Non-triviality of the vacuum in light-front quantization

John Collins (Penn State)

- A commonly stated advantage of light-front quantization is triviality of the vacuum.
- If that is true, an important implication is the vanishing of vacuum bubbles and hence a solution of the cosmological constant problem. (Brodsky & Shrock, PNAS 108,45 (2011))
- However, vacuum triviality is false, as known since Chang & Ma (1969).
- Here I give a simple account of the issues.

(See JCC, arXiv:1801.003960)

Summary

- Paradox
- View within Feynman graph methods

  But do Feynman graphs miss something (cf. Brodsky & Shrock)?
- View within pure light-front methods
- Implications
- [LF wave functions]

Background

- Light-front coordinates $(x^+, x^-, \boldsymbol{x}_\mathsf{T}) = ((t+z)/\sqrt{2}, (t-z)/\sqrt{2}, \boldsymbol{x}_\mathsf{T})$.
- Use Heisenberg picture:
  - States time-independent;
  - Fields are $\phi(x)$, etc, with all t, x^+ dependence;
  - Lagrangian density specifies theory;
  - Equations of motion for fields in space-time;
  - CRs specified on quantization surface (fixed x^+ or fixed t or . . .).

  Then can treat equal-x^+ and equal-t quantization in same framework, given a solution of theory.
- Light-front analysis and annihilation and creation operators:

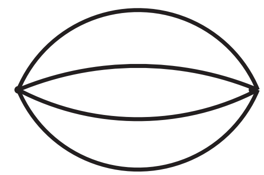
$$\phi(x) = \int \frac{\mathrm{d}k^+ \, \mathrm{d}\boldsymbol{k}_\mathsf{T}}{(2\pi)^3} e^{-ik^+x^- + i\boldsymbol{k}_\mathsf{T}\cdot\boldsymbol{x}_\mathsf{T}} \; \widetilde{\phi}(x^+; k^+, \boldsymbol{k}_\mathsf{T})$$

$$= \int \frac{\mathrm{d}k^+ \, \mathrm{d}\boldsymbol{k}_\mathsf{T}}{2k^+(2\pi)^3} \theta(k^+) \left[e^{-ik^+x^- + i\boldsymbol{k}_\mathsf{T}\cdot\boldsymbol{x}_\mathsf{T}} a_k(x^+) + e^{ik^+x^- - i\boldsymbol{k}_\mathsf{T}\cdot\boldsymbol{x}_\mathsf{T}} a_k(x^+)^\dagger \right].$$

- x^+-ordered perturbation theory: intermediate states with on-shell particles and "energy" (P^-) denominators.

Paradox I

- Standard statement: Vacuum bubble:



  is zero in LF quantization: External $P^+ = 0$, intermediate states have $k^+ > 0$.

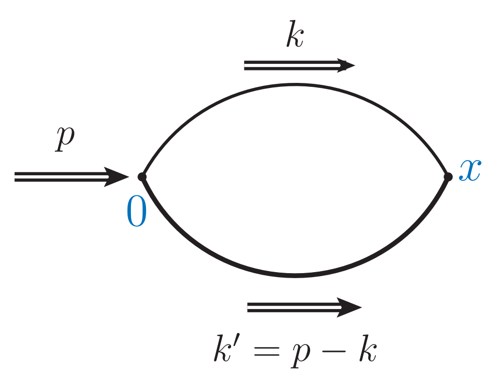
- Same rationale applies to *any* loop graph with imposed external $P^+ = 0$.

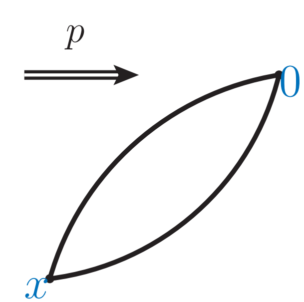
- Example: Green function

$$\Pi(p^2) \stackrel{\text{def}}{=} \int d^2x\; e^{ip\cdot x} \langle 0|T \tfrac{1}{2}\phi^2(x) \tfrac{1}{2}\phi^2(0)|0\rangle_{\text{connected}}$$

  in free scalar QFT in $1+1$ dimensions.

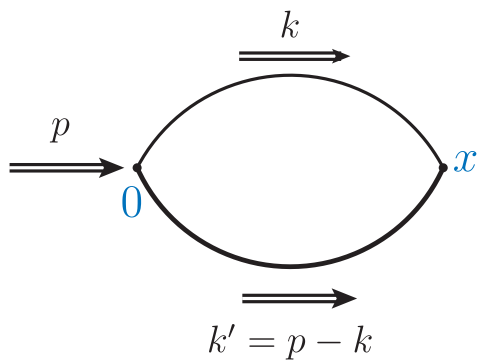
- x^+-ordered graphs:

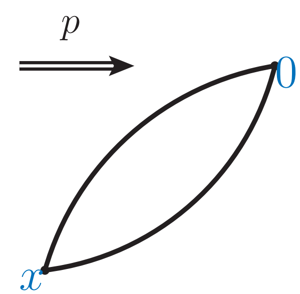




Paradox II

- x^+-ordered graphs:





- When $p^+ > 0$:

$$\Pi(p^2) = \frac{1}{4\pi} \int_0^{p^+} \frac{dk^+}{4k^+(p^+ - k^+)} \frac{i}{p^- - \frac{m^2}{2k^+} - \frac{m^2}{2(p^+ - k^+)} + i\epsilon}$$

$$= \frac{i}{8\pi} \int_0^1 d\xi \, \frac{1}{p^2 \xi(1-\xi) - m^2 + i\epsilon},$$

Agrees with Feynman-graph calculation.

- When $p^+ = 0$: Standard statement is that $\Pi(0) = 0$.
- But $\Pi(p^2)$ is analytic at all p^2 (except $p^2 = 4m^2$).

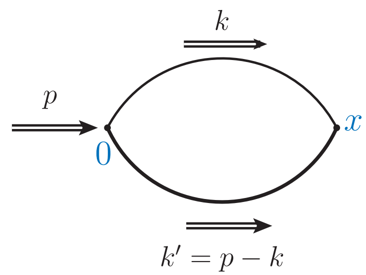
Feynman graph view

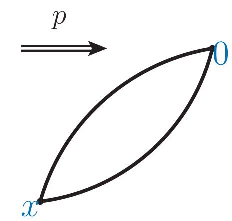
- Chang & Ma (1969):
  - Derivation of x^+-ordered perturbation theory from Feynman perturbation theory by integrating over k^- loop momenta.
  - Derivation needs correction at external $p^+ = 0$ by delta-function terms: A kind of zero mode contribution

- But Brodsky & Shrock raise issue that LF quantization may give different results than equal-time quantization and standard Feynman graphs.

- Therefore address issue within derivation of LF methods . . .

Light-front calculation: its derivation





$$\Pi(p^2) = \frac{1}{8\pi^2} \int_{-\infty}^{\infty} \frac{dk^+}{2k^+} \int_{-\infty}^{\infty} \frac{dk'^+}{2k'^+} \int_{-\infty}^{\infty} dx^- e^{ix^-(p^+ - k^+ - k'^+)}$$

$$\left[\theta(k^+)\theta(k'^+) \frac{i}{p^- - \frac{m^2}{2k^+} - \frac{m^2}{2k'^+} + i\epsilon} + \theta(-k^+)\theta(-k'^+) \frac{i}{-p^- + \frac{m^2}{2k^+} + \frac{m^2}{2k'^+} + i\epsilon} \right]$$

Get delta-function for momentum conservation by

$$\int dx^- e^{ix^-(p^+ - k^+ - k'^+)} = 2\pi\delta(p^+ - k^+ - k'^+),$$

BUT only if integrated with function continuous at $p^+ = k^+ + k'^+$.

Theorem fails at $p^+ = 0$.

Get (correct and non-zero) result by change of variable to ξ and K^+, with $k^+ = \xi K^+$ and $k'^+ = (1 - \xi)K^+$.

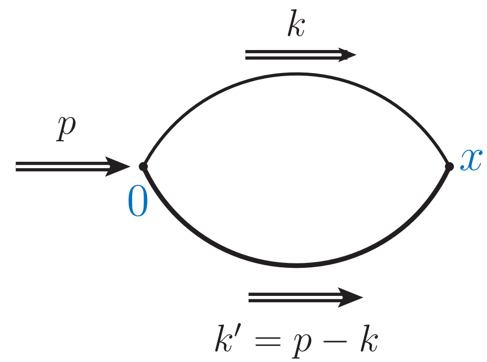
Conclusions

- Method that appears to show vacuum bubbles are zero gives wrong result for loop in ordinary Green function at zero external p.
- Derivation of LF perturbation theory fails there because derivation of delta function for momentum conservation fails.
- Correct results are given by rules of Chang & Ma and of Yan, and agrees with Feynman perturbation theory.
- Issue is confined to modes with $y \to \infty$, so most simplifications in light-front quantization still apply: A kind of zero-mode issue.
- That allows definition of LF wave functions.
- Warning about solvable problems: UV renormalization; rapidity divergences in gauge theories.
- There are implications not yet analyzed.

—EXTRAS—

—FEYNMAN GRAPH VIEW—

Feynman graph I



$$\Pi(p^2) = -\frac{1}{8\pi^2}\int dk^+ dk^- \frac{1}{[2k^+k^- - m^2 + i\epsilon][2(k^+ - p^+)(k^- - p^-) - m^2 + i\epsilon]}$$

At $p = 0$

$$\Pi(0) = -\frac{1}{8\pi^2}\int dk^+ dk^- \frac{1}{[2k^+k^- - m^2 + i\epsilon]^2}$$

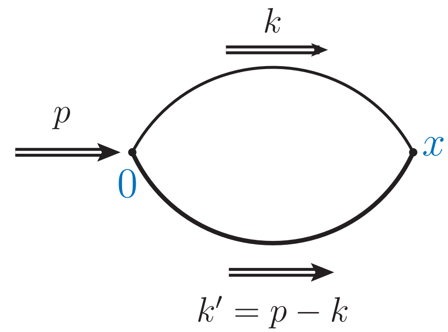
Then

– When $k^+ > 0$ close in upper half plane and get zero.
– When $k^+ < 0$ close in lower half plane and get zero.

But convergence at large $|k^-|$ is non-uniform.

Non-zero contribution from $|k^-| \to \infty$, $|k^+| = O(1/k^-)$, but only if $p^+ = 0$

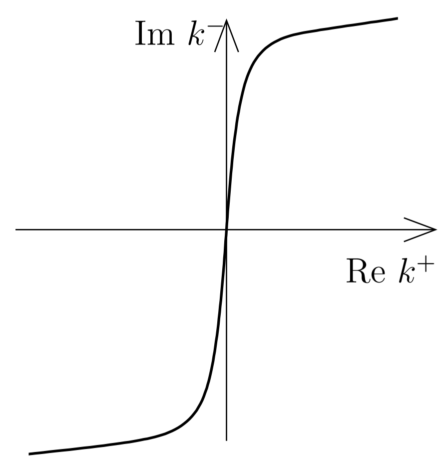
Feynman graph II: Problem of break in contour



$$\Pi(0) = -\frac{1}{8\pi^2}\int dk^+ dk^- \frac{1}{[2k^+k^- - m^2 + i\epsilon]^2}$$

When $k^+ \gtrless 0$ close in upper/lower half plane.

Need to fill in break in contour:



—WAVE FUNCTIONS—

Statements of properties of light-front wave functions

- Define light-front wave functions by

$$\psi((k_1^+, \boldsymbol{k}_{1\,\mathsf{T}}), \dots) = \langle 0| \prod_{j=1}^{N} a_{k_j} |\psi\rangle.$$

- That works in super-renormalizable non-gauge theories. (E.g., Yukawa theory in $2+1$ dimensions.

- Need to check completeness, and e.g.,

$$|\psi\rangle = \sum_N \frac{1}{N!} \int \prod_{j=1}^{N} a_{k_j}^\dagger |0\rangle \psi_{k_1,\dots,k_N},$$

- Complications:
  - UV renormalization: $a_{k,\text{ren.}} = \frac{1}{\sqrt{Z}} a_{k,\text{unren.}}$, with $Z \to 0$ as UV cutoff removed.
  - Rapidity divergences.

- Rapidity divergences:
  - Were found by CSS in closely related case of TMD partonic functions; finite modified definitions were constructed.
  - Corresponding analysis for the LF wave functions themselves was provided by Ma & Wang (PLB 642, 232 (2006), arXiv:hep-ph/0605075).
  - A modernized treatment was given by Li & Wang (JHEP 06, 013 (2015), arXiv:1410.7274).