QUARKS AND MESONS IN THE COVARIANT SPECTATOR THEORY

Elmar P. Biernat

Centro de Física Teórica de Partículas, Instituto Superior Técnico, Universidade de Lisboa

May 14, 2018

Light Cone 2018, JLab, May 14-18, 2018

COLLABORATORS AND WORKS

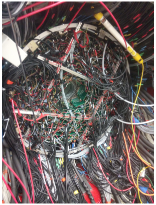
- IST Lisboa: **Teresa Peña**, **Sofia Leitão**, **Alfred Stadler** (U. Évora)
- JLab and C. William & Mary: **Franz Gross**

PAPERS:
Phys. Lett. B764, 38 (2017)
Phys. Rev. D 96, 074007 (2017)
Phys. Rev. D 90, 096008 (2014)
Phys. Rev. D 89, 016005 (2014)

OUTLINE

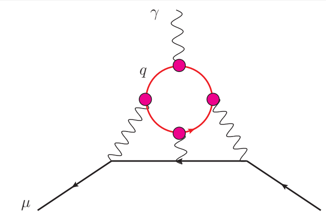
MESON PHENOMENOLOGY — MOTIVATION

- upcoming experiments
  GlueX (JLab), PANDA (FAIR-GSI):
  meson properties
  search exotic mesons
- need better understanding of
  conventional **$q\bar{q}$ mesons**

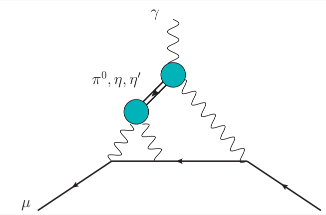


- study **production** mechanisms and transition **form factors**
  e.g. hadronic **light-by-light scattering** in prediction of muon g-2:
  search for new physics

dressed quark **current** & **propagator**



meson **transition form factors**



SOME APPROACHES

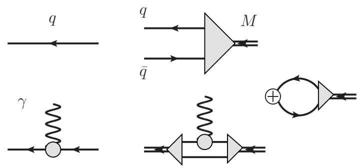
- Based on Dirac's forms of dynamics:
  - Light-front Dynamics ($\hookrightarrow$ talks by Brodsky, Vary, Ji, Mulders, Bakker, Frederico, Karmanov, Salme, de Melo, Glazek, etc.)
  - Point Form
  - Instant Form (Polyzou)
- Lattice QCD ($\hookrightarrow$ talks by Constantinou, etc...)
- Dyson-Schwinger/Bethe Salpeter approach ($\hookrightarrow$ talks by Eichmann, etc...)
- Covariant Spectator Theory

OBJECTIVES

- unified description for **all** $q\bar{q}$ mesons from pion (0.14 GeV) to $b\bar{b}$ ($>$ 10 GeV)
  ✓ already achieved for **heavy** and **heavy-light** mesons
  Leitão, Stadler, Peña, EB, PLB (2017), PRD (2017)
  × to do: **light** mesons
- spectrum and decay properties ⇒ information about the Lorentz structure of the **confining** interaction
- describe mass-generation mechanism of **dynamical chiral-symmetry breaking**

Calculation of

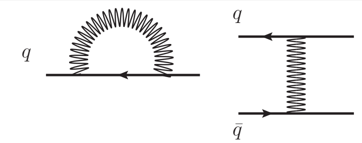
- dynamical quark mass function
- vertex functions
- quark-photon vertex
- meson form factors
- meson decay properties



COVARIANT SPECTATOR THEORY

Guiding principles

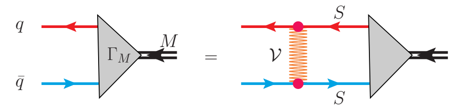
- manifest Lorentz **covariance**
- work in **Minkowski** space
- **confinement**: covariant interaction kernel that reduces to linear+Coulomb potential in nonrelativistic limit
- **dynamical chiral-symmetry breaking**:
  massless pion in chiral limit
  quark masses dynamically generated through self-interaction with $q\bar{q}$ interaction



GROSS (CST) EQUATION

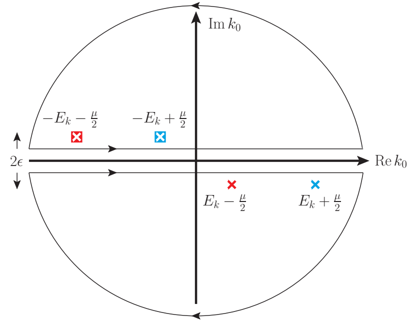
GROSS, PR 186 (1969)

Bethe-Salpeter equation (BSE)



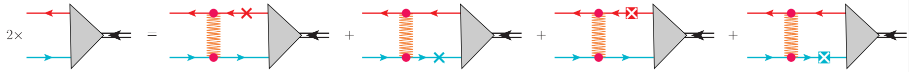
- assume: $\exists$ **real** quark-mass poles
- keep only **quark pole** contributions
- **3D** loop integrations, but covariant
- correct **1-body** Dirac limit

quark poles in complex k_0 plane

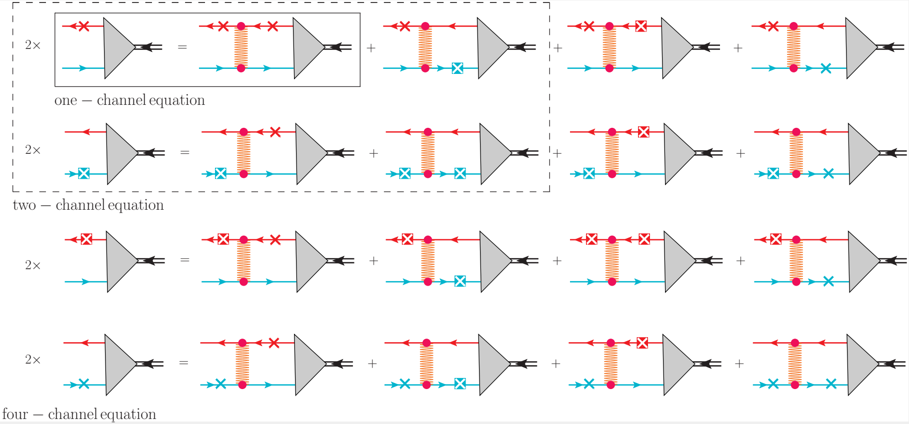


charge-conjugation symmetric Gross equation

SAVKLI, GROSS PRC (2001), EB, GROSS, PEÑA, STADLER PRD (2014)



4-CHANNEL EQUATIONS



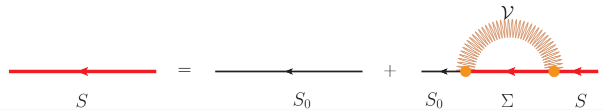
- 1-channel equation: ✓ unequal quark masses, × $\mathcal{C}$-symmetry, × light mesons
- 2-channel equation: ✓ $\mathcal{C}$-symmetry, × light mesons
- 4-channel equation: ✓ all mesons, also light mesons (pion)

All have smooth 1-body (Dirac) and nonrelativistic (Schrödinger) limits

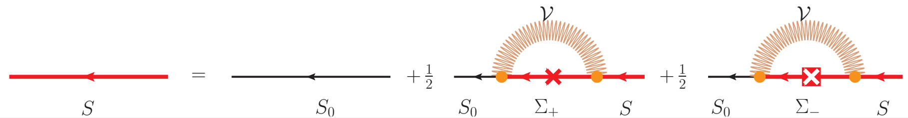
CST DYSON EQUATION

EB, Gross, Peña, Stadler, PRD (2014)

Dyson equation for **dressed** quark propagator



CST Dyson equation



- $S_0(p) = \frac{1}{m_0 - \not{p} - \mathrm{i}\epsilon} \longrightarrow S(p) = \frac{1}{m_0 + \Sigma(p) - \not{p} - \mathrm{i}\epsilon} \equiv \frac{Z(p^2)[M(p^2)+\not{p}]}{M^2(p^2) - p^2 - \mathrm{i}\epsilon}$

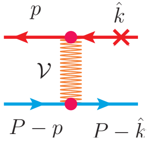
  quark self energy

  $$\Sigma(p) = A(p^2) + \not{p}B(p^2) = \frac{1}{2}\int_{\mathbf{k}} (m + \hat{\not{k}})\mathcal{V}(p,\hat{k}) + \frac{1}{2}\int_{\mathbf{k}} (m - \hat{\not{k}})\mathcal{V}(p,-\hat{k})$$

  dressed quark **mass function** $M(p^2) = \frac{A(p^2)+m_0}{1-B(p^2)}$
- **constituent quark mass** $m = M(p^2 = m^2)$

INTERACTION KERNEL

$$\mathcal{V}(p,k) = \underbrace{\frac{3}{4}\boldsymbol{F}_1 \cdot \boldsymbol{F}_2}_{=1 \text{ (color singlets)}} \sum_K \underbrace{V_K(p,k)}_{\text{momenta}} \underbrace{\Theta_1^K \otimes \Theta_2^K}_{\text{Lorentz } \Theta^K = \mathbf{1}, \gamma^5, \gamma^\mu}$$



Covariant off-shell generalization of 'linear+Coulomb+constant' potential:
$\boldsymbol{q}^2 \to -q^2 = -(p-\hat{k})^2$

- CST 'linear **confinement**': $\int_{\mathbf{k}} V_{\mathrm{L}}(p,\hat{k})\psi(\hat{k}) = -\sigma \int_{\mathbf{k}} \frac{\psi(\hat{k})-\psi(\hat{k}_R)}{q^4}$

  confinement: meson vertex function vanishes if **both** *quarks are on-shell!*

  Savkli, Gross PRC (2001)

  $\int_{\mathbf{k}} V_L(p,\hat{k}) = 0$ corresponds to $V_L^{nr}(r=0) = 0$

- CST **one-gluon** exchange: $V_{\mathrm{G}}(p,\hat{k}) = \frac{\alpha_s}{M_{\mathrm{G}}^2(q^2)-q^2}\left[\mathrm{g}^{\mu\nu} - (1-\xi)\frac{q^\mu q^\nu}{q^2}\right]\gamma_\mu \otimes \gamma_\nu$

  $M_{\mathrm{G}}(m_{\mathrm{G}}^2) = m_{\mathrm{G}}$ dressed constituent gluon mass, ξ gauge parameter

- CST "**constant**":

  $V_C(p,\hat{k}) = C\frac{E_k}{m}\delta^3\left(\vec{k} - \frac{m}{\sqrt{p^2}}\vec{p}\right) h(q^2)\left[\mathrm{g}^{\mu\nu} - (1-\xi)\frac{q^\mu q^\nu}{q^2}\right]\gamma_\mu \otimes \gamma_\nu$

  $h(0) = 1$

AVWTI AND LORENTZ STRUCTURE

EB, Peña, Ribeiro, Stadler, Gross PRD (2014)

- Consistency with chiral symmetry and its breaking: **axial-vector Ward-Takahashi identity** (AVWTI)

  $$-\mathrm{i}(p_1 - p_2)_\mu \Gamma^{5\mu}(p_1, p_2) + 2m_0 \Gamma^5(p_1, p_2) \equiv \Gamma^A(p_1, p_2) = S^{-1}(p_1)\gamma_5 + \gamma_5 S^{-1}(p_2)$$

  ✓ **automatically** satisfied for $\{\Theta^K, \gamma^5\} = 0$
  can also be satisfied for Θ^K in $\mathcal{V}_\mathrm{L}$ for which $\{\Theta^K, \gamma^5\} \neq 0$, because $\int V_\mathrm{L} = 0$

- constrains scalar, pseudoscalar and tensor structures of kernel

  $$\mathcal{V}(p,k) = V_L(p,k)\Big[\lambda_S(\mathbf{1}\otimes\mathbf{1}) + \lambda_S(\gamma^5\otimes\gamma^5) + \lambda_V(\gamma^\mu\otimes\gamma_\mu) + \lambda_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu) + \frac{\lambda_T}{2}(\sigma^{\mu\nu}\otimes\sigma_{\mu\nu})\Big] + V_{C,G}(p,k)\Big[\kappa_V(\gamma^\mu\otimes\gamma_\mu) + \kappa_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu)\Big]$$

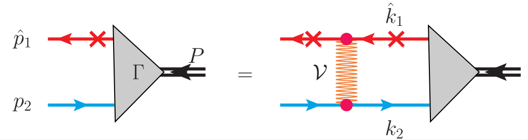
  ⇒ *if* $\mathcal{V}_L$ *has* **scalar**, *it should also have equally-weighted* **pseudoscalar** *structure!*

- ‘soft pion’ limit $P \to 0$: $\Gamma^A(p_1, p_2) \to \Gamma_\pi^\chi(p,p) \sim A(p^2)\gamma^5$

  ⇒ π becomes **massless** if quark mass is generated dynamically ✓

HEAVY AND HEAVY-LIGHT MESONS

1-channel equation (with fixed constituent quark masses)



$$\Gamma(\hat{p}_1, p_2) = -\frac{1}{2}\int_{\mathbf{k}_1} \mathcal{V}(\hat{p}_1, \hat{k}_1)(m_1 + \hat{\not{k}}_1)\Gamma(\hat{k}_1, k_2)\left(\frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon}\right)$$

- particularly suited for heavy-light and heavy mesons
- numerically easier than 2- or 4-channel equations
- $\mathcal{C}$-parity splitting small in heavy quarkonia
- use fixed quark masses
- gauge-parameter ξ independent because $\bar{u}(\boldsymbol{p})\not{q}u(\boldsymbol{k}) = 0$

Kernel

$$\mathcal{V}(\hat{p}, \hat{k}) = [(1-y)(\mathbf{1}_1 \otimes \mathbf{1}_2 + \gamma_1^5 \otimes \gamma_2^5) - y\gamma_1^\mu \otimes \gamma_{\mu 2}]V_{\rm L}(\hat{p}, \hat{k}) - \gamma_1^\mu \otimes \gamma_{\mu 2}[V_{\rm G}(\hat{p}, \hat{k}) + V_{\rm C}(\hat{p}, \hat{k})]$$

mixing parameter y in $V_{\rm L}$: $y = 0$ (pure S+PS)... $y = 1$ (pure V)

RELATIVISTIC PARTIAL WAVE COMPONENTS

Dirac-spinor decomposition of projector and propagator

$m + \hat{\not p} \sim \sum_{\text{spin}} u(\boldsymbol{p})\bar{u}(\boldsymbol{p})$ and $\frac{m+\not p}{m^2-p^2-i\epsilon} \sim \sum_{\text{spin}} \left[-\frac{v(-\boldsymbol{p})\bar{v}(-\boldsymbol{p})}{E_p+p^0-i\epsilon} + \frac{u(\boldsymbol{p})\bar{u}(\boldsymbol{p})}{E_p-p^0-i\epsilon} \right]$

Pseudoscalar $J^{\mathcal{P}} = 0^-$:

- $\bar{u}_{\sigma_1}(\boldsymbol{p})\Gamma(\hat{p}_1, p_2)v_{\sigma_2}(-\boldsymbol{p}) \sim$ S-wave and intrinsic $\mathcal{P}$ odd
- $\bar{u}_{\sigma_1}(\boldsymbol{p})\Gamma(\hat{p}_1, p_2)u_{\sigma_2}(\boldsymbol{p}) \sim$ P-wave and intrinsic $\mathcal{P}$ even

$\Rightarrow$ S-wave and **relativistic** P-wave component consistent with total $\mathcal{P}$ odd ✓

Scalar $J^{\mathcal{P}} = 0^+$: S- and P-wave roles interchanged

Vector $J^{\mathcal{P}} = 1^-$

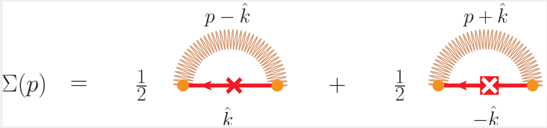
- $\bar{u}\Gamma v \sim$ S- and D-waves and intrinsic $\mathcal{P}$ odd
- $\bar{u}\Gamma u \sim$ **relativistic** spin-singlet and triplet P-waves and intrinsic $\mathcal{P}$ even

Axial-vector $J^{\mathcal{P}} = 1^+$: {S, D} and {Ps, Pt} roles interchanged

MORE & RESULTS: ↪ SOFIA LEITÃO's talk Friday afternoon!

CST QUARK SELF ENERGY

Consistency with 2-body calculations $\Rightarrow$ take *only* quark pole contributions:



BUT: $\Sigma(p) = A(p^2) + \not{p}B(p^2) \xrightarrow{p\to 0} \infty$ integral diverges because $(p \pm \hat{k})^2 \xrightarrow{p\to 0} m^2$

$\Rightarrow M(p^2) = \frac{A(p^2)+m_0}{1-B(p^2)} \xrightarrow{p\to 0} 0 \Rightarrow$ behaviour not in agreement with LQCD $\times$

reason: neglecting kernel poles

FIX

- Linear confining part: $y = 0$ (consistent with meson spectrum)
  $\Rightarrow \mathcal{V}_\text{L} = [\mathbf{1} \otimes \mathbf{1} + \gamma_5 \otimes \gamma_5] V_\text{L}$ does no contribute to self energy ✓
- OGE: must use regularization **form factor** $h_\text{G}(p, k)$ and choose particular **gauge** ✓

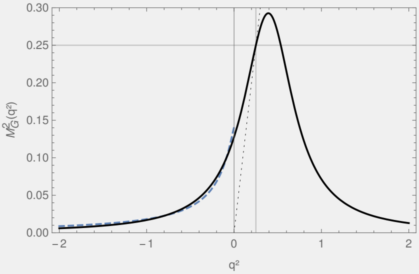
PARAMETERS

Quark self energy from $\mathcal{V}_{\mathrm{C}}$ and $\mathcal{V}_{\mathrm{G}}$

- $C \sim 0.2$ GeV, $\alpha_s \sim 0.5$ **fixed** from meson spectrum (↪ Sofia Leitão's talk)
- $\Lambda_C \sim 3m$, $g^2 \sim -3$ reasonable values in $h_C(q^2) = \frac{\Lambda_C^4}{\Lambda_C^4 + q^4 + m^2(g^2-4)q^2}$

- **running** gluon mass $M_{\mathrm{G}}^2(q^2) = m_{\mathrm{G}}^2 \left[\frac{L_{\mathrm{G}}^4 + (q_0^2 - m_{\mathrm{G}}^2)^2}{L_{\mathrm{G}}^4 + (q_0^2 - q^2)^2}\right]$ with $m_{\mathrm{G}} = 0.5$ GeV $< 2m_\chi$

  fit at spacelike q^2 to RG result gives $L_{\mathrm{G}} = 0.587$ GeV, $q_0 = 0.627$ GeV



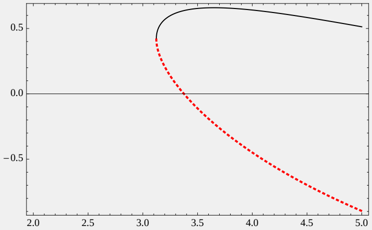
Solid: ours; Dashed: RG [Aquilar, Binosi, Papavassiliou PRD (2014)]

PARAMETERS AND "CST GAUGE"

- Adjust cutoff $\Lambda_G \gtrsim 2m$ and $n \geq 3$ in $h_G(p,k) = \left(\frac{\Lambda_G^4 + m^2}{\Lambda_G^4 + \frac{(p\cdot k)^2}{p^2 k^2}} \right)^n$

  $\Rightarrow$ fix at $\Lambda_G \gtrsim 2.5m$ and $n = 4 \Rightarrow B_G$ not too large

- self energy ξ **dependent**
- Feynman ($\xi = 1$) and Landau ($\xi = 0$) gauge: $\nexists$ real solution of $M(m^2) = m$ $\times$
- $B(p^2)$ still too large at $p^2 = 0$ $\times$



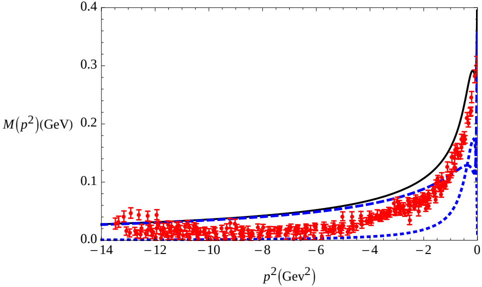
C/m vs. ξ

BUT: B **very** sensitive on ξ, A not $\Rightarrow$ fix $\xi = 4$ ("CST gauge") makes B reasonable ✓

MASS AND Z FUNCTIONS – RESULTS

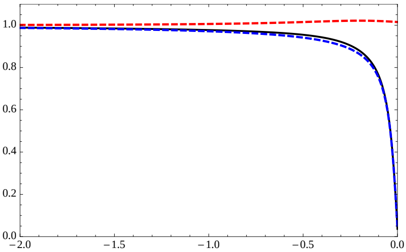
Quark mass function and Z function in chiral limit with $m_\chi = 0.3$ GeV

PRELIMINARY!



$M(p^2)$ vs. p^2, total (solid), OGE (dashed), constant (dotted)

lattice data: Bowman *et al* PRD (2005)



$Z(p^2) = \frac{1}{1-B(p^2)}$ vs. p^2, total (solid), OGE (dashed), constant (dashed)

⇒ **reasonable** behaviour similar to LQCD **without** further adjusting of parameters! ✓

possible explanation: higher-order terms neglected by CST-DE small in "CST gauge"?

SUMMARY AND OUTLOOK

- Covariant Spectator Theory: dynamical quark model in Minkowski space with **confinement** and **dynamical chiral-symmetry breaking**
- 4-channel Gross equation to describe **all mesons**
- 1-channel Gross equation sufficient for **heavy and heavy-light meson**
  ↪ Sofia Leitão (Friday afternoon)
- reasonable first results for dressed **quark mass function** in "CST gauge"

Outlook and work in progress:

1. calculate mass function in timelike region
2. mass function for finite bare quark masses
3. include running quark-gluon coupling
4. include vector structures for V_L in mass function calculation
5. include quark mass function into bound-state calculations

ACKNOWLEDGEMENTS/SUPPORT

CFTP

TÉCNICO LISBOA

FCT
Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Jefferson Lab

THANK YOU!

RELATIVISTIC PARTIAL WAVE COMPONENTS

Pseudoscalar $J^{\mathcal{P}} = 0^-$

- **Covariant** basis: $\Gamma(\hat{p}_1, p_2) = G_1(m_1^2, p_2^2)\gamma^5 + G_2(m_1^2, p_2^2)\gamma^5(m_2 + \not{p}_2)$
- **Partial-wave** basis: eigenstates of L and S (useful for spectroscopic identification)

  Dirac-spinor decomposition of propagator

  $\frac{m_2+\not{p}_2}{m_2^2-p_2^2-i\epsilon} \sim \sum_{\text{spin}} \left[-\frac{v(-\mathbf{p})\bar{v}(-\mathbf{p})}{E_2+p_2^0-i\epsilon} + \frac{u(\mathbf{p})\bar{u}(\mathbf{p})}{E_2-p_2^0-i\epsilon} \right]$

  $\Rightarrow$ spinor matrix elements:

  - $\bar{u}_{\sigma_1}(\mathbf{p})\Gamma(\hat{p}_1, p_2)v_{\sigma_2}(-\mathbf{p}) \sim \psi_S(p) Y_{00} C^{00}_{\frac{1}{2}\sigma_1 \frac{1}{2}\sigma_2} C^{00}_{0000}$
  - $\bar{u}_{\sigma_1}(\mathbf{p})\Gamma(\hat{p}_1, p_2)u_{\sigma_2}(\mathbf{p}) \sim \psi_P(p) \sum_{\mu_L} Y_{1\mu_L}(\hat{\mathbf{p}}) C^{1-\mu_L}_{\frac{1}{2}\sigma_1 \frac{1}{2}\sigma_2} C^{00}_{1-\mu_L 1\mu_L}$

  normalization: $\int_0^\infty p^2 \mathrm{d}p \left[\psi_S^2(p) + \psi_P^2(p)\right] = 1$

$\Rightarrow$ **dominant** S-wave and purely **relativistic** P-wave consistent with $\mathcal{P}$ odd ✓

Scalar $J^{\mathcal{P}} = 0^+$

- $\bar{u}\Gamma v \sim$ dominant, nonrelativistic P-wave
- $\bar{u}\Gamma u \sim$ relativistic S-wave

RELATIVISTIC PARTIAL WAVE COMPONENTS

Vector $J^{\mathcal{P}} = 1^-$

- Covariant basis
  $\Gamma^\mu(\hat{p}_1, p_2) = G_1(m_1^2, p_2^2)\gamma^\mu + G_2(m_1^2, p_2^2)\gamma^\mu(m_2 + \not{p}_2) + G_5(m_1^2, p_2^2)(\hat{p}_1 + p_2)^\mu + G_6(m_1^2, p_2^2)(\hat{p}_1 + p_2)^\mu(m_2 + \not{p}_2)$
- Partial-wave basis
  - **Dominant** S- and D-waves $\bar{u}_{\sigma_1}(\boldsymbol{p})\xi^\nu_{\mu_J}\Gamma_\nu(\hat{p}_1, p_2)v_{\sigma_2}(-\boldsymbol{p}) \sim$
    $\psi_{\mathrm{S}}(p)Y_{00}C^{1\mu_J}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}C^{1\mu_J}_{1\mu_J00} + \psi_{\mathrm{D}}(p)\sum_{\mu_L,\mu_S} Y_{2\mu_L}(\hat{\boldsymbol{p}})C^{1\mu_S}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}C^{1\mu_J}_{1\mu_S2\mu_L}$
  - **Relativistic** spin-singlet and triplet P waves $\bar{u}_{\sigma_1}(\boldsymbol{p})\xi^\nu_{\mu_J}\Gamma_\nu(\hat{p}_1, p_2)u_{\sigma_2}(\boldsymbol{p}) \sim$
    $\psi_{\mathrm{Ps}}(p)\sum_{\mu_L} Y_{1\mu_L}(\hat{\boldsymbol{p}})C^{00}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}C^{1\mu_J}_{001\mu_L} + \psi_{\mathrm{Pt}}(p)\sum_{\mu_L,\mu_S} Y_{1\mu_L}(\hat{\boldsymbol{p}})C^{1\mu_S}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}C^{1\mu_J}_{1\mu_S1\mu_L}$

normalization: $\int_0^\infty p^2 \mathrm{d}p \left[\psi_{\mathrm{S}}^2(p) + \psi_{\mathrm{D}}^2(p) + \psi_{\mathrm{Ps}}^2(p) + \psi_{\mathrm{Pt}}^2(p)\right] = 1$

Axial-vector $J^{\mathcal{P}} = 1^+$

- $\bar{u}\xi \cdot \Gamma v \sim$ dominant nr spin-singlet and triplet P-waves,
- $\bar{u}\xi \cdot \Gamma u \sim$ relativistic S- and D-waves

MORE & RESULTS: $\hookrightarrow$ Sofia Leitão's talk Friday afternoon!