1)$ and $A(1,0)$ become essential.

Cross sections

Because the Bethe-Heitler and VCS processes are coherent, the amplitudes must be added when the cross section for the process $e + {}^4\text{He} \to e' + {}^4\text{He} + \gamma$ is calculated. Then the complete cross section can be split into a Bethe-Heitler cross section, a VCS cross section and a part that is obtained by the interference of the two amplitudes:

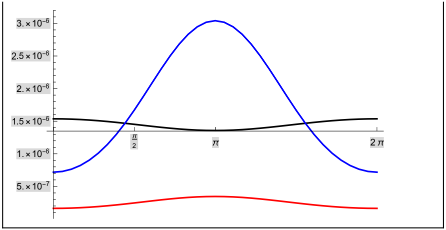
$$\begin{aligned} |A_{\text{tot}}|^2 &= |A_{\text{BH}} + A_{\text{VCS}}|^2 \\ &= |A_{\text{BH}}|^2 + |A_{\text{VCS}}|^2 + A^*_{\text{BH}} A_{\text{VCS}} + A^*_{\text{BH}} A_{\text{VCS}} + A_{\text{BH}} A^*_{\text{VCS}}. \end{aligned}$$

The ϕ dependence of this interference part gives acces to the CFFs. In the following slides we show this ϕ dependence of the Bethe-Heitler cross section, the VCS cross section and the complete cross section.

The values of the quantities Q^2, x_{Bj}, and t_{had} are taken from the proposal W.R. Armstrong *et al.*, arXiv:1708.00888v2 [nucl-ex] 5 Aug 2017.

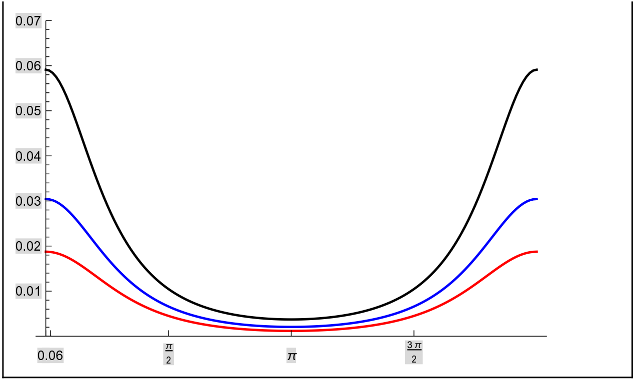
VCS cross section and CFFs

Including the leptonic part of the VCS amplitudes, we calculate the physical VCS cross section. The **black** curve represents for small scattering angle the ϕ dependence of the VCS cross-section at $Q^2 = 1.5$ GeV^2 and $x\text{Bj} = 0.22/4$. The **blue** curve corresponds to the case where $\mathcal{H}_0 = \mathcal{H}_0^{tree}$ and $\mathcal{H}_1 = 0$, and the **red** curve is for the case that $\mathcal{H}_0 = 0$ and $\mathcal{H}_1 = \mathcal{H}_1^{tree}$



Clearly, leaving out the “sub-dominant” $\mathcal{H}_1$ gives a completely wrong result. Note that $\mathcal{H}_0$ and $\mathcal{H}_1$ have opposite signs at tree level.

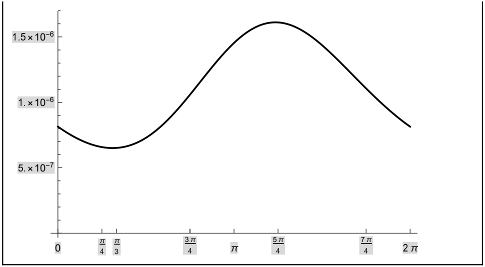
Cross section including Bethe Heitler



Bethe-Heitler plus VCS cross sections. The beam energy is 11 GeV, $Q^2 = 1.5$ GeV. The hadronic $t = -0.1$ GeV^2.

Color coding: $xBj = 0.18/4$, $xBj = 0.20/4$, $xBj = 0.22/4$.

When the CFFs are **real**, the beam-spin asymmetry is supposed to vanish. Therefore, we modified the CFF $\mathcal{H}_0$ by multiplication with $1 + i$. Then the cross section becomes dependent on ϕ



Bethe-Heitler plus VCS cross sections. The beam energy is 11 GeV, $Q^2 = 1.5$ GeV. The hadronic $t = -0.1$ GeV^2, $x_{\rm Bj} = 0.22/4$.

Color coding: $xBj = 0.18/4$, $xBj = 0.20/4$, $xBj = 0.22/4$.

Beam-spin asymmetry

Whenever there is an interference term, it can used to determine the CFFs. This term will show up in the single-spin asymmetry A_{LU}, which is defined as

$$A_{\mathrm{LU}} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}.$$

It is well known that in case the CFFs are real, this observable vanishes. Thus in the case we use the tree-level CFFs, $A_{\mathrm{LU}} = 0$. This is borne out in our calculation. However, If we again introduce in the CFF $\mathcal{H}_0$ an imaginary part:

$$\mathcal{H}_0 \to \mathcal{H}_0(1+i),$$

then the asymmetry shows up, as expected. ($\mathcal{H}_1$ is kept real).

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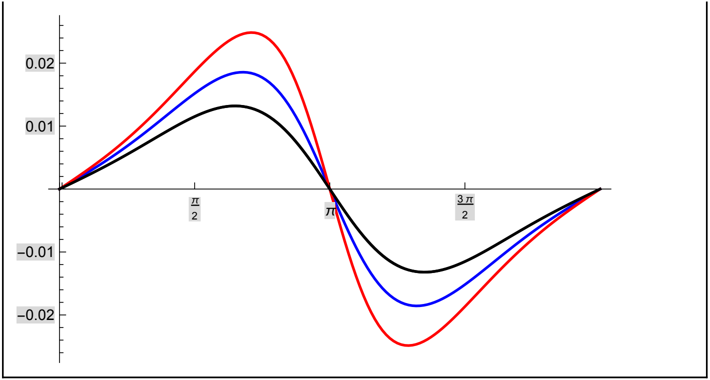

Bethe-Heitler-plus-VCS A_{LU}. The beam energy is 11 GeV, $Q^2 = 1.5$ GeV. The hadronic $t = -0.1$ GeV^2.

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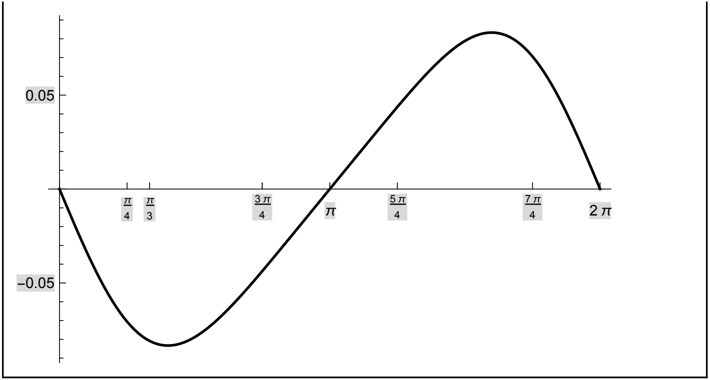

Because all CFFs contribute to the VCS amplitudes, VCS also may show a single-spin asymmetry.

$xBj = 0.22/4$

Summary and conclusions

- We have discussed the number of Compton Form factors for a scalar target. This number is **three**.
- **We have presented a model-independent form of the Compton tensor, containing all three CFFs**.
- We have shown that in the forward direction and for large Q the number of CFFs reduces effectively to two.
- To estimate the relative importance of the three CFFS, one should include the leptonic part and the Bethe-Heitler amplitude.

- For illustration, we have used the tree-level Compton tensor and modifications. Adding the Bethe-Heitler amplitudes to the VCS ones, we calculated the cross section as wel as the single-spin asymmetry A_{LU}.
- We verified that A_{LU} must vanish for real CFFs, but becomes finite for complex CFFs.
- **A surprising result is that even without interference of the Bethe-Heitler process, there occurs a single-spin symmetry in VCS.** This result is obtained because the VCS amplitude is the coherent sum of two parts, one related to the CFF $\mathcal{H}_0$, the other to $\mathcal{H}_1$.