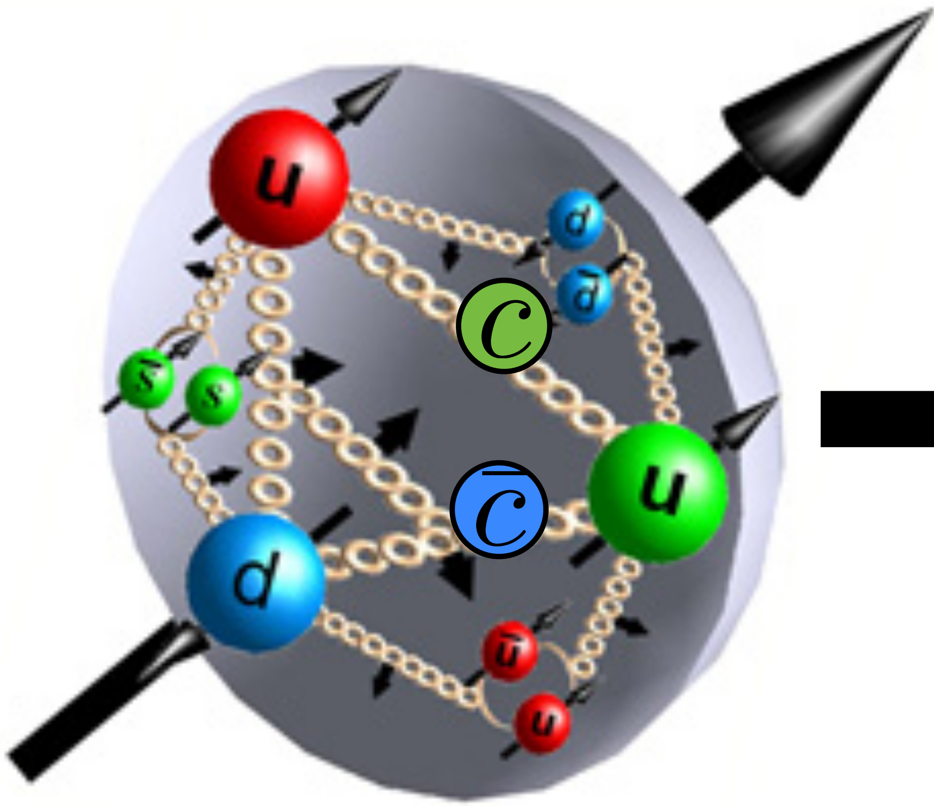
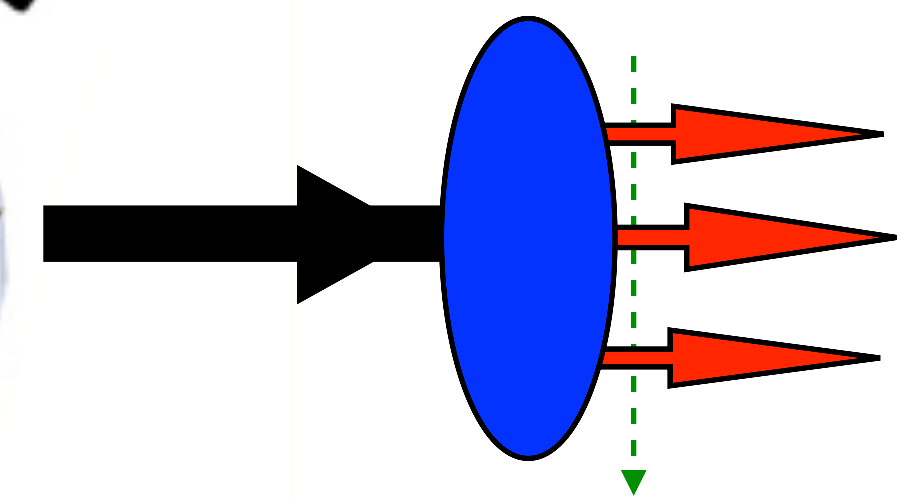
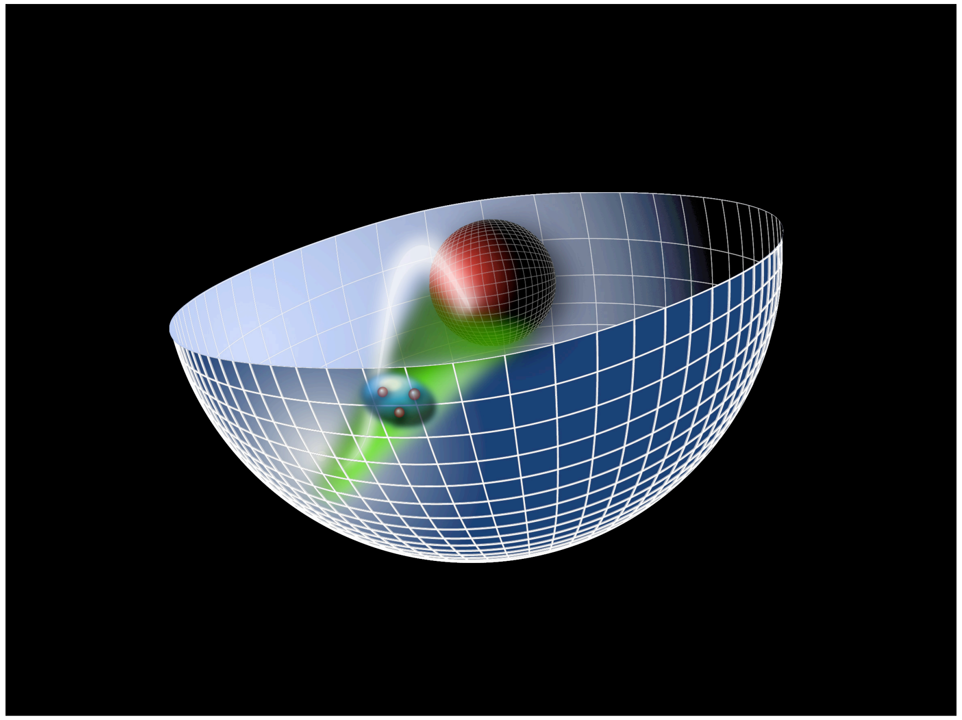
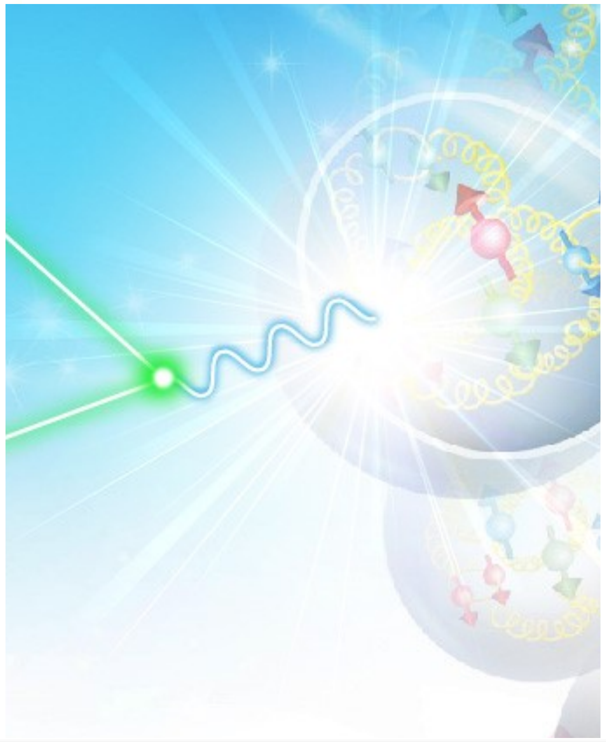
Hadron Physics on the Light-Front: Hadron Dynamics, Spectroscopy and Vacuum Structure from Light-Front Holography and Superconformal Algebra



Fixed $\tau = t + z/c$







Stan Brodsky

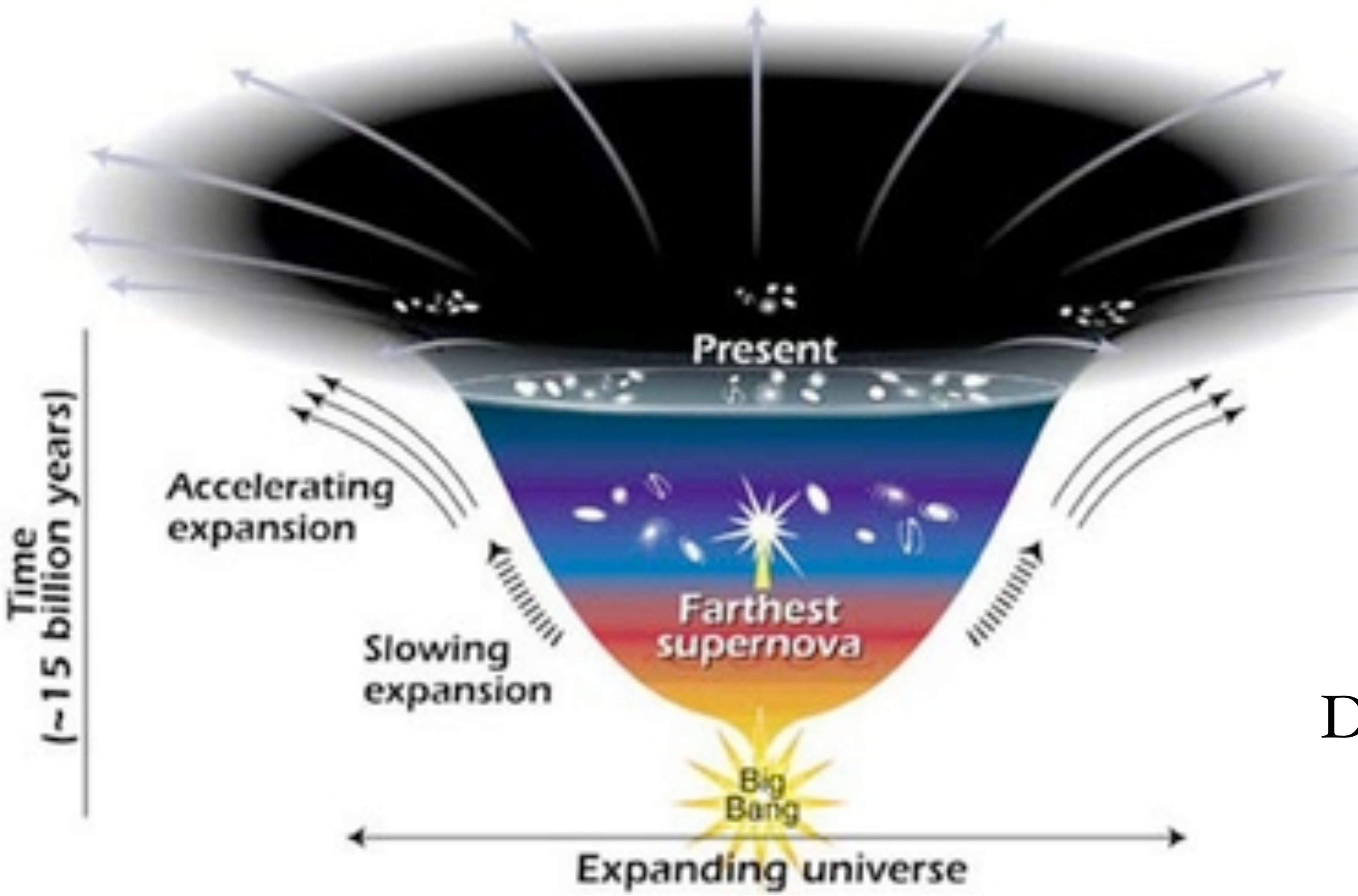
LC2018, May 15, 2018

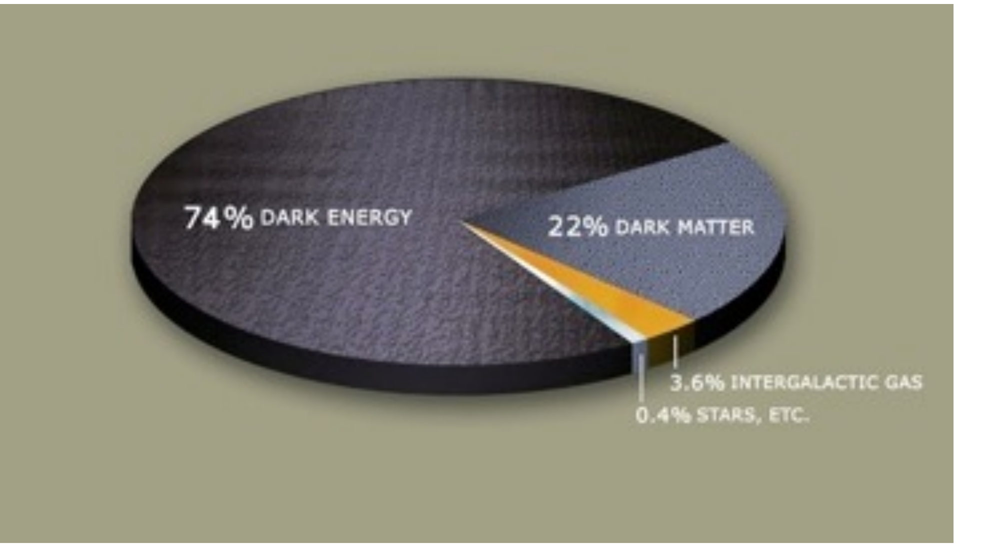
Jefferson Lab

SLAC NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY

with Guy de Tèramond, Hans Günter Dosch, R. Shrock, C. Roberts, P. Tandy, C. Lorcè, M. Nielsen, Prem Srivistava, R. S. Sufian, A. Deur





$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = (8\pi G_N) T_{\mu\nu}$$

↑

Dark energy/cosmological constant causes accelerating expansion

$$\frac{1}{a}\frac{d^2}{dt^2} a = \Lambda/3 = (8\pi) G_N \rho_\Lambda / 3$$

If the vacuum energy ρ is due to QCD condensates

$$\rho_\Lambda^{\mathrm{QCD}} \simeq M_{\mathrm{QCD}}^4 \simeq 10^{45} \rho_\Lambda^{\mathrm{obs}} \; !$$

$$\Omega_\Lambda = \frac{\rho_\Lambda^{\mathrm{obs}}}{\rho_c} \simeq 0.76 \qquad \rho_c = \frac{3H_0^2}{8\pi G_N}$$

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

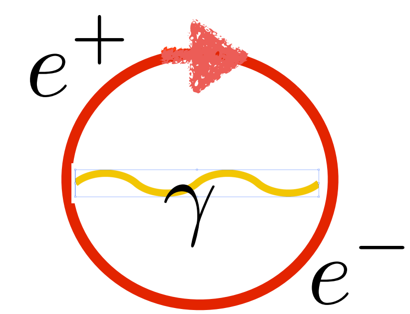
$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:
(A) Light-Front Quantization: causal, frame-independent vacuum
(B) New understanding of QCD "Condensates"
(C) Higgs Light-Front Zero Mode

Instant-Form Vacuum in QED



- Loop diagrams of all orders contribute
- Huge vacuum energy: $\rho_\Lambda^{QED} \simeq 10^{120} \rho_\Lambda^{Observed}$
- $\frac{E}{V} = \int \frac{d^3k}{2(2\pi)^3} \sqrt{\vec{k}^2 + m^2}$ Cut off the quadratic divergence at M_{Planck}
- Frame-dependent, acausal
- Divide S-matrix by disconnected vacuum diagrams
- In Contrast: Light-Front Vacuum trivial since plus momenta are positive and conserved: $k^+ = k^0 + k^3 > 0$

- Why is the cosmological constant so small, $\Lambda < 10^{-120}$ in Planck density units ?

" ***Most embarrassing observation in physics*** – ***that's the only quick thing I can say about dark energy that's also true.***"   **-- Edward Witten**

String Theory Landscape

Renata Kallosh

Metaphysics of the Vacuum

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0>= E_0|\psi_0>, E_0 = min\{E_i\}$$

Eigenstate defined at one time t over all space; Acausal! Frame-Dependent

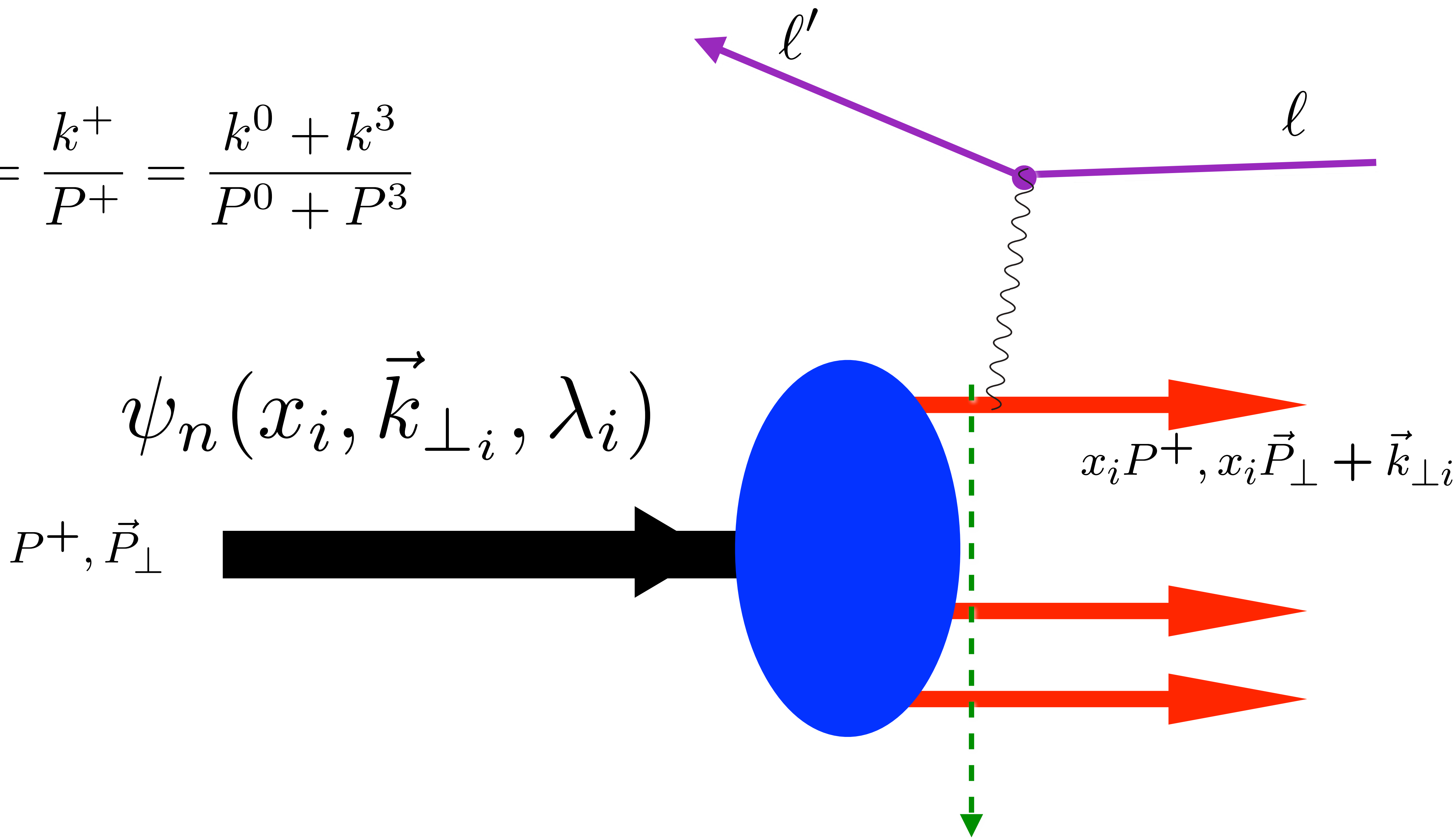
Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

$$H_{LF}|\psi_0>_{LF}= M_0^2|\psi_0>_{LF}, M_0^2 = 0.$$

Frame-independent eigenstate at fixed LF time τ = t+z/c within causal horizon

Frame-independent description of the causal physical universe!

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

Each element of flash photograph illuminated at same Light-Front time

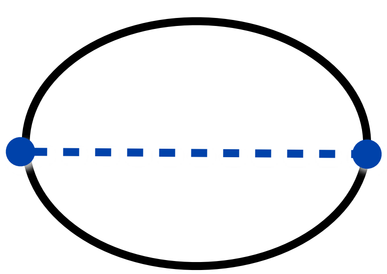
$$\tau = t + z/c$$

Compton and electron scattering like a flash photograph

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Causal, Trivial Vacuum



zero !!

Zero Cosmological Constant

We view the universe as light reaches us along the light-front at fixed

$$\tau = t + z/c$$

Front-Form Vacuum Describes the Empty, Causal Universe

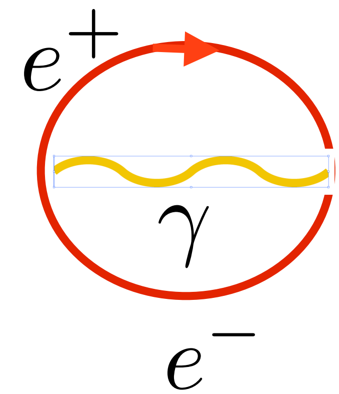
Roberts, Shrock, Tandy, sjb

"Essence of the vacuum quark condensate," Phys. Rev. C **82**, 022201 (2010)

"Confinement contains condensates," Phys. Rev. C **85**, 065202 (2012)

Front-Form Vacuum in QED

$P^+ = 0$



$\sum_i k_i^+ \neq P^+ = 0$

$k_i^+ > 0$

- **All Light-Front Perturbative Vacuum Loop Amplitudes Vanish!**
- **Light-Front Vacuum is trivial since all plus momenta are positive and conserved.**
- **Zero modes ($k^+=0$) in vacuum allowed in some theories**
- **Zero contribution to Λ from QED LF Vacuum**
- **Instant Form gives same result if one normal-orders.**

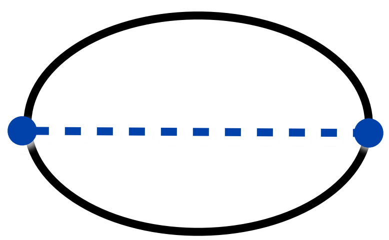
Front-Form Vacuum ($P^\mu=0$)

All LF propagators have positive k^+

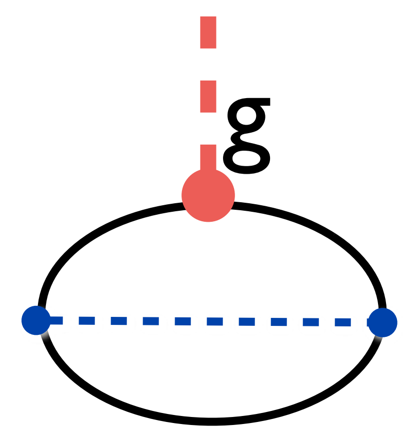
$$k^+ = k^0 + k^3 \geq 0 \text{ since } |\vec{k}| \leq k^0$$

P^+ Momentum Conserved

$P^+ = 0$



zero !!



zero !!

$$< 0|T^{\mu\nu}|0 >= 0$$

Graviton does not couple to LF vacuum!

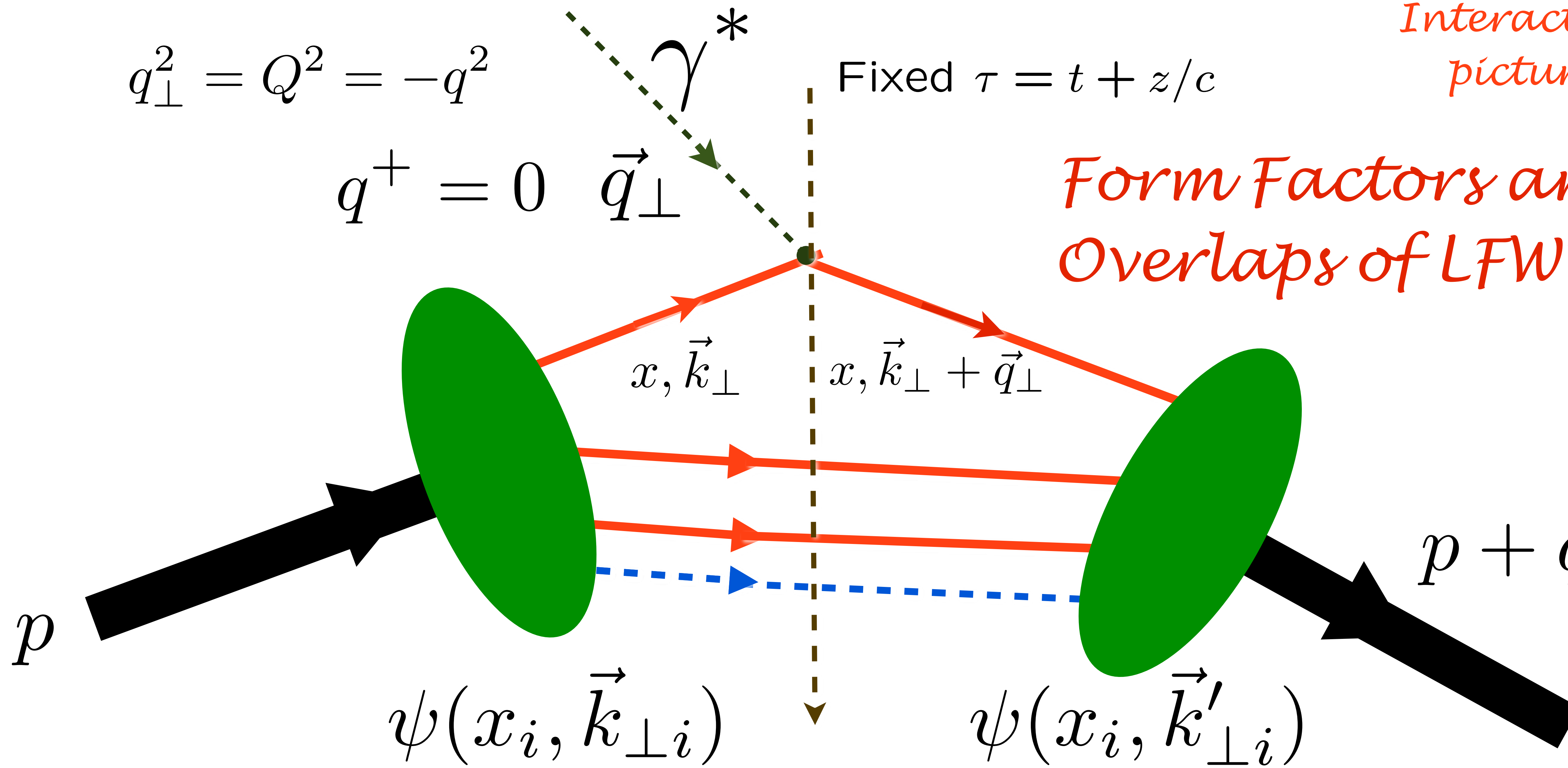
Vanishing graviton coupling even in presence of zero modes

$$< p+q|j^+(0)|p >= 2p^+ F(q^2)$$

Front Form

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Gravitational Form Factors

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \overline{u}(P')\Big[A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P)\ ,$$

where $q^\mu = (P' - P)^\mu,\ \overline{P}^\mu = \frac{1}{2}(P' + P)^\mu,\ a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

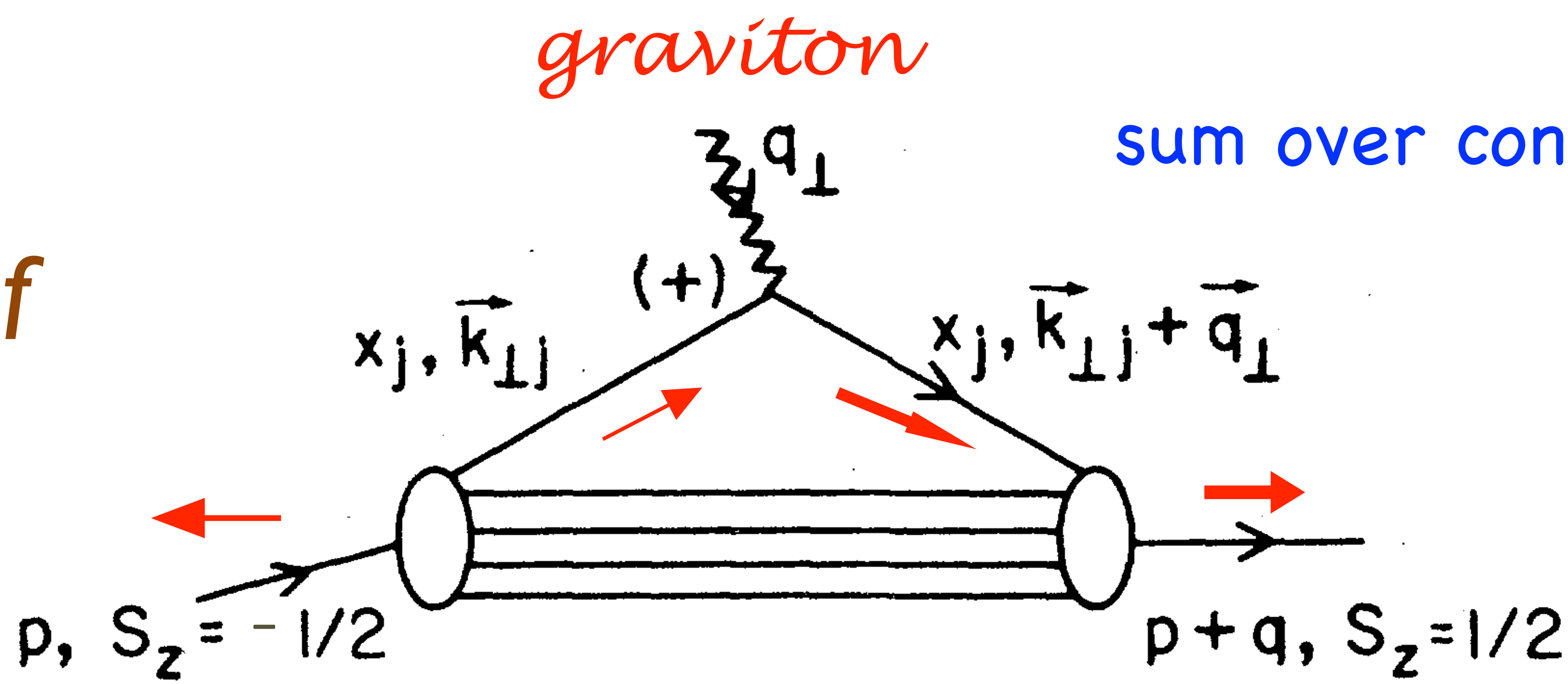
$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\uparrow\right\rangle = A(q^2)\ ,$$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\downarrow\right\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M}\ .$$

Dae Sung Hwang, Bo-Qiang Ma, Ivan Schmidt, sjb

Terayev, Okun: *B(0) Must vanish because of Equivalence Theorem*

LF Proof



sum over constituents

B(0) = 0

Each Fock State

Vanishing Anomalous gravitomagnetic moment B(0)

Light-Front vacuum can simulate empty universe

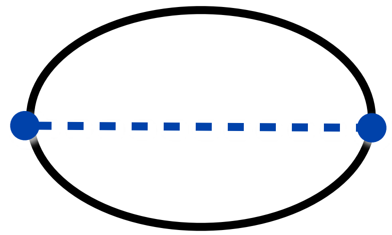
Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state M= 0.
- Trivial up to $k^+=0$ zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: "In-hadron"condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no vacuum loops
- Zero cosmological constant from QED, QCD, EW

Front Form Vacuum Describes the Empty, Causal Universe

- $P^+ = \sum_i p_i^+,\ p_i^+ > 0$: LF vacuum is the state with $P^+ = 0$ and contains no particles: all other states have $P^+ > 0$ (usual vacuum bubbles are kinematically forbidden in the front form !)

- Frame independent definition of the vacuum within the causal horizon



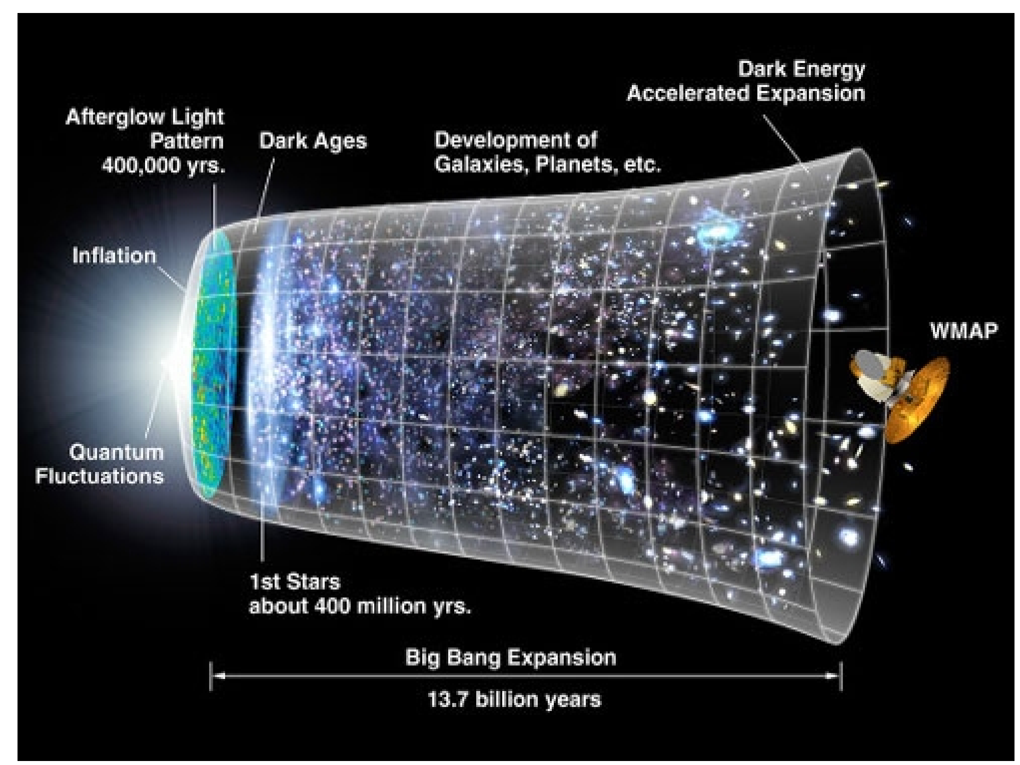
zero !!

$$P^2|0\rangle = 0$$

(LF vacuum also has zero quantum numbers and $P^+ = 0$)

- LF vacuum is defined at fixed LF time $x^+ = x^0 + x^3$ over all $x^- = x^0 - x^3$ and $\mathbf{x}_\perp$, the expanse of space that can be observed within the speed of light

Roberts, Shrock, Tandy, sjb



- Causality is maintained since LF vacuum only requires information within the causal horizon

- The front form is a natural basis for cosmology: universe observed along the front of a light wave

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

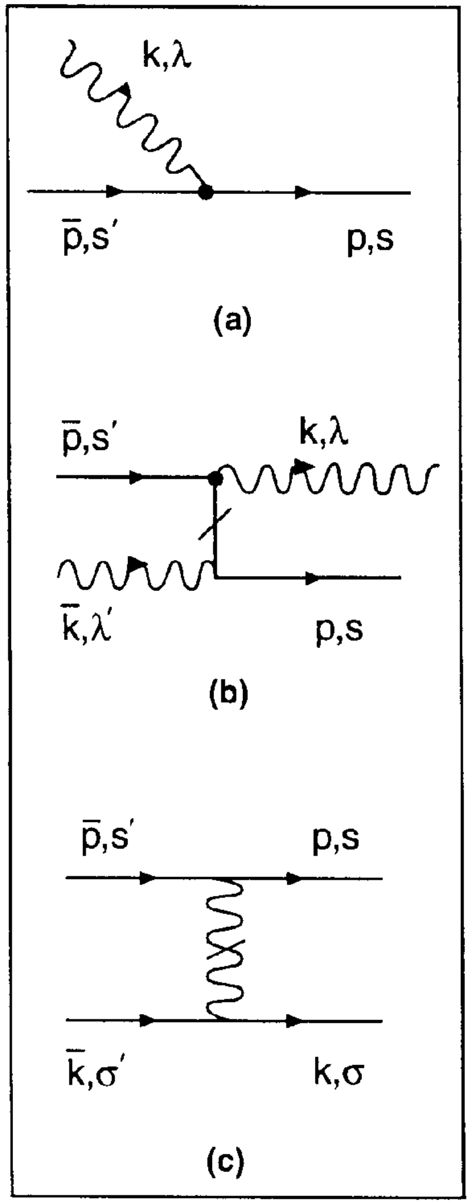
$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

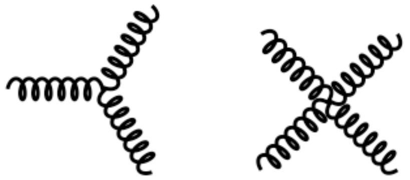
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



LFWFs: Off-shell in P- and invariant mass

H^{int}_{LF}

Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + \cdots$$

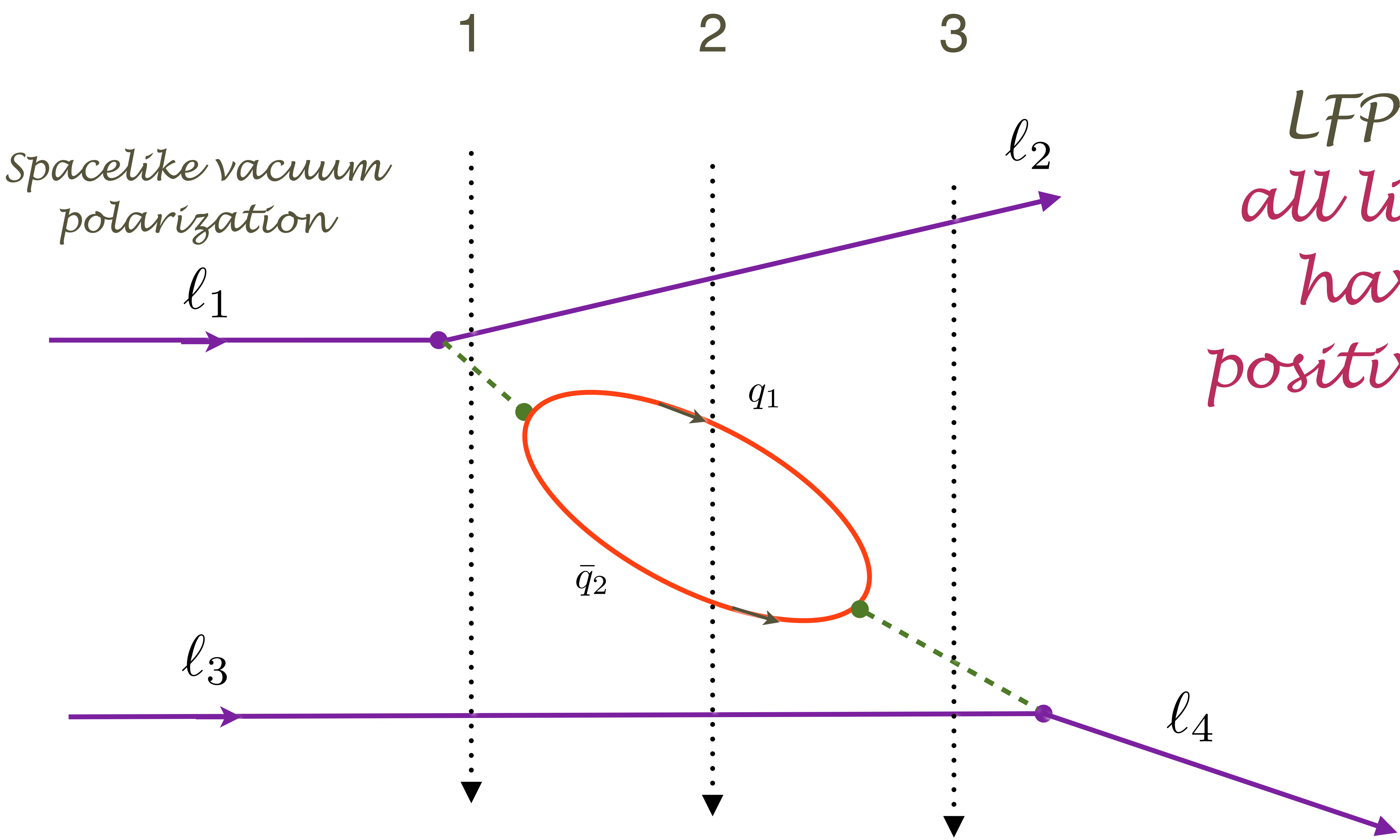
- *"History" : Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!*
- *Wick Theorem applies, but few amplitudes since all $k^+ > 0$.*
- *J_z Conservation at every vertex* $| \sum_{initial} S^z - \sum_{final} S_z | \leq n$ *at order g^n*
- *Unitarity is explicit*

K. Chiu, sjb

- *Loop Integrals are 3-dimensional* $\int_0^1 dx \int d^2 k_\perp$
- *hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Roskies, Suaya, sjb (two loops)

P. Lowdon, K. Chiu, sjb



LFPth
all lines have positive k+

$$< \ell_4 \ell_2 | H_I | \gamma_2 \ell_3 \ell_2 > \frac{1}{D_3} < \gamma_2 \ell_2 \ell_3 | H_I | q_1 \bar{q}_2 \ell_3 \ell_2 > \frac{1}{D_2} < \ell_2 \ell_3 q_1 \bar{q}_2 | H_I | \ell_3 \ell_2 \gamma_1 > \frac{1}{D_1} < \gamma_1 \ell_2 \ell_3 | H_I | \ell_1 \ell_3 >$$

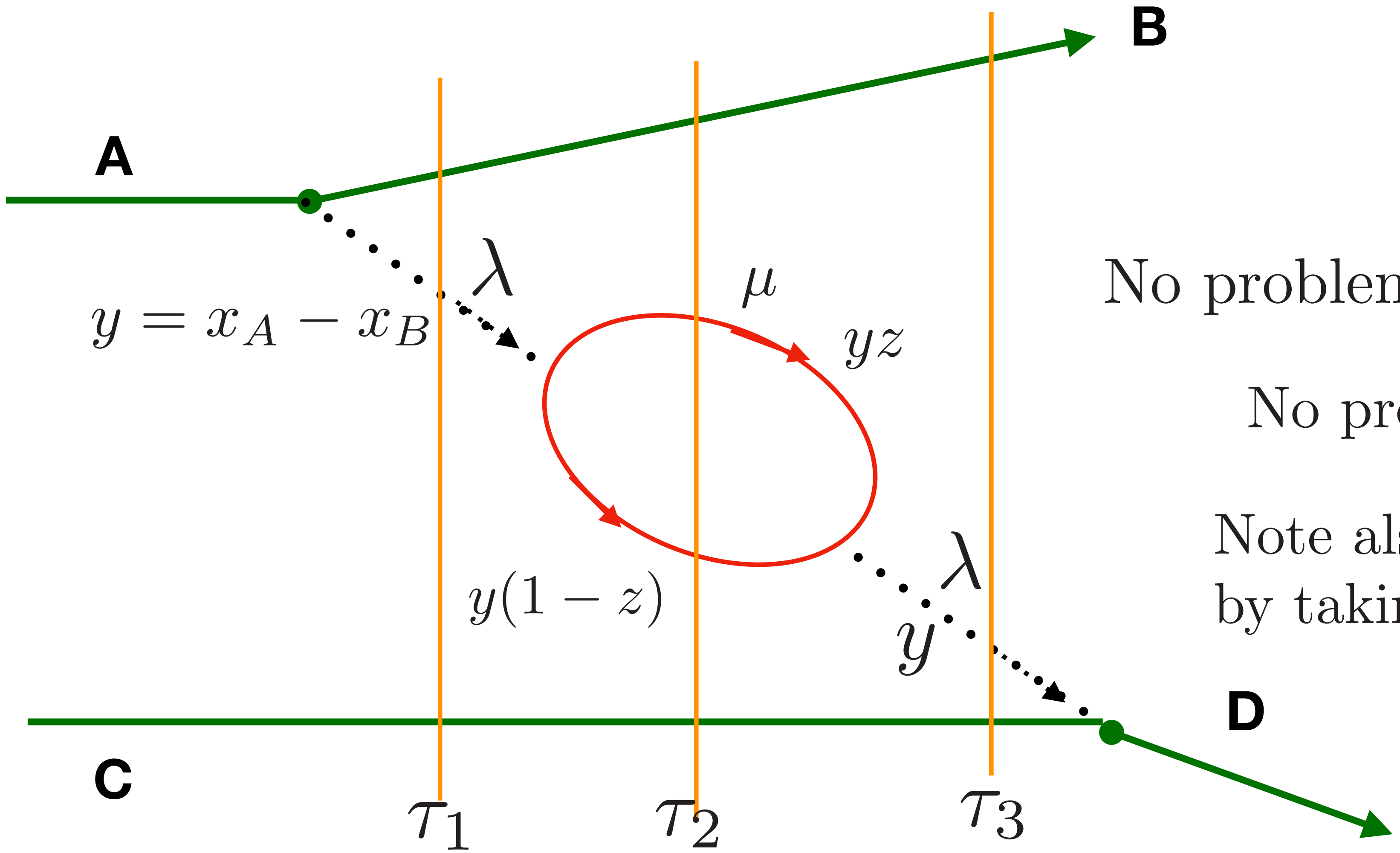
- **Initial or Final Leptons always present in each intermediate state**
- **Sum over LF time-orderings gives covariant result for $\Pi(Q^2)$**

No zero modes appear!

$$D_1 \times (x_A - x_B) = [\frac{m_A^2}{x_A} - \frac{m_B^2}{x_B} - \frac{\lambda^2}{x_A - x_B}](x_A - x_B) = (\frac{m_A^2}{x_A} - \frac{m_B^2}{x_B})(x_A - x_B) - \lambda^2 \equiv t - \lambda^2$$

$$D_2 \times yzy(1-z) = [\frac{m_A^2}{x_A} - \frac{m_B^2}{x_B} - \frac{\mu^2}{yz} - \frac{\mu^2}{y(1-z)}]yzy(1-z) = y[tz(1-z) - \mu^2]$$

$$D_3 = D_1$$



No problem for $y = x_A - x_B \to 0$

No problem for $-t \to 0$

Note also $-t \to 0$ at finite $y > 0$ by taking $m_A^2 = m_B^2 = 0$

$$T = \frac{g}{t-\lambda^2} \times \int_0^1 dz \frac{g^2}{tz(1-z)-\mu^2} \times \frac{g}{t-\lambda^2}$$

Analytically correct in t

PHYSICAL REVIEW D **66**, 045019 (2002)

Light-front formulation of the standard model

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(Received 20 February 2002; published 20 August 2002)

Light-front (LF) quantization in the light-cone (LC) gauge is used to construct a renormalizable theory of the standard model. The framework derived earlier for QCD is extended to the Glashow-Weinberg-Salam (GWS) model of electroweak interaction theory. The Lorentz condition is automatically satisfied in LF-quantized QCD in the LC gauge for the free massless gauge field. In the GWS model, with the spontaneous symmetry breaking present, we find that the 't Hooft condition accompanies the LC gauge condition corresponding to the massive vector boson. The two transverse polarization vectors for the massive vector boson may be chosen to be the same as found in QCD. The nontransverse and linearly independent third polarization vector is found to be parallel to the gauge direction. The corresponding sum over polarizations in the standard model, indicated by $K_{\mu\nu}(k)$, has several simplifying properties similar to the polarization sum $D_{\mu\nu}(k)$ in QCD. The framework is unitary and ghost free (except for the ghosts at $k^+=0$ associated with the light-cone gauge prescription). The massive gauge field propagator has well-behaved asymptotic behavior. The interaction Hamiltonian of electroweak theory can be expressed in a form resembling that of covariant theory, plus additional instantaneous interactions which can be treated systematically. The LF formulation also provides a transparent discussion of the Goldstone boson (or electroweak) equivalence theorem, as the illustrations show.

Standard Model on the Light-Front

- **Same phenomenological predictions**
- **Higgs field has three components**
- **Real part creates Higgs particle**
- **Imaginary part (Goldstone) become longitudinal components of W, Z**
- ***Higgs VEV of instant form becomes $k^+=0$ LF zero mode !***
- **Analogous to a background static classical Zeeman or Stark Fields**
- **Zero contribution to $T^\mu{}_\mu$; zero coupling to gravity**

P. Srivastava, sjb

Abelian U(1) LF Model with Spontaneous Symmetry Breaking

$$\mathcal{L} = \partial_+\phi^\dagger\partial_-\phi + \partial_-\phi^\dagger\partial_+\phi - \partial_\perp\phi^\dagger\partial_\perp\phi - \mathcal{V}(\phi^\dagger\phi)$$

where $V(\phi^\dagger\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$ with $\lambda > 0,\ \mu^2 < 0$

Constraint equation: $\int d^2x_\perp dx^- \left[\partial_\perp\partial_\perp\phi - \frac{\delta V}{\delta\phi^\dagger}\right] = 0$

$$\phi(\tau, x^-, x_\perp) = \omega(\tau, x_\perp) + \varphi(\tau, x^-, x_\perp)$$

$\omega(\tau, x_\perp)$ is a $k^+ = 0$ zero mode

$\omega = v/\sqrt{2}$ where $v = \sqrt{-\mu^2/\lambda}$

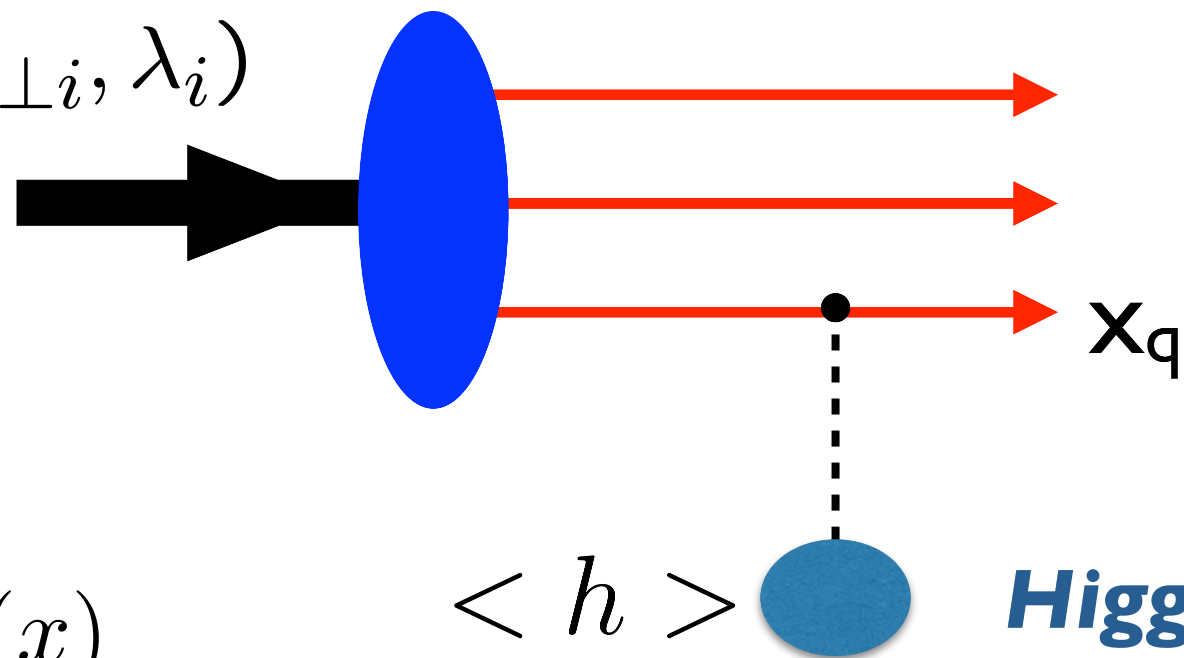
Thus a c-number in LF replaces conventional Higgs VEV

No coupling to gravity!

Possibility: $\partial_\perp\omega \neq 0$ **in curved universe**

Coupling of confined quarks to Higgs Zero Mode <h>

$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$



$g_q \bar{\psi}_q(x) \psi_q(x) h(x)$

$< h >$ **Higgs Zero Mode!**

Yukawa Higgs coupling of confined quark to Higgs zero mode gives

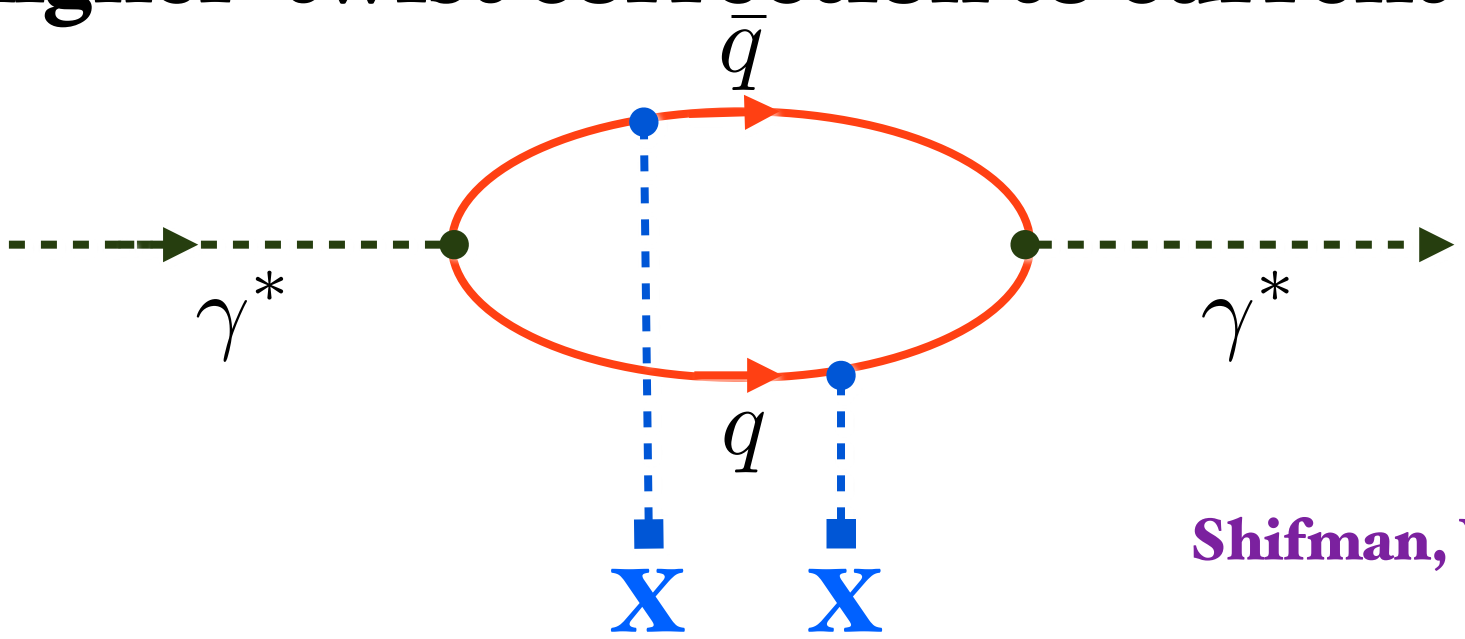
$$\bar{u}u \; g_q < h > = \frac{m_q}{x_q} m_q = \frac{m_q^2}{x_q}$$

$$H_{LFKE} = \sum_i \left[\frac{\vec{k}_\perp^2 + m_q^2}{x_q}\right]_i = \mathcal{M}^2 = \left[\sum_i k_q^\mu\right]^2$$

Is there empirical evidence for a gluon vacuum condensate?

$$< 0|\frac{\alpha_s}{\pi}G^{\mu\nu}(0)G_{\mu\nu}(0)|0 >$$

Look for higher-twist correction to current propagator



Shifman, Vainshtein, Zakharov

$$e^+e^- \to X,\ \tau\ \text{decay},\ Q\bar{Q}\ \text{phenomenology}$$

$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \frac{\alpha_s}{\pi}\frac{\Lambda^4_{\rm QCD}}{s^2} + \cdots)$$

Determinations of the vacuum Gluon Condensate

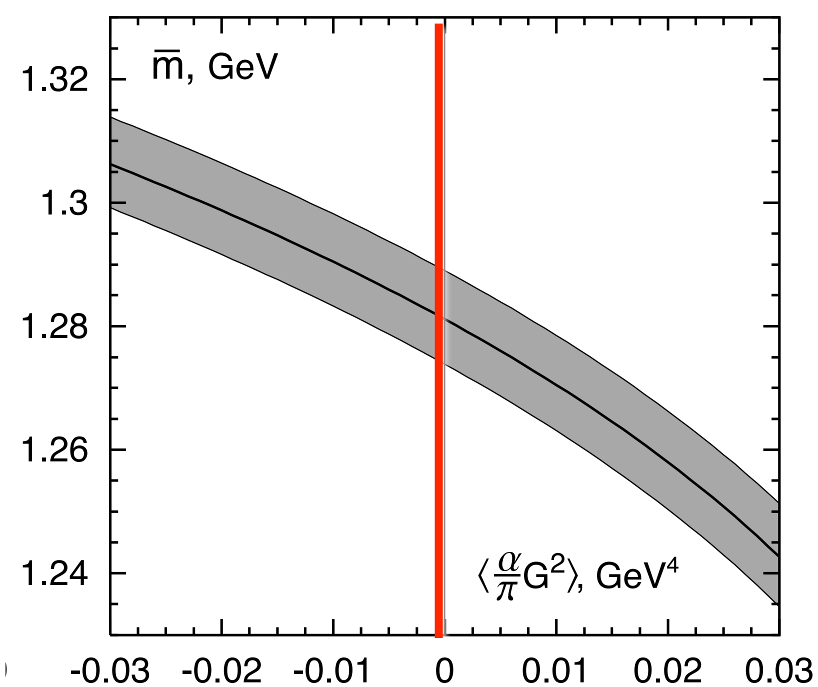
$$< 0|\frac{\alpha_s}{\pi}G^2|0 > [\mathrm{GeV}^4]$$

-0.005 ± 0.003 from τ decay. Davier et al.

$+0.006 \pm 0.012$ from τ decay. Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk

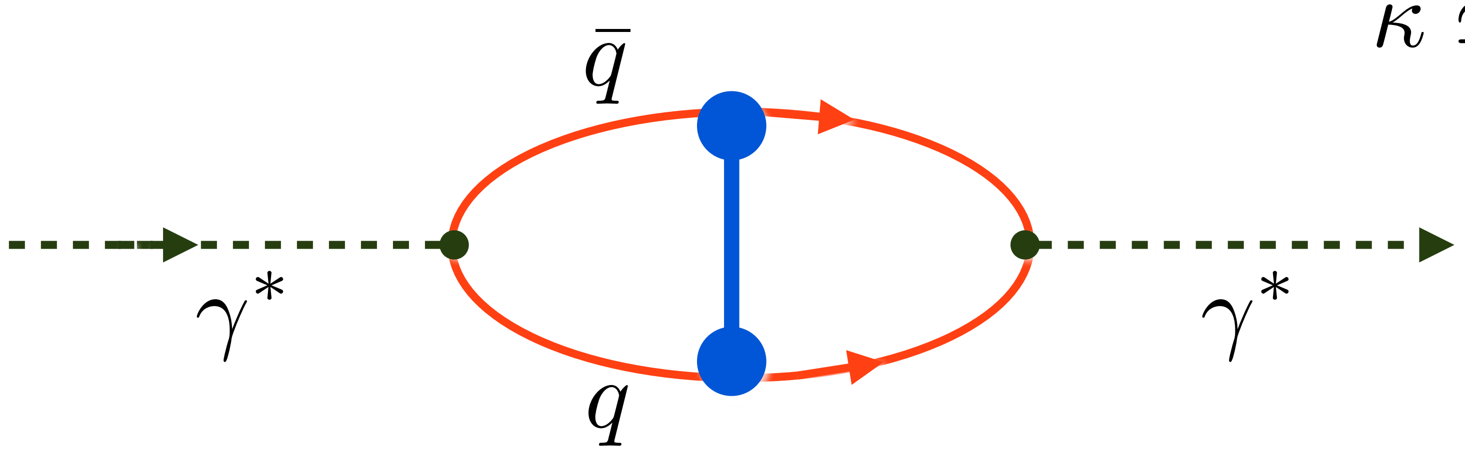


Consistent with zero vacuum condensate

Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator

$$M^2 = 4\kappa^2(n + L + S/2)$$ light-quark meson spectra

$\kappa \simeq 0.5$ GeV



$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots)$$

mimics dimension-4 gluon condensate $< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$ *in*

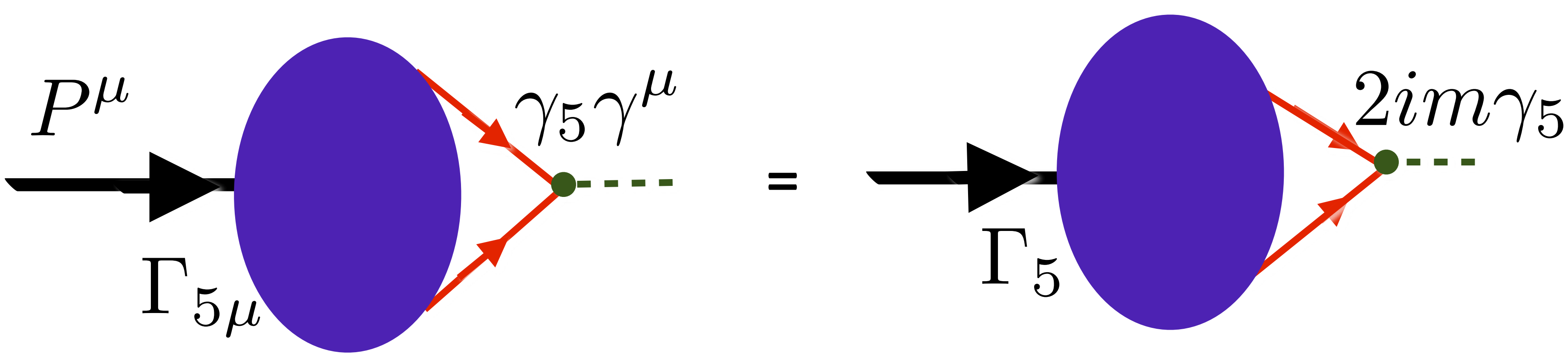
$e^+e^- \to X$, τ decay, $Q\bar{Q}$ phenomenology

Ward-Takahashi Identity for axial current

GMOR satisfied, no VEV

$$P^\mu \Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



plus non-pole

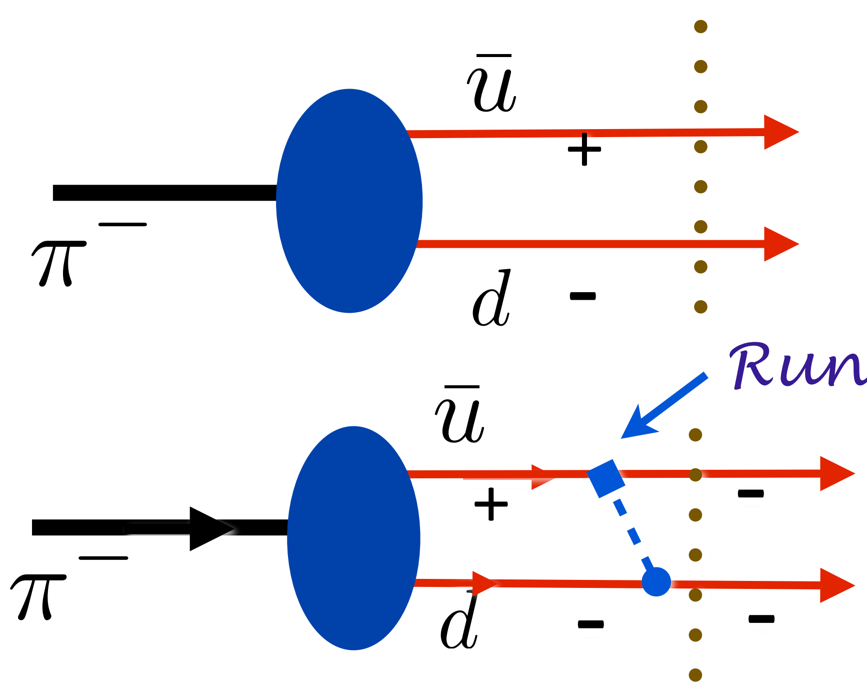
Identify pion pole at $P^2 = m_\pi^2$

$$P^\mu < 0|\bar{q}\gamma_5\gamma^\mu q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_\pi m_\pi^2 = -(m_u + m_d)\rho_\pi$$

Light-Front Pion Valence Wavefunctions

$S^z_{\bar u} + S^z_d = +1/2 - 1/2 = 0$



Couples to

$$L^z = 0, S^z = 0 \quad < \pi|\bar{\gamma}^\mu q \gamma_5 q|0 > \ \sim f_\pi$$

Running constituent mass at vertex

Couples to

$$L^z = +1, S^z = -1 \quad < \pi|\bar{q} \gamma_5 q|0 > \ \sim \rho_\pi$$

$S^z_{\bar u} + S^z_d = -1/2 - 1/2 = -1$

Angular Momentum Conservation

$$J^z = \sum_i^n S_i^z + \sum_i^{n-1} L_i^z$$

Revised Gell Mann-Oakes-Renner Formula in QCD

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

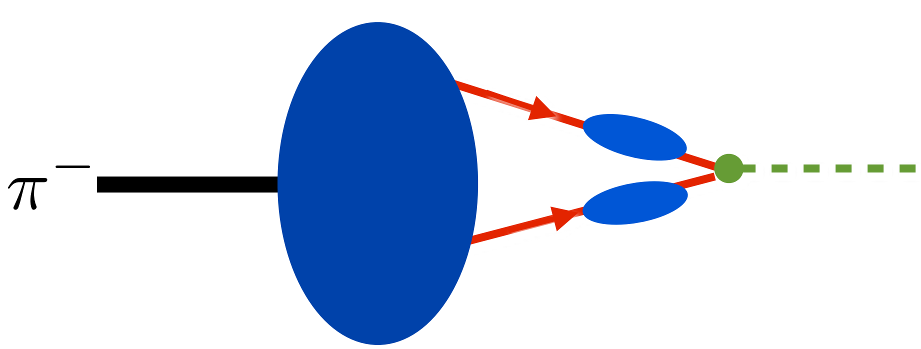
current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

No VEV!

vacuum condensate actually is an "in-hadron condensate"



$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind **Maris, Roberts, Tandy** **Shrock and sjb**

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:

(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD "Condensates"

(C) Higgs Light-Front Zero Mode

"One of the gravest puzzles of theoretical physics"

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$
$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$
$$\Omega_\Lambda = 0.76(expt)$$

QCD gives Λ=zero if Quark and Gluon condensates reside within hadrons, not vacuum!

Electroweak contribution gives Λ=zero from Zero Mode solution to Higgs Potential

Electroweak Problem also could be solved in technicolor-- condensates within technihadrons

$$(\Omega_\Lambda)_{QCD} = 0 \qquad (\Omega_\Lambda)_{EW} = 0$$

Central Question: What is the source of Dark Energy?

$$\Omega_\Lambda = 0.76(expt)$$

Higgs Zero-Mode Curvature?

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

Fundamental Question: Origin of the QCD Mass Scale

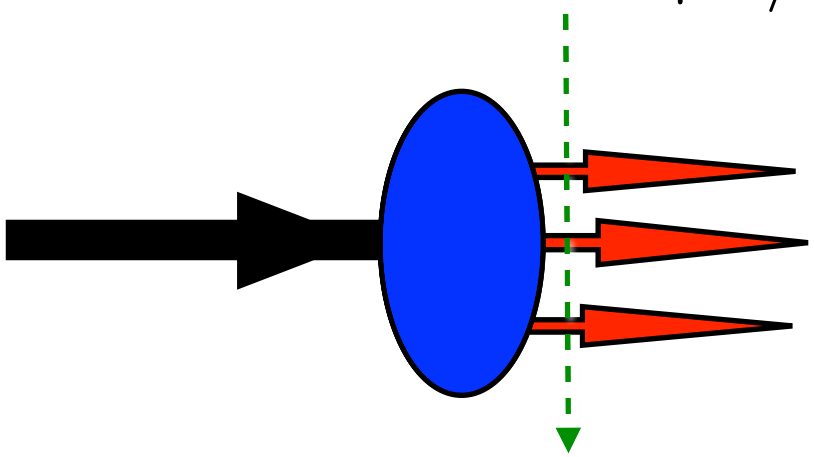
- ***Pion massless for m_q=0***
- ***What sets the mass of the proton when m_q=0 ?***
- ***QCD: No knowledge of MeV units:***
  Only ratios of masses can be predicted
- ***Novel proposal by de Alfaro, Fubini, and Furlan (DAFF):***
  Mass scale κ can appear in Hamiltonian leaving the action conformal!
- ***Unique Color-Confinement Potential*** $\kappa^4\zeta^2$
- ***Eigenstates of Light-Front Hamiltonian determine hadronic mass spectrum and LF wavefunctions*** $\psi_H(x_i, \vec{k}_{\perp i}, \lambda_i)$
- ***Superconformal algebra: Degenerate meson, baryon, and tetraquark mass spectrum***
- ***Running QCD Coupling at all scales:*** Predict $\frac{\Lambda_{\overline{MS}}}{m_p}$

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}^{QCD}_{LF} |\psi> = M^2 |\psi>$$

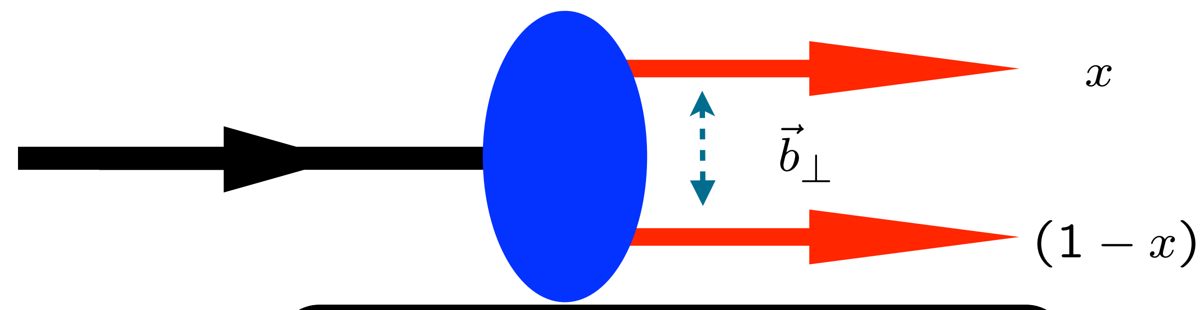
Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x) b_\perp^2$$

$$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\rm eff}^{LF}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Azimuthal Basis

ζ, ϕ

$m_q = 0$

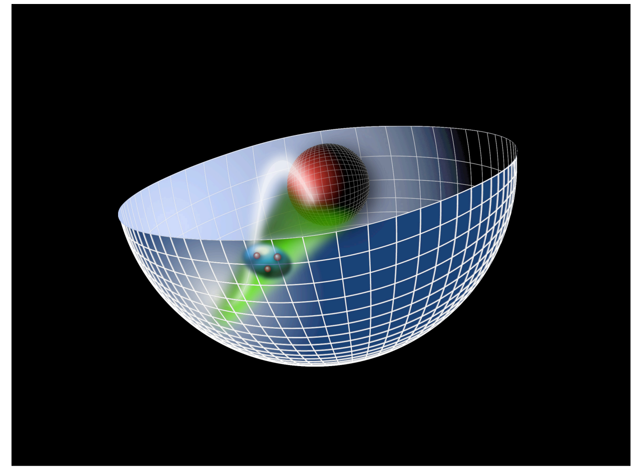
Single variable! **AdS/QCD:**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique
Confinement Potential!

Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

IL NUOVO CIMENTO Vol. 34 A, N. 4 21 Agosto 1976

Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)

Conformal Invariance in Quantum Mechanics.

V. de Alfaro

Istituto di Fisica Teorica dell'Università - Torino
Istituto Nazionale di Fisica Nucleare - Sezione di Torino

S. Fubini and G. Furlan (*)

CERN - Geneva

(ricevuto il 3 Maggio 1976)

Summary. — The properties of a field theory in one over-all time dimension, invariant under the full conformal group, are studied in detail. A compact operator, which is not the Hamiltonian, is diagonalized and used to solve the problem of motion, providing a discrete spectrum and normalizable eigenstates. The role of the physical parameters present in the model is discussed, mainly in connection with a semi-classical approximation.

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

New term

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

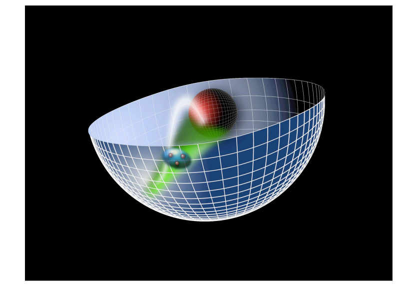
$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

- **Dosch, de Teramond, sjb**

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

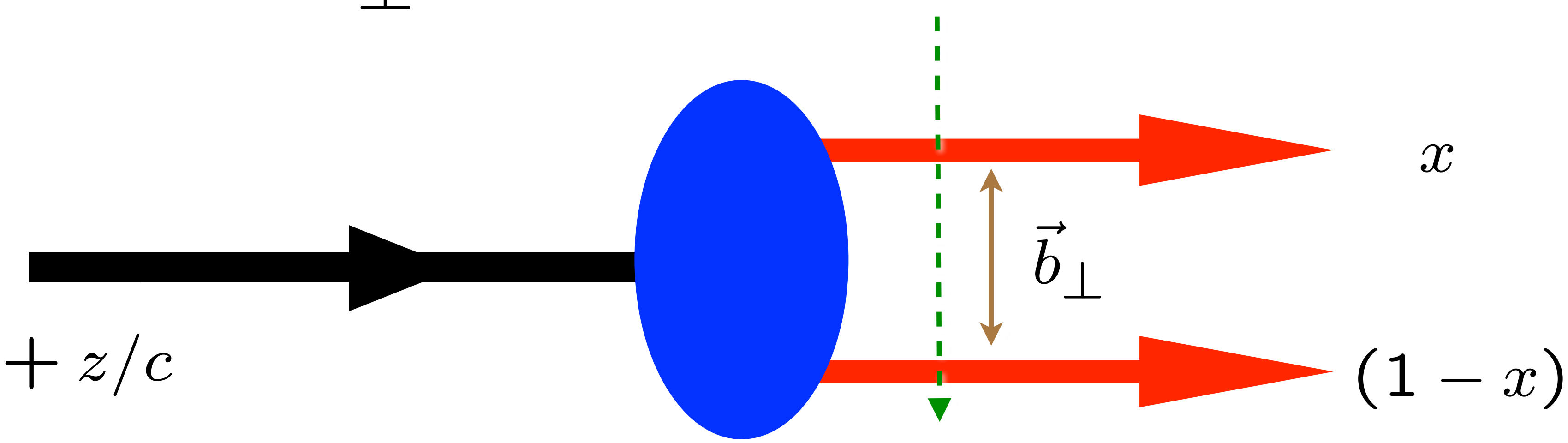
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \quad \longleftrightarrow \quad z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (J-1)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2 \left(n + \frac{J+L}{2}\right)$$

$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

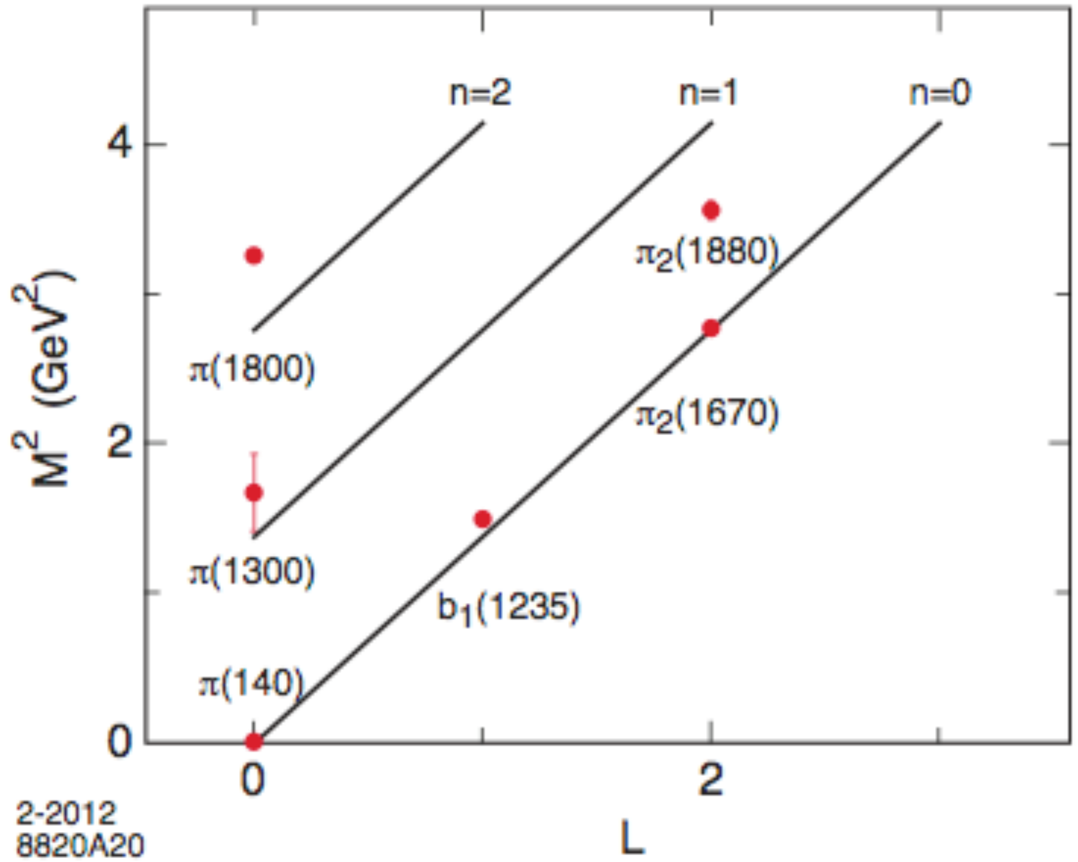
G. de Teramond, H. G. Dosch, sjb

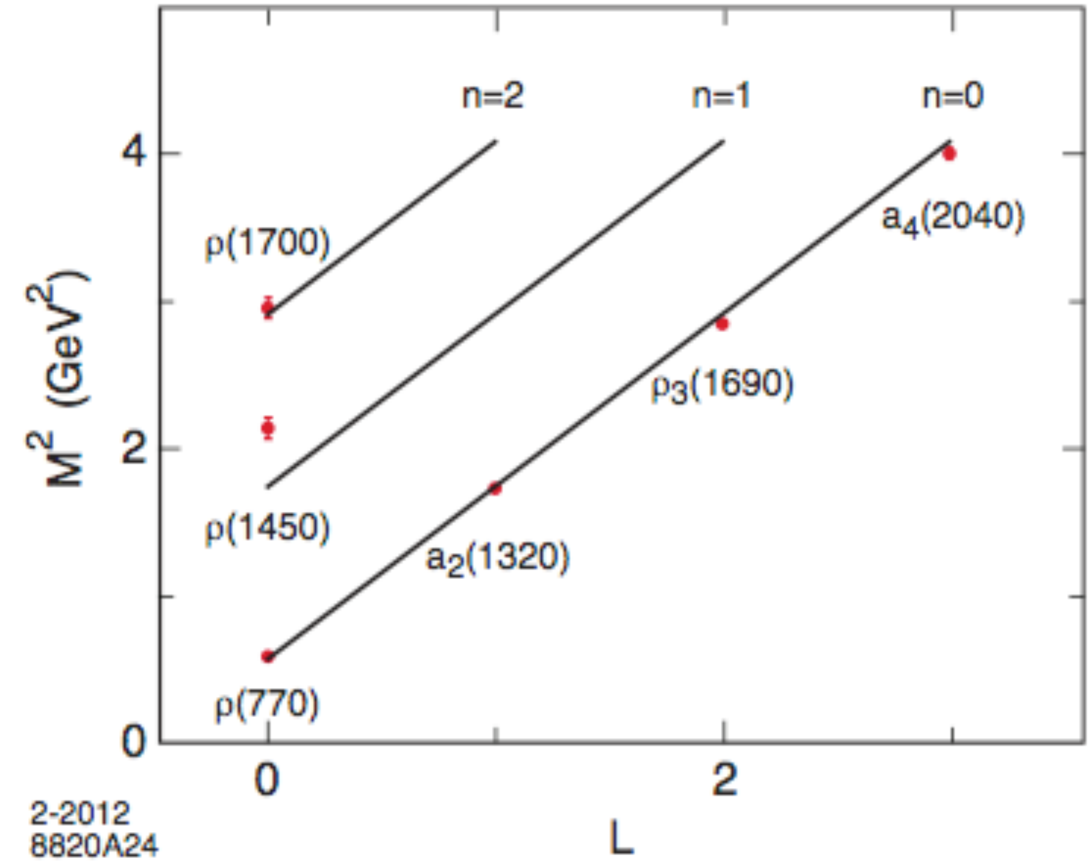
- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2\,(n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$m_q = 0$

Massless pion in Chiral Limit! ***Same slope in n and L!***





I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1$, $L = 1$, $J = 0, 1, 2$, vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

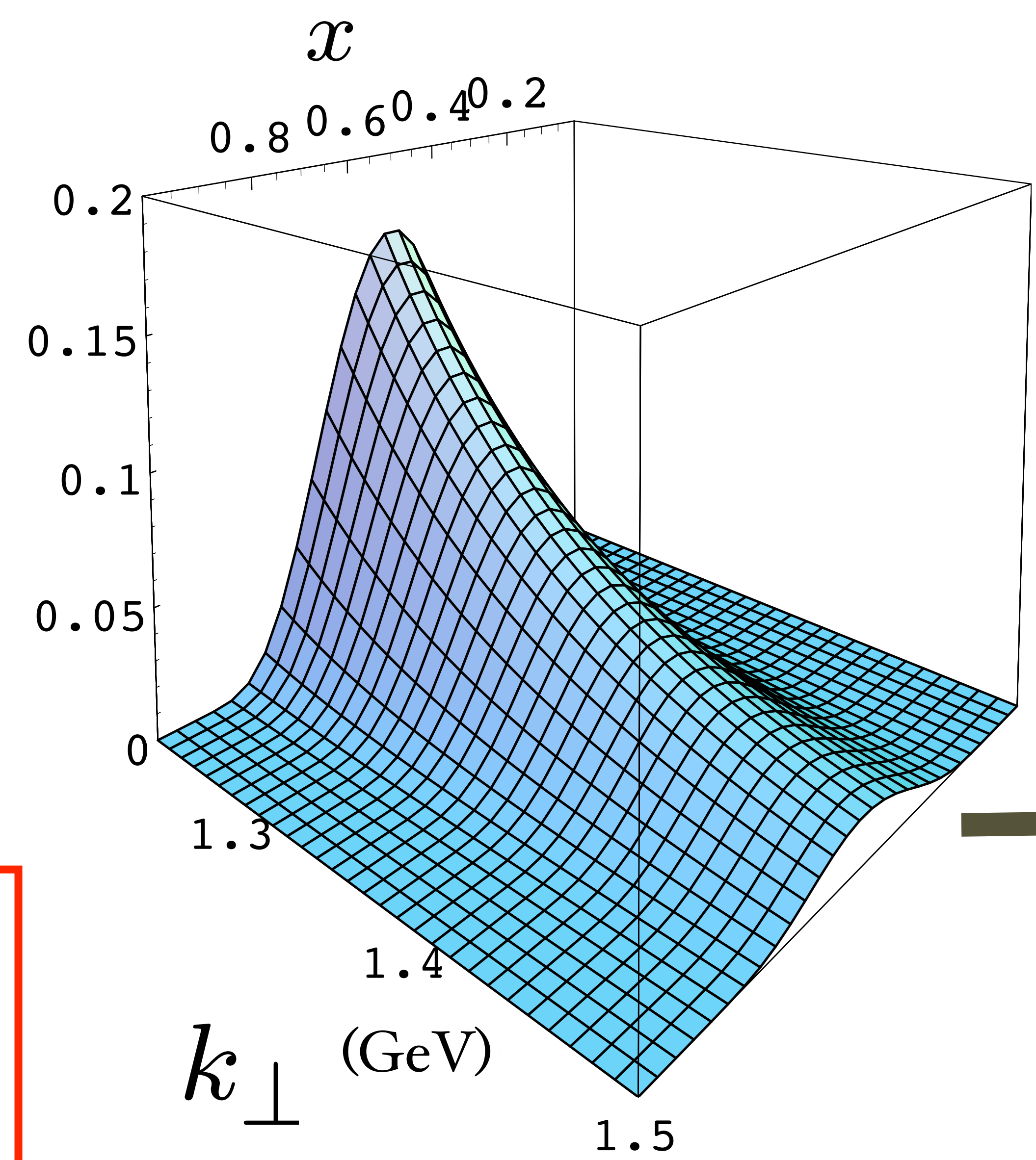
Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

Prediction from AdS/QCD: Meson LFWF

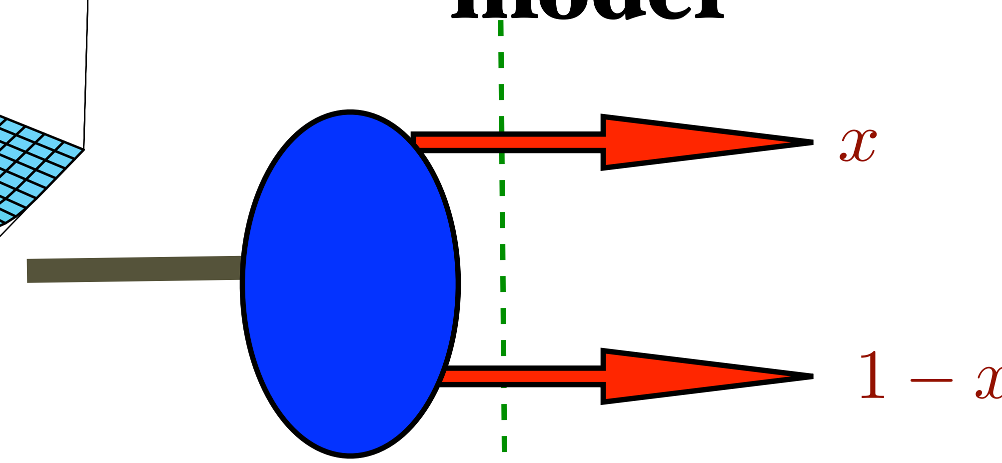
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

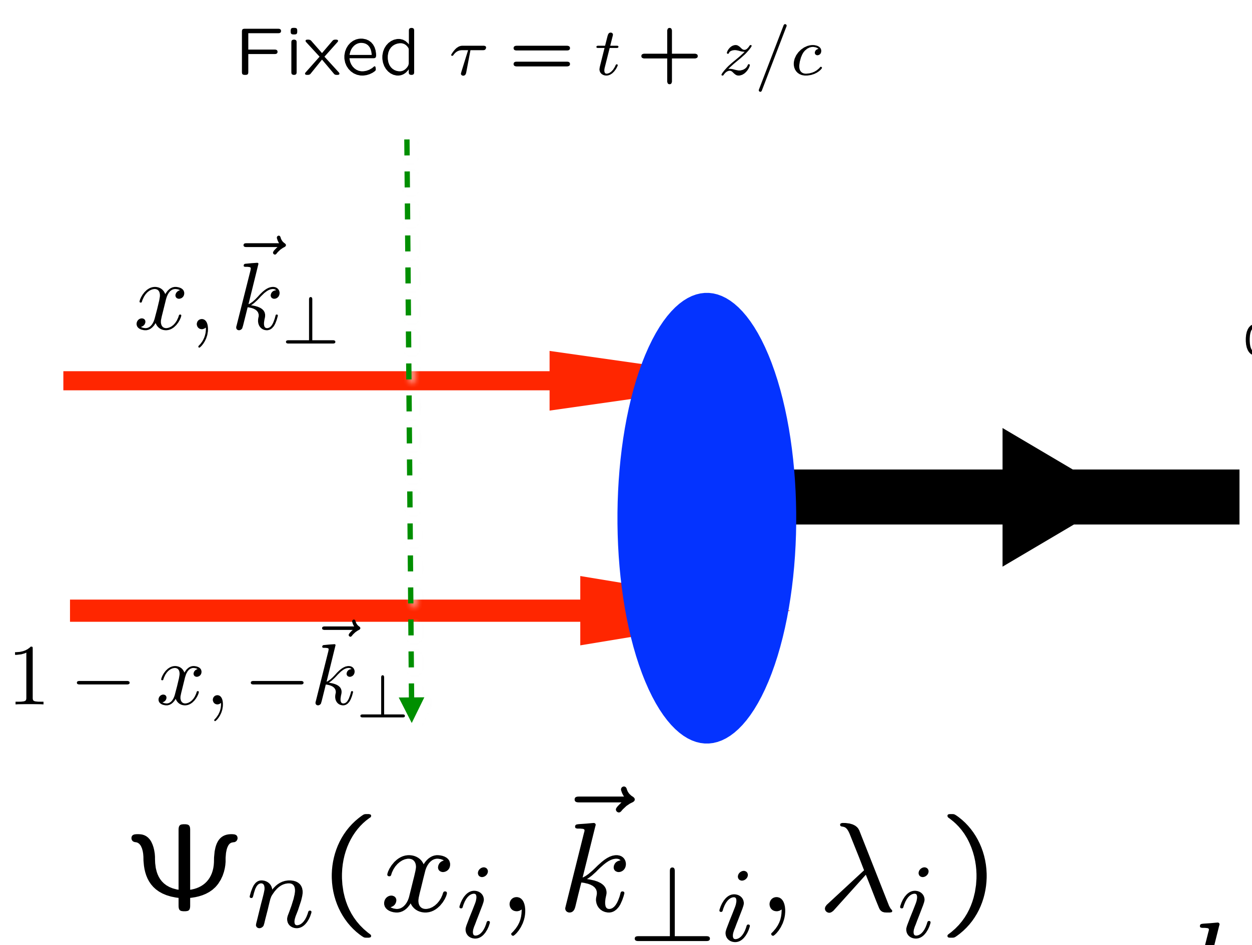
$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

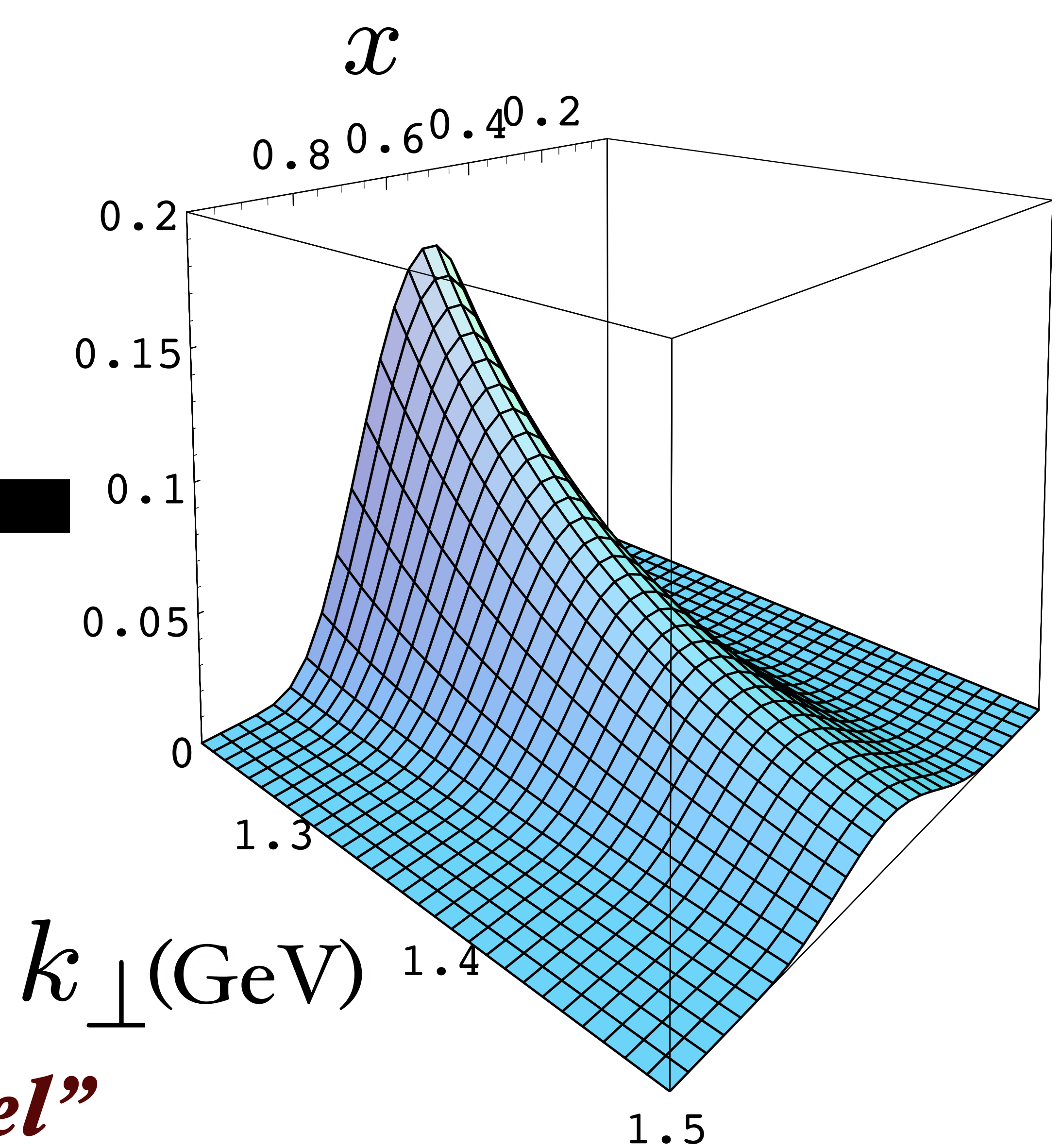
Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$

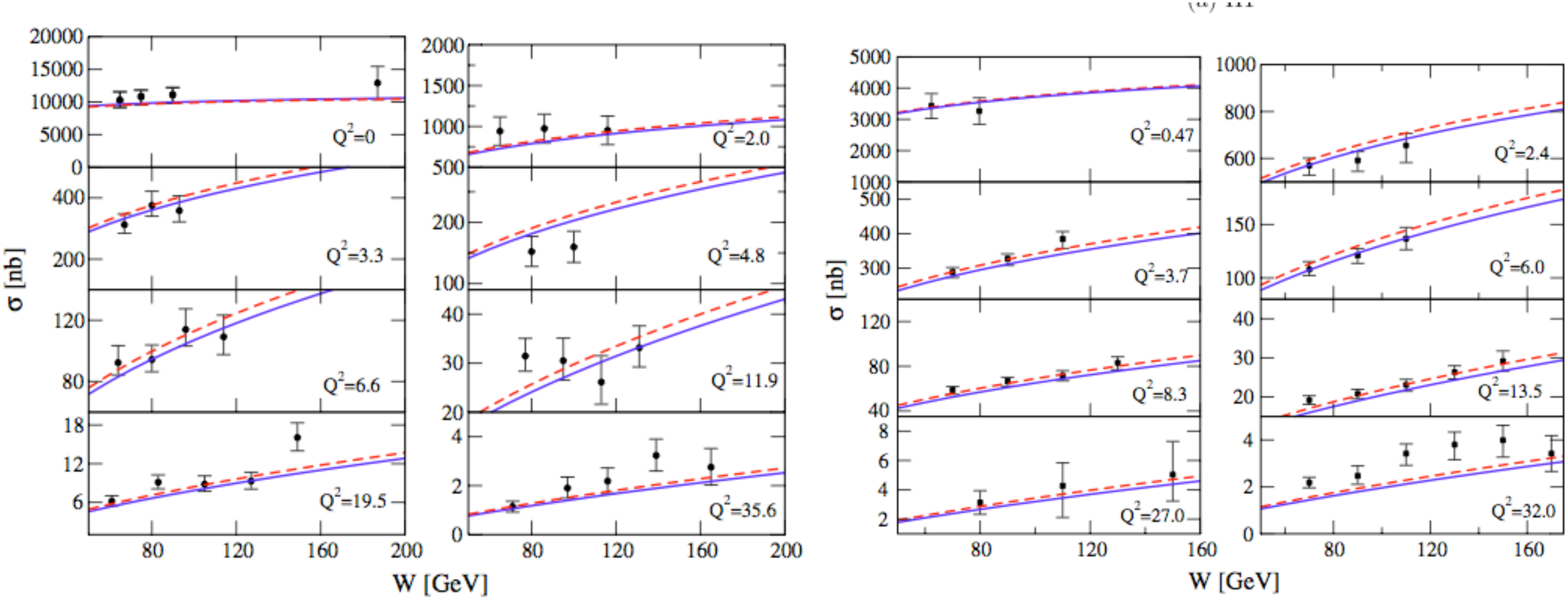




"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

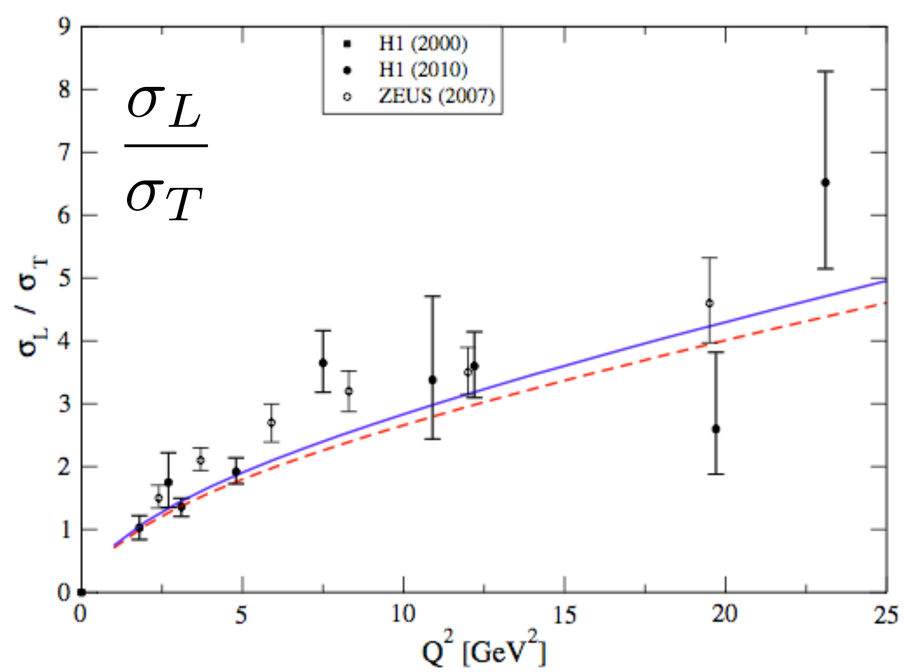


(a) H1

(b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \rightarrow \rho^0 p'$$

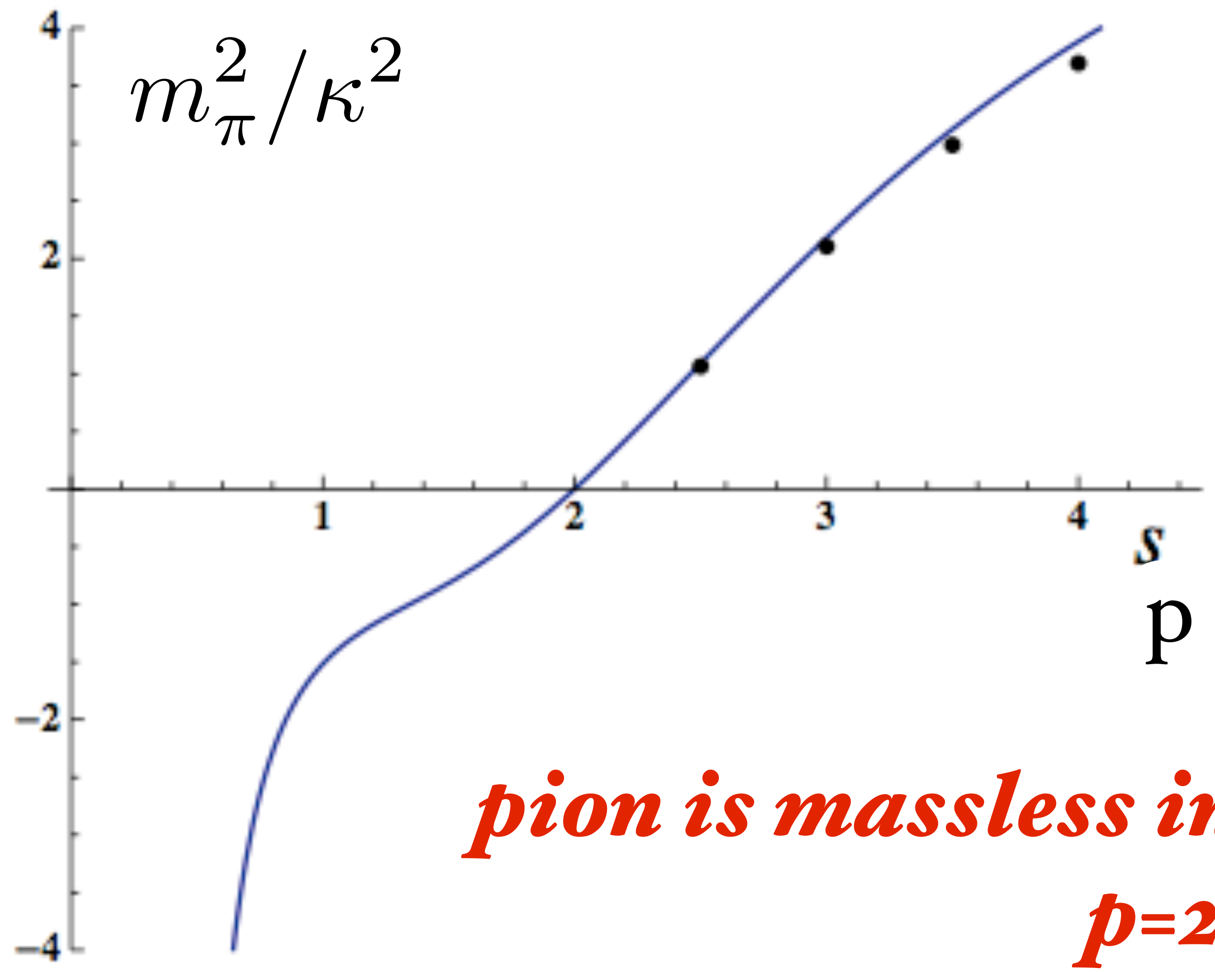


$$\frac{\sigma_L}{\sigma_T}$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

Uniqueness of Dilaton

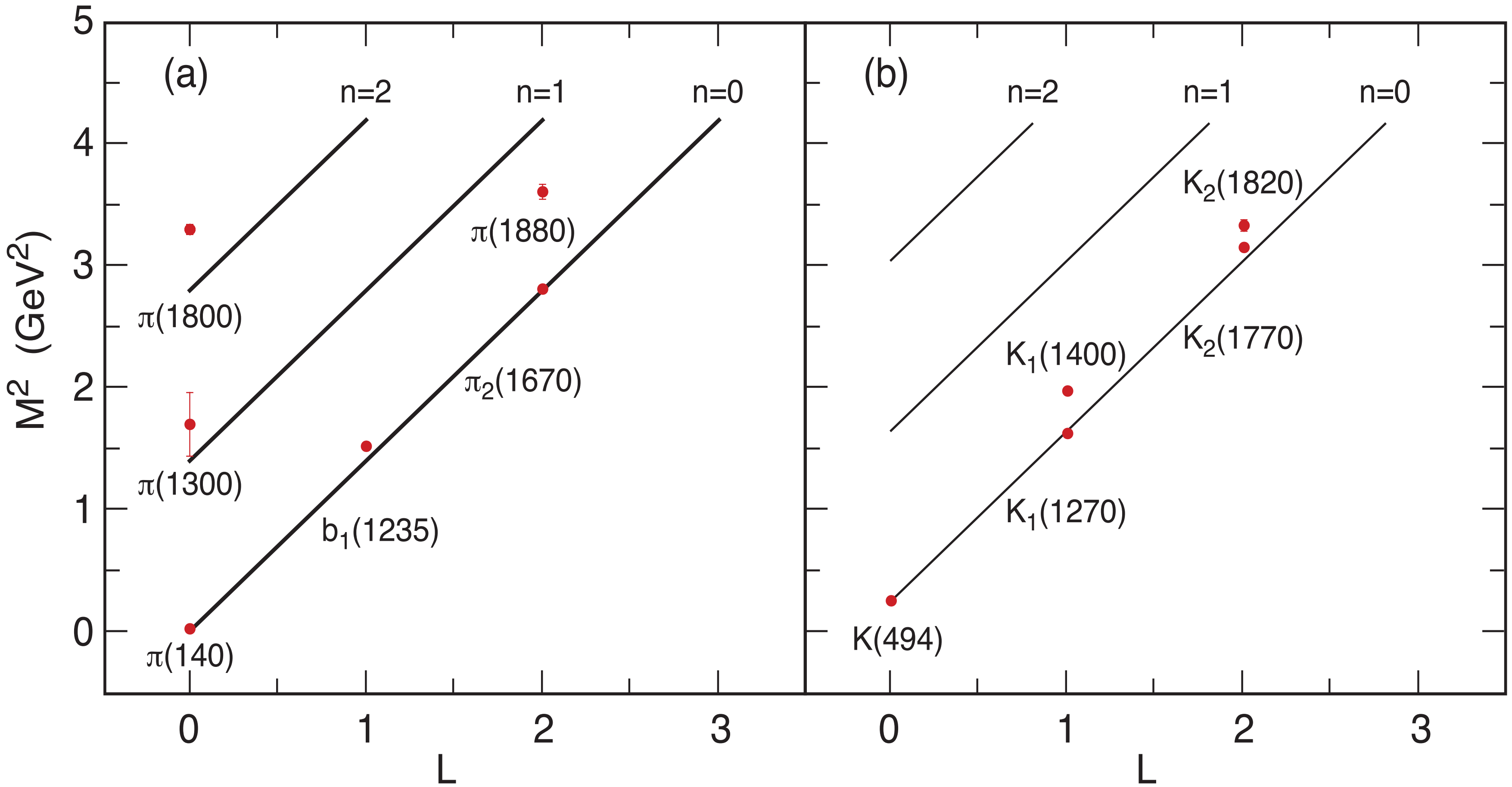
$$\varphi_p(z) = \kappa^p z^p$$

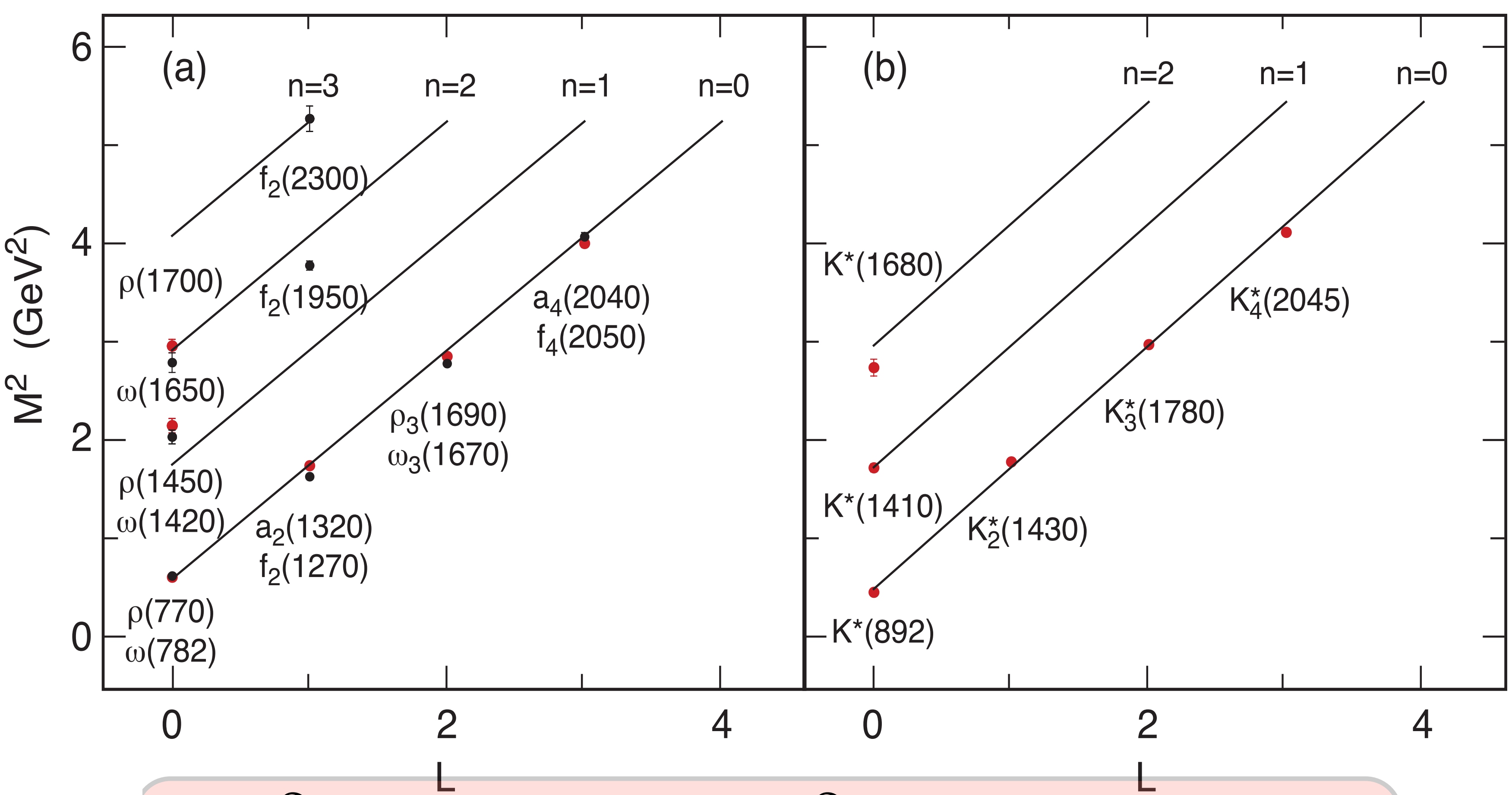


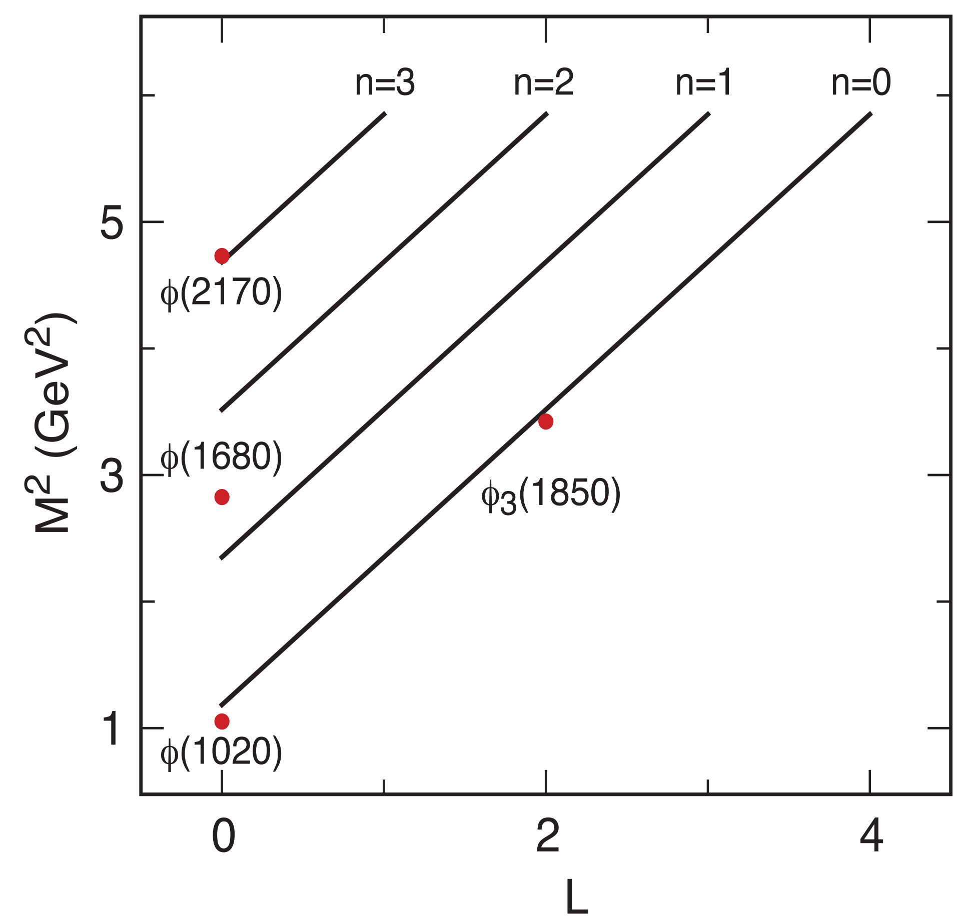
pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**







$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L

Quark separation increases with L

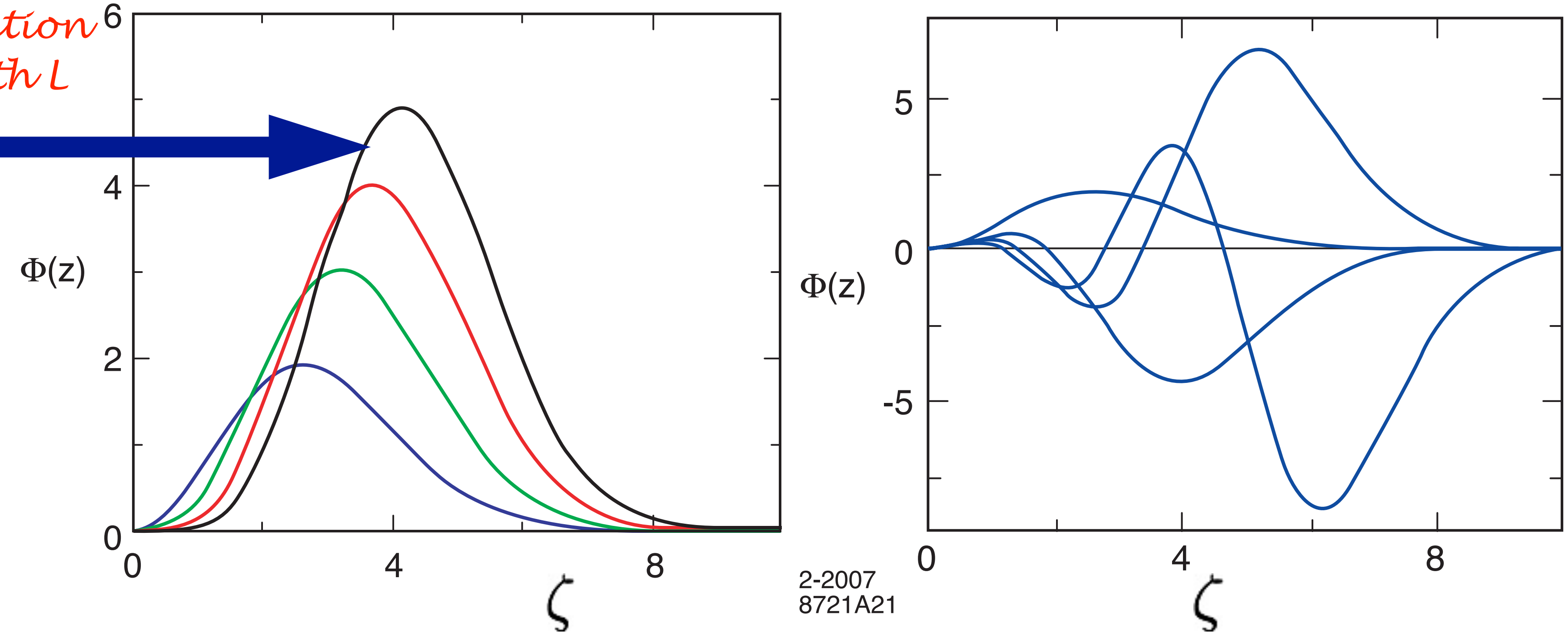
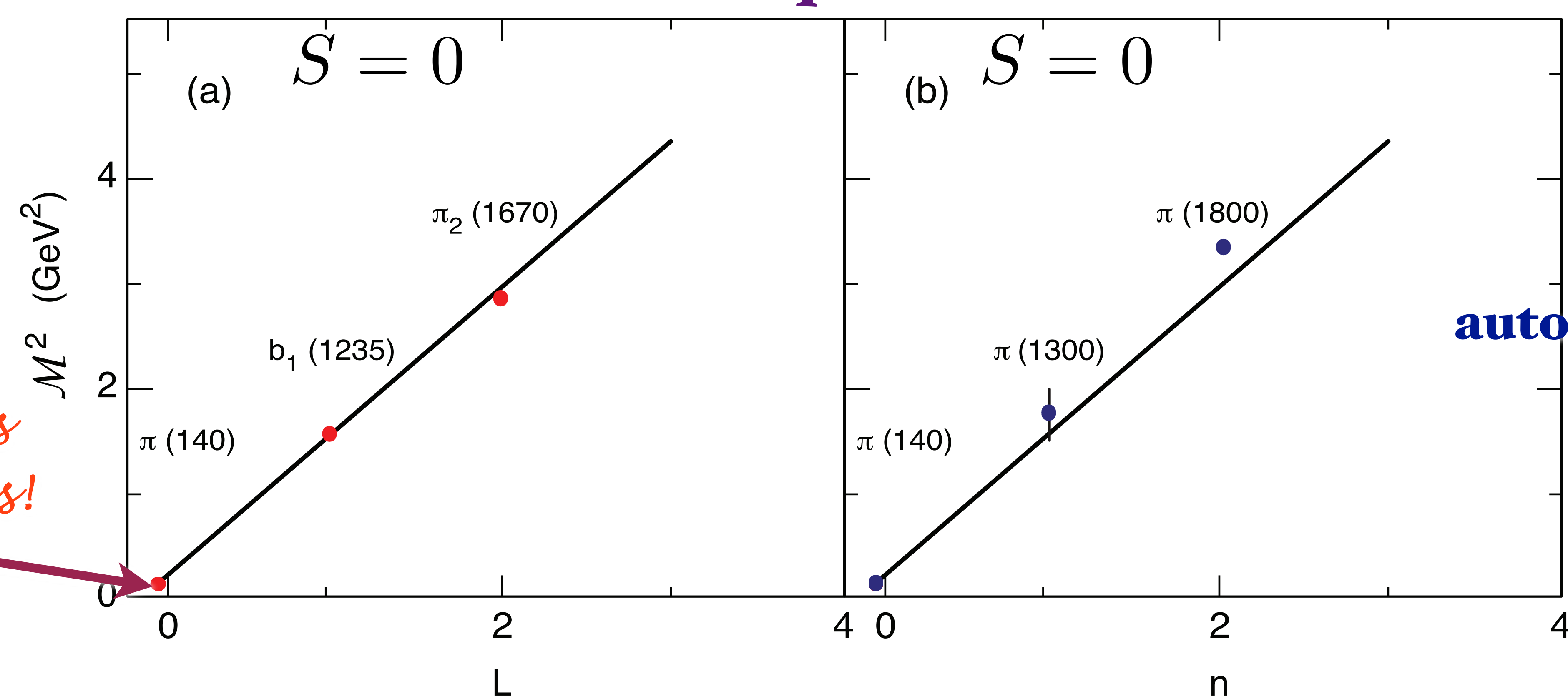


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!



Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

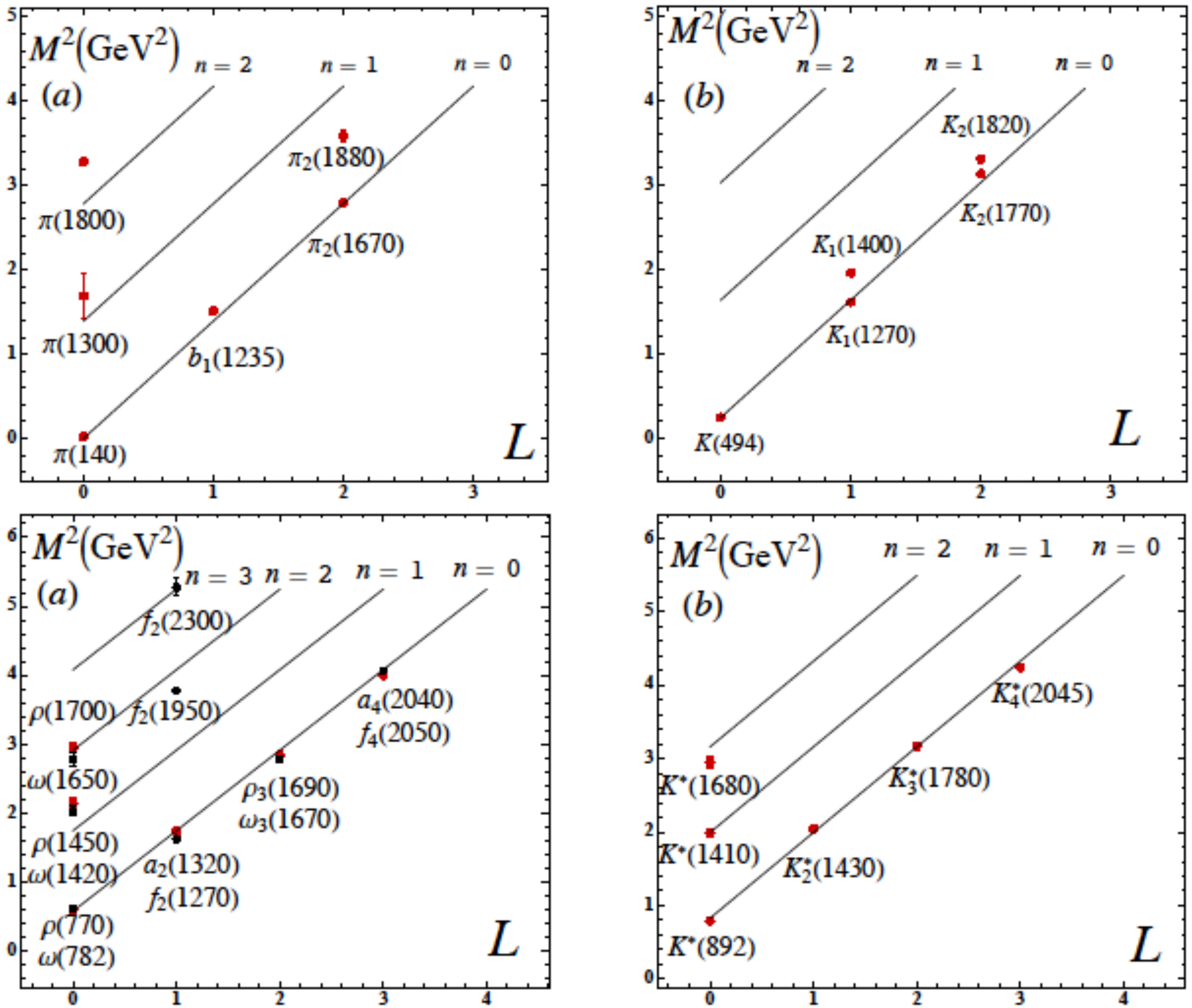
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

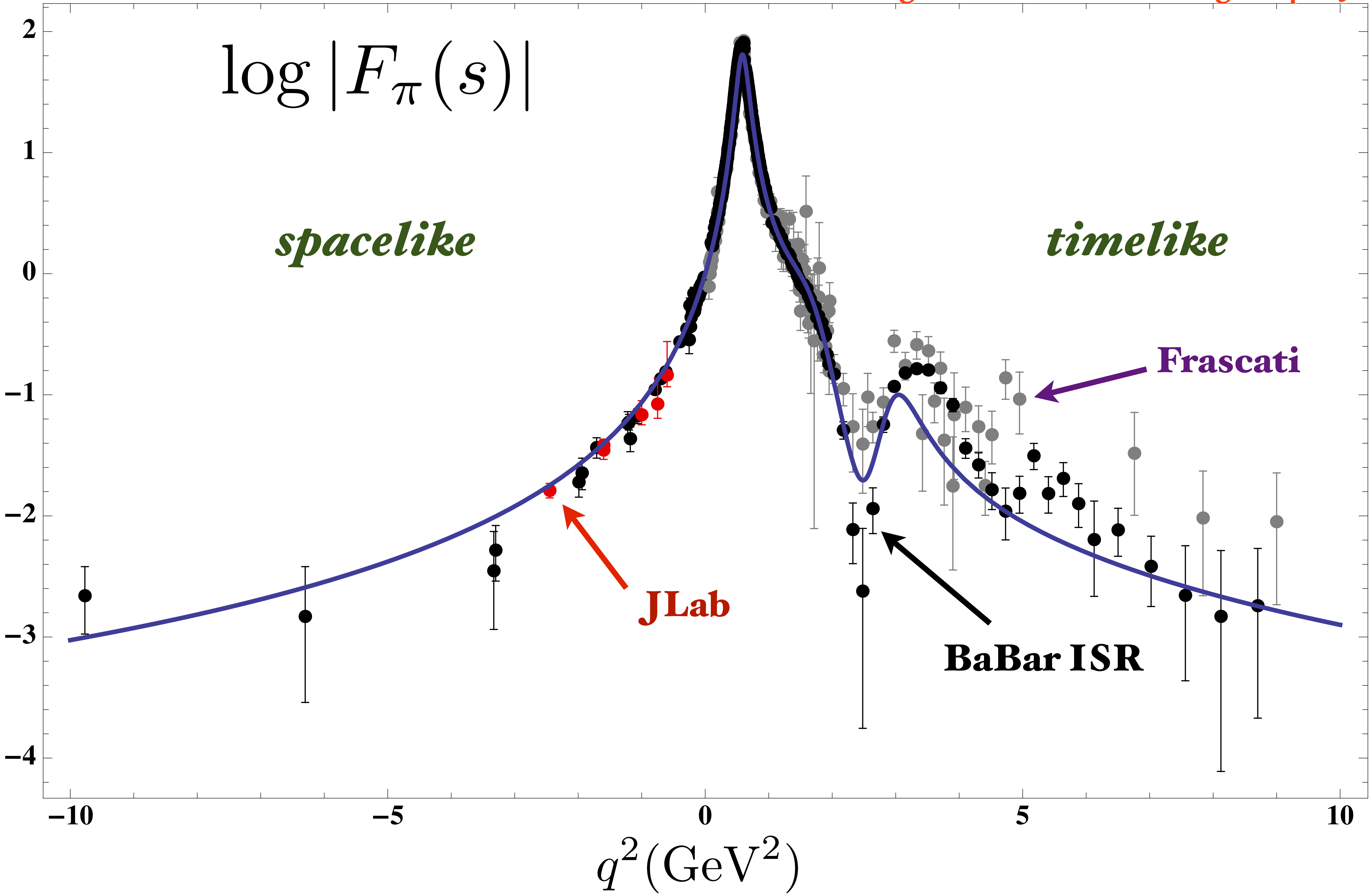
from LF Higgs mechanism



Effective mass from $m(p^2)$

Tandy, Roberts, et al

Pion Form Factor from AdS/QCD and Light-Front Holography



Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

↕

Harmonic Oscillator $U(\zeta) = \kappa^4\zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

Consider $R_w = Q + wS;$

w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \big(- \partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \big)$$

$$G_{22} = \big(- \partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \big)$$

Identify $f - \frac{1}{2} = L_B \ , \quad w = \kappa^2$ $\qquad \lambda = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ *!*

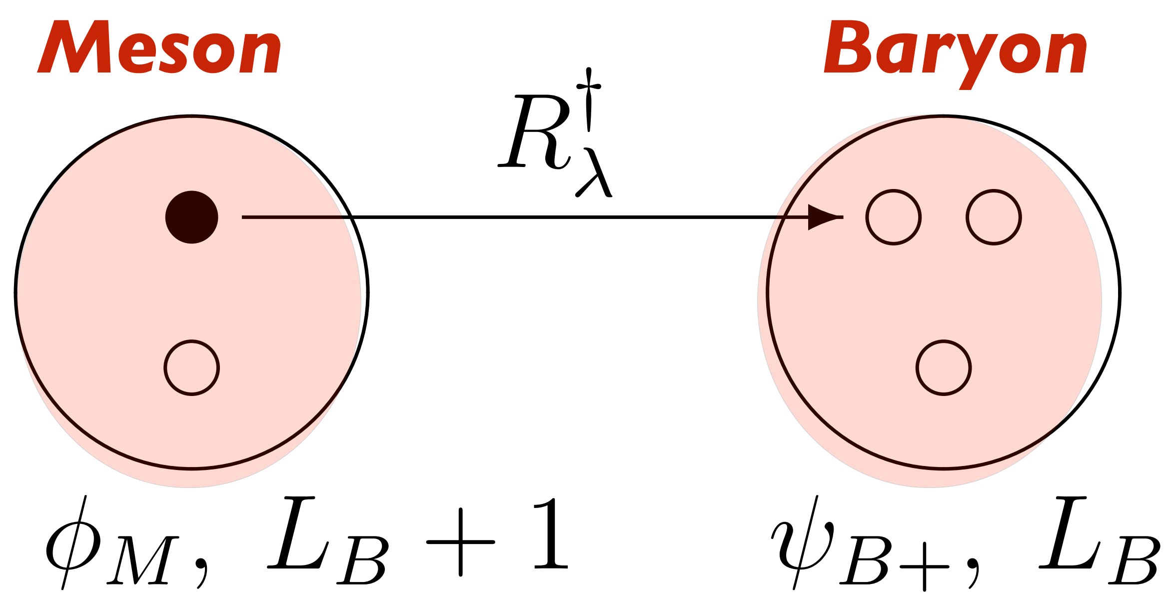
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

Superconformal Algebra

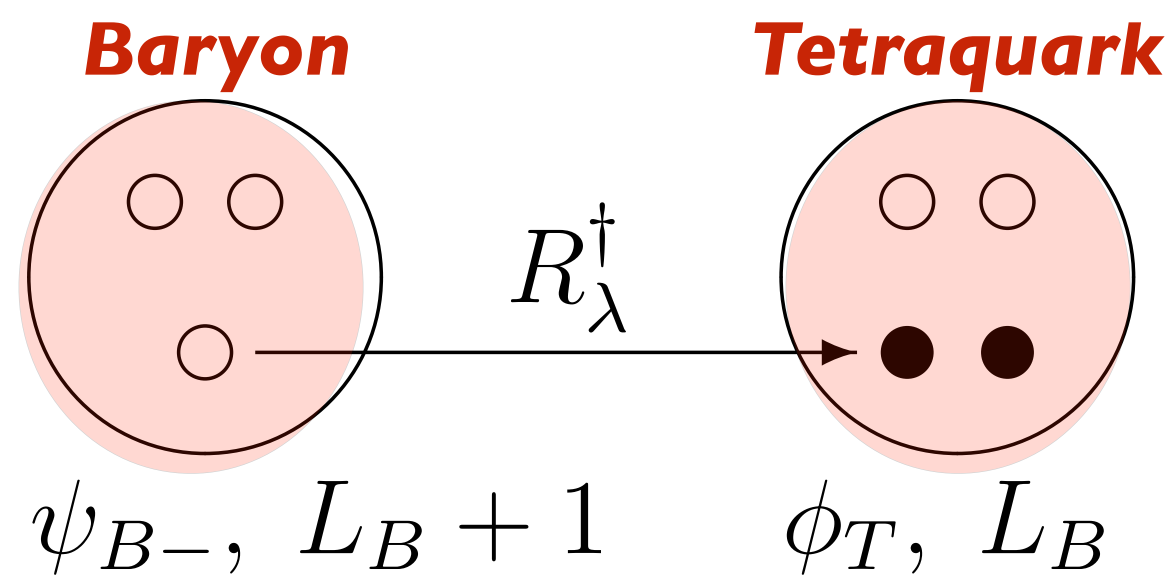
2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



$$R^\dagger_\lambda \; \bar{q} \to [qq]$$

$$\bar{3}_C \to \bar{3}_C$$



$$R^\dagger_\lambda \; q \to [\bar{q}\bar{q}]$$

$$3_C \to 3_C$$

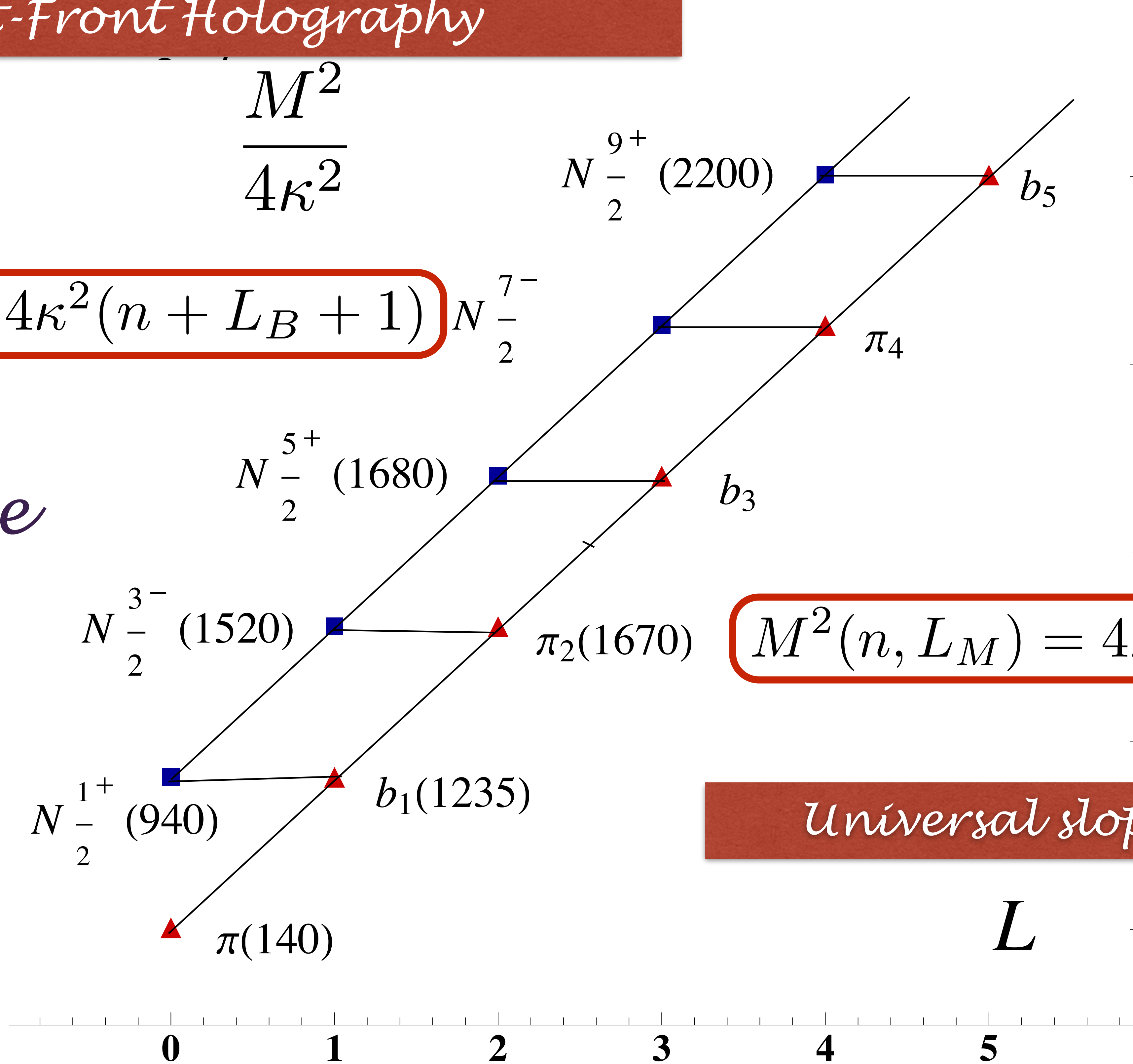
Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Superconformal Quantum Mechanics Light-Front Holography

de Tèramond, Dosch, Lorce, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

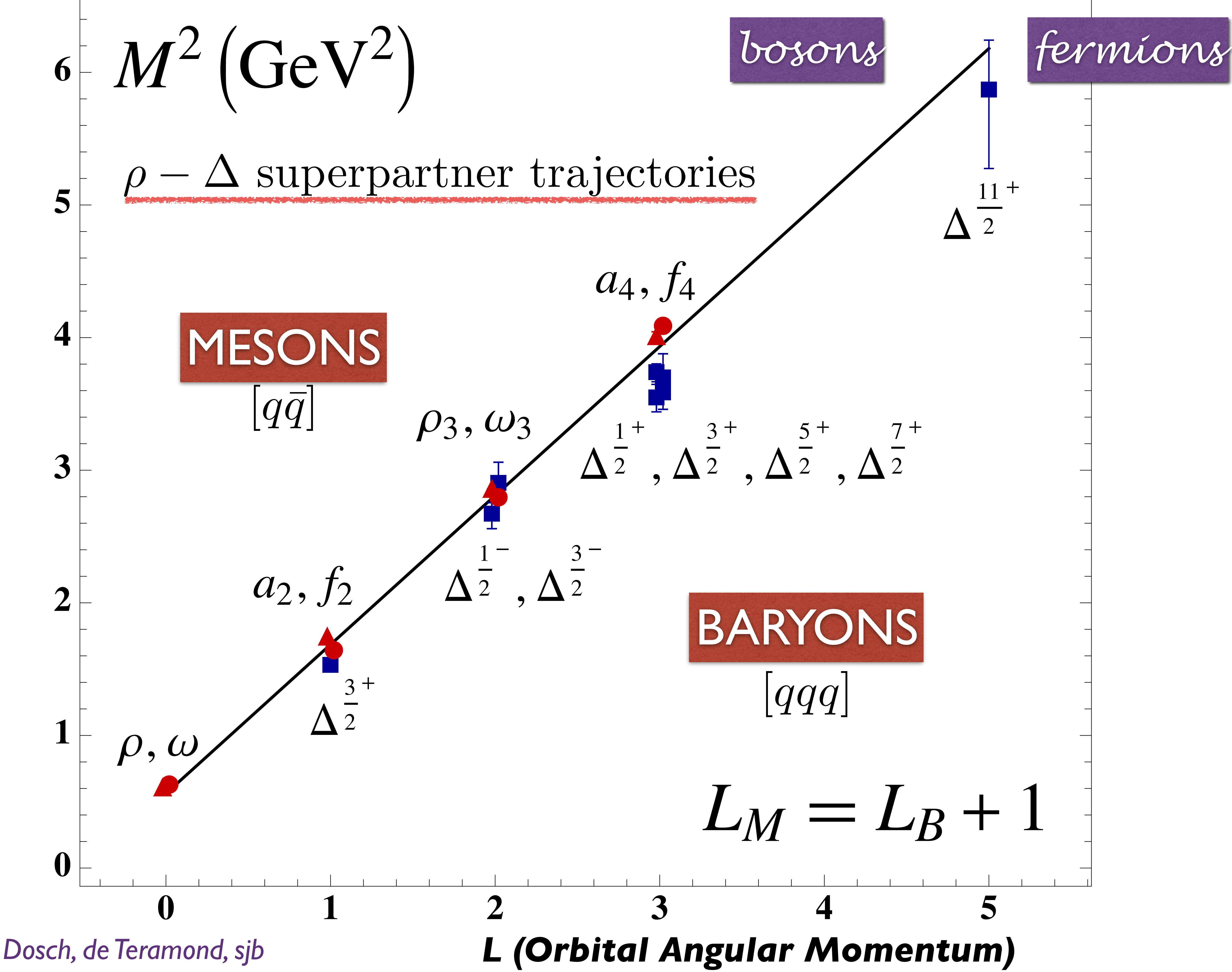


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

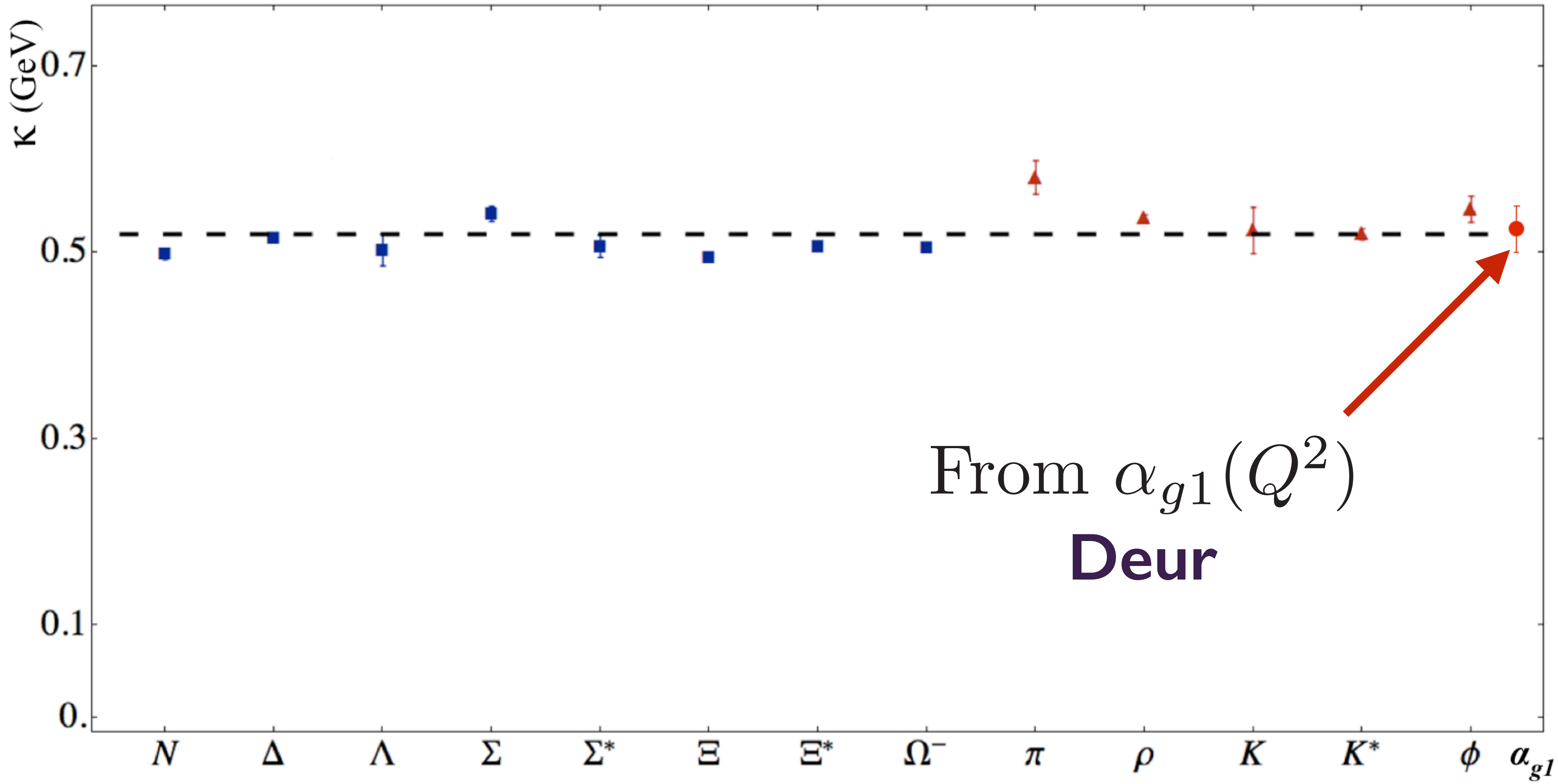

$M^2\,(\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta \frac{3}{2}^+$
$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$
$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$
$\Delta \frac{11}{2}^+$
$L_M = L_B + 1$
L (Orbital Angular Momentum)
0
1
2
3
4
5
6
Dosch, de Teramond, sjb

$\lambda = \kappa^2$

de Tèramond, Dosch, Lorce', sjb

$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$

κ

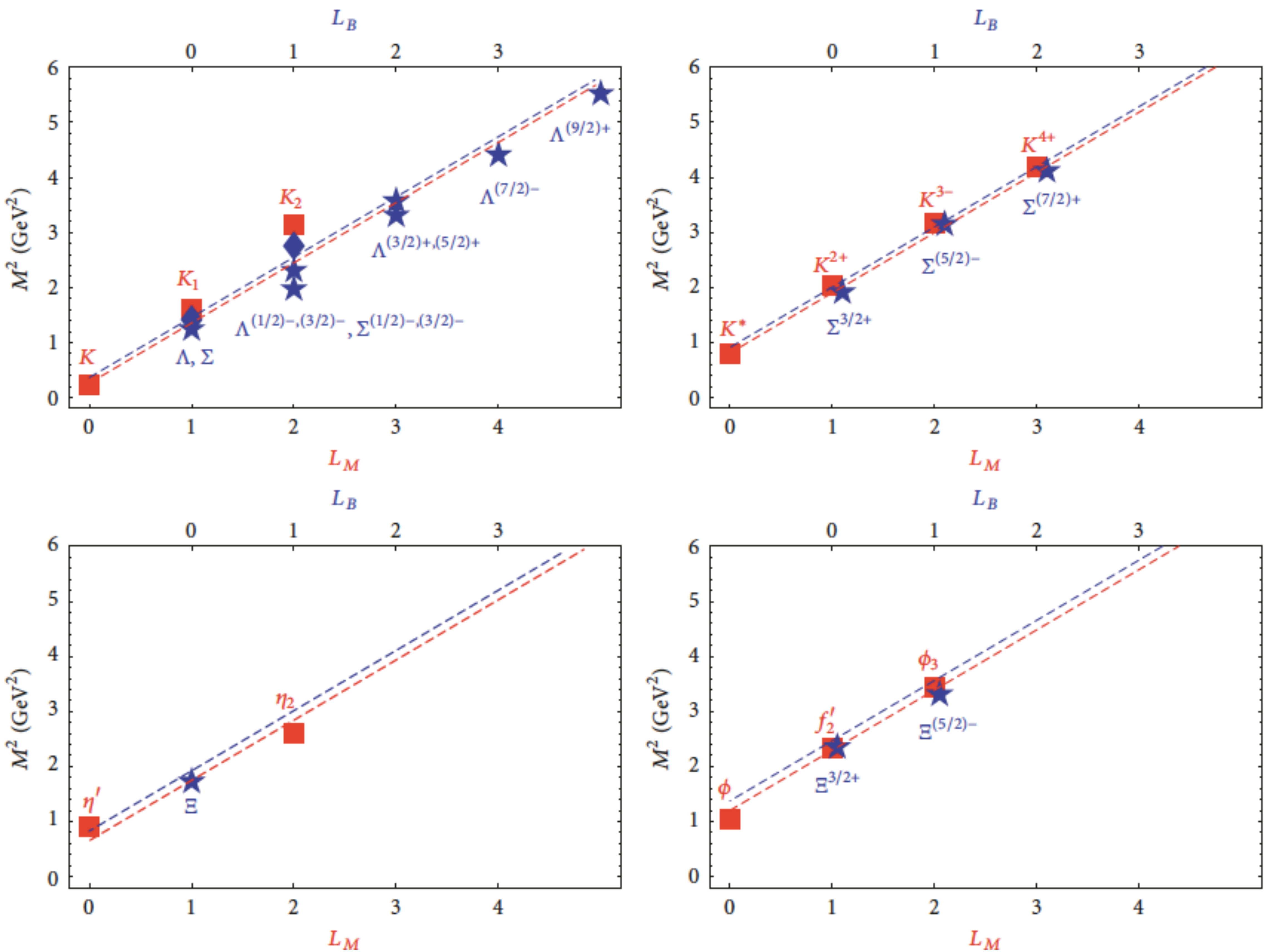


Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

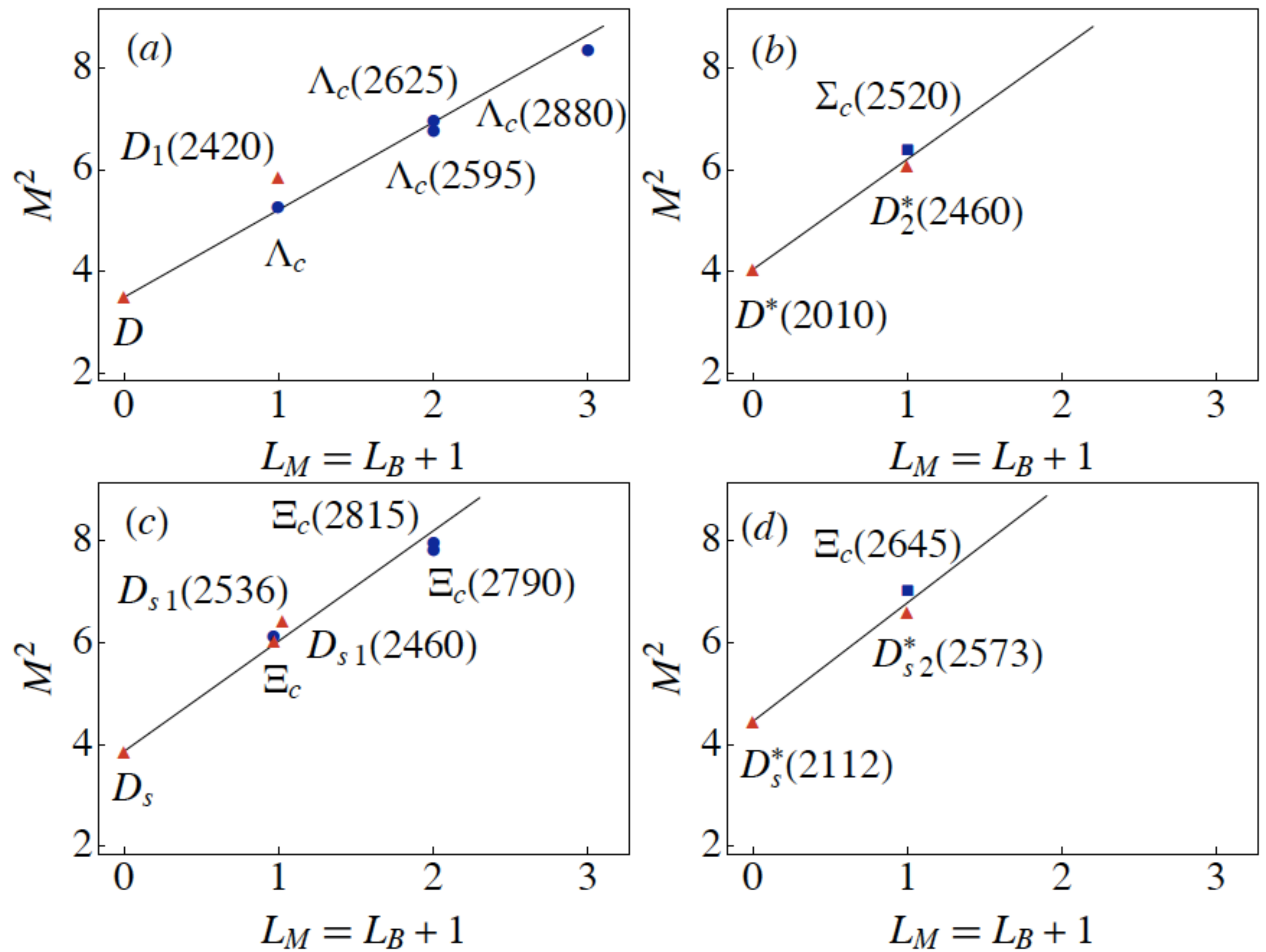
Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770),\ \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320),\ f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690),\ \omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040),\ f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)?$
$\bar{q}s$	$0^{-(+)}$	$\bar{K}(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$\bar{K}_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$\bar{K}_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$\bar{K}_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)?$
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)?$
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)?$
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)?$
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)?$

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen

Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

predictions

beautiful agreement!

Guy de Tèramond, Hans Günter Dosch, sjb

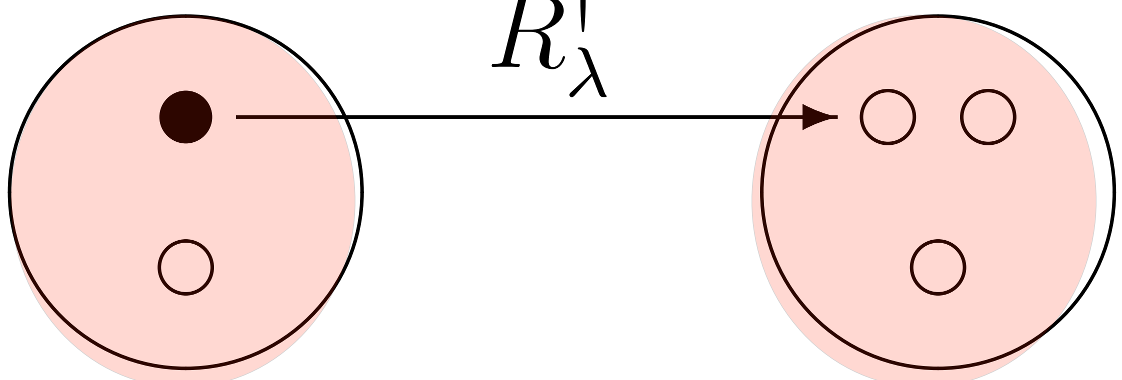
Superconformal Algebra

2X2 Hadronic Multiplets: 4-Plet

Bosons, Fermions with Equal Mass!

Meson **Baryon**

$R^{\dagger}_{\lambda}$



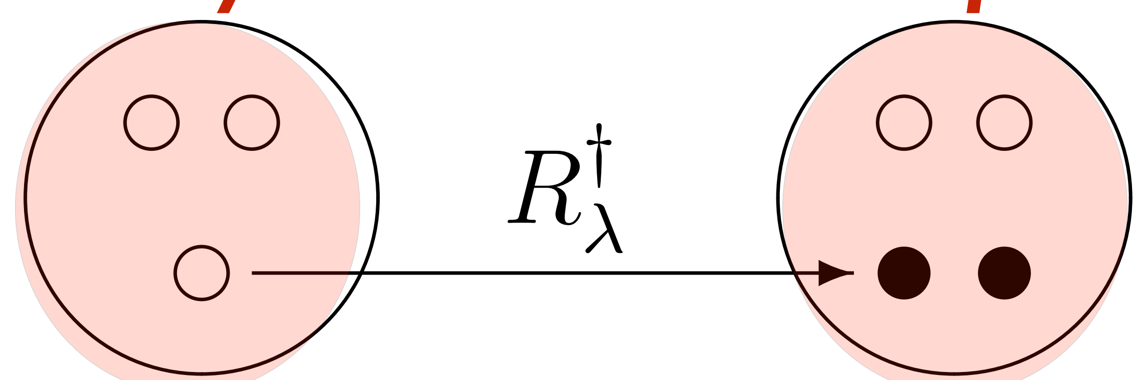
$R^{\dagger}_{\lambda} \; \bar{q} \to [qq]$

$\bar{3}_C \to \bar{3}_C$

$\phi_M, \; L_B + 1$ $\psi_{B+}, \; L_B$

Baryon **Tetraquark**

$R^{\dagger}_{\lambda}$



$R^{\dagger}_{\lambda} \; q \to [\bar{q}\bar{q}]$

$3_C \to 3_C$

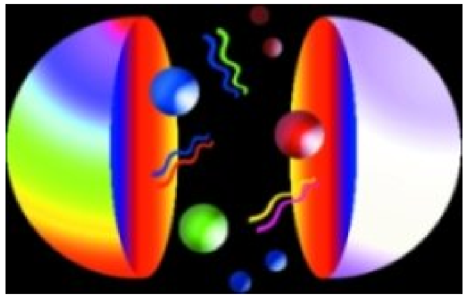
$\psi_{B-}, \; L_B + 1$ $\phi_T, \; L_B$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int_0^\infty d\zeta \int_0^1 dx \psi_+^2(\zeta^2, x) = \int_0^\infty d\zeta \int_0^1 dx \psi_-^2(\zeta^2, x) = \frac{1}{2}$$

Quark Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Nucleon: Equal Probability for L=0,1

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad ({F_1}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

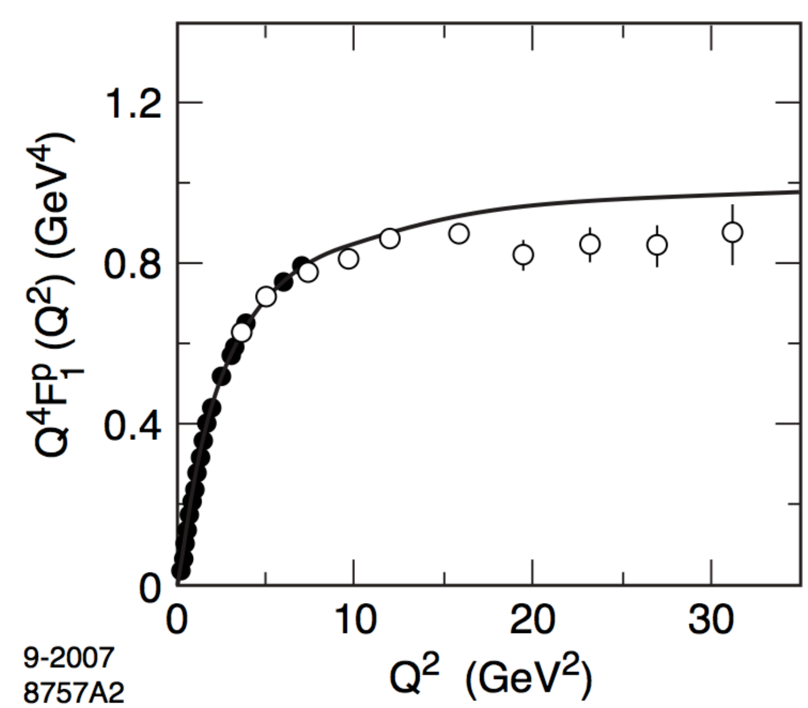
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$

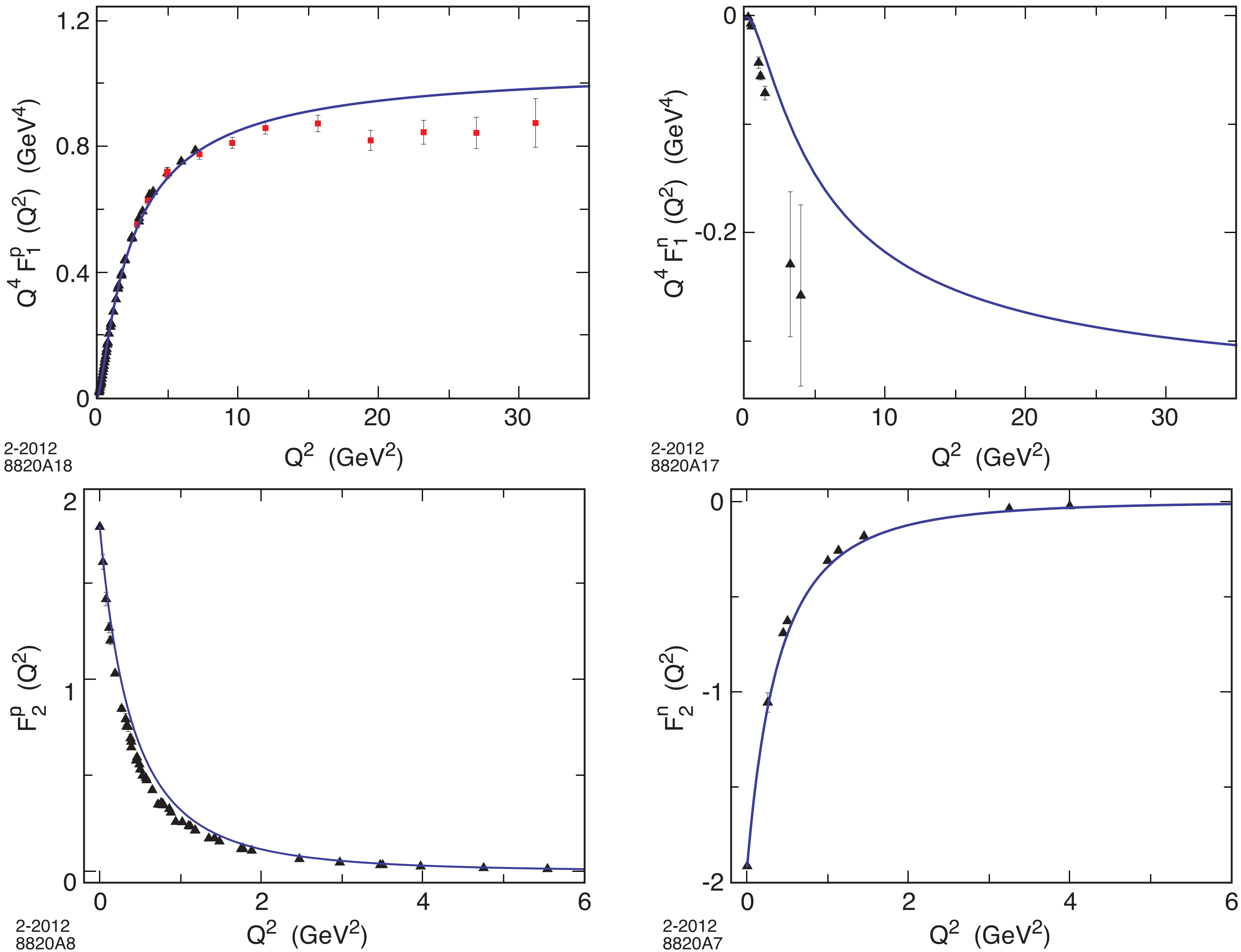
- Find

$$F_1^p(Q^2) = \frac{1}{\Big(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\Big)\Big(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\Big)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n + 1/2)$

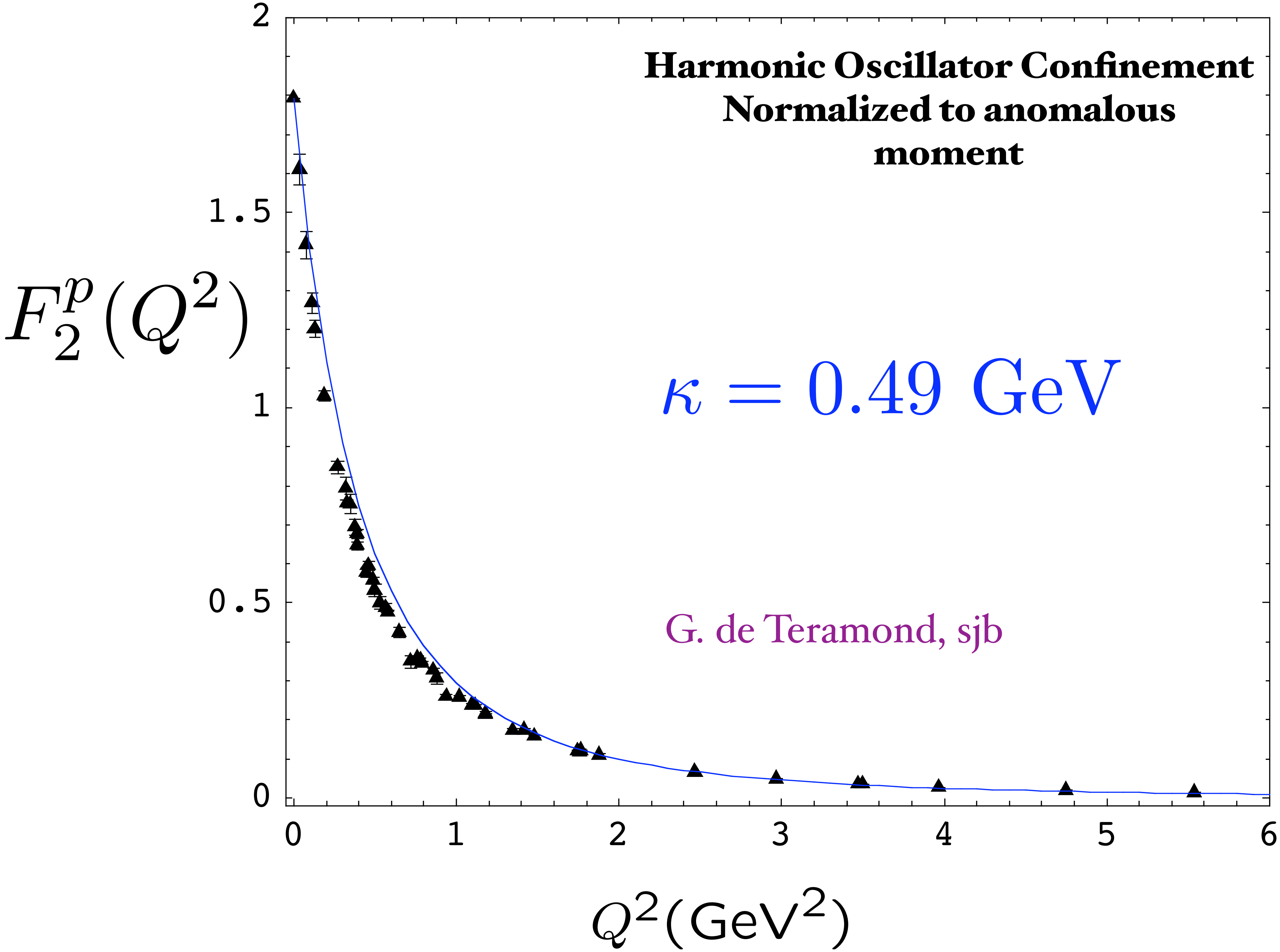


Using $SU(6)$ flavor symmetry and normalization to static quantities

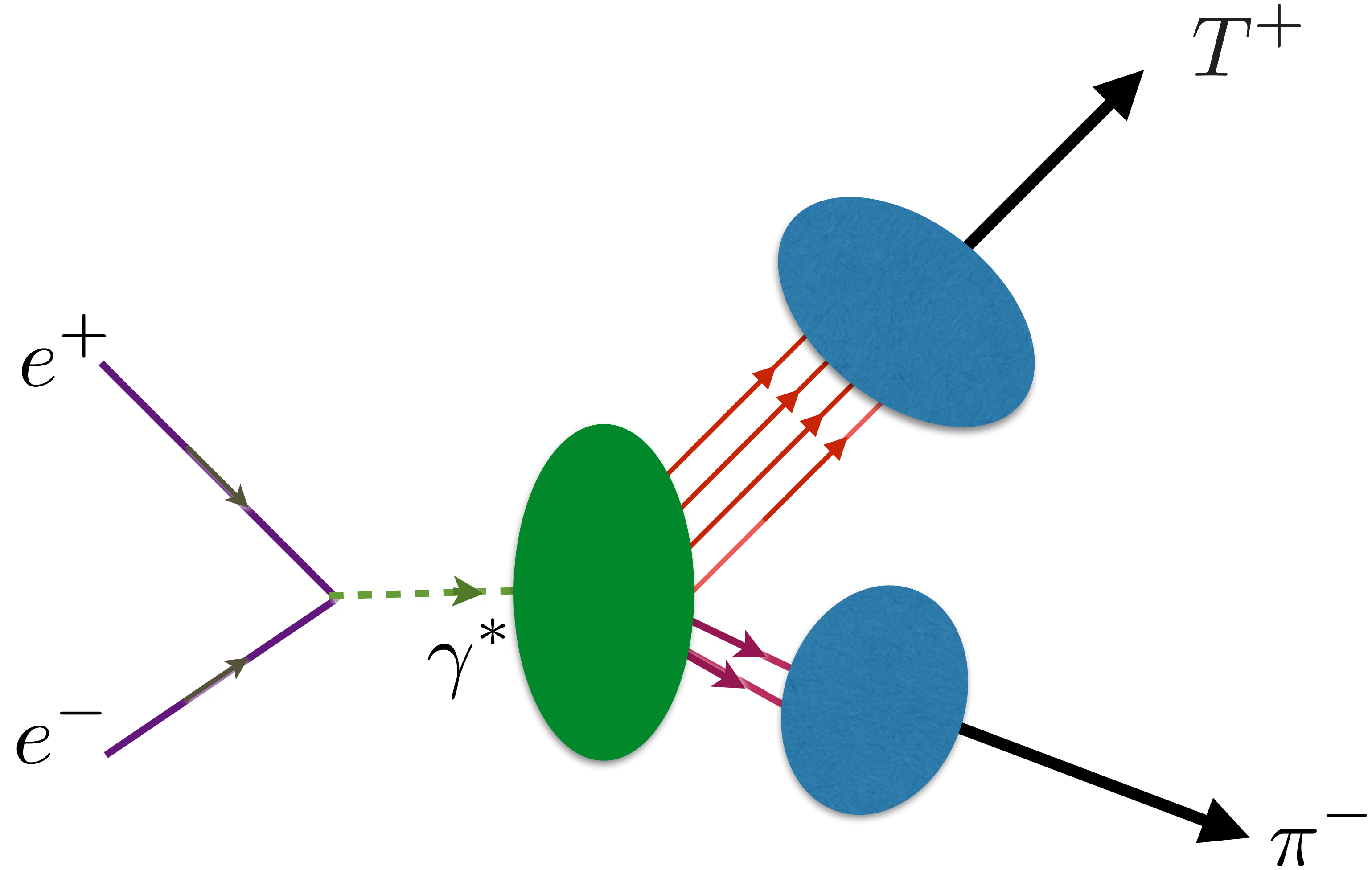


Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



$$\sigma(e^+e^- \to MT) \propto \frac{1}{s^{N-1}} \quad N = 6$$



Use counting rules to identify composite structure

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)}\frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

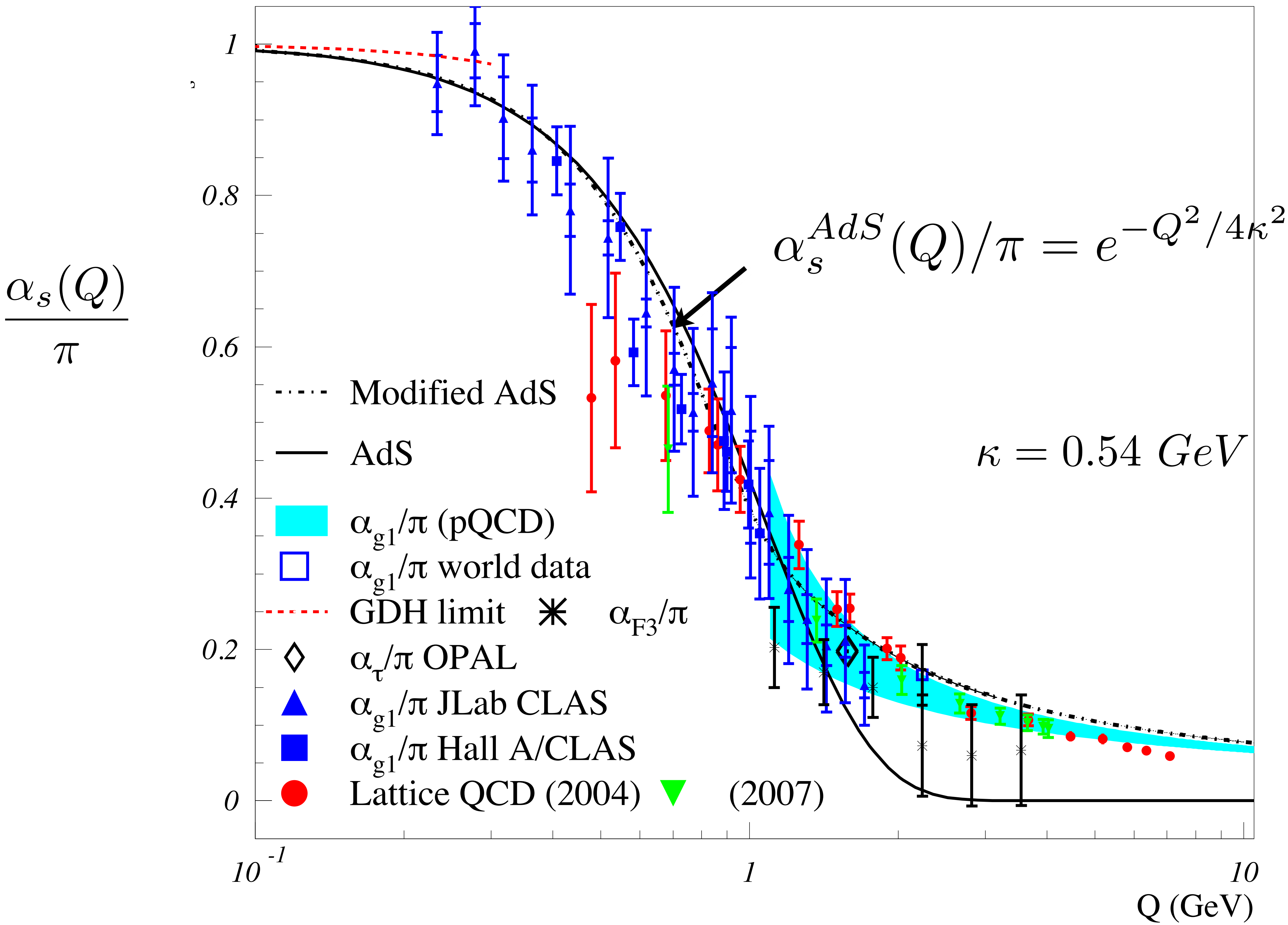
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

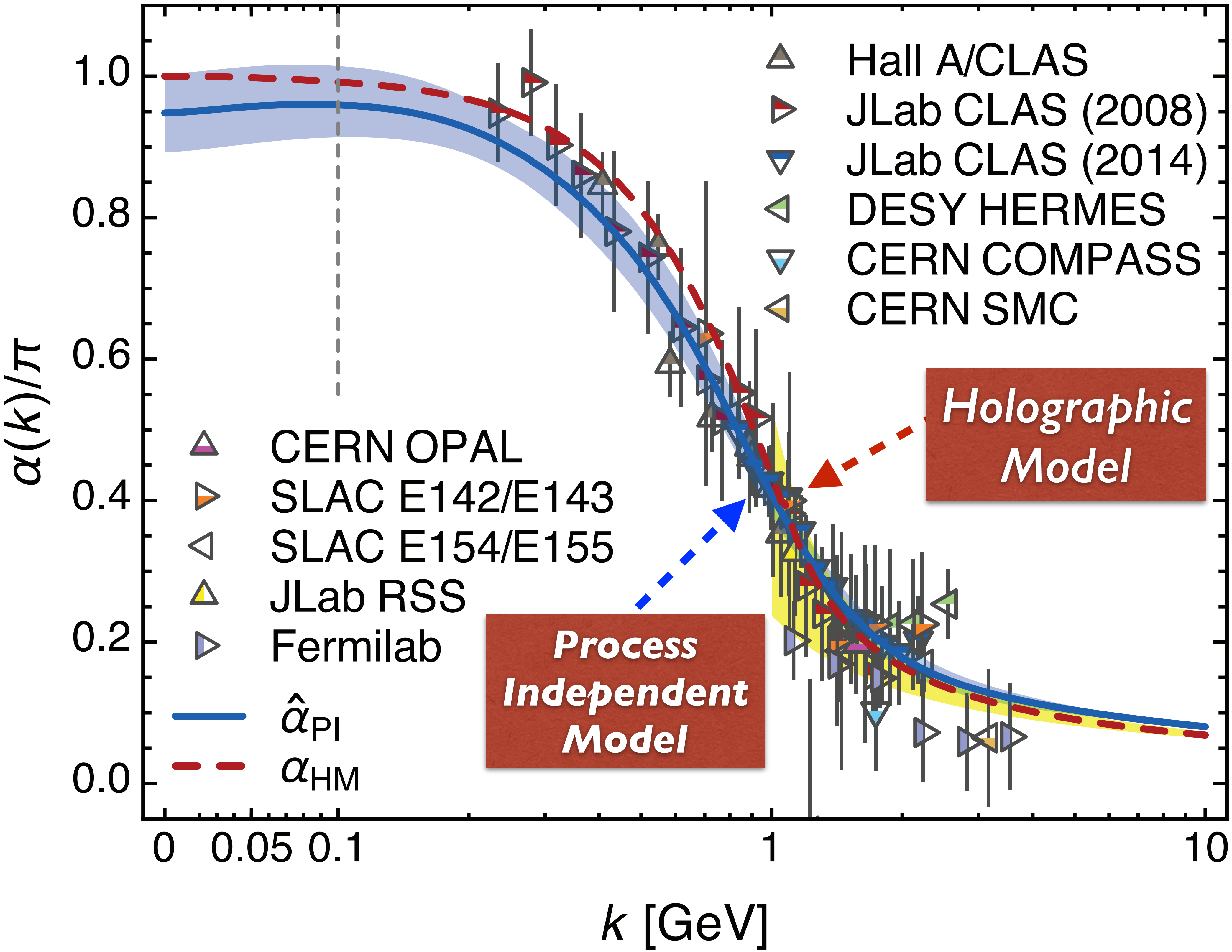
Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb



Process-independent strong running coupling

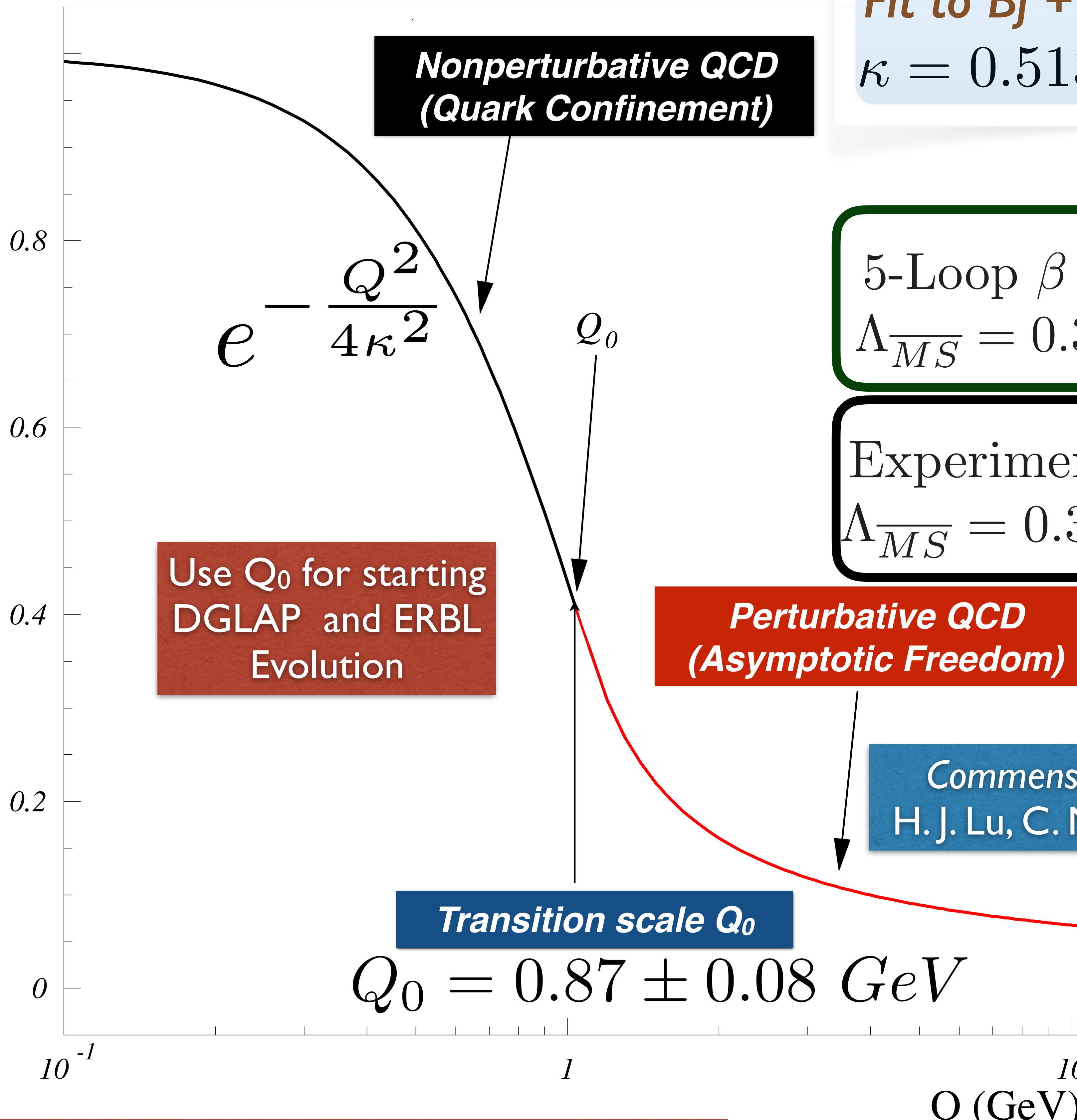
Daniele Binosi,[1] Cédric Mezrag,[2] Joannis Papavassiliou,[3] Craig D. Roberts,[2] and Jose Rodríguez-Quintero[4]

$$m_\rho = \sqrt{2}\kappa$$
$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Tèramond, sjb

Fit to Bj + DHG Sum Rules:
$\kappa = 0.513 \pm 0.007\ GeV$



5-Loop β Prediction:
$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$

Experiment:
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$

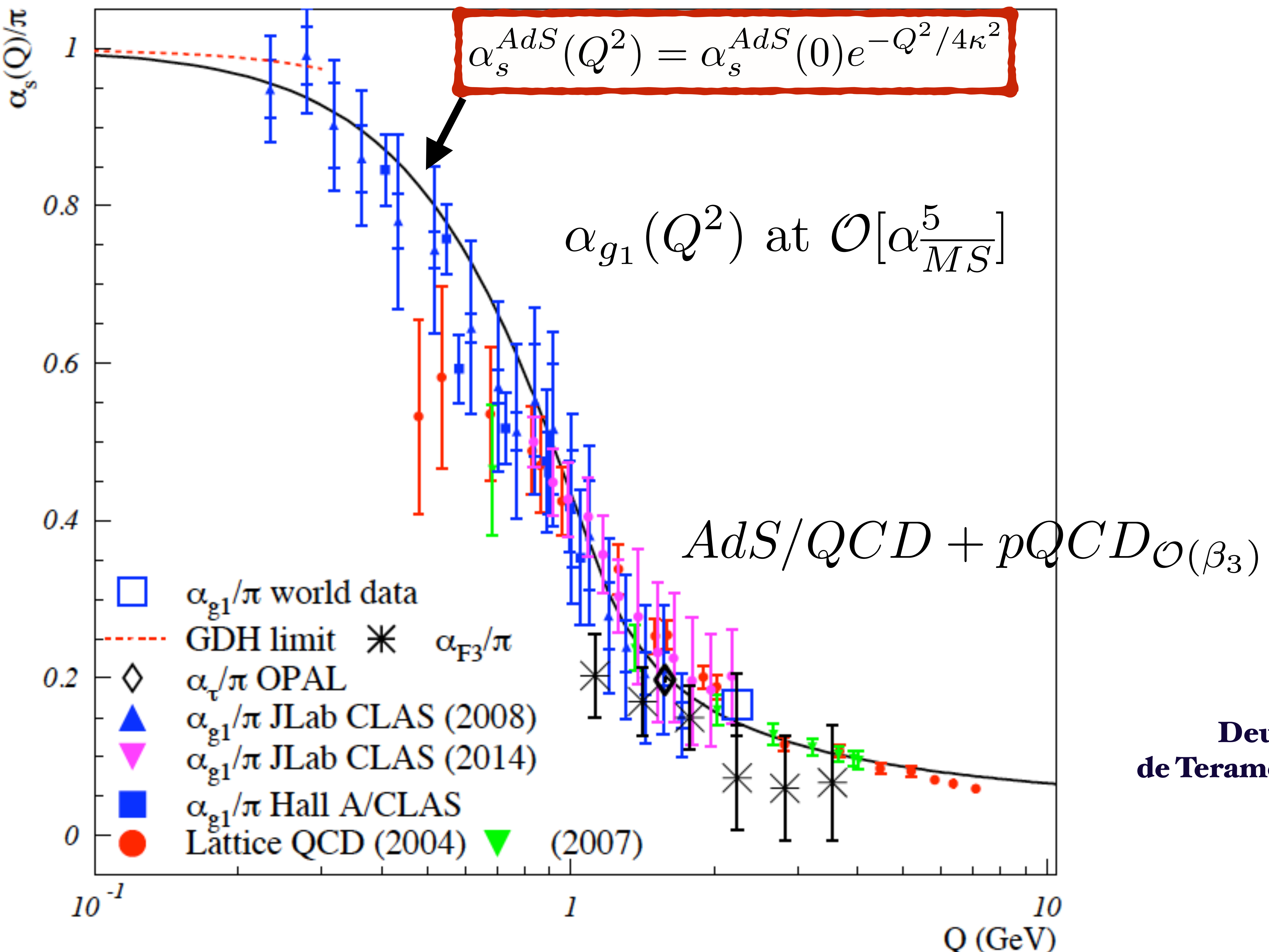
Commensurate Scale Relations
H. J. Lu, C. Merino, J. R. Pelàez, sjb

$\overline{MS}$ *scheme*

$\lambda \equiv \kappa^2$

Reverse Dimensional Transmutation!

$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$



Deur, de Teramond, sjb

Features of LF Holographic QCD

- Color Confinement, Analytic form of confinement potential
- Massless pion bound state in chiral limit
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincare' Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Analytic First Approximation to QCD
- Systematically improvable: Basis LF Quantization (BLFQ)

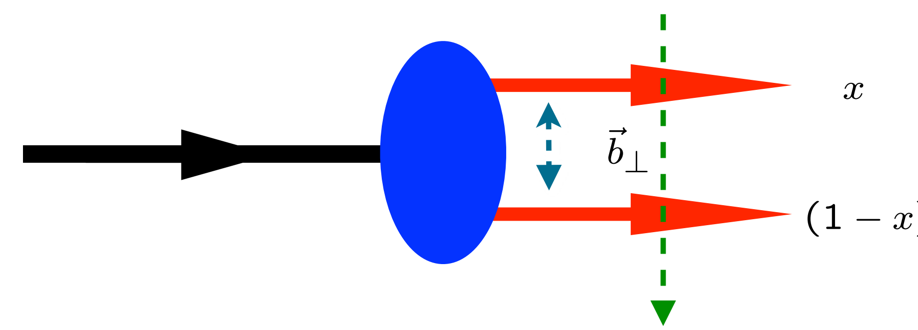
Many phenomenological tests

Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame*
- *Quantization at Fixed Light-Front Time* τ
- *Causality: Information within causal horizon*
- *Light-Front Holography: AdS_5 = LF (3+1)*

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$

Fixed $\tau = t + z/c$



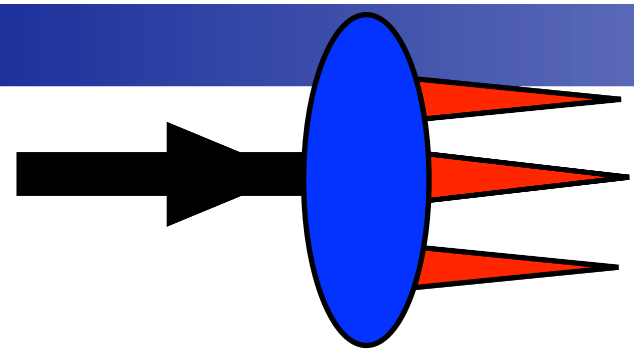
$$\zeta^2 = x(1-x)b_\perp^2$$

- *Single fundamental hadronic mass scale κ: but retains the Conformal Invariance of the Action (dAFF)!*
- *Unique color-confining LF Potential!* $U(\zeta^2) = \kappa^4 \zeta^2$
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

Light-Front Wavefunctions underly hadronic observables

Lorce, Pasquini



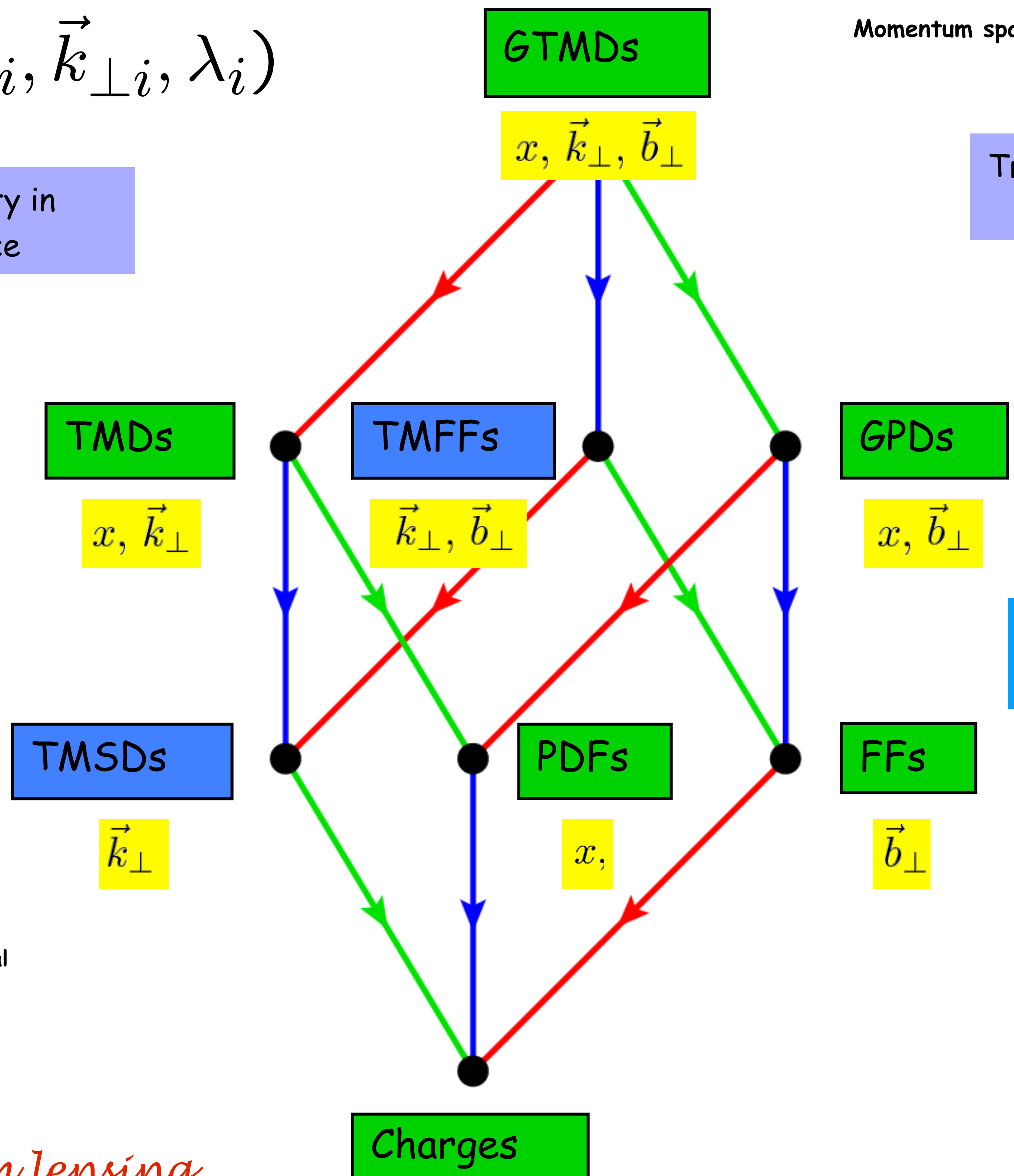
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

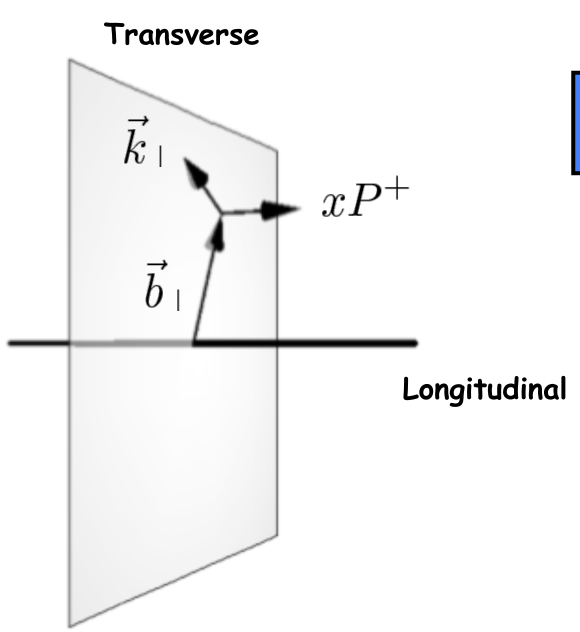
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in momentum space

Transverse density in position space



DGLAP, ERBL Evolution Factorization Theorems



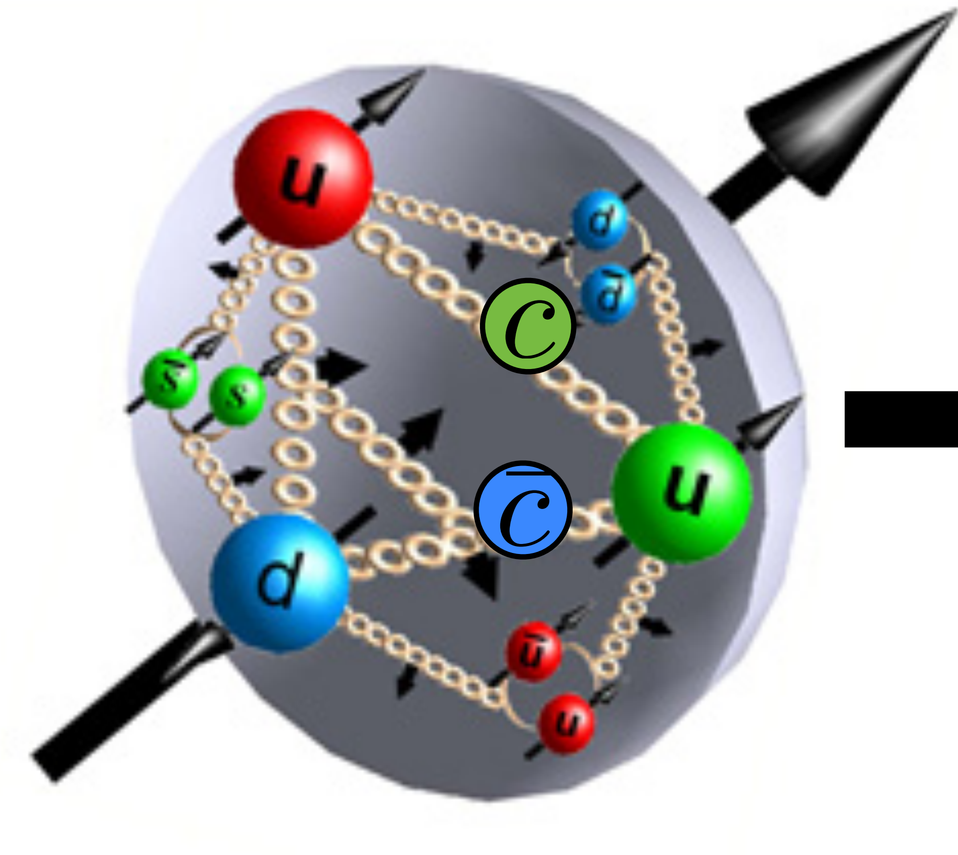
Sivers, T-odd from lensing

red: $\int d^2 b_\perp$

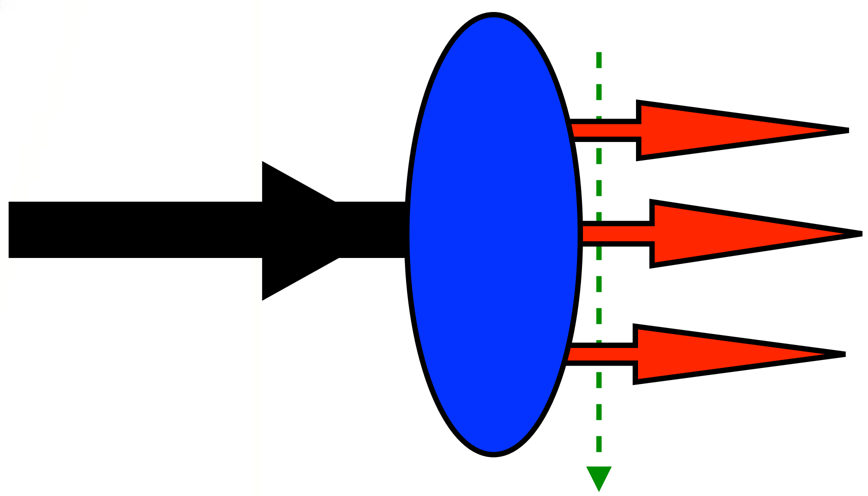
blue: $\int dx$

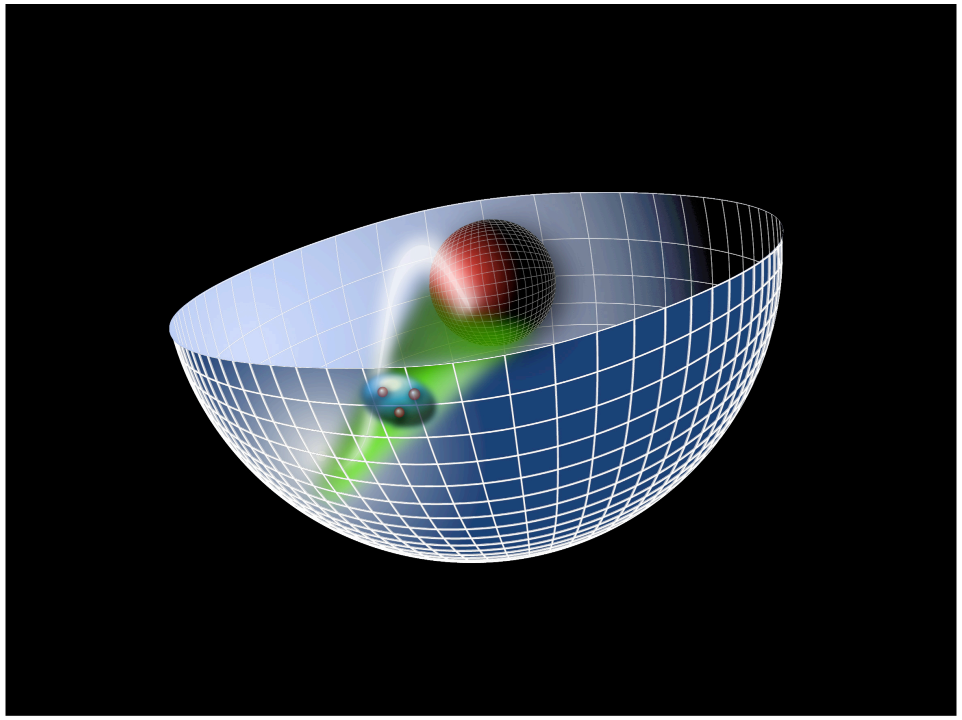
green: $\int d^2 k_\perp$

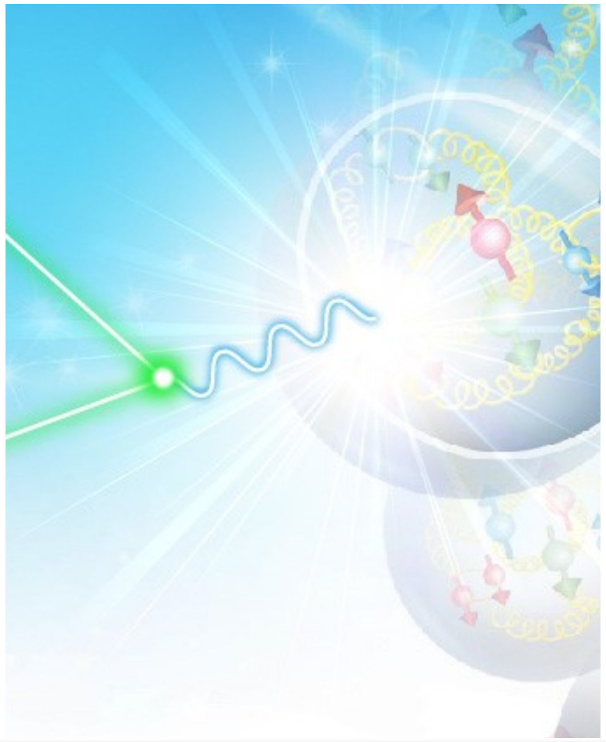
Hadron Physics on the Light-Front: Hadron Dynamics, Spectroscopy and Vacuum Structure from Light-Front Holography and Superconformal Algebra



Fixed $\tau = t + z/c$







Stan Brodsky

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LC2018, May 15, 2018

with Guy de Tèramond, Hans Günter Dosch, R. Shrock, C. Roberts, P. Tandy, C. Lorcè, M. Nielsen, Prem Srivistava, R. S. Sufian, A. Deur