Correspondence between IFD and LFD: Vacuum and related issues

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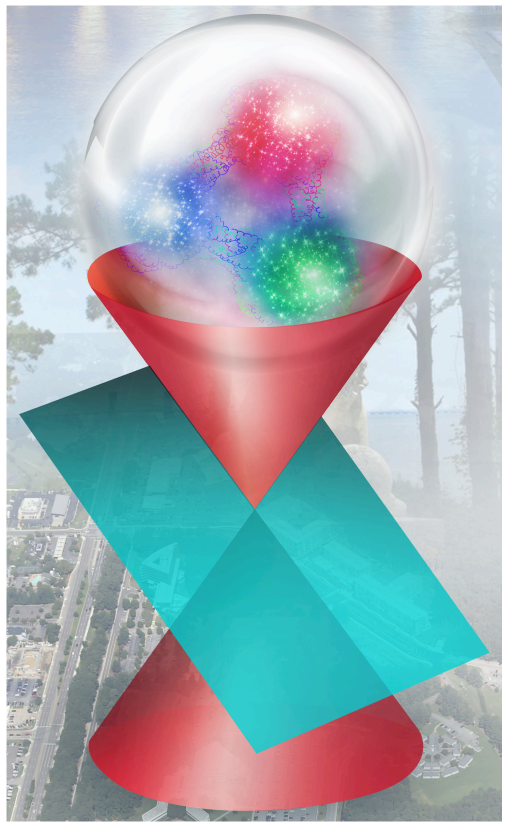
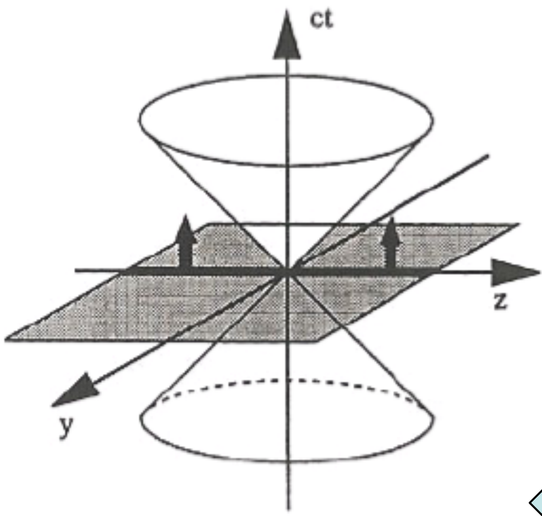


Jefferson Lab, May 15, 2018

Outline

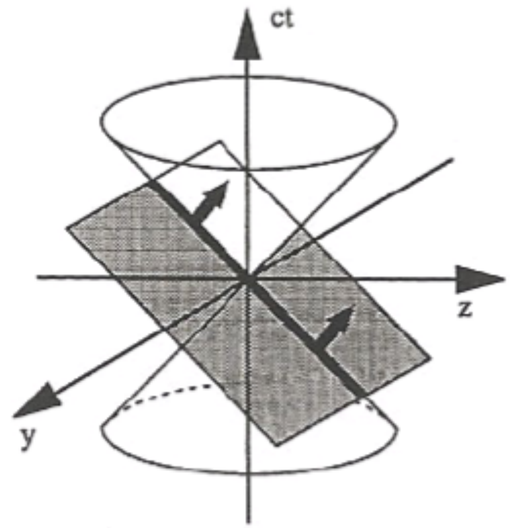
- Laying out the vacuum and related issues
  - Dirac's Proposition of IFD and LFD
- Correspondence between IFD and LFD
  - Symmetry Breaking and Vacuum Condensation
  - LF Zero-mode (LFZM)
  - IMF approach vs. LFD
- Summary and Outlook

Dirac's Proposition



The instant form

1949



The front form

Can they be linked?

Traditional approach
evolved from NR dynamics

Innovative approach
for relativistic dynamics

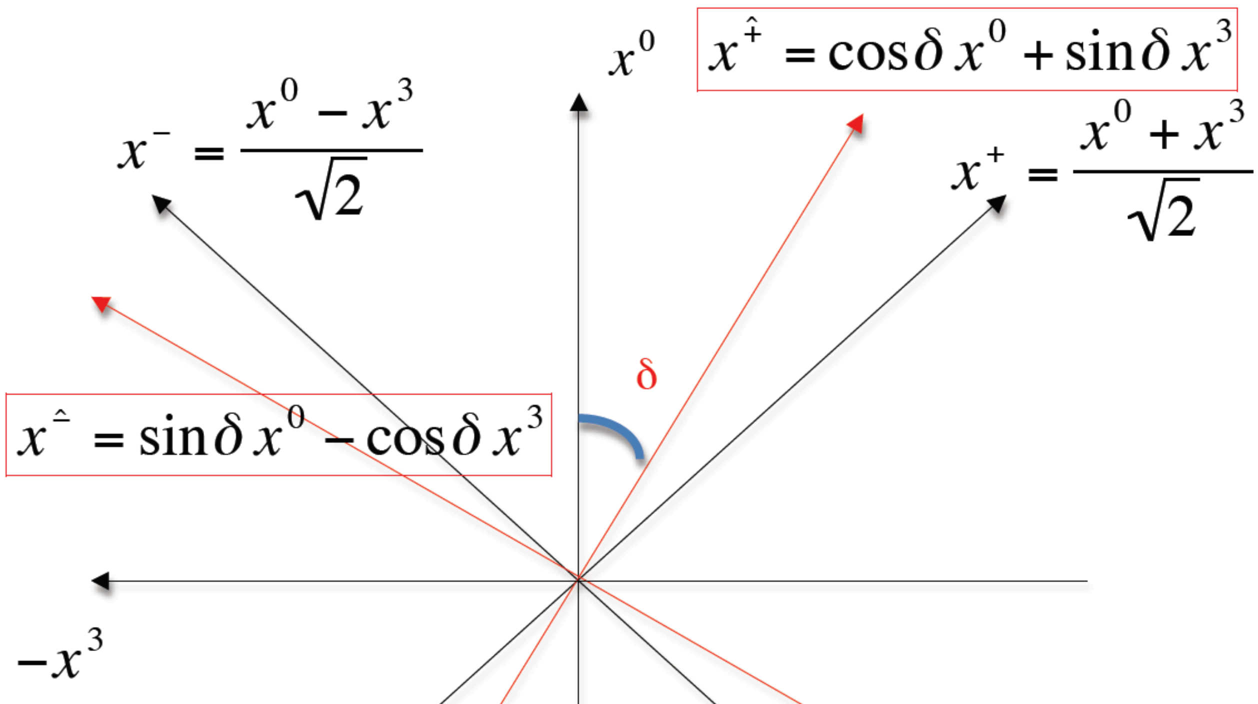
Close contact with
Euclidean space

Strictly in Minkowski space

T-dept QFT, LQCD, IMF, etc.

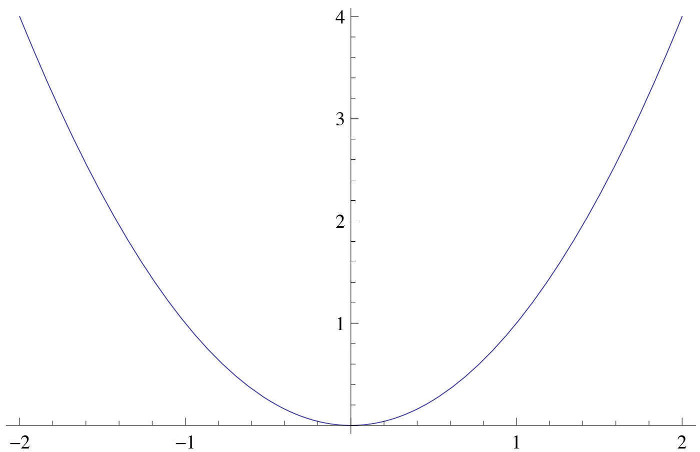
DIS, PDFs, DVCS, GPDs, etc.

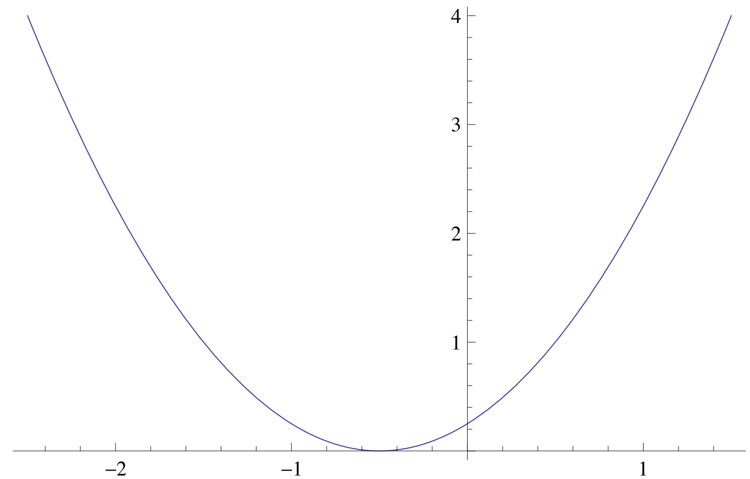
Interpolation between Instant and Front Forms



K. Hornbostel, PRD45, 3781 (1992) – RQFT
C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly
C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps
C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges
Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors
C.Ji, Z.Li, B.Ma and A.Suzuki, submitted to PRD – Fermion Prop.

Symmetry Breaking





$$\phi \rightarrow \phi' = \phi + \nu$$

$$L \rightarrow L' = L - m^2 \nu \phi - \frac{1}{2} m^2 \nu^2$$

$$P_{\hat{+}} \rightarrow P'_{\hat{+}} = P_{\hat{+}} + \frac{\left(m^3 \ell\right)^{1/2} \nu}{C^{1/4}} \left(a_0 + a_0^{+}\right)$$, where $C = \cos 2\delta$

Nontrivial Vacuum State

$$|0> \rightarrow |\Omega>$$

Translation in scalar field: $\phi \rightarrow \phi' = \phi + v$

$$|\Omega> = \exp\left(i \int_{-\ell}^{+\ell} dx^{\hat{-}} v\, \pi(x^{\hat{-}}) \right) |0>$$

$$\pi(x^{\hat{+}} = 0, x^{\hat{-}}) = -i \sum_{n=-\infty}^{\infty} \left(\frac{\pi}{\ell}\right) \sqrt{\frac{\omega_n}{4\pi}} \left[a_n e^{-i\left(\frac{n\pi}{\ell}\right)x^{\hat{-}}} - a_n^{+} e^{i\left(\frac{n\pi}{\ell}\right)x^{\hat{-}}} \right]$$

$$|\Omega> = \exp\left[-(C^{1/2} m\ell) \frac{v^2}{2} \right] \exp\left[-(C^{1/2} m\ell)^{1/2} v a_0^{+} \right] |0>$$

Condensation of Zero-Modes

Vacuum Energy

$$P_{\hat{+}} \,|\Omega> = E_{\Omega} \,|\Omega>$$

$$a\, e^{\alpha a^{+}} \,|0> = \alpha\, e^{\alpha a^{+}} \,|0>$$

$$P_{\hat{+}} \,|\Omega> \longrightarrow \left[\frac{m\nu}{C^{1/2}} a_0^{+} a_0 + \frac{\left(m^{3}\ell\right)^{1/2} \nu}{C^{1/4}} (a_0 + a_0^{+}) \right] \exp\left[-(C^{1/2} m\ell)^{1/2} \nu a_0^{+} \right] |0>$$

$$= (-m^{2}\nu^{2}\ell) \exp\left[-(C^{1/2} m\ell)^{1/2} \nu a_0^{+} \right] |0>$$

$$E_{\Omega} = -m^{2}\nu^{2}\ell = \int_{-\ell}^{+\ell} (-\frac{1}{2} m^{2}\nu^{2})\, dx^{\hat{-}}$$

Independent of interpolation angle!

Recovery of Trivial Vacuum in LFD?

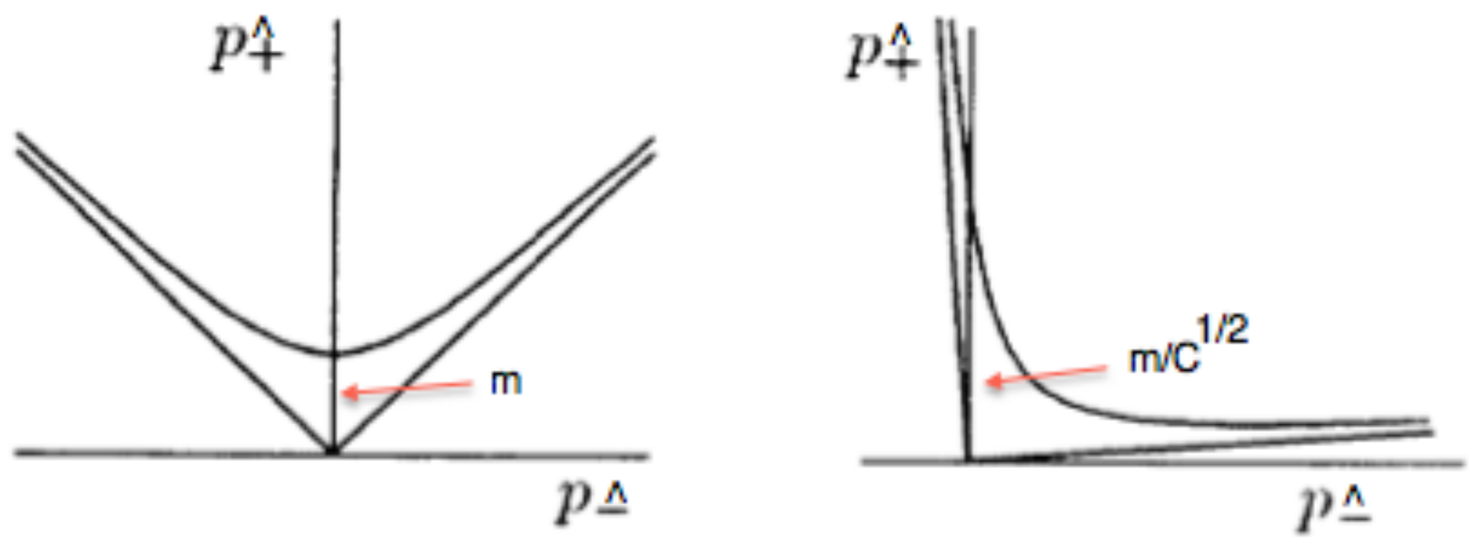
$$|\Omega> = \exp\left[-(C^{1/2}m\ell)\frac{\nu^2}{2}\right]\exp\left[-(C^{1/2}m\ell)^{1/2}\nu a_0^+\right]|0>$$

$$|\Omega> \to |0> \quad as \quad C \to 0$$

However, E_Ω and $<\Omega|\phi(x)|\Omega> = -\nu$

are still independent of interpolation angle!

What is going on?



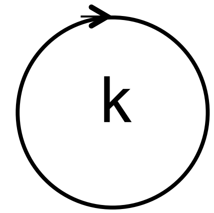
$$< \Omega | \phi(x) | \Omega >$$

$$= < 0 | \exp\left[(C^{1/2} m\ell)^{1/2} \nu (a_0^+ - a_0) \right] \left(\frac{a_0 + a_0^+}{2(C^{1/2} m\ell)^{1/2}} \right) \exp\left[-(C^{1/2} m\ell)^{1/2} \nu (a_0^+ - a_0) \right] | 0 >$$

$$= -\nu$$

Complication is transferred from vacuum to operator.

Vaccum Bubble in 1+1 Dim



$$I = \int d^2k \frac{1}{k^2 - m^2 + i\varepsilon}$$

$$= \int dk_{\hat{-}}\, dk_{\hat{+}} \frac{1}{C(k_{\hat{+}}^2 - k_{\hat{-}}^2) + 2Sk_{\hat{+}}k_{\hat{-}} - m^2 + i\varepsilon}$$

$$= -i\pi \int dk_{\hat{-}} \frac{1}{\sqrt{k_{\hat{-}}^2 + Cm^2} - i\frac{\varepsilon}{2}C}$$

$$= i\pi \log m^2 \quad \text{mod. log } \mathit{divergence}$$

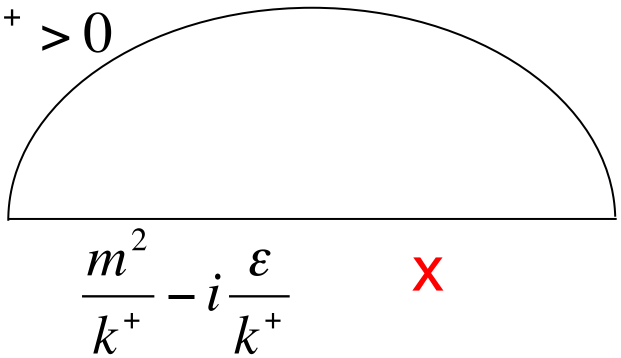
✕ $$k_{\hat{+}}^{pole(+)} = -\frac{S}{C}k_{\hat{-}} - \frac{\sqrt{k_{\hat{-}}^2 + Cm^2}}{C} + i\varepsilon$$

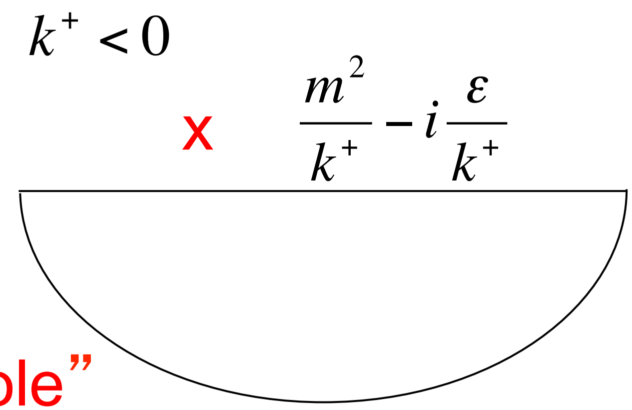
$$(C = \cos 2\delta,\ S = \sin 2\delta)$$

✕ $$k_{\hat{+}}^{pole(-)} = -\frac{S}{C}k_{\hat{-}} + \frac{\sqrt{k_{\hat{-}}^2 + Cm^2}}{C} - i\varepsilon$$

LFD

$$I = \frac{1}{2}\int dk^+ dk^- \frac{1}{k^+k^- - m^2 + i\varepsilon} = \frac{1}{2}\int \frac{dk^+}{k^+}\int dk^- \frac{1}{k^- - \frac{m^2}{k^+} + i\frac{\varepsilon}{k^+}}$$



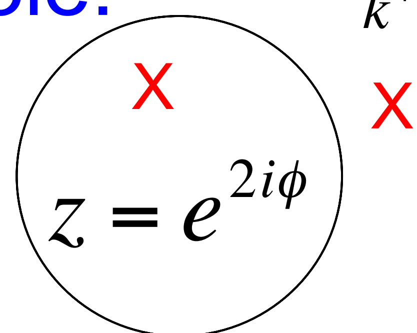


``Moving Pole"

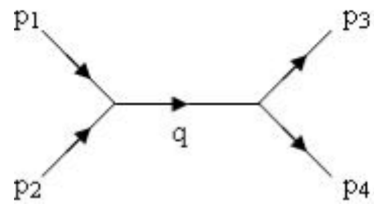
B.Bakker, M. DeWitt, Y. Mischchenko, C.Ji, PRD72, 076005)(2005)

Capture the pole!

$k^+ = r\cos\phi \quad k^- = r\sin\phi$



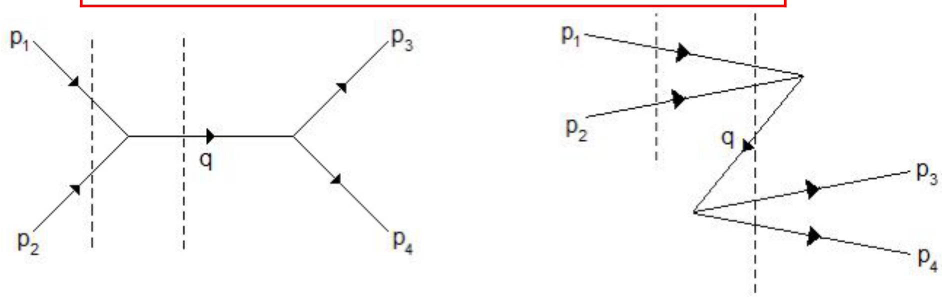
$$I = \int_0^\infty \frac{dr}{r}\oint dz \frac{2}{\left[z-(i\alpha+\sqrt{1-\alpha^2}+\varepsilon)\right]\left[z-(i\alpha-\sqrt{1-\alpha^2}+\varepsilon)\right]} \Rightarrow i\pi\log m^2$$



$$\delta = 0$$
$$p_0 = p^0$$
$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$
$$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$$
$$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$$

$$\delta = \pi/4$$
$$p_+ = p^-$$
$$p_- = p^+$$



(a) (b)

$$\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}}\right)$$

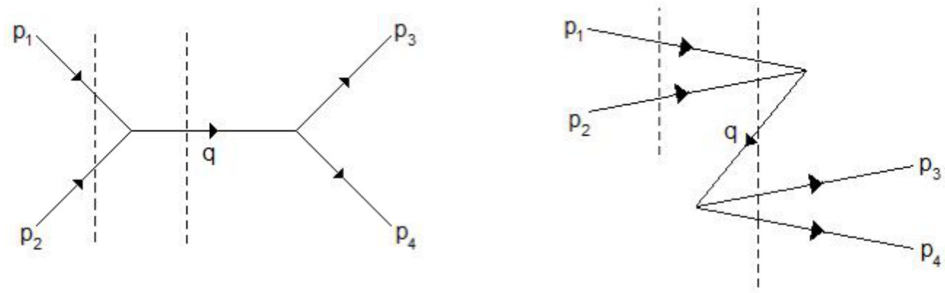
$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2\right)}$$

$$\mathbb{C} = \cos 2\delta$$

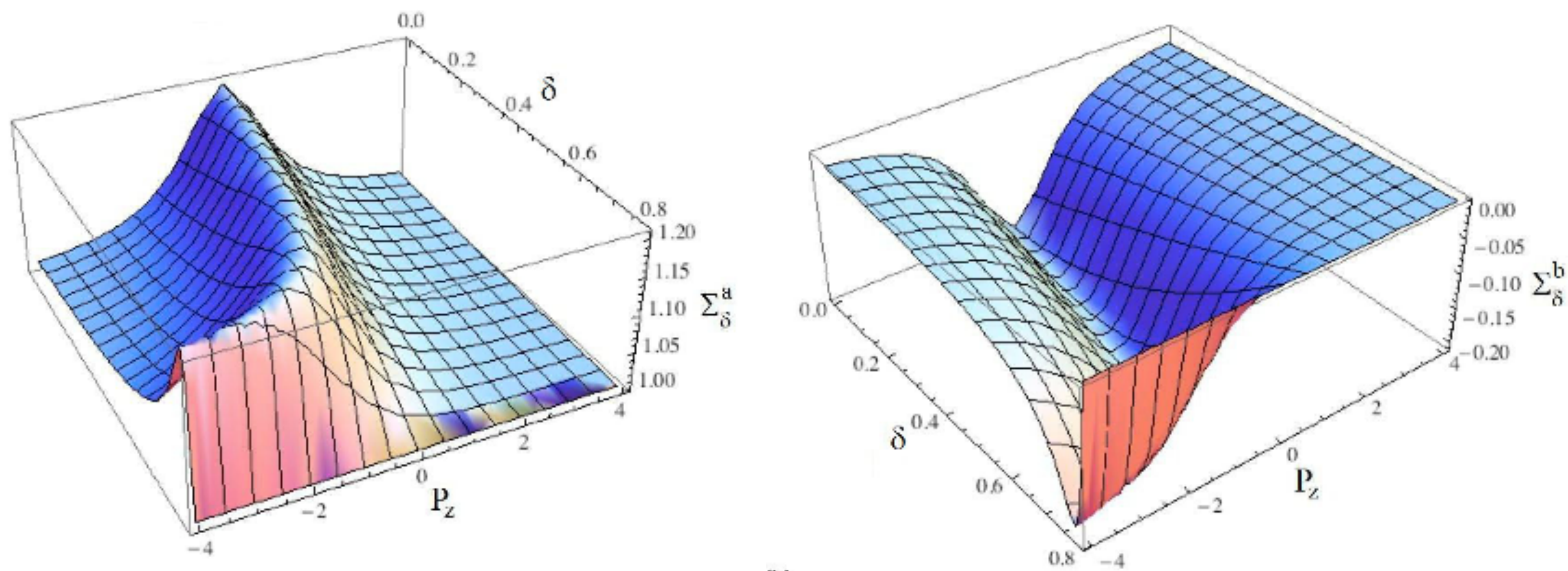
$$\mathbb{S} = \sin 2\delta$$

$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{\mathbf{P}}_\perp^2 + m^2)}{2P^+}\right\}}$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \to \frac{2}{\mathbb{C}} - \frac{\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$
$$\to \infty \quad \text{as } \mathbb{C} \to 0$$



(a) (b)

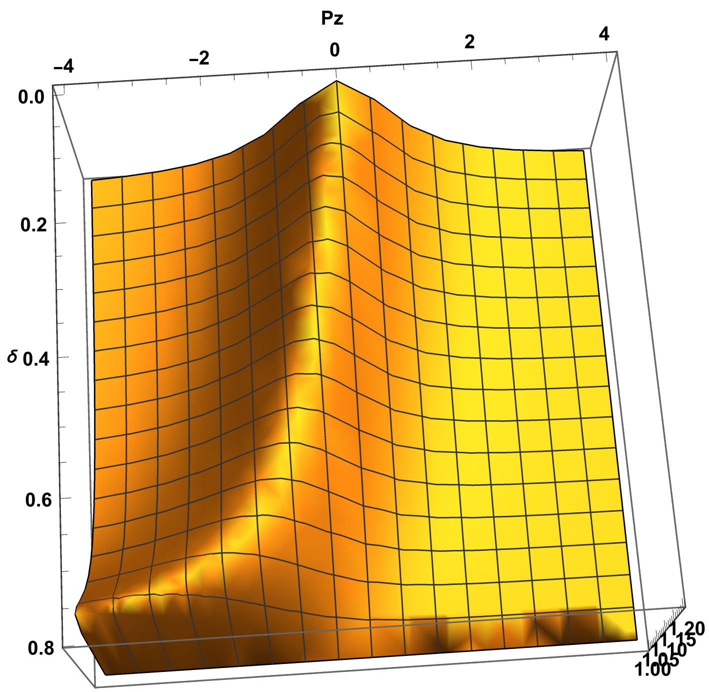


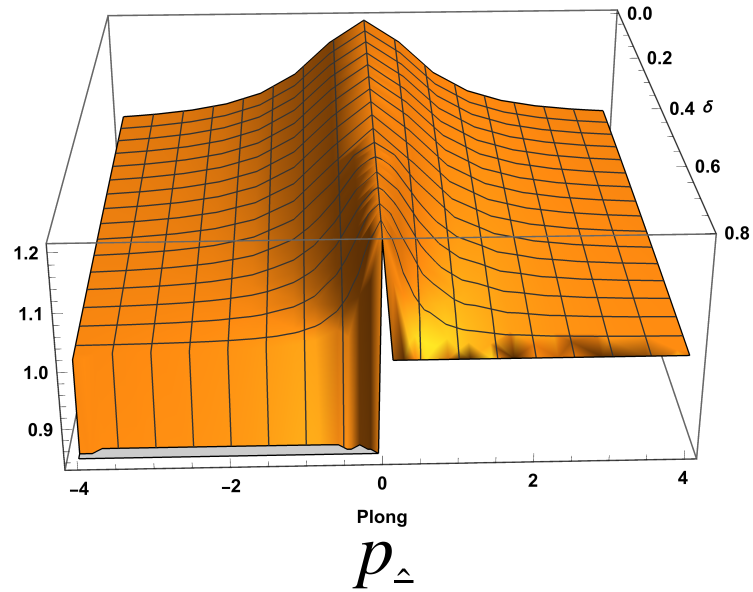
(a) (b)

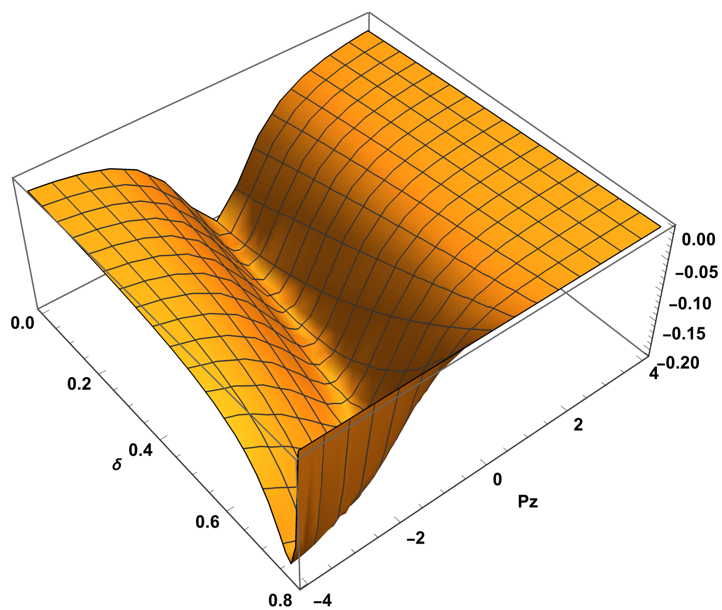
$\Sigma(a)+\Sigma(b)=1/(s-m^2)$; $s=2\ \mathrm{GeV}^2$, $m=1\mathrm{GeV}$

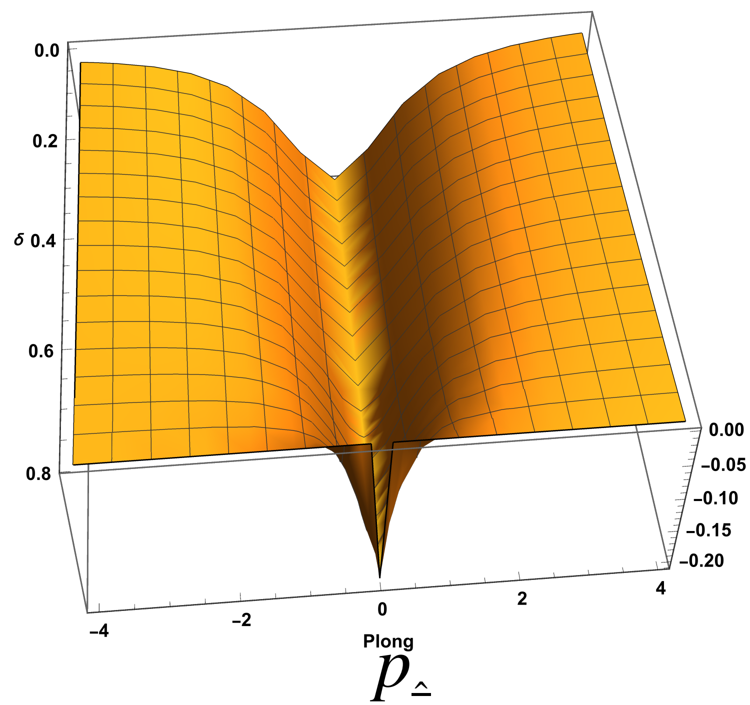
J-shape peak & valley : $P_z = -\sqrt{\frac{s(1-C)}{2C}}$; $C = \cos(2\delta)$

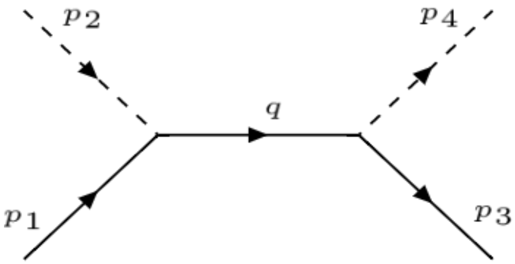
As $C \rightarrow 0$, $P^+ = P^0 + P_z \rightarrow 0$ leads to LF Zero-modes.


Pz
-4
-2
0
2
4
0.0
0.2
δ 0.4
0.6
0.8
1.20
1.00


0.0
0.2
0.4 δ
0.6
0.8
1.2
1.1
1.0
0.9
-4
-2
0
2
4
Plong


0.00
-0.05
-0.10
-0.15
-0.20
0.0
0.2
δ 0.4
0.6
0.8
-4
-2
0
2
4
Pz

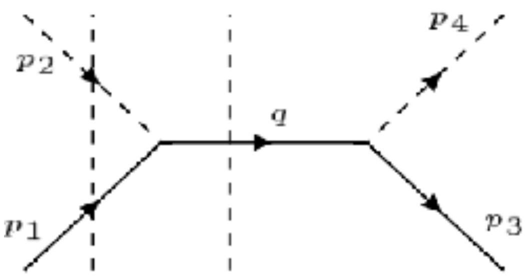

0.0
0.2
δ 0.4
0.6
0.8
0.00
-0.05
-0.10
-0.15
-0.20
-4
-2
0
2
4
Plong



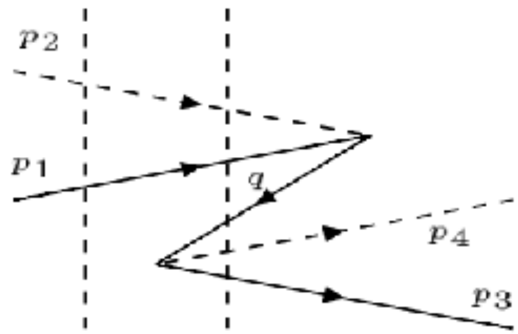
Bailing Ma's Talk on Thursday session 4B

Constrained Degrees of Freedom of Fermion in Light-Front Dynamics

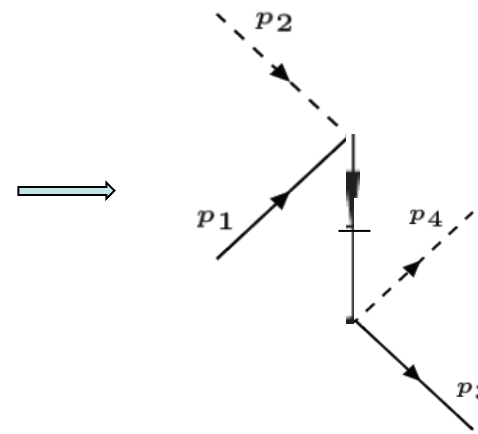
Fermion Propagator



(a)



(b)



$$
\begin{aligned}
\Sigma_a^{\text{IFD}} + \Sigma_b^{\text{IFD}} &= \frac{1}{2q^0_{on}} \left(\frac{\not{q} + m}{q^0 - q^0_{on}} - \frac{\not{q} + m}{q^0 + q^0_{on}} \right) \\
&= \frac{1}{2q^0_{on}} \frac{2q^0_{on}(\not{q} + m)}{(q^0)^2 - (q^0_{on})^2} \\
&= \frac{\not{q} + m}{q^2 - m^2}
\end{aligned}
$$

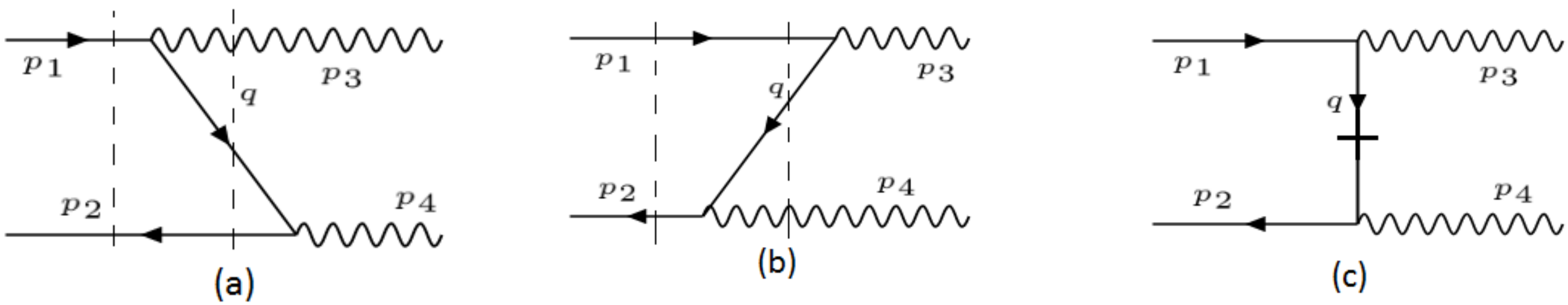
$$\Sigma_{a,\delta\to\frac{\pi}{4}} = \frac{\not{q}_{on} + m}{q^2 - m^2}$$

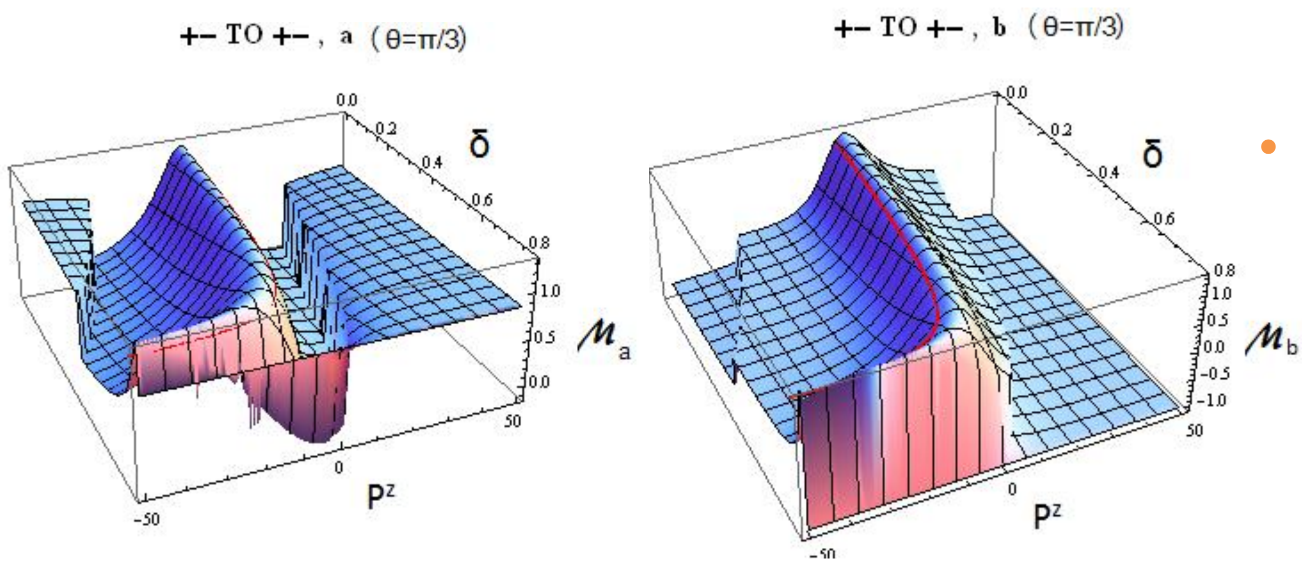
$$\Sigma_{b,\delta\to\frac{\pi}{4}} = \frac{\gamma^+}{2q^+}$$

S.-J.Chang and T.-M.Yan, PRD7,1147(1973)

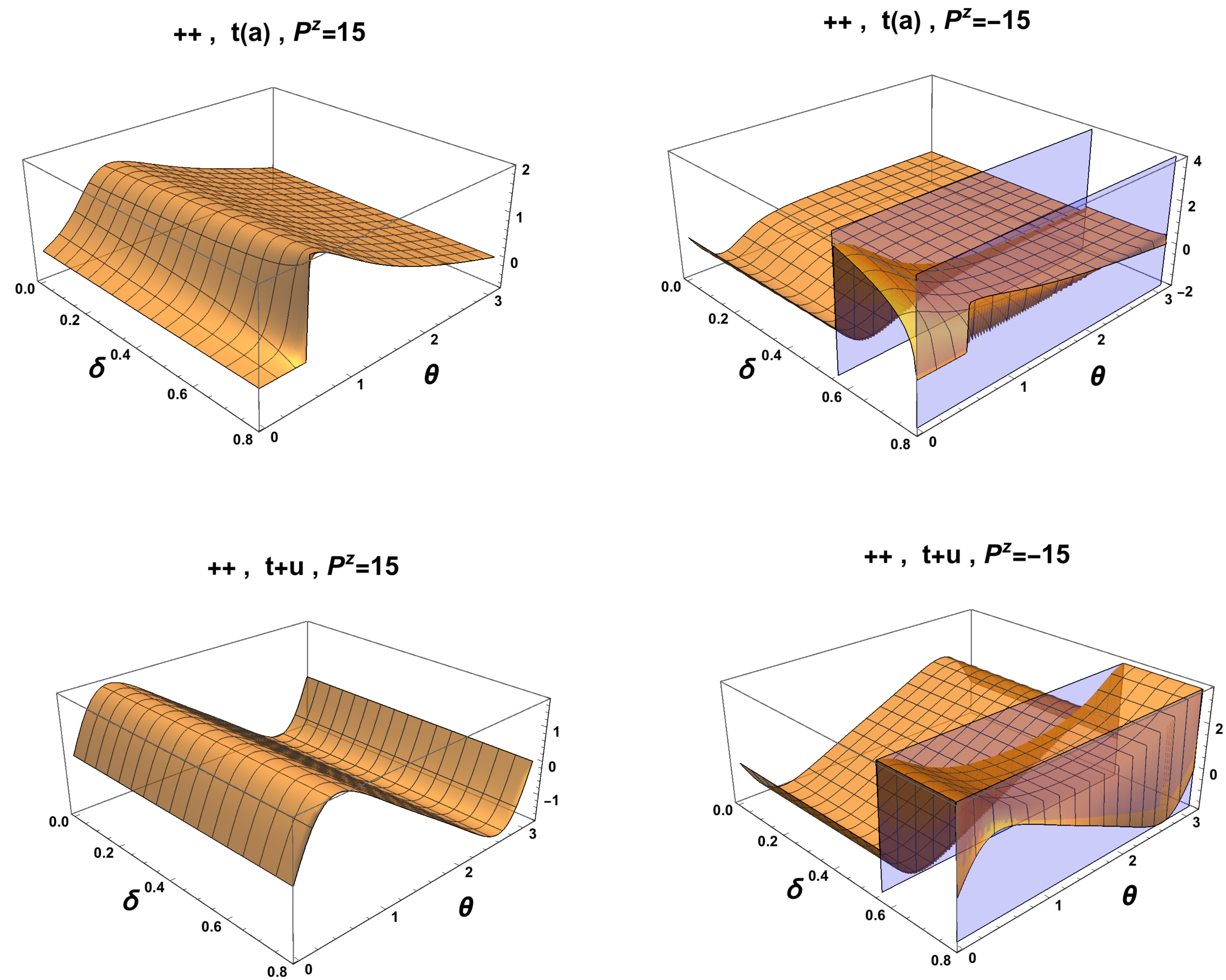
$$\frac{1}{\not{q} - m} = \frac{\sum_s u(q,s)\bar{u}(q,s)}{q^2 - m^2} + \frac{\gamma^+}{2q^+}$$

Example Application: The Annihilation of Electron- positron Pair into Two Photons





- Diagram (c) only exists in LFD
- Only one of (a) and (b) is allowed in LFD and the other one changes to instantaneous interaction in LFD


++ , t(a) , P^z=15
δ
θ
++ , t(a) , P^z=−15
δ
θ
++ , t+u , P^z=15
δ
θ
++ , t+u , P^z=−15
δ
θ

Standard Model

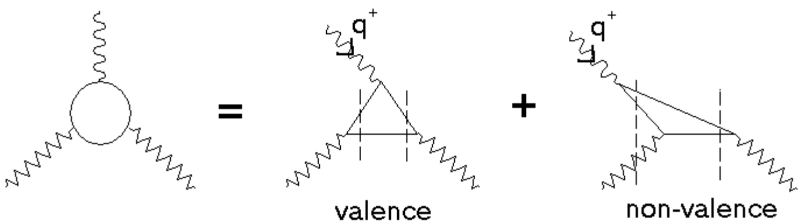
$$
\begin{array}{cccc}
2/3 & \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} & \begin{pmatrix} t \\ b \end{pmatrix} \\
-1/3 & & & \\
0 & \begin{pmatrix} \nu_e \\ e \end{pmatrix} & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \\
-1 & & &
\end{array}
$$

$$\sum_f Q_f = 0 \quad (Anomaly - Free \ Condition)$$

LFD Results

$$G^{+}_{h'h} = \left\langle h', p' | J^{+} | h, p \right\rangle \quad in \quad q^{+} = 0 \quad frame \quad with \quad \eta = Q^2/4M_W^2 \quad (Q^2 = -q^2),$$

$$G^{+}_{++} = 2p^{+}(F_1 + \eta F_3), G^{+}_{+0} = p^{+}\sqrt{2\eta}(2F_1 + F_2 + 2\eta F_3), G^{+}_{+-} = -2p^{+}\eta F_3, G^{+}_{00} = 2p^{+}(F_1 - 2\eta F_2 - 2\eta^2 F_3)$$

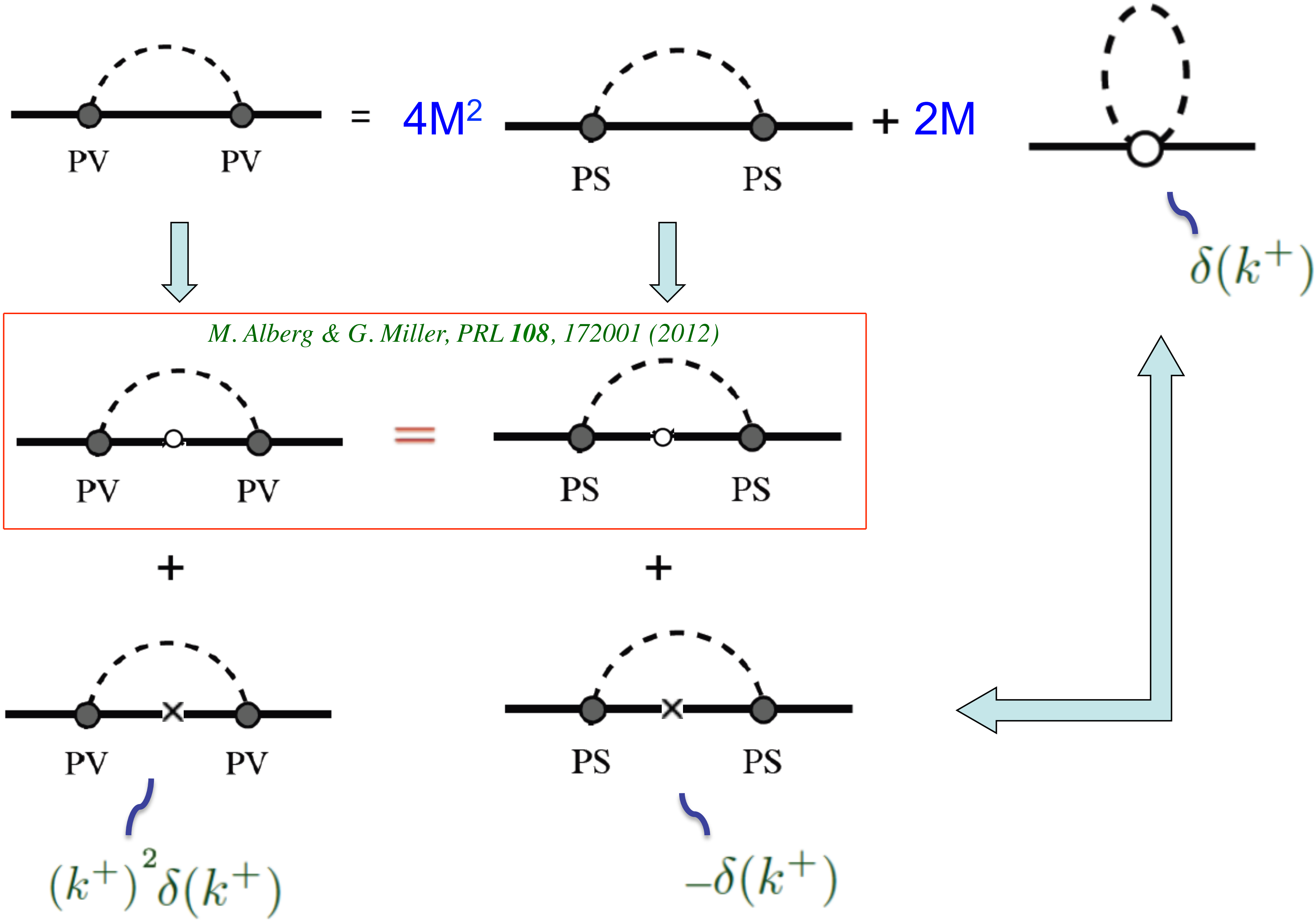


$$\left(G^{+}_{00}\right)_{Z.M.} = \frac{g^2 Q_f p^{+}}{2\pi^3 M_W^2}\int_0^1 dx \int d^2k_\perp \frac{k_\perp^2 + m_1^2 - x(1-x)Q^2}{k_\perp^2 + m_1^2 + x(1-x)Q^2} \neq 0$$

$$\left(F_2 + 2F_1\right)^{+0} = \frac{1}{p^{+}}\left[\frac{G^{+}_{+0}}{\sqrt{2\eta}} + G^{+}_{+-}\right], \quad \left(F_2 + 2F_1\right)^{00} = \frac{1}{4p^{+}\eta}\left[(1+2\eta)G^{+}_{++} - G^{+}_{00} + (1+4\eta)G^{+}_{+-}\right]$$

$$(F_2 + 2F_1)^{+0}_{DR2} = (F_2 + 2F_1)_{DR4} + \frac{g^2 Q_f}{4\pi^2}\left(\frac{1}{6}\right)$$

$$(F_2 + 2F_1)^{00}_{DR2} = (F_2 + 2F_1)_{DR4} - \frac{g^2 Q_f}{4\pi^2}\left(\frac{1}{2\eta}\right)\left(\frac{1}{3} + \frac{2\eta}{9}\right)$$



*C.Ji, W.Melnitchouk, A.W.Thomas, PRL **110**, 179191 (2013)*

Connection with QCD?

- Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with "Sullivan" result

$$\langle x^n \rangle_{u-d} = a_n \left(1 + \frac{(3g_A^2+1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \right) + \mathcal{O}(m_\pi^2)$$

cf. $4g_A^2$ in "Sullivan", via moments of $f_\pi(y)$

Chen, X. Ji, PLB ***523****, 107 (2001)*
Arndt, Savage, NPA ***692****, 429 (2002)*

→ is there a problem with application of ChPT or "Sullivan process" to DIS?

→ is light-front treatment of pion loops problematic?

→ investigate relation between *covariant*, *instant-form*, and *light-front* formulations

→ consider simple test case: nucleon self-energy

Nonanalytic behavior

$$\mathcal{M}_N^{(p)} \xrightarrow{\text{LNA}} 1 - \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(p)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_N^{(n)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(n)} \xrightarrow{\text{LNA}} -\frac{(3g_A^2+1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

→ no pion corrections to isoscalar moments

→ isovector correction agrees with ChPT calculation

$$\mathcal{M}_N^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{(4g_A^2+[1-g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \xrightarrow{\text{LNA}} \frac{(4g_A^2+[1-g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

PS ("on-shell") contribution

δ-function contribution

C.Ji, W. Melnitchouk, and A. W. Thomas, Phys. Rev. D 347 88, 076005 (2013).

Summary and Outlook

- Whole landscape between IFD and LFD has been revealed in QED by interpolating spinors,gauge bosons,their propagators.
- Vacuum structure in LFD is still not fully uncovered.
- Interpolating quantum field theory appears useful in resolution of theoretical issues, e.g. LFZM.
- Prevailing notion of equivalence between the IMF approach and the LFD should be clarified.