Pion off-shell electromagnetic form factors

Light Cone 2018
Thomas Jefferson National Accelerator Facility
May 14-18, 2018 Newport News, VA

J. Pacheco B. C. de Melo[a]

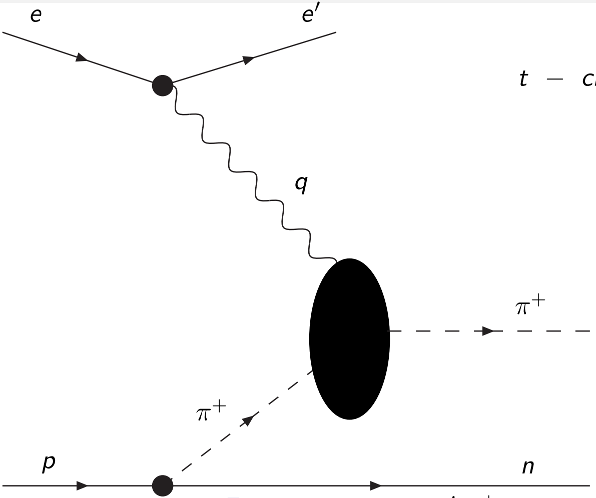
[a]Laboratório de Física Téorica e Computacional-LFTC, UCS (Brazil)
Collaborators: T. Frederico (ITA, Brazil) and Chueng -R. Ji (NCSU, USA)

May 15, 2018

Motivations

- Hadronic Form factors: **Important Sources Informations for Hadrons Strucuture**
- **Off-Shell Effects: Electroproduction** $\Rightarrow$ $^{1}H(e, e'\pi^{+})n$
- **Cross section's:** $\sigma_L, \sigma_T, \sigma_{LL}, \sigma_{TT}$
- **Pion Electromagnetic Form Factors:** $F_1(Q^2, t)$ **and** $F_2(Q^2, t)$
  $\Longrightarrow$ **Extracted from Cross sections**

Data from Experiments: Electroproduction



$t \; - \; channel$

Figure : $e \; p \; \longrightarrow \; e' \; \pi^+ \; n$ scattering.

Electromagnetic Form Factors

$\Longrightarrow$ **Form factors are essential for our understanding of internal hadron structure and the dynamics**

- **Off-shell Case**

$\Longrightarrow$ **Two Electromagnetic Form Factors:** F_1 **and** F_2

- **Most Simple Structure:**

$$< p'|\mathcal{O}|P >= (p' + p)^{\mu} F_1(Q^2, t) + (p' - p)^{\mu} F_2(Q^2, t)$$

- **Most General Structure**

$$\Gamma_\mu \;=\; e[(p+p')_\mu \; G^+ + (p-p')_\mu \; G^-]$$

- $\Longrightarrow \quad G^+ = F_1(p^2, p'^2, q^2)$ **and** $G^- = F_2(p^2, p'^2, q^2)$

and, with Ward-Takahashi Identity-WTI

$$q^\mu \Gamma_\mu \;=\; e\Delta_0^{-1}[\Delta(p) - \Delta(p')]\Delta_0^{-1}(p),$$

where, $\Delta_0(p) \;=\; \frac{1}{p^2 - m^2 + \imath\epsilon}$**, and,** $\Delta(p) \;=\; \frac{1}{p^2 - m^2 - \Sigma(p^2) + \imath\epsilon}$**.**

- **Assuming (Standard renormalization)**

$$\Longrightarrow \quad \{ \;\; \Sigma(m^2) = 0$$

- **Half On-shell limit** $\Longrightarrow$ $p'^2 = m^2$

$$\begin{aligned} \Delta_o^{-1}(p')\Delta(p') &\longrightarrow 1 \\ \Delta_0^{-1}(p')\Delta(p) &\longrightarrow 0 \end{aligned}$$

So that, $(m^2 - p^2)G^+ + q^2 G^- = m^2 - p^2$

- **With the normalization** $\Longrightarrow$ $G^+(m^2, m^2, q^2 = 0) = 1$
- **This gives us a relation between** G^+**, and** G^-**:**

$$q^2 G^-(p'^2, p^2, q^2)) = (m^2 - p^2)\left[1 - G^+(p'^2, p^2, q^2)\right]$$

- **We get**

$$\Gamma_{\mu}^{half-off-shell} = e[(p' + p)_{\mu} \; G^{+} + (p - p')_{\mu} \frac{(m^2 - p^2)}{q^2} \; (1 - G^{+})] \; .$$

- **vs**

$$\Gamma_{\mu}^{on-shell} = e[(p' + p)_{\mu} \; G^{+}] \; .$$

Extraction of the pion electromagnetic form factors, F_π^1 and F_π^2 from the experimental cross-section

• **The electromagnetic pion form factor is extracted from the longitudinal cross section**

$$\frac{d\sigma}{dt} \propto f(Q^2, W, t)$$

• **Longitudinal cross-section and pion form factor**

$$\frac{d\sigma_L}{dt} = \frac{16\pi}{137N} \frac{-tQ^2}{(t - m_\pi^2)^2} G^2_{\pi NN}(t) F_\pi^2(Q^2, t)$$

• **Factor** $N = 32\pi \left(W^2 - m_p^2\right) \sqrt{(W^2 - m_p^2)^2 + Q^4 + 2Q^2(W^2 + m_p^2)}$

• **Invariant mass of virtual-photon-nucleon W**

$W = \sqrt{M_p^2 + 2M_p^2\omega - Q^2}$, **and** $q_\gamma = (\omega, \vec{q})$, $t = (p_\pi - q)^2$

Ref. G. M. Huber *et al.* [Jefferson Lab Collaboration]. Phys. Rev. C78 (2008) 045203

- **Pion electromagnetic form factor, $F_\pi(Q^2,t) = F_\pi^1$ in terms of the cross-section**

$$F_1^2(Q^2,t) = \frac{137N}{16\pi}\frac{1}{G_{\pi NN}^2(t)}\frac{(t-m_\pi^2)^2}{-Q^2t}\frac{d\sigma_L}{dt}$$

- **From the Cross section we can extract the function H**

$$[F_1.G]^2 = H^2(Q^2,t) = \frac{137N}{16\pi}\frac{(t-m_\pi^2)^2}{-tQ^2}\frac{d\sigma_L}{dt}$$

- **Pion electromagnetic form factor $F_2(Q^2,t)$**

$$F_2(Q^2,t) = \frac{(p'^2-p^2)}{q^2}\left[1.0 - F_\pi^1(Q^2,t)\right]$$

$\Longrightarrow \quad F_1(m^2,m^2,q^2=0)=1$ **Normalization**

Pion - Nucleon Form factor $G_{\pi NN}(t)$

- **Two possible models**
- **Choice I**

$$G_{\pi NN}(t) = G_{\pi NN}(m_\pi^2)\left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t}\right)^n$$

- **Choice II**

$$G_{\pi NN}(t) = G_{\pi NN}(m_\pi^2)\frac{1}{\left(1 - \frac{t}{\Lambda^2}\right)^n}$$

Ref.: • **G. M. Huber** *et al.* **[Jefferson Lab Collaboration]. Phys. Rev. C78 (2008) 045203.**

• **R. Machleidt, K. Holinde, Ch. Ester, Phy. Rep. 149 (1987) 1.**

Light-Front Model: Non-Symmetric Vertex

- Electromagnetic current

$$J^{\mu} = -\imath 2e\frac{m^2}{f_\pi^2}N_c\int\frac{d^4k}{(2\pi)^4}Tr\Big[S(k)\gamma^5 S(k-p')\gamma^{\mu}S(k-p)\gamma^5\Big]\Gamma(k,p')\Gamma(k,p)$$

- **Quark propagator** $\Longrightarrow$ $S(p) = \frac{1}{\not{p}-m+\imath\epsilon}$
- **Numbers Colors** $\Longrightarrow$ $N_c = 3$ **and Factor 2 from the isospin algebra**
- **Non-symmetric Vertex Function**

$$\Gamma^{NSY}(k,p) = \left[\frac{N}{((p-k)^2 - m_R^2 + \imath\epsilon)}\right]$$

$\Rightarrow$ **Ref.: de Melo, Naus, H. and Frederico,T., Phy. Rev.C59 (1999) 2278**

- **Kinematics** $\Longrightarrow$ **Off-shell** **(with light-front quadrimomentum)**
- **Definition:**

$$a^\mu = (a^+ = a^0 + a^3, a^- = a^0 - a^3, a_\perp = (a_1, a_2))$$

$$\begin{aligned}
p^\mu . p_\mu &= t^2 \ , \ p^- p^+ = t^2 + \frac{q^2}{4}, \\
p_0 &= \frac{p^+ + p^-}{2} \ , \ p^+ = p'^+, \\
p^- &= p_0'^+ \ , \ p'^+ = p^0, \\
p'^0 &= m_\pi^2 + \frac{q^2}{4}, \\
p'^\mu . p_{\prime\mu} &= m_\pi^2 \ , \ q^+ = 0 \ , q_x = q, \ q_y = 0
\end{aligned}$$

- **Some Results from LFCQM -(with NSY-Vertex)**

$\Longrightarrow$ **Parameters:** $m_R = 1.00\ GeV$, $m_u = m_d = 0.220\ GeV$, $m_s = 0.510\ GeV$

$$f_\pi = 93.1\ \mathrm{MeV} \qquad (f_\pi^{\mathrm{exp.}} = 92.42\ \mathrm{MeV})$$
$$f_K = 126.9\ \mathrm{MeV} \qquad (f_K^{\mathrm{exp.}} = 110.4\ \mathrm{MeV})$$
$$\langle r_\pi \rangle = 0.679\ \mathrm{fm} \qquad (f_\pi^{\mathrm{exp.}} = 0.672\ \mathrm{fm})$$
$$\langle r_K \rangle = 0.636\ \mathrm{fm} \qquad (f_\pi^{\mathrm{exp.}} = 0.560\ \mathrm{fm})$$

Ref.:

- **de Melo, Naus, H., Frederico, T., Phy. Rev. C59 (1999) 2278.**
- **E. Silva, de Melo, B. El-Bennich, V. S. Filho, Phy. Rev. C86 (2012) 038202**
- **The Review of Particle Physics (2017), C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)**

Results

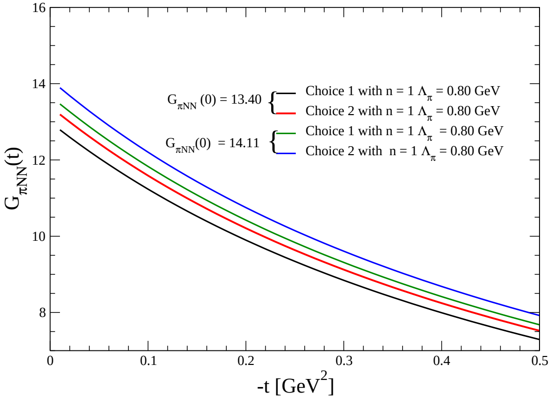


Figure : The form factors $G_{\pi NN}(t)$ I and II

.

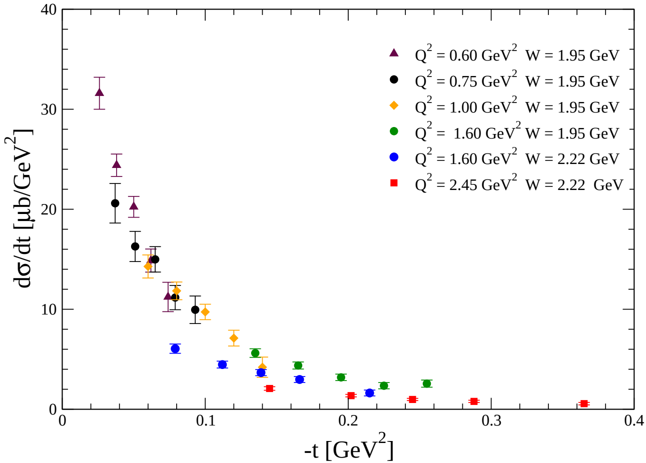


Figure : Experimental cross section, σ_l in function of t.
Data from Block et al., PRC78 (2008) 04520 2.

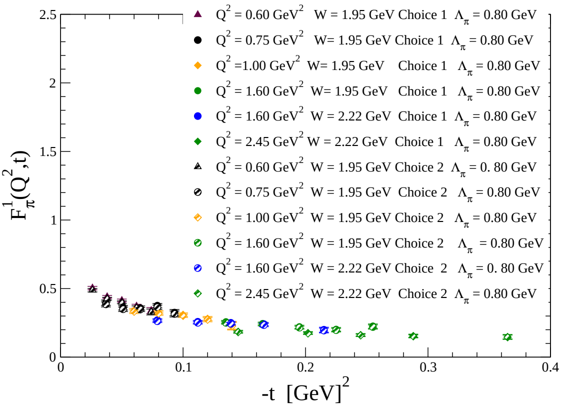


Figure : The experimental pion electromagnetic form factor, $F_\pi^{(1)}$, and $F_\pi^{(2)}$, extract from cross-section data, in terms of t, and Q^2, with the Choice I for $G_{\pi NN}(t)$, with $G_{\pi NN}(0) = 13.4$ and $\Lambda_\pi = 0.80$, and compared with the LFCQM, with W, Q^2 and t dependence.

Data from Block et al., PRC78 (2008) 045202.

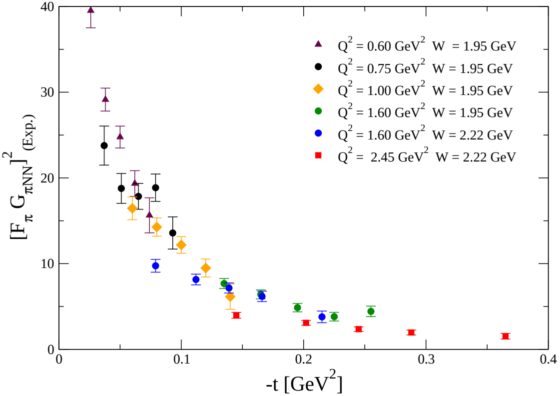


Figure : The pion electromagnetic form factor F_1, times $G_{\pi NN}(t)$ extract from cross-section data, in terms of t.

Data from Block et al., PRC78 (2008) 045202.

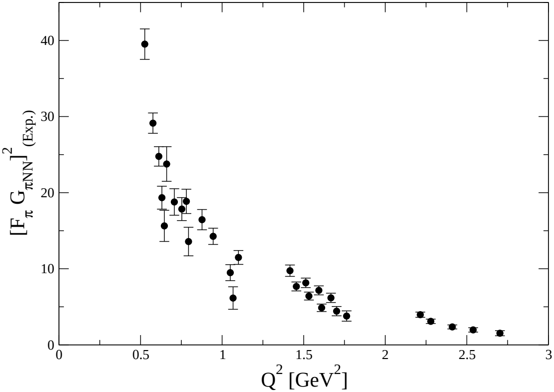


Figure : The pion electromagnetic form factor F_1, times $G_{\pi NN}(t)$ extract from cross-section data, in terms of Q^2.
Data from Block et al., PRC78 (2008) 045202.

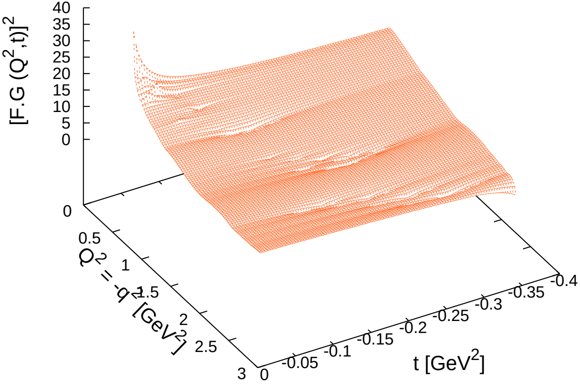


Figure : The pion electromagnetic form factor F_1, times $G_{\pi NN}(t)$ extract from cross-section data, in terms of t, and Q^2 .

Data from Block et al., PRC78 (2008) 045202.

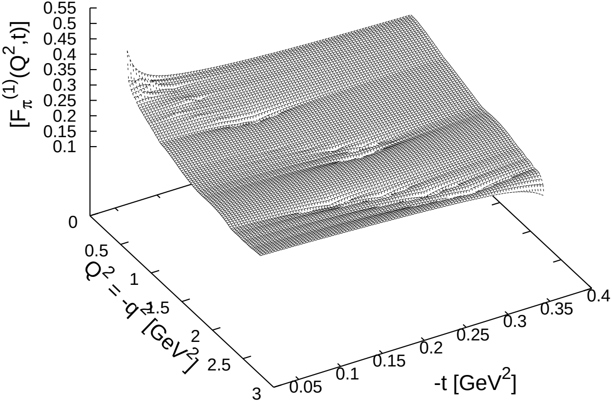


Figure : The experimental pion electromagnetic form factor, $F_{\pi}^{(1)}$, extract from cross-section data, in terms of t, and Q^2, with Choice I for $G_{\pi NN}(t)$, with $G_{\pi NN}(0) = 13.4$ and $\Lambda_{\pi} = 0.80$.

Data from Block et al., PRC78 (2008) 045202.

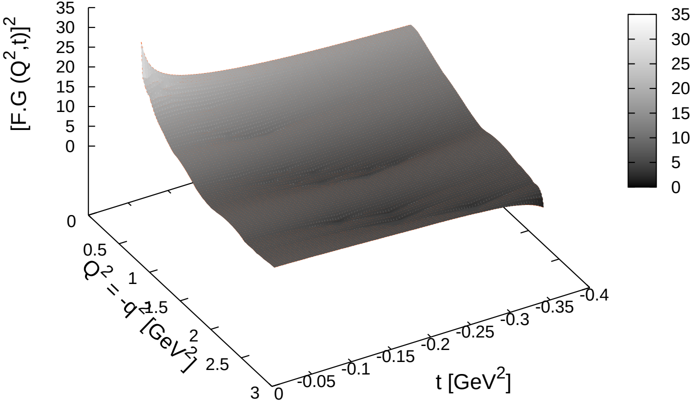


Figure : The pion electromagnetic form factor F_1, times $G_{\pi NN}(t)$, calculate with LFCQM non-symmetric vertex, in terms of t, and Q^2.

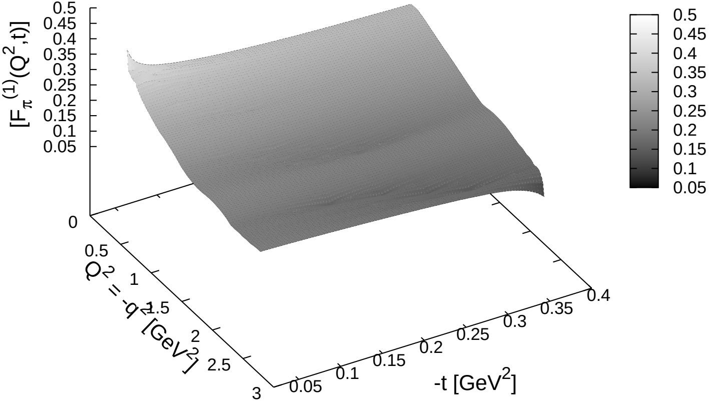


Figure : The pion electromagnetic form factor, $F_\pi^{(1)}$, calculated with LFCQM, in terms of t, and Q^2, for $G_{\pi NN}(t)$, with $G_{\pi NN}(0) = 13.4$ and $\Lambda_\pi = 0.80$.

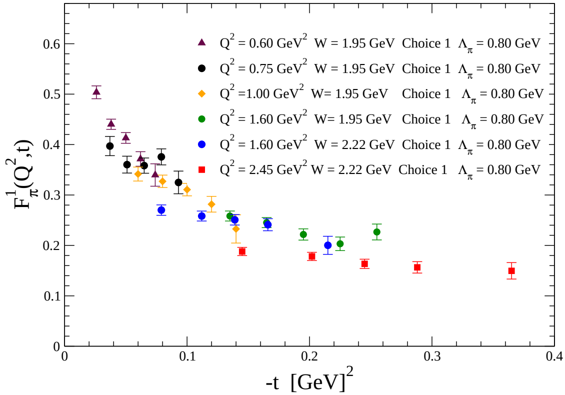


Figure : Experimental pion form factor, $F_1(Q^2, t^2)$, extracted from cross section, σ_l in function of t, with the choice 1 for $G_{\pi NN}(t)$, with $G_{\pi NN}(m_\pi^2) = 13.40$, and, $\Lambda_\pi = 0.80\ GeV$.

Exp. Data: Block et al., [TJLAB] PRC78 (08)045202.

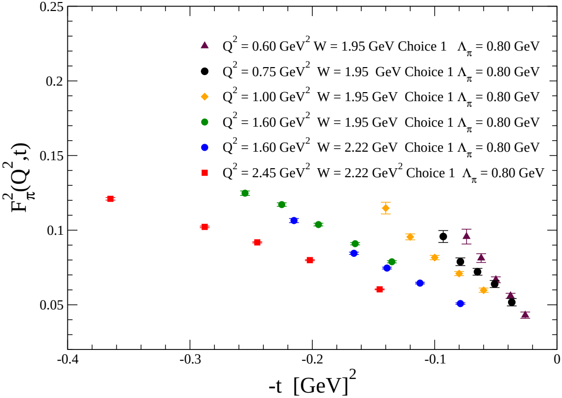


Figure : Experimental pion form factors, $F_2(Q^2, t)$, extracted from cross section, σ_I in function of t, with the choice 1 for $G_{\pi NN}(t)$, with $G_{\pi NN}(m_\pi^2) = 13.40$, and, $\Lambda_\pi = 0.80\ GeV$.

Exp. Data: Block et al., [TJLAB] PRC78 (08)045202.

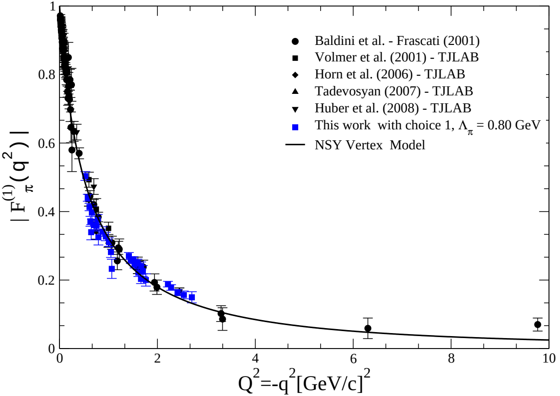


Figure : (Blue points) Experimental pion form factors, $F_1(Q^2, t^2)$, extracted from cross section data, σ_I in function of Q^2, with the choice 1, choice 1 for $G_{\pi NN}(t)$, with $G_{\pi NN}(m_\pi^2) = 13.40$, and, $\Lambda_\pi = 0.80$ GeV, compared with the experimental previus literature; and, also, with a light-front constituient quark model (LFQCM).
Exp. Data: Block et al., [TJLAB] PRC78 (08)045202.

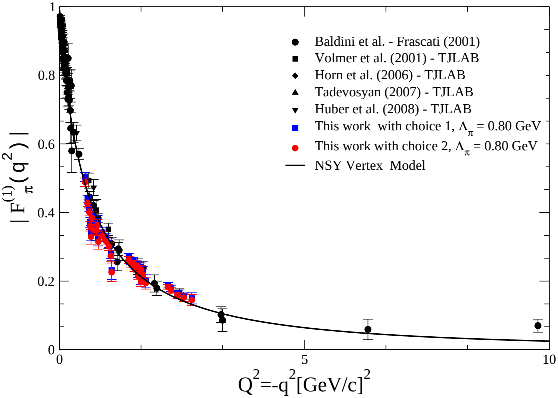


Figure : Experimental pion form factors, $F_1(Q^2, t^2)$, extracted from cross section, σ_I in function of Q^2, with the choice 1 and 2, for both form factors, with $G_{\pi NN}(m_\pi^2) = 13.40$, and, $\Lambda_\pi = 0.80\ GeV$, compared with the experimental previus literature; and, also, with a light-front constituient quark model (LFQCM). **Exp. Data: Block et al., [TJLAB] PRC78 (08)045202.**

- **Also: Contribuition from pion resonance // Preliminary Results!!**

$$F_\pi^{(2)}(q^2,t) = \frac{m_\pi^2 - t}{q^2}\left[1 - \left(F^{(1)-onshell)}(q^2) + \frac{t - m_\pi^2}{t - m_\pi^{*2}}\frac{G^*_{\pi NN}(t)}{G_{\pi NN}(t)}\; F_{\pi^*\pi}(Q^2,t)\right)\right]$$

$$F_\pi^{(1)(off-shell)}(q^2,t) = F_\pi^{(1)(on-shell}(q^2) + \frac{t - m_\pi^2}{t - m_\pi^{*2}}\frac{G_{\pi^* NN}(t)}{G_{\pi NN}(t)} F_{\pi^*\pi}(Q^2,t)$$

- **We can extracted the following function**

$$H_2(Q^2,t) = \frac{G_{\pi^* NN}(t)}{G_{\pi NN}(t)}\;.\; F_{\pi^* NN}(Q^2,t) = \left[F_\pi^{(1-off)} - F_\pi^{(1-on)}\right]\frac{(t - m_\pi^{*2})}{(t - m_\pi^2)}$$

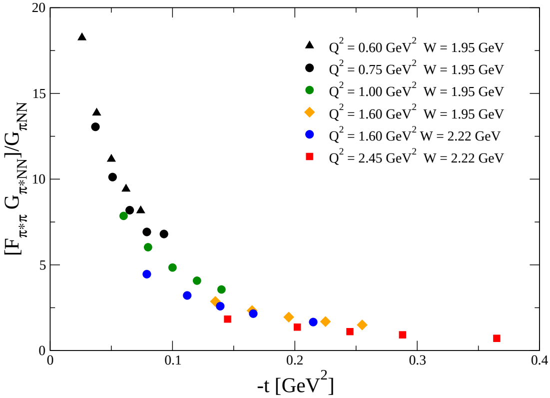


Figure : Function $H_2(Q^2, t)$ times t, extract with the experimental data, and, also, with a light-front LFCQM.

Exp. Data: Block et al., [TJLAB] PRC78 (08)045202.

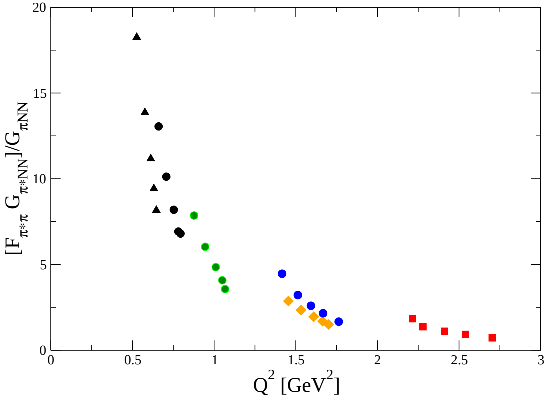


Figure : Function $H_2(Q^2, t)$ times Q^2, extract with the experimental data, and, also, with a light-front LFCQM.

Exp. Data: Block et al., [TJLAB] PRC78 (08)045202.

Summary

- Show $F_1(Q^2,t)\ X\ G_{\pi NN}(t)$ from experiments
- Use $F_1(Q^2,t)$ and $F_2(Q^2,t)$ relationship to extract $F_2(Q^2,t)$
- Choose $G_{\pi NN}(t)$ with two differents parametrizations
- Show $F_2(Q^2,t) \neq 0$ no matter what $G_{\pi NN}(t)$ is used
- Correction from m_{π^*} and "some" parametrization to $G^*_{\pi^* NN}(t) \Rightarrow$ extract $F^*_{\pi^* NN}(Q^2,t)$ from data?
- Compute $F_{\pi^* NN}$ from some LFCQM

Thanks to the Organizers
Light-Cone 2018 - TJLAB - Virginia - USA

Support LFTC and Brazilian Agencies

- **FAPESP , CNPq and CAPES**

Thanks (Obrigado)!!