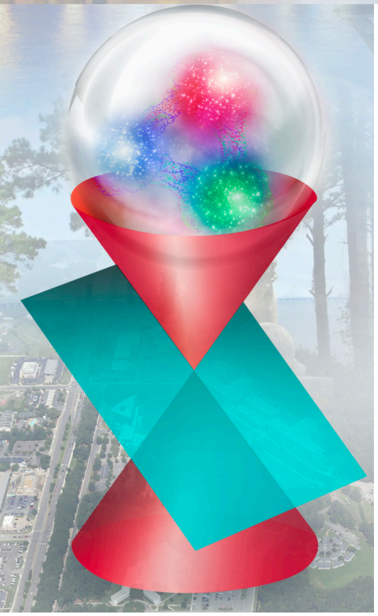


Matching the TMD and collinear factorization frameworks

Leonard Gamberg

May 14, 2018

PennState
Berks

Overview comments

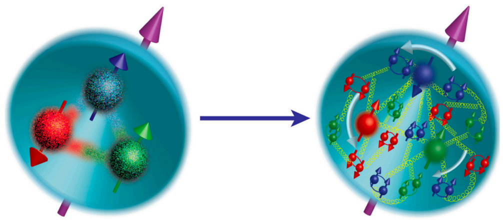
- Report implementation for combining TMD factorization & collinear factorization in studying nucleon structure in SIDIS

- Using enhanced version of Collins-Soper-Sterman (CSS) framework, able to re-derive at @ “LO” the well-known relation between the unpolarized TMD & $f_1(x, \mu)$, (TMD) Sivers function and the (collinear twist-3) Qiu-Sterman function
  nb … power counting remains open question

    - **Phys.Rev. D (2016) Collins, Gamberg, Prokudin, Sato, Rogers, Wang**

    - **Phys. Lett B (2018) Gamberg , Metz, Pitonyak, Prokudin**

- Relies on a modification of the so called *W+Y* construction used to “match” the cross section as a function of q_T point-by-point, from small $q_T \sim m$ (m-typical hadronic mass scale), to large $q_T \sim Q$



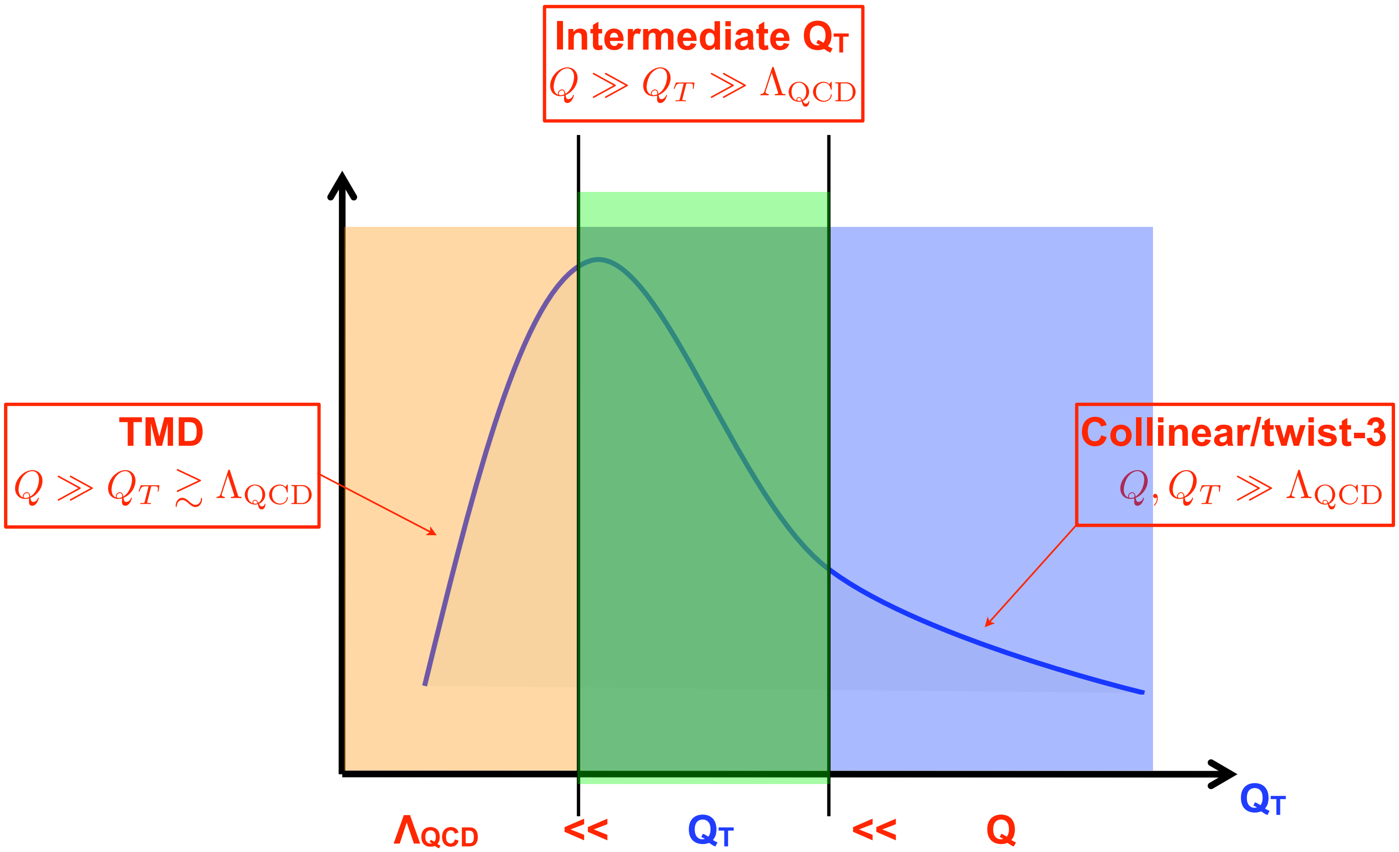
Overview comments

- We modify the "*standard matching prescription*" traditionally used in CSS formalism relating low & high q_T behavior cross section @ moderate Q in particular where studies of TMDs are relevant

Phys.Rev. D (2016) Collins, Gamberg, Prokudin, Sato, Rogers, Wang

Phys. Lett B (2018) Gamberg , Metz, Pitonyak, Prokudin

Phys. Lett B (2018) Echevarria, Kasemets, Lansberg, Pisano, Signori



"Matching-1" $W + Y$-schematic

- Collins Soper Sterman NPB 1985
- Collins 2011 Cambridge Press
- Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD (2016)

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- The $W + Y$ construction of cross section arise from applying **approximators** T_{TMD} and T_{coll} to cross section in "design" regions $m \sim q_T \ll Q$ and $m \ll q_T \sim Q$ respectively, in the process of extracting the leading factorized contributions to the TMD & collinear contributions to the cross section using subtractive formalism to prevent double counting; resulting in the combination $W + Y$ having a relative error $O(m/Q)^c$ in the range $m \lesssim q_T \lesssim Q$
- "Designed" with the aim to have a formalism valid to leading power in m/Q uniformly in q_T, where m is a typical hadronic mass scale

Matching $W + Y$-schematic

✦Collins Soper Sterman NPB 1985

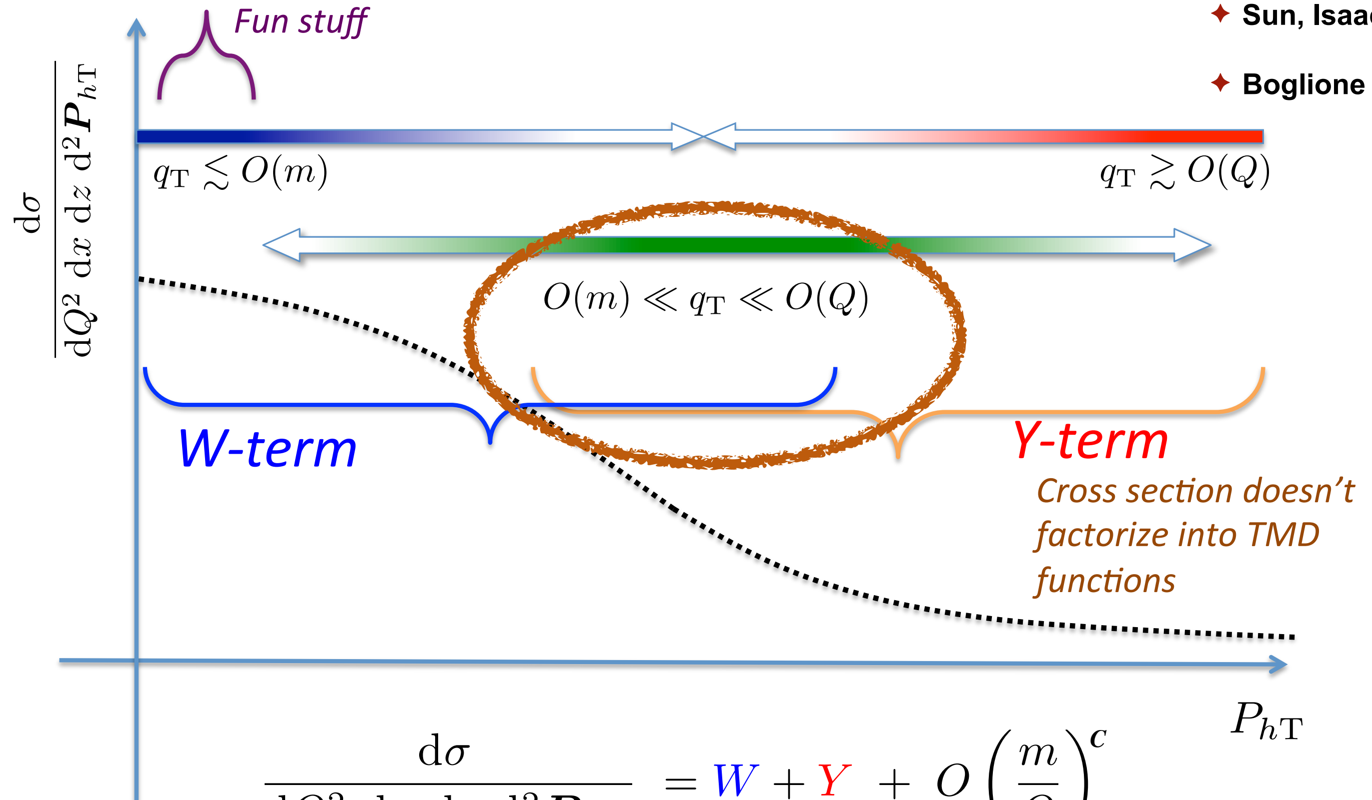
✦Collins 2011 Cambridge Press

- *Designed* with the aim to have a formalism valid to leading power in *m/Q* uniformly in q_T, where *m* is a typical hadronic mass scale
- **&** where broad intermediate range transverse momentum s.t. $m \ll q_T \ll Q$

Implementations/studies

- ✦ **Nadolsky Stump C.P. Yuan PRD 1999 HERA data**
- ✦ **Y. Koike, J. Nagashima, W. Vogelsang NPB (2006) eRHIC**
- ✦ **Sun, Isaacson, C. -P. Yuan , F Yuan arXiv 2014**
- ✦ **Boglione Gonzalez Melis Prokudin JHEP 2015 ….**

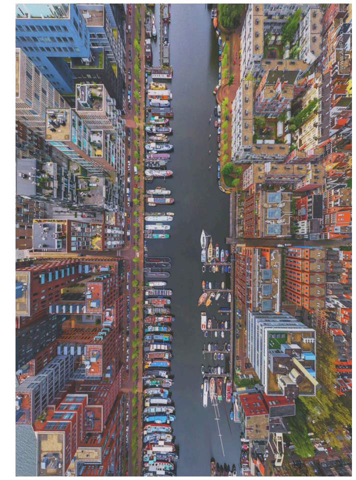
From Ted Rogers



$$\frac{d\sigma}{dQ^2\, dx\, dz\, d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)^c$$

note $P_{hT} = zq_T$

Review of CSS $W + Y$ definition—Birds eye view



- Collins Soper Sterman NPB 1985
- Collins 2011 Cambridge Press
- Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD (2016)

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- ***W*** describes the small transverse momentum behavior $q_T \ll Q$ and an additive correction term Y accounts for behavior at $q_T \sim Q$

Review of CSS $W + Y$ definition—Birds eye view

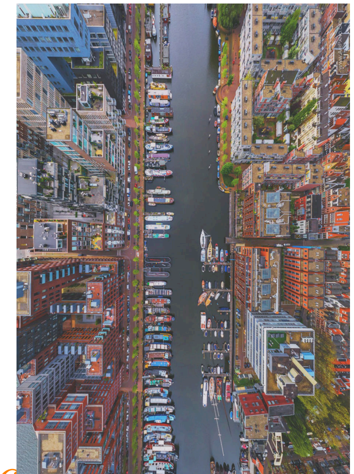
- Collins Soper Sterman NPB 1985
- Collins 2011 Cambridge Press
- Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD (2016)

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- ***W*** describes the small transverse momentum behavior $q_T \ll Q$ and an additive correction term Y accounts for behavior at $q_T \sim Q$
- ***W*** is written in terms of TMD pdfs and/or TMD ffs and is constructed to be an accurate description in the limit of $q_T/Q \ll 1$. It includes all non-perturbative transverse momentum dependence

Review of CSS $W + Y$ definition—Birds eye view

- Collins Soper Sterman NPB 1985
- Collins 2011 Cambridge Press
- Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD (2016)



$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- ***W*** describes the small transverse momentum behavior $q_T \ll Q$ and an additive correction term Y accounts for behavior at $q_T \sim Q$
- ***W*** is written in terms of TMD pdfs and/or TMD ffs and is constructed to be an accurate description in the limit of $q_T / Q \ll 1$. It includes all non-perturbative transverse momentum dependence
- The " ***Y*** -term " is described in terms of "collinear approximation"to the cross section: it is the correction term for large $q_T \sim Q$

One finds the definition of the Y term via "approximators"

$$Y(q_T,Q) \equiv T_{coll}\, d\sigma(q_T,Q) - T_{coll}T_{TMD}\, d\sigma(q_T,Q)$$

$$Y(q_T,Q) = FO(q_T,Q) - ASY(q_T,Q)$$

- It is the difference of the cross section calculated with collinear pdfs and ffs at fixed order FO and the asymptotic contribution of the cross section

- ***nb At small q_T the FO and ASY are dominated by the same diverging terms***

$$\frac{1}{q_T^2} \quad \text{and} \quad \frac{1}{q_T^2}\log\frac{Q^2}{q_T^2}$$

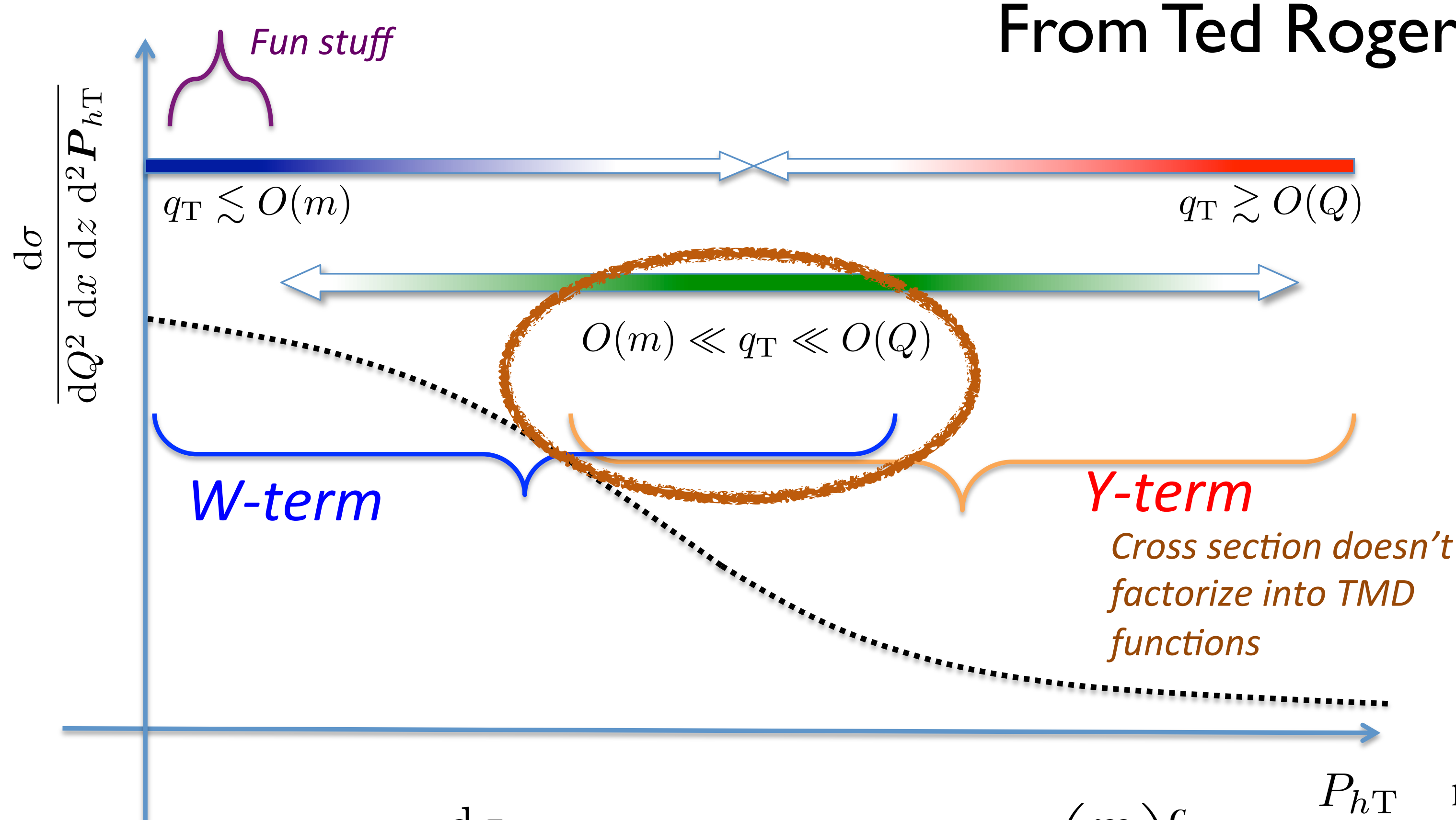
- ***Thus its expected that the Y term is small or zero leaving***

$$d\sigma(q_T \ll Q, Q) \approx W(q_T, Q)$$

“Matching-1” and $W + Y$-schematic

- However at lower phenomenologically interesting values of Q, neither of the ratios q_T/Q or m/q_T are necessarily very small and matching can be problematic—small “matching region” & resulting in differences of large quantities

From Ted Rogers



note $P_{hT} = z q_T$

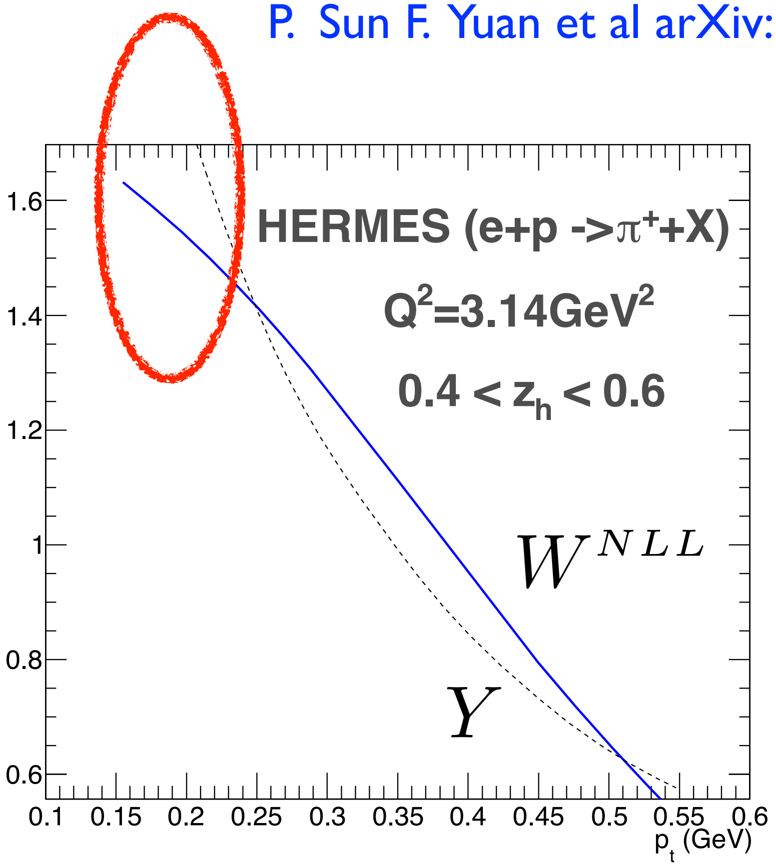
$$\frac{d\sigma}{dQ^2\, dx\, dz\, d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)^c$$

Matching and $W + Y$ -studies low q_T

- At small q_T the Y term is in principle suppressed: it is the difference of the FO perturbative calculation of the cross section and the asymptotic contribution of W for small q_T

- But there can be a difference of of large terms and truncation errors are augmented: **Here the *Y* term is larger than *W* ?!**

P. Sun F. Yuan et al arXiv: 1406.3073



$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

Matching and $W + Y$ -studies large q_T

Compass Example

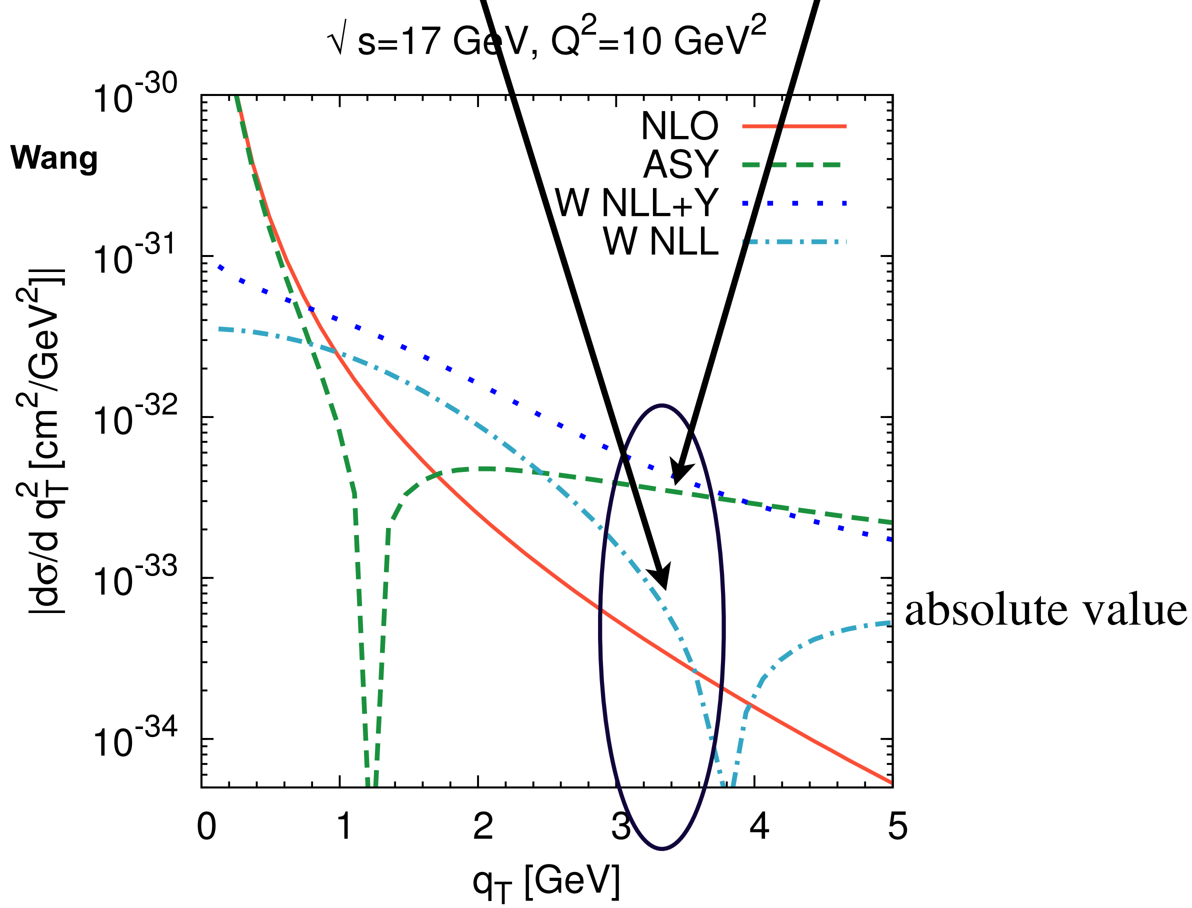
Boglione Prokudin et al. JHEP 2015

- When q_T is above some small fraction of Q, W deviates alot from $d\sigma(q_T, Q)$
- Then it becomes negative and "asymptotes" to $\frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$

Nadolsky et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, NPB744, 59 (2006)

- At large q_T $W+Y$ is then difference of large terms and *truncation errors* can be augmented (ASY!)

PRD 94 2016 Collins, Gamberg, Prokudin, Sato, Rogers, Wang



Matching becomes a challenge COMPASS/Jlab like energies

Matching and *W + Y to collinear Factorization*

$$\int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\,\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_\mathrm{T}\ldots} = \int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\,W + \int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\,Y$$

A second/third issue is the problem of matching the TMD factorized cross section integrated over q_T to the collinear factorization formalism.

CSS and W term

TMD factorization/evolution CSS in b space; (will drill down on this …)

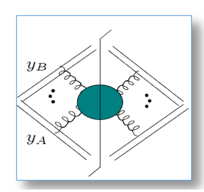
$$\frac{d\sigma}{dq_T^2\, dQ^2} \approx \frac{4\pi^3\alpha_w}{3s} e^2 \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T\cdot b_T} \sum_i \tilde{W}_i(b_T, Q)$$

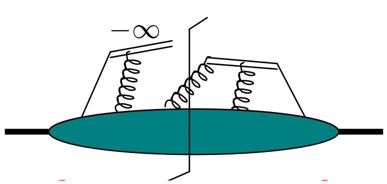
- ***Collins Soper, NPB 1982***
- ***Collins Soper Sterman NPB 1985***
- ***Aybat Rogers PRD 2011***
- ***Collins 2011 Cambridge Press***

$$\tilde{W}_i(b_T, Q) = H_i(Q) \left(\tilde{C}_i^{pdf}(x_A/\hat{x}, b_T) \otimes \tilde{f}_{i/A}(\hat{x}, \mu_b) \right) \left(\tilde{C}_j^{pdf}(x_B/\hat{z}, b_T) \otimes \tilde{\tilde{f}}_{j/B}(\hat{x}, \mu_b) \right) e^{-S(b_T, Q)}$$

TMD factorization

This expression contains the OPE of the Fourier transforms of the TMDs with soft factor in evolution exponent. Ted Rogers pics…





$$\tilde{f}^{\rm sub}_{j/H}(x, b_T; \mu, y_n) = \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \underbrace{\tilde{f}^{\rm unsub}_{j/H}(x, b_T; \mu, y_P - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}} \times UV_{renorm}$$

$$\Updownarrow$$

$$\tilde{f}^{\rm unsub}_{j/H}(x, b_T; \mu, y_P - y_B) = \int \frac{db^-}{2\pi} e^{-ixP^+b^-} \langle P|\bar{\psi}(0)\gamma^+ \mathcal{U}_{[0,b]}\psi(b)|P\rangle\big|_{b^+=0}$$

CSS and W term

TMD factorization/evolution CSS in b space; (will drill down on this …)

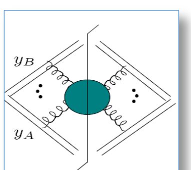
$$\frac{d\sigma}{dq_T^2 dQ^2} \approx \frac{4\pi^3 \alpha_w}{3s} e^2 \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \sum_i \tilde{W}_i(b_T, Q)$$

- ✦ *Collins Soper, NPB 1982*
- ✦ *Collins Soper Sterman NPB 1985*
- ✦ *Aybat Rogers PRD 2011*
- ✦ *Collins 2011 Cambridge Press*

$$\tilde{W}_i(b_T, Q) = H_i(Q) \left(\tilde{C}_i^{pdf}(x_A/\hat{x}, b_T) \otimes \tilde{f}_{i/A}(\hat{x}, \mu_b) \right) \left(\tilde{C}_j^{pdf}(x_B/\hat{z}, b_T) \otimes \tilde{\tilde{f}}_{j/B}(\hat{x}, \mu_b) \right) e^{-S(b_T, Q)}$$

TMD factorization

This expression contains the OPE of the Fourier transforms of the TMDs with soft factor in evolution exponent. Ted Rogers pics…



Matching and $W + Y$ *to collinear Factorization*

$$\int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\, \frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_\mathrm{T}\ldots} = \int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\, W + \int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\, Y$$

A second/third issue is the problem of matching the TMD factorized cross section integrated over q_T to the collinear factorization formalism.

LHS, In QCD the cross section integrated over all q_T; it is of the form of factors of collinear parton densities and/or fragmentation functions at scale Q convoluted with hard scattering that is expanded in powers of $\alpha_S(Q)$

RHS

1) Integral

$$\int d^2\boldsymbol{q}_\mathrm{T}\, W(\boldsymbol{q}_\mathrm{T}, Q, S) = \tilde{W}_\mathrm{UU}(b_T \to 0, Q)$$

$$\sim b_T^a \times (\text{log corrections}) = 0\,,$$

$$a = 8C_F/\beta_0, \quad \beta_0 = 11 - 2n_f/3$$

2) Using collinear factorization the Y term "starts" at NLO α_s^1

b-Dependence driven by perturbative part of ev. Kernel

$$\exp\left[\int_{\mu_{b*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - 2\ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right]$$

$$\begin{aligned}
\tilde{W}(b_T \to 0, Q) \sim \exp\left[\frac{C_F}{\pi\beta_0}\int_{\ln\mu_b^2}^{\ln\mu_Q^2} \ln\mu'^2\right] &= \exp\left[-\frac{C_F}{\pi\beta_0}\ln\left(\frac{\mu_b^2}{\mu_Q^2}\right)\right] \\
&= \exp\left[-\frac{C_F}{\pi\beta_0}\ln\left(\frac{C_1^2}{b_T^2\mu_Q^2}\right)\right] \\
&= b_T^a \quad \text{where, } a = 2C_F/(\pi\beta_0) > 0 \\
&\to 0
\end{aligned}$$

Collinear Matching and $W + Y$

$$\int \mathrm{d}^2\boldsymbol{q}_{\mathrm{T}} \, \frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_{\mathrm{T}} \ldots} = \int \mathrm{d}^2\boldsymbol{q}_{\mathrm{T}} \, W + \int \mathrm{d}^2\boldsymbol{q}_{\mathrm{T}} \, Y$$

Paradox? A mismatch of orders in $\alpha_S(Q)$ between the LHS & RHS

Parton Model interpretation is lost

- Parton Model (expectation) W-term

$$W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) \int d^2k_T f_{j'/A}(x, k_T) d_{B/i'}(z, q_T + k_T)$$

$$\int d^2q_T \, W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) f_{j'/A}(x) d_{B/i'}(z)$$

Underlies Model building
w/ and w/o evolution using TMD
and collinear evolution approach
Anselmino et al. 2005-2016

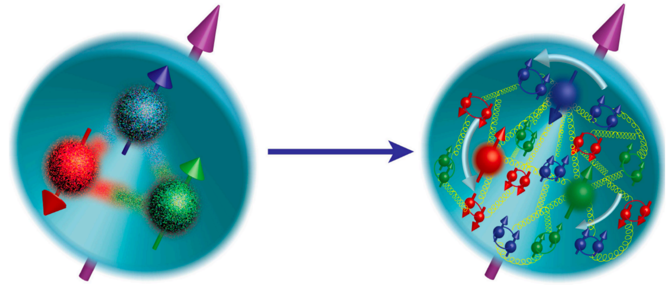
- Standard CSS W-term

$$W_{CSS}(q_T, Q) = \int \frac{d^2b_T}{(2\pi)^2} e^{iq_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$

$$\int d^2q_T W_{CSS}(q_T, Q) = 0 \quad !$$

Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

TMD to collinear PDFs



EIC White Paper

$W(x, b_T, k_T)$

Wigner distributions

$\int d^2 b_T$ $\int d^2 k_T$

$f(x, k_T)$

transverse momentum distributions (TMDs)

semi–inclusive processes

$f(x, b_T)$

impact parameter distributions

$\int d^2 k_T$ $\int d^2 b_T$

$f(x)$

parton densities

inclusive and semi–inclusive processes

TMD to collinear

nb CSS TMD factorisation carried out in coordinate space: then FT back to momentum space

Must consider UV and IR Divergences and TMD evolution

Collins 2011 Cambridge Press, Aybat Rogers 2011 PRD

Parton Model interpretation is lost

$$f_{CSS}(k_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i k_T \cdot b_T} \tilde{f}_{CSS}(b_T, Q)$$

$$\int d^2 k_T f_{CSS}(k_T, Q) = \int \delta^2(b_T)\, b_T^a \times \text{logarithmic corrections}$$

$$\int d^2 k_T f_{CSS}(k_T, Q) = 0$$

$$\neq f(x, \mu) \quad !$$

J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang PRD 2016
Gamberg , Metz, Pitonyak, Prokudin PLB 2018

TMD Evolution-Solution for unpolarised

With $\mu_b = C_1/b_*$ as hard scale, the b dependence of TMDs is calculated in perturbation theory and related to their collinear parton distribution (PDFs), fragmentation functions (FFs), or multiparton correlation functions , … OPE

$$\tilde{\boldsymbol{f}}_{\mathbf{1}}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu^2_{b_*}, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1}(\hat{\boldsymbol{x}}; \boldsymbol{\mu_{b_*}})\right)$$

Collins (2011); ...

$$\times \ \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S^{f_1}_{NP}(b_T, Q)\right]$$

Also relation to Parton Model?

Turn off α_s *don't get back parton model*

Collins 2011 QCD Aybat Rogers PRD 2011

$$\tilde{D}_{H/f}(z, b_T; \zeta_F, \mu) \to \frac{1}{z^2} d_{H/f}(z) \quad \times \exp\left\{-\left[g_1' + g_2 z^2 \ln\frac{Q}{2Q_0}\right]\frac{b_T^2}{2z^2}\right\}.$$

TMD Evolution-Solution for unpolarised & Sivers

- TMD/CSS Evolution/Factorization carried out in *b*-space "Bessel transforms"

Boer Gamberg Musch Prokudin 2011 JHEP
Collins Aybat Rogers Qiu 2012 PRD

$$\tilde{\Phi}^{[\gamma^+]}(x, \boldsymbol{b}_T) \;=\; \tilde{f}_1(x, \boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)$$

Correlator obeys CSS equation so,

$$\boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} \;\sim\; \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)$$

Collins (2011); ...

$$\times\; \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

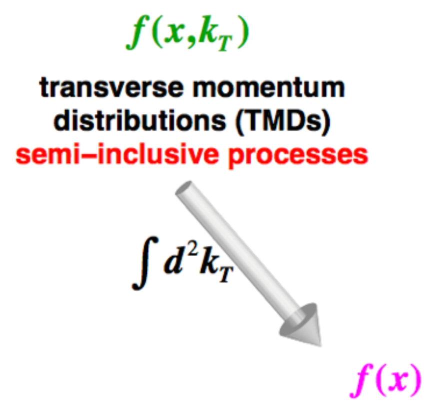
Qiu & Sterman PRL 1991

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)} \;\sim\; \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1, \hat{x}_2, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{T_F(\hat{x}_1, \hat{x}_2; \mu_{b_*})}\right)$$

$$\times\; \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T, Q)\right]$$

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

Collinear limit Original CSS



- *Collins, Soper, Sterman NPB 1985*
- *Ji Ma Yuan, PRD 2005*
- *Collins 2011 Cambridge Press*

Consequence is that physical interpretation of integrated TMDs as collinear pdfs is at odds with parton model intuition in original version of CSS

$$\int d^2k_T\, f_1(x,k_T;Q^2,\mu_Q) = \tilde{f}_1(x,b_T\to 0;Q^2,\mu_Q) = 0\,!$$

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q) \equiv f_{1T}^{\perp(1)}(x;Q^2,\mu_Q) = \tilde{f}_{1T}^{\perp(1)}(x,b_T\to 0;Q^2,\mu_Q) = 0\,!$$

Phys. Lett B (2018) Gamberg , Metz, Pitonyak, Prokudin

TMDs lose their physical interpretation in the "Original CSS" formalism!

Collinear limit Original CSS

- *Collins, Soper, Sterman NPB 1985*
- *Ji Ma Yuan, PRD 2005*
- *Collins 2011*

Consequence is that physical interpretation of integrated TMDs as collinear pdfs is at odds with parton model intuition in original version of CSS

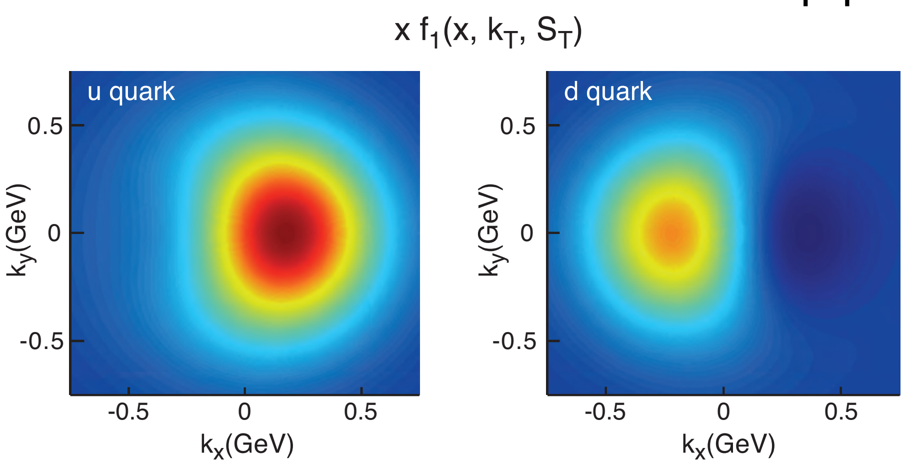
TMDs lose their physical interpretation in the "Original CSS" formalism!

$$\langle k_T^i(x)\rangle_{UT} = \int d^2k_T\, k_T^i \left(-\frac{\vec{k}_T \times \vec{S}_T}{M} f_{1T}^{\perp}(x,k_T)\right)$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target

Boer Mulders Teryaev PRD 1998
Burkhardt 2004,2013 PRD
Metz et al. 2013 PRD
And others …

Prokudin 2015 EIC White paper



- To understand this lets unpack perturbative part of CSS TMD evolution Kernel

Can we preserve generalised parton model as an approximation to TMD evolution?

YES!

J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang PRD 2016
Gamberg , Metz, Pitonyak, Prokudin PLB 2018

QCD for Unpolarized FT-TMD

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes f_1(\hat{x}; \mu_{b_*})\right)$$

$$\times \; \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

perturbative Sudakov factor

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

non-perturbative Sudakov factor

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

different for each TMD

universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

Note: $b_*(0) = 0$ and $(\mu_{b_*})_{b_* \to 0} = \infty$ ➡ problematic large logarithms in S_{pert}

(Bozzi, Catani, de Florian, Grazzini (2006); Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

bQ <<1 contributions to the *W term*

- Issue has been addressed "q_T resummation" by Bozzi, Catani, de Florian, Grazzini, (2006) NPB, & "TMD CSS analysis" Collins, Gamberg, Prokudin, Rogers, Sato, Wang PRD 2016 studying the Fourier transform of the W term in the $W+Y$ matching in q_T of the SIDIS cross section from coordinate b-space to q_T momentum space
- In order to regulate the large logs(Q^2b^2) at small b in the FT they Bozzi et al. , replace logs(Q^2b^2) with logs(Q^2b^2+1) cutting off the $b << 1/Q$ contribution
- Also Kulesza,Sterman,Vogelsang PRD 2002 in threshold resummation studies

We address these large logs by placing another boundary condition on now small b_T

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))*

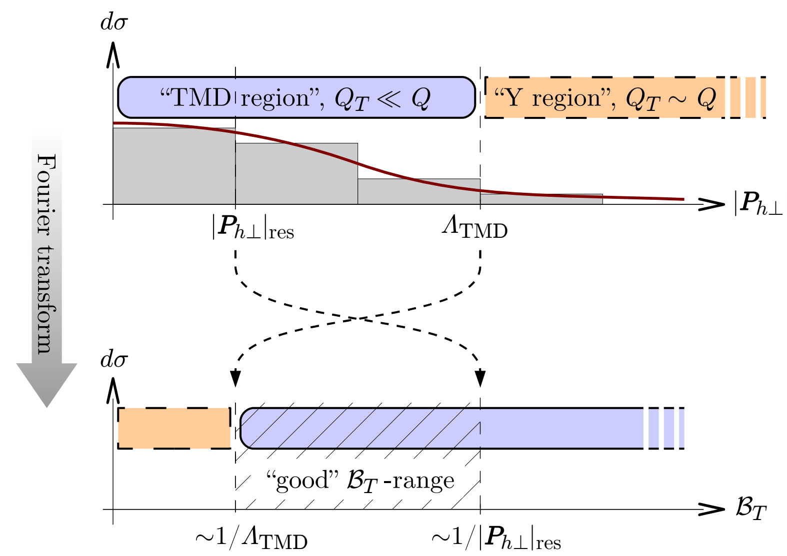
Place a lower cut-off on b_T: $b_T \rightarrow b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$$\Rightarrow \mu_{b_*} \rightarrow \bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))} \text{ so } \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

Modifications to CSS

a) B.C. Introduce small b-cuttoff Similar to Catani et al. NPB 2006 &

"Bessel Weighting" ppr. Boer LG Musch Prokudin JHEP 2011



$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \implies b_c(0) \sim 1/Q$$

Regulate unphysical divergences from in W term

b) Introduce large q_T-cuttoff so that $W_{\rm New}$ vanishes at large $q_{\rm T}$ Similar to Nadolsky et al. PRD 1999

$$\Xi\left(\frac{q_T}{Q},\eta\right) = \exp\left[-\left(\frac{q_T}{\eta Q}\right)^{a_\Xi}\right]$$

$$\tilde{W}_{New}(q_T, Q; \eta, C_5) = \Xi\left(\frac{q_T}{Q},\eta\right)\int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T\cdot b_T}\tilde{W}^{OPE}\left(b_*(b_c(b_T)), Q\right)\tilde{W}_{NP}(b_c(b_T)), Q; b_{max})$$

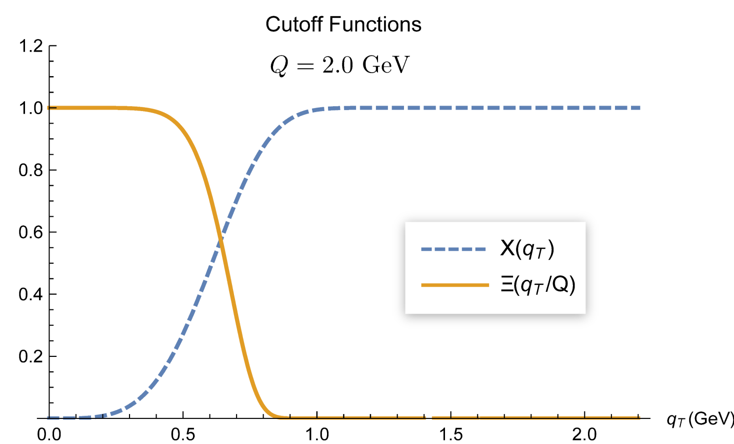
Generalized B.C.

$$b_*(b_c(b_{\rm T})) \longrightarrow \begin{cases} b_{\rm min} & b_{\rm T} \ll b_{\rm min} \\ b_{\rm T} & b_{\rm min} \ll b_{\rm T} \ll b_{\rm max} \\ b_{\rm max} & b_{\rm T} \gg b_{\rm max}\,. \end{cases}$$

Now Y term is further modified

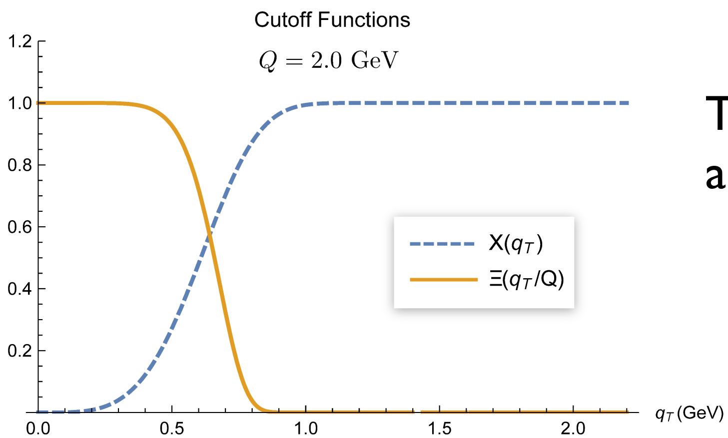
$$Y_{New}(q_T, Q) = [T_{coll}\, d\sigma(q_T, Q) - T_{coll} T_{TMD}^{New}\, d\sigma(q_T, Q)]\, X(q_T/\lambda)$$

$$= [FO(q_T, Q) - ASY_{New}(q_T, Q)]\, X(q_T/\lambda)$$



(a)

Switching functions



(a)

The cutoff functions in for low q_T/lambda (blue dashed line) and large q_T/Q (brown solid line) for Q = 20.0 GeV

See also Altarelli et al NPB1984, Catani et al NPB 2015 Arnold and Kauffman 1991, Alternative approach Berger Qiu & Zhang PRL 2001

Enhanced CSS—Putting all together

$$d\sigma(q_T, Q) \approx T^{New}_{TMD} d\sigma(q_T, Q) + T_{coll}\left[d\sigma(q_T, Q) - T^{New}_{TMD} d\sigma(q_T, Q)\right]$$

$$+ O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

or

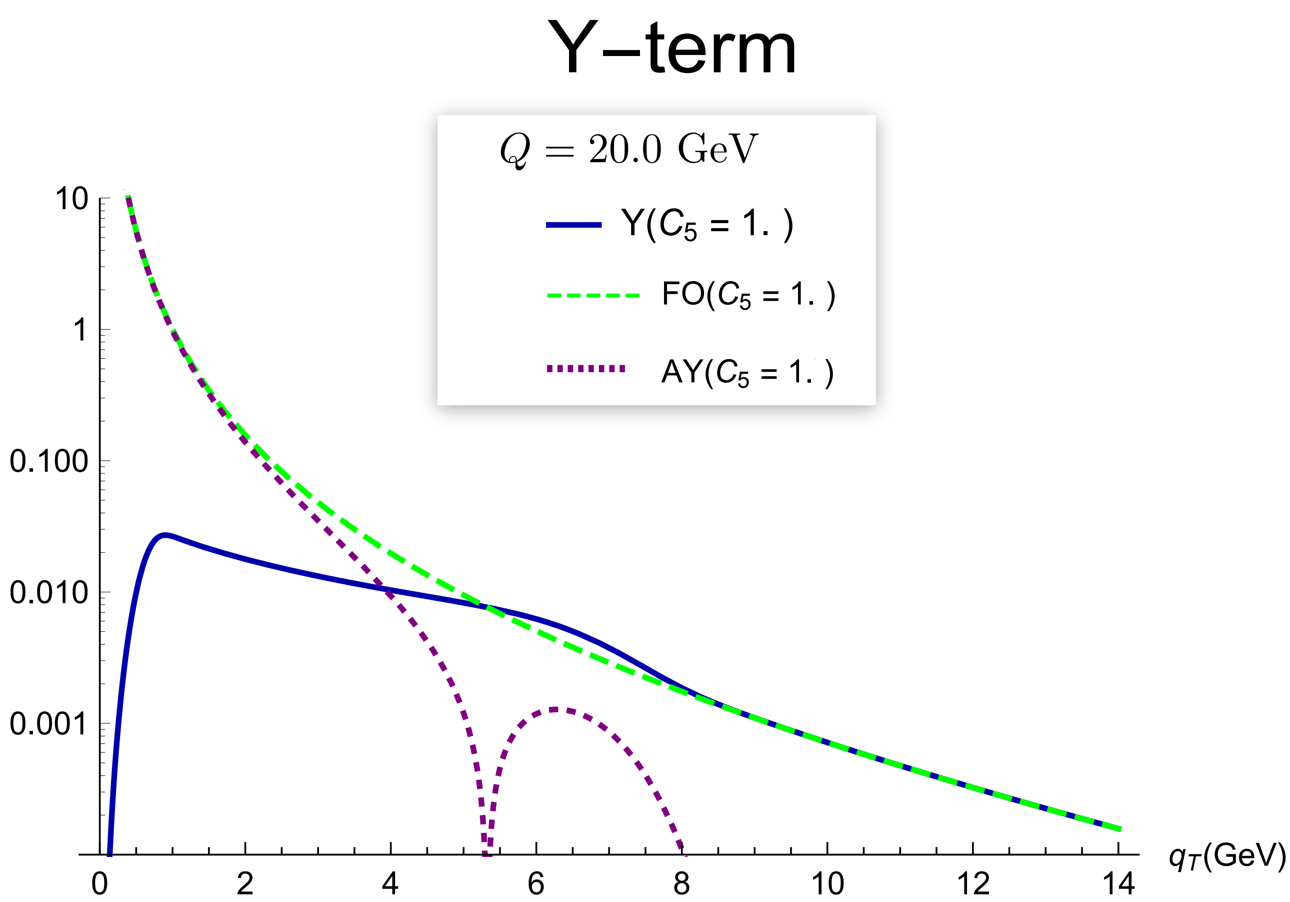
$$d\sigma(q_T, Q) \approx W_{New}(q_T, Q) + Y_{New}(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

✦ **Phys.Rev. D (2016) Collins, Gamberg, Prokudin, Sato, Rogers, Wang**

Putting all together demonstration

Illustration: we have performed sample calculations of the Y-term using analytic approximations for the collinear pdfs and collinear ffs. We consider only the target up-quark gamma *q -> q+g* channel, and for the running *alpha*$_s$ we use the two-loop beta function f = 3 since we are mainly interested in the transition to low Q.
Thus we use $\Lambda_{QCD} = 0.330$

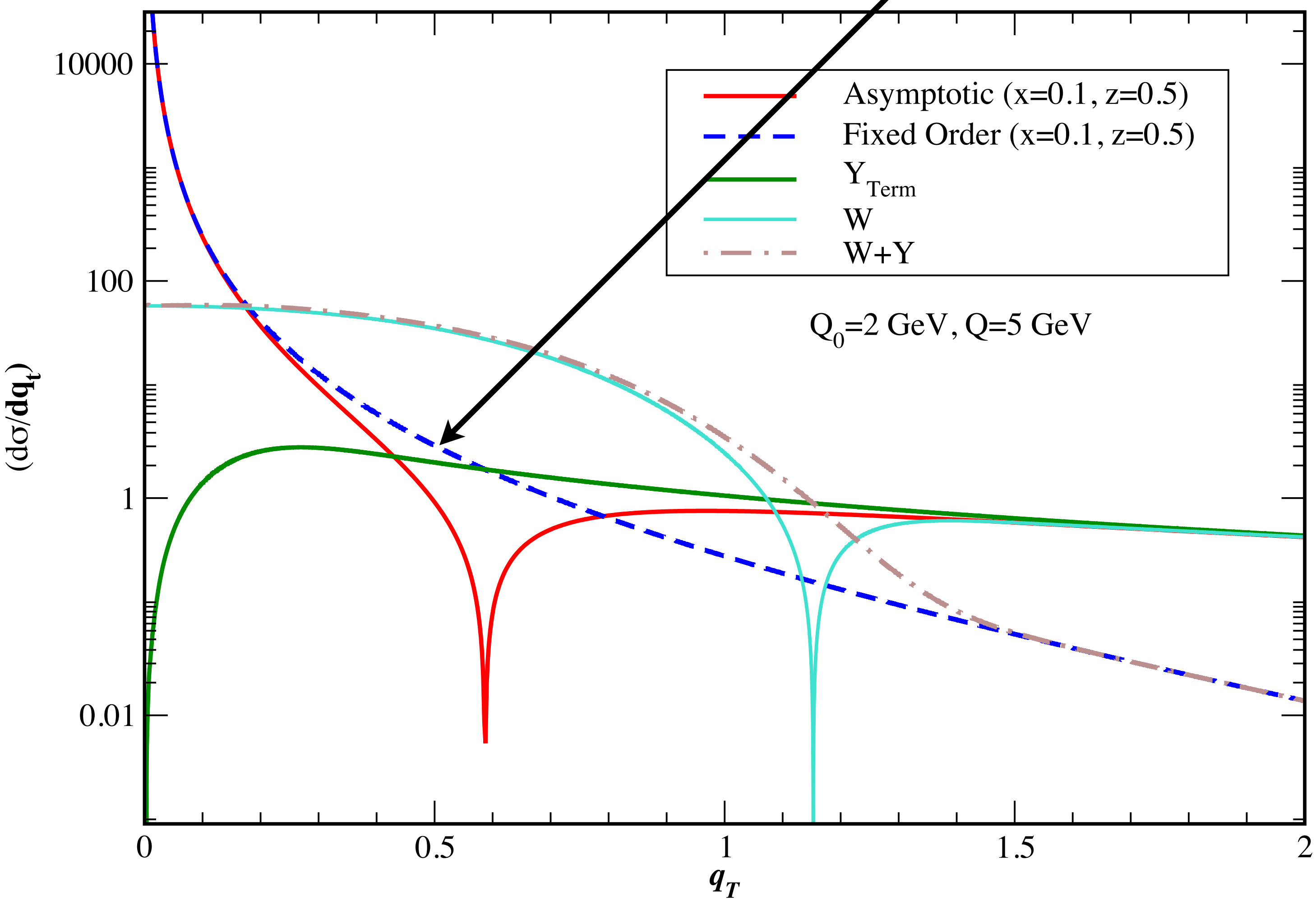
To further simplify our calculations, we use analytic expressions for the collinear correlation functions, taken from appendix A1 of GRV ZPC 1992 for the up-quark pdf and from Eq. (A4) of KKP NPB 2001 for the up-quark-to-pion fragmentation function.



Implementation of Collins, Gamberg, Prokudin, Sato, Rogers, Wang

- Now we can extend the power suppression error estimate down to $q_T = 0$ to get

$$d\sigma(q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$



Use analytic expressions for the collinear correlation functions, from GRV ZPC 1992 for up-quark pdf and from KKP NPB 2001 for the up-quark-to-pion ffs.

Modified FT-TMD from enhanced CSS

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \rightarrow b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$$\Rightarrow \quad \mu_{b_*} \rightarrow \bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))} \text{ so } \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

$$\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes f_1(\hat{x}; \bar{\mu})\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, to appear soon)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{f}_1(x, b_T; Q^2, \mu_Q) - iM\epsilon^{ij} b_T^i S_T^j \tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)$$

b_T -> $b_c(b_T)$

NO b_T -> $b_c(b_T)$ replacement – kinematic factor NOT associated with the scale evolution

b_T -> $b_c(b_T)$

Enhanced CSS definitions of TMDs

$$f_1^j(x, k_T; Q^2, \mu_Q; C_5) \equiv \int \frac{db_T}{2\pi}\, b_T J_0(k_T b_T) \tilde{f}_1^j(x, b_c(b_T); Q^2, \mu_Q)\,,$$

$$D_1^j(z, p_T; Q^2, \mu_Q; C_5) \equiv \int \frac{db_T}{2\pi}\, b_T J_0(p_T b_T)\, \tilde{D}_1^{h/j}(z, b_c(b_T); Q^2, \mu_Q)\,,$$

$$\frac{k_T^2}{2M_P^2}\, f_{1T}^{\perp j}(x, k_T; Q^2, \mu_Q; C_5) \equiv k_T \int \frac{db_T}{4\pi} b_T^2 J_1(k_T b_T)\, \tilde{f}_{1T}^{\perp(1) j}(x, b_c(b_T); Q^2, \mu_Q)\,.$$

Which leads to well defined collinear pdfs

$$\int d^2\vec{k}_T\, \boldsymbol{f_1(x, k_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{f}_1(x, b_c(0); Q^2, \mu_Q)} = \boldsymbol{f_1(x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, \boldsymbol{D_1(z, p_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{D}_1(z, b_c(0); Q^2, \mu_Q)} = \boldsymbol{D_1(z; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, \boldsymbol{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(0); Q^2, \mu_Q)} = -\frac{\boldsymbol{T_F(\hat{x}_1, \hat{x}_2; \mu_{b_*})}}{\boldsymbol{2M_P}} + O(\alpha_s(Q)) + O((m/Q)$$

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^2}{2z^2 M_h^2}\, \boldsymbol{H_1^{\perp}(z, p_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_c(0); Q^2, \mu_Q)} = \boldsymbol{H_1^{\perp(1)}(z; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

At LO in the “Improved CSS” formalism we recover the relations one expects from the “naïve” operator definitions of the functions

The “Improved CSS” formalism (approximately) restores the physical interpretation of TMDs!

Agreement between TMD and Collinear results

- **Relies on further modifications of W+Y construction see**
- **Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD 2016**

$$\frac{d\sigma}{dxdyd\phi_S\,dz} \equiv 2z^2 \int d^2\boldsymbol{q}_\mathrm{T}\,\Gamma(\boldsymbol{q}_\mathrm{T}, Q, S) = 2z^2\,\tilde{W}^{\mathrm{OPE}}_{\mathrm{UU}}(b'_{min}, Q)_{\mathrm{LO}} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$= \frac{2\alpha_{em}^2}{yQ^2}(1 - y + y^2/2) \sum_j e_j^2\, f_1^j(x;\mu_c)\, D_1^{h/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

- **Gamberg , Metz, Pitonyak, Prokudin … 2017**

$$\frac{d\langle P_{h\perp}\,\Delta\sigma(S_T)\rangle}{dxdydz} = -4\pi z^3 M_P\,\tilde{W}^{\mathrm{siv,OPE}}_{\mathrm{UT}}(b'_{min}, Q)_{\mathrm{LO}} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

$$= \frac{2\pi\, z\, \alpha_{em}^2}{yQ^2}(1 - y + y^2/2)\sum_j e_j^2\, T_F^j(x, x;\mu_c)\, D_1^{h/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

Agrees with collinear twist-3 result at leading order

Z.-B.Kang,Vitev, Xing,PRD(2013)

Comments

- With our method, the redefined W term allowed us to construct a relationship between integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two being suppressed by powers of 1/Q
- Importantly, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations. We preserve transverse-coordinate space version of the W term, but only modify the way in which it is used
- **We have a new now applied to transverse polarized phenomena**
- We are able to recover the well-known relations between TMD and collinear quantities expected from the leading order parton model picture operator definition
- We recover the LO collinear twist 3 result from a weighted q_T integral of the differential cross section and derive the well known relation between the TMD Sivers function and the collinear twist 3 Qiu Sterman function from iCSS approach