Semi-Inclusive DIS and TMDs

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Light Cone 2018

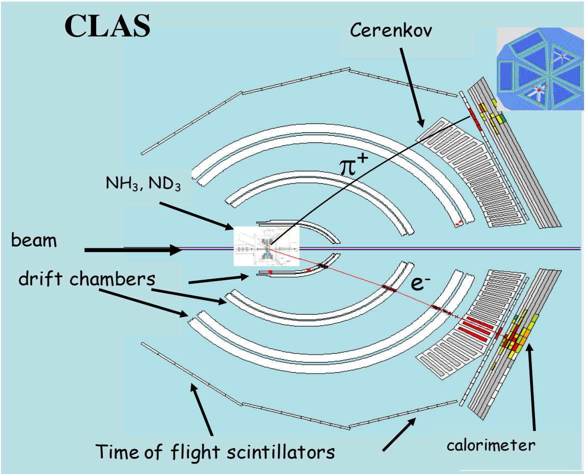
Factorization

- Observables

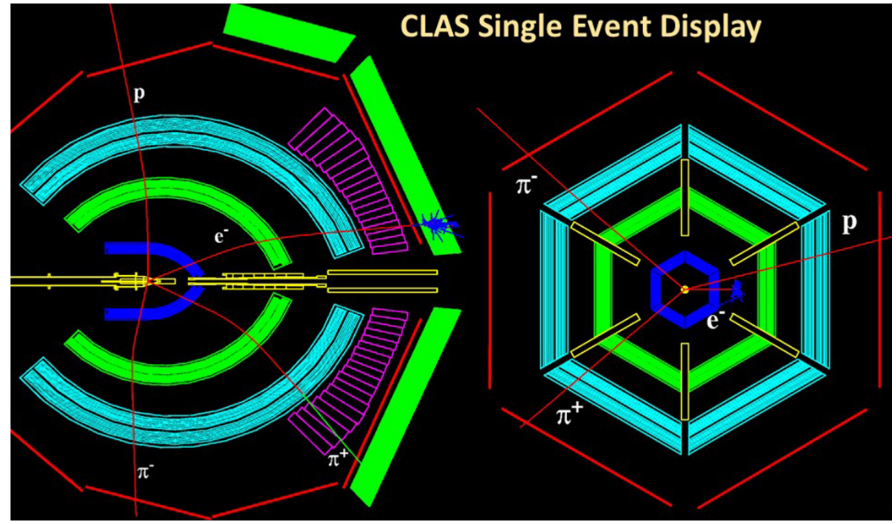
$$\mathrm{d}\sigma = \frac{1}{\Phi}\sum\int |\langle P_H, P_a, P_b \cdots | P, l_e\rangle|^2$$

- Theory objects: Specific correlation functions:
  - E.g., parton distributions, fragmentation functions, TMDs, fracture functions etc…

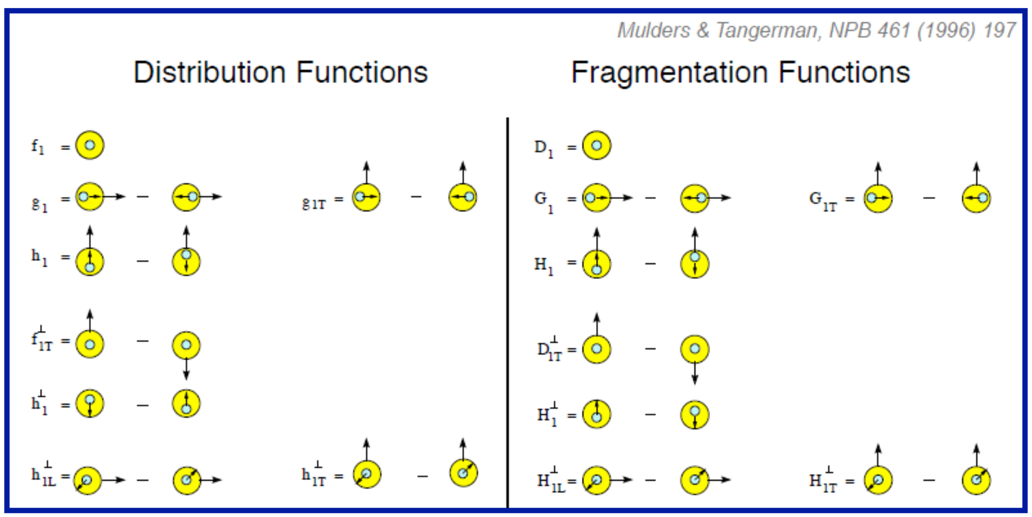
$$\int \frac{dw^-}{(2\pi)} e^{-i\xi P^+ w^-} \langle P| \bar{\psi}_0(0, w^-, \mathbf{0}_t) \frac{\gamma^+}{2} \psi_0(0, 0, \mathbf{0}_t) |P\rangle$$



Point A



Point B



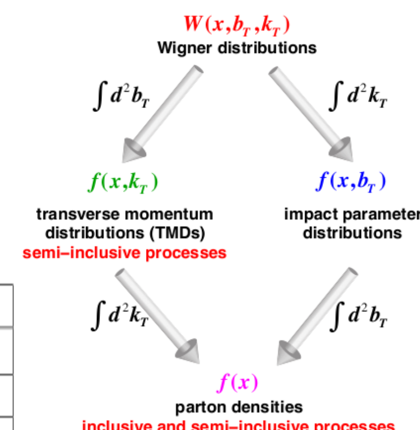
Wigner Distributions

	U	L	T
U	F_{11}	G_{11}	H_{11}, H_{12}
L	F_{14}	G_{14}	H_{17}, H_{18}
T	$F_{12}, \bar{F}_{13}$	$\bar{G}_{12}, \bar{G}_{13}$	$\bar{H}_{13}, H_{14}$, $\bar{H}_{15}, \bar{H}_{16}$

Fracture Functions

	U	L	T
U	M	$M_L^{\perp,h}$	$M_T^h, M_T^{\perp}$
L	$\Delta M^{\perp,h}$	ΔM_L	$\Delta M_T^h, \Delta M_T^{\perp}$
T	$\Delta_T M_T^h, \Delta_T M_T^{\perp}$	$\Delta_T M_L^h$, $\Delta_T M_L^{\perp}$	$\Delta_T M_T, \Delta_T M_T^{hh}$, $\Delta_T M_T^{\perp\perp}, \Delta_T M_T^{\perp h}$

EIC White Paper



From Experiment

$$\mathrm{d}\sigma = \frac{1}{\Phi}\sum\int |\langle P_H, P_a, P_b \cdots | P, l_e\rangle|^2$$

Structure functions with QED and electromagnetic corrections

$$\bar{A}_0\left(\frac{\Lambda^2}{Q^2}\right)^0 + \bar{A}_1\left(\frac{\Lambda^2}{Q^2}\right)^1 + \bar{A}_2\left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$$

Large Q?

Partonic degrees of freedom? Target/ Hadron masses?

Factorization for inclusive DIS

$$A_0\left(\frac{\Lambda^2}{Q^2}\right)^0 + A_1\left(\frac{\Lambda^2}{Q^2}\right)^1 + A_2\left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$$

Small $P_{H,T}$?

Or,

Large $P_{H,T}$?

$$D_0\left(\frac{P_T^2}{Q^2}\right)^0 + D_1\left(\frac{P_T^2}{Q^2}\right)^1 + D_2\left(\frac{P_T^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{P_T^2}{Q^2}\right)^N\right)$$

$$E_0\left(\frac{\Lambda^2}{P_T^2}\right)^0 + E_1\left(\frac{\Lambda^2}{P_T^2}\right)^1 + E_2\left(\frac{\Lambda^2}{P_T^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{P_T^2}\right)^N\right)$$

Current Region?

Central/Soft Region?

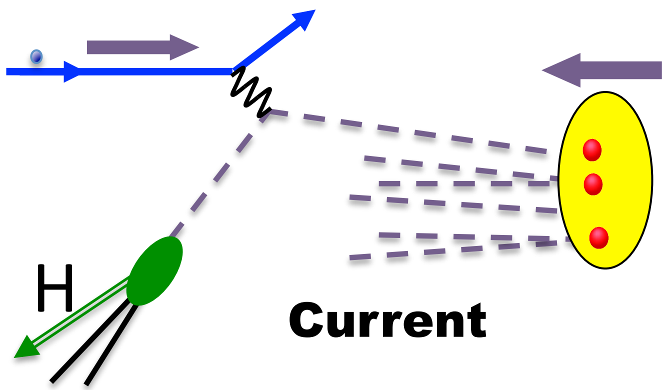
Target Region?

$$F_0 + F_1\left(e^{y_H}\right)^1 + \cdots O({e^{y_H}}^N)$$

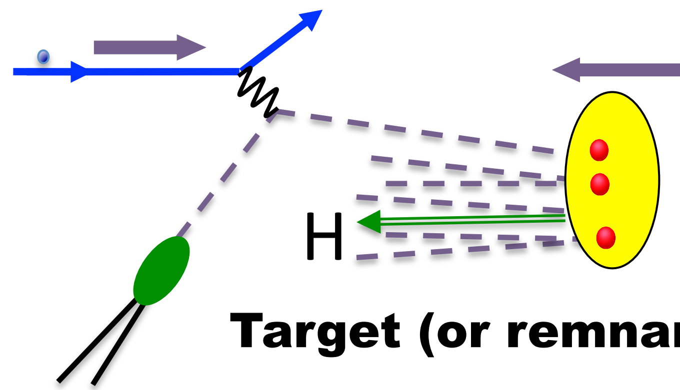
$$G_0 + G_1\left(e^{-y_H}\right)^1 + \cdots O({e^{-y_H}}^N)$$

Begin Classification/Analysis Of Theory Objects

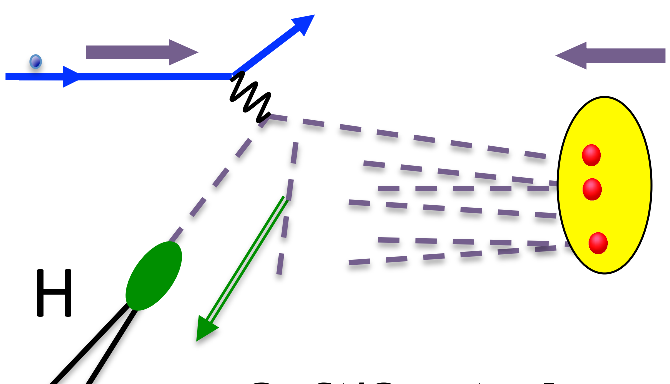
Deeply Inelastic Region: Sub-Regions



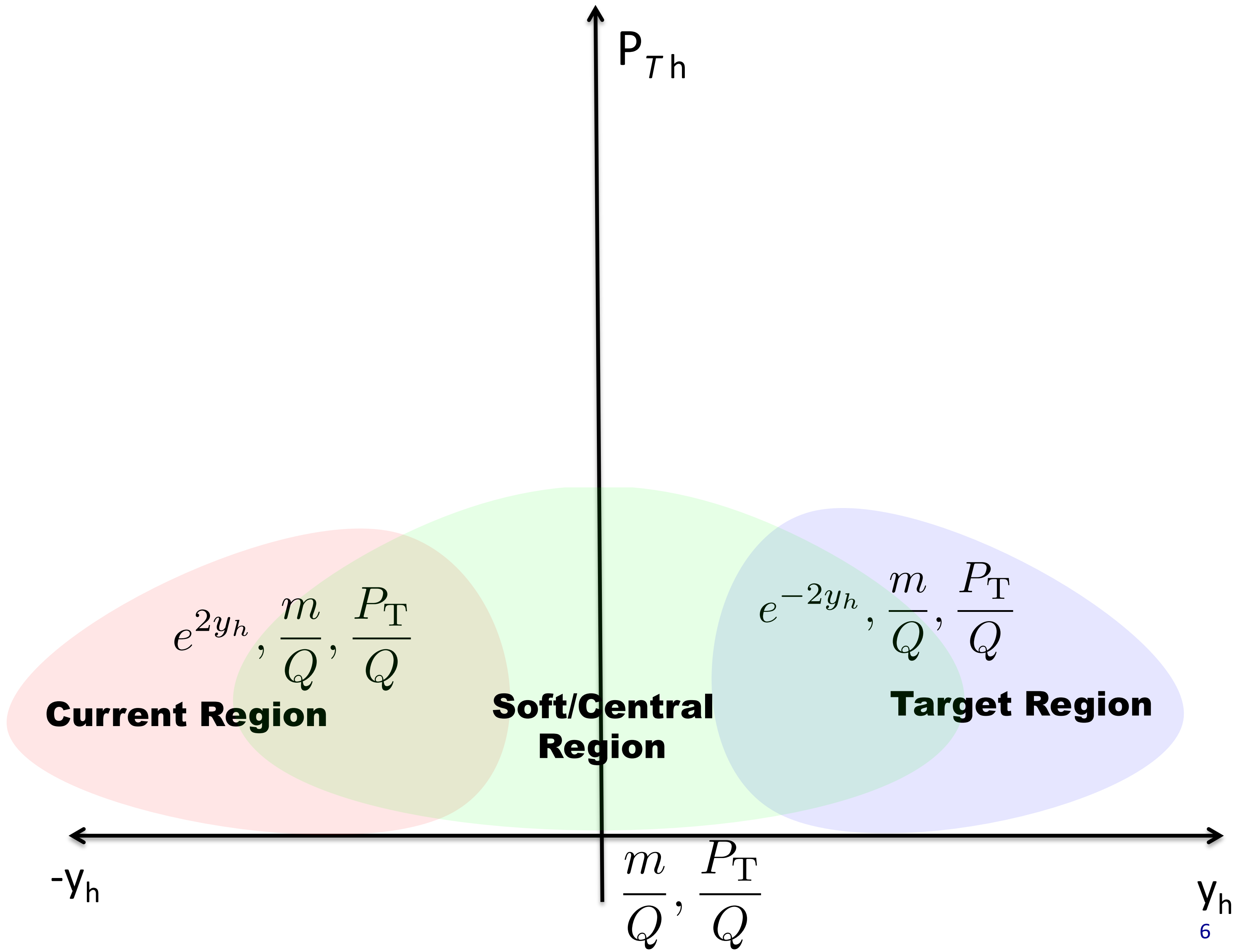
Current



Target (or remnant)



Soft/Central



P_{Th}
$e^{2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Current Region
Soft/Central Region
$e^{-2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Target Region
$-y_h$
$\frac{m}{Q}, \frac{P_T}{Q}$
y_h

From Experiment

$$d\sigma = \frac{1}{\Phi}\sum\int |\langle P_H, P_a, P_b \cdots | P, l_e\rangle|^2$$

Structure functions with QED and electromagnetic corrections

$$\bar{A}_0\left(\frac{\Lambda^2}{Q^2}\right)^0 + \bar{A}_1\left(\frac{\Lambda^2}{Q^2}\right)^1 + \bar{A}_2\left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$$

Large Q?

Partonic degrees of freedom? Target/ Hadron masses?

Factorization for inclusive DIS

$$A_0\left(\frac{\Lambda^2}{Q^2}\right)^0 + A_1\left(\frac{\Lambda^2}{Q^2}\right)^1 + A_2\left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$$

Small $P_{H,T}$?

Or,

Large $P_{H,T}$?

$$D_0\left(\frac{P_T^2}{Q^2}\right)^0 + D_1\left(\frac{P_T^2}{Q^2}\right)^1 + D_2\left(\frac{P_T^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{P_T^2}{Q^2}\right)^N\right)$$

$$E_0\left(\frac{\Lambda^2}{P_T^2}\right)^0 + E_1\left(\frac{\Lambda^2}{P_T^2}\right)^1 + E_2\left(\frac{\Lambda^2}{P_T^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{P_T^2}\right)^N\right)$$

Current Region?

Central/Soft Region?

Target Region?

$$F_0 + F_1\left(e^{y_H}\right)^1 + \cdots O({e^{y_H}}^N)$$

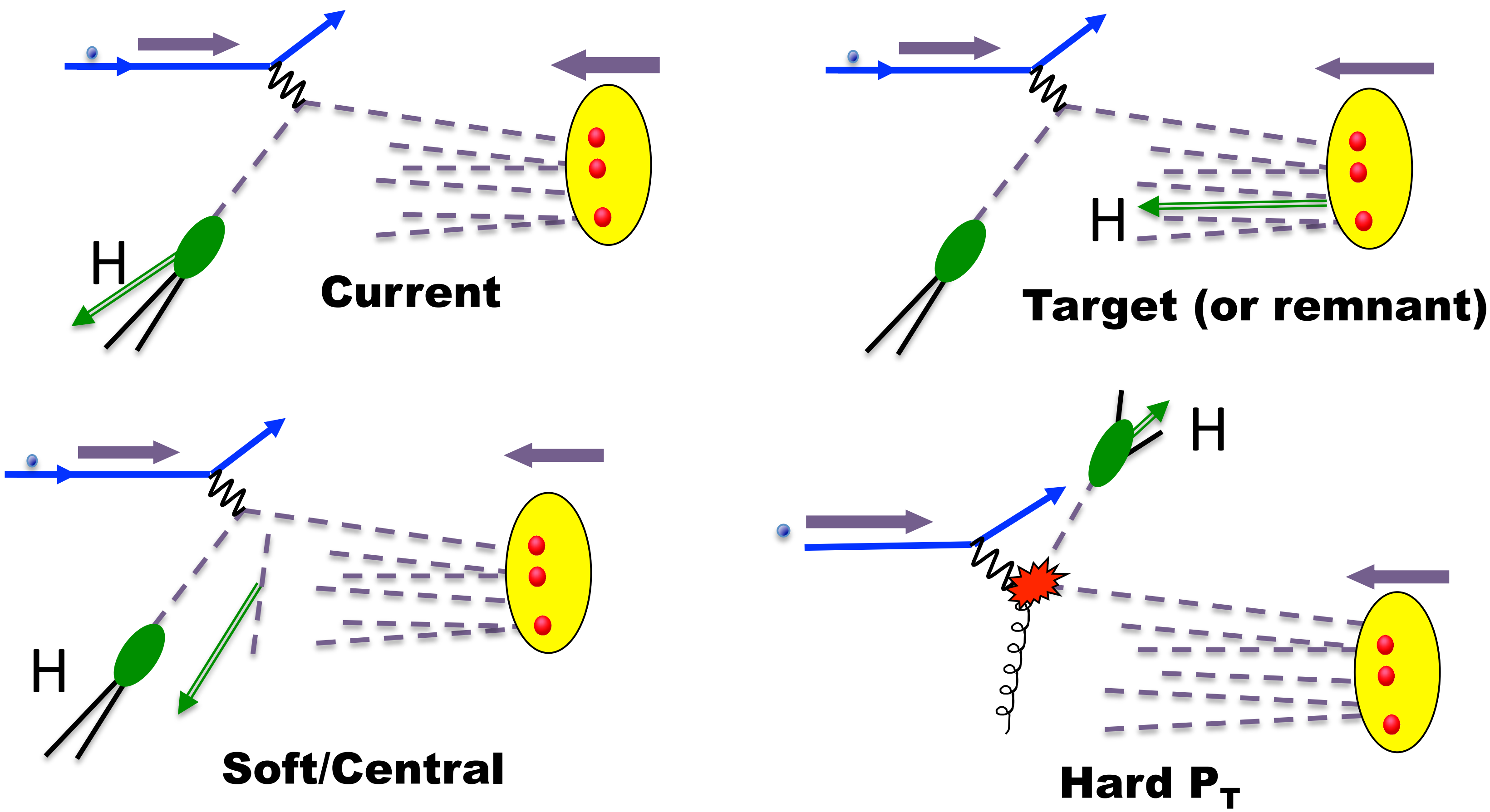
$$G_0 + G_1\left(e^{-y_H}\right)^1 + \cdots O({e^{-y_H}}^N)$$

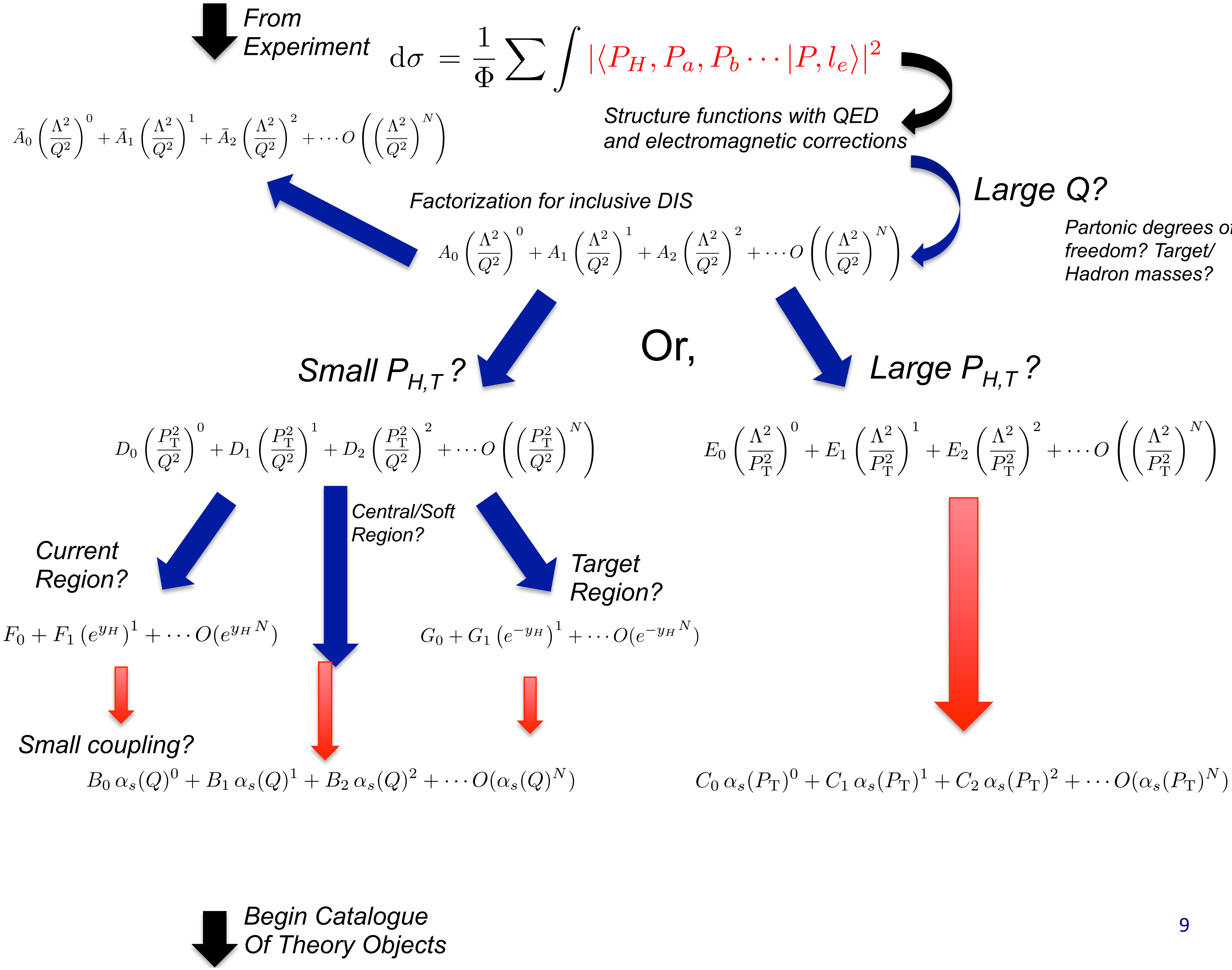
Small coupling?

$$B_0\,\alpha_s(Q)^0 + B_1\,\alpha_s(Q)^1 + B_2\,\alpha_s(Q)^2 + \cdots O(\alpha_s(Q)^N)$$

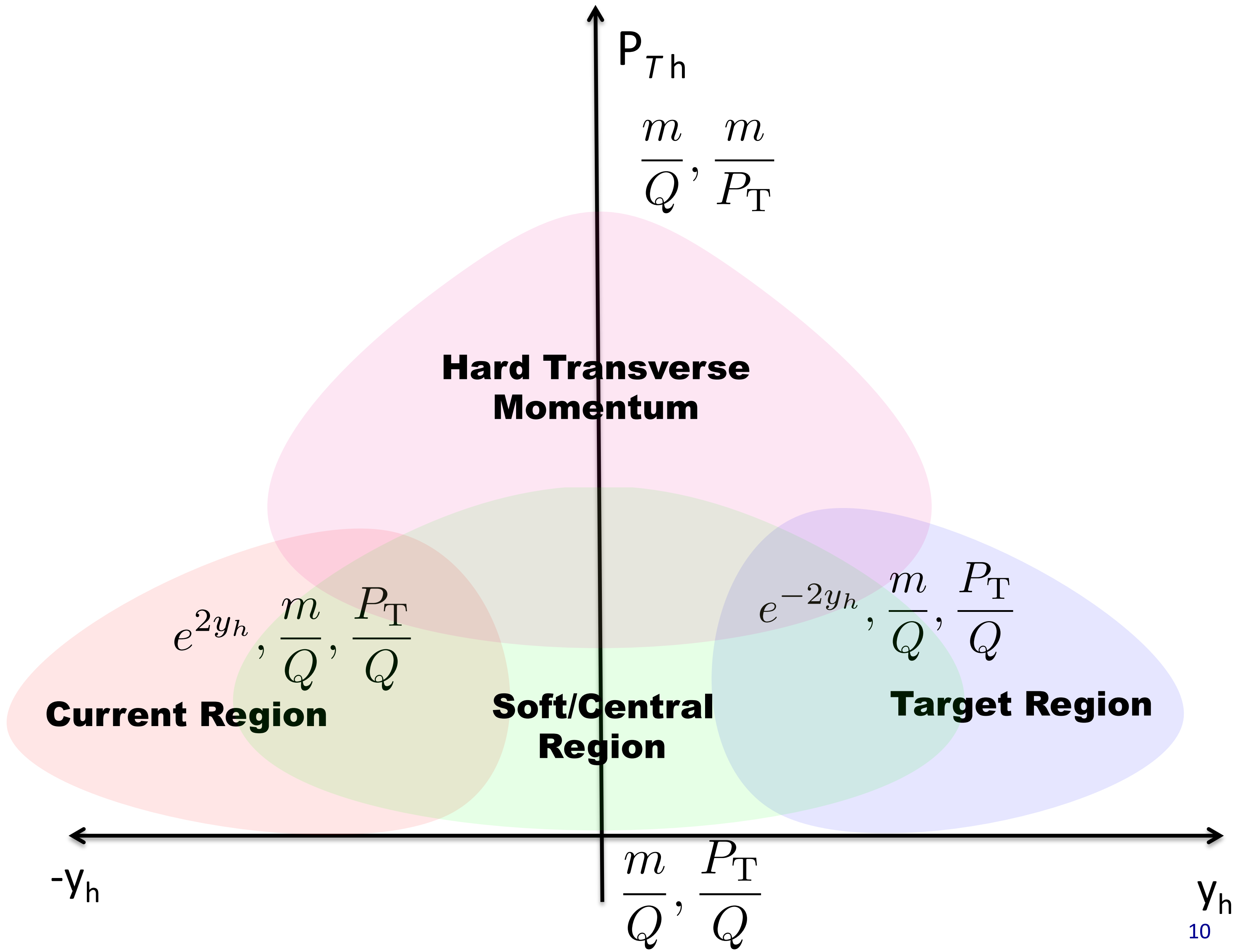
Begin Classification/Analysis Of Theory Objects

Deeply Inelastic Region: Sub-Regions

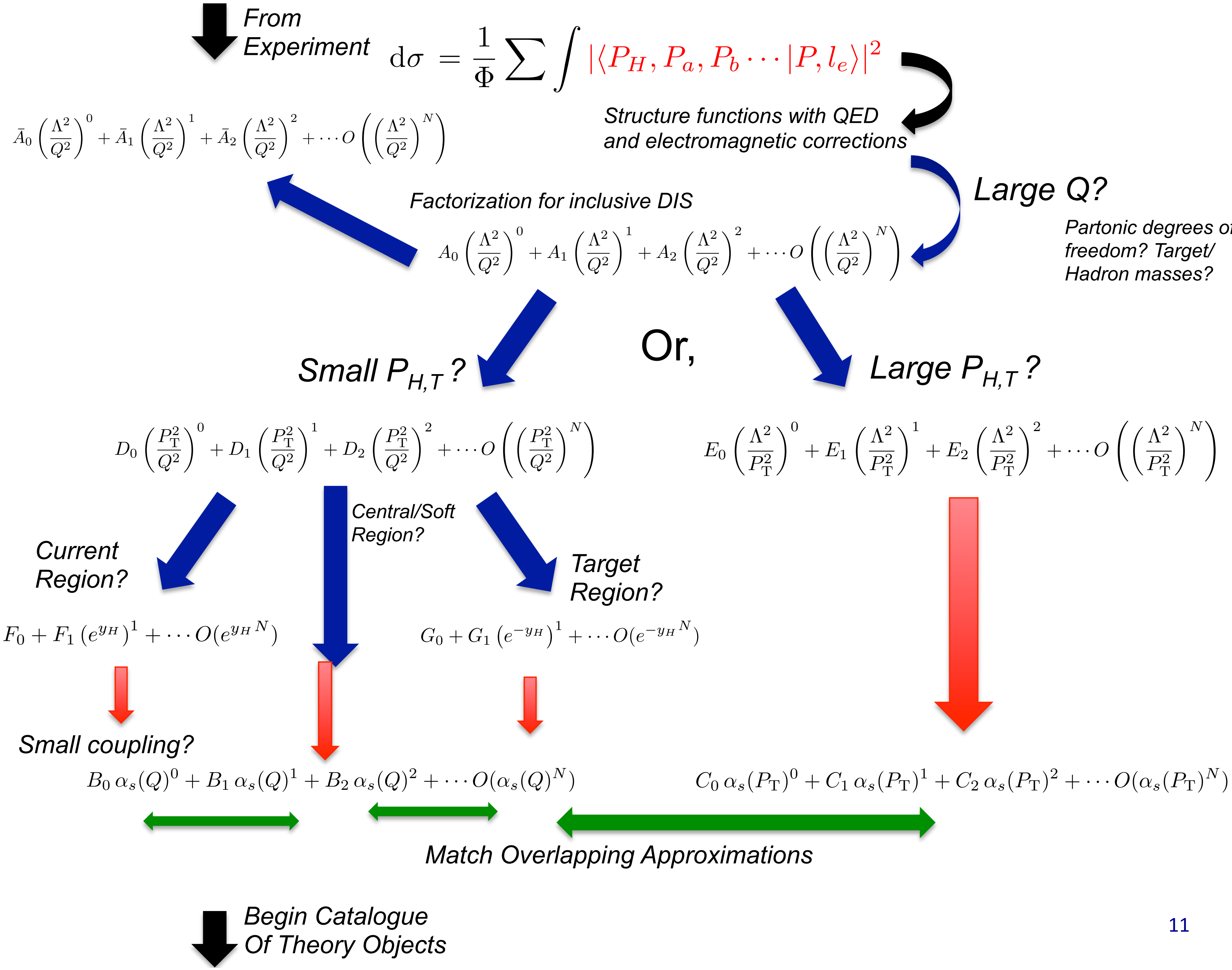


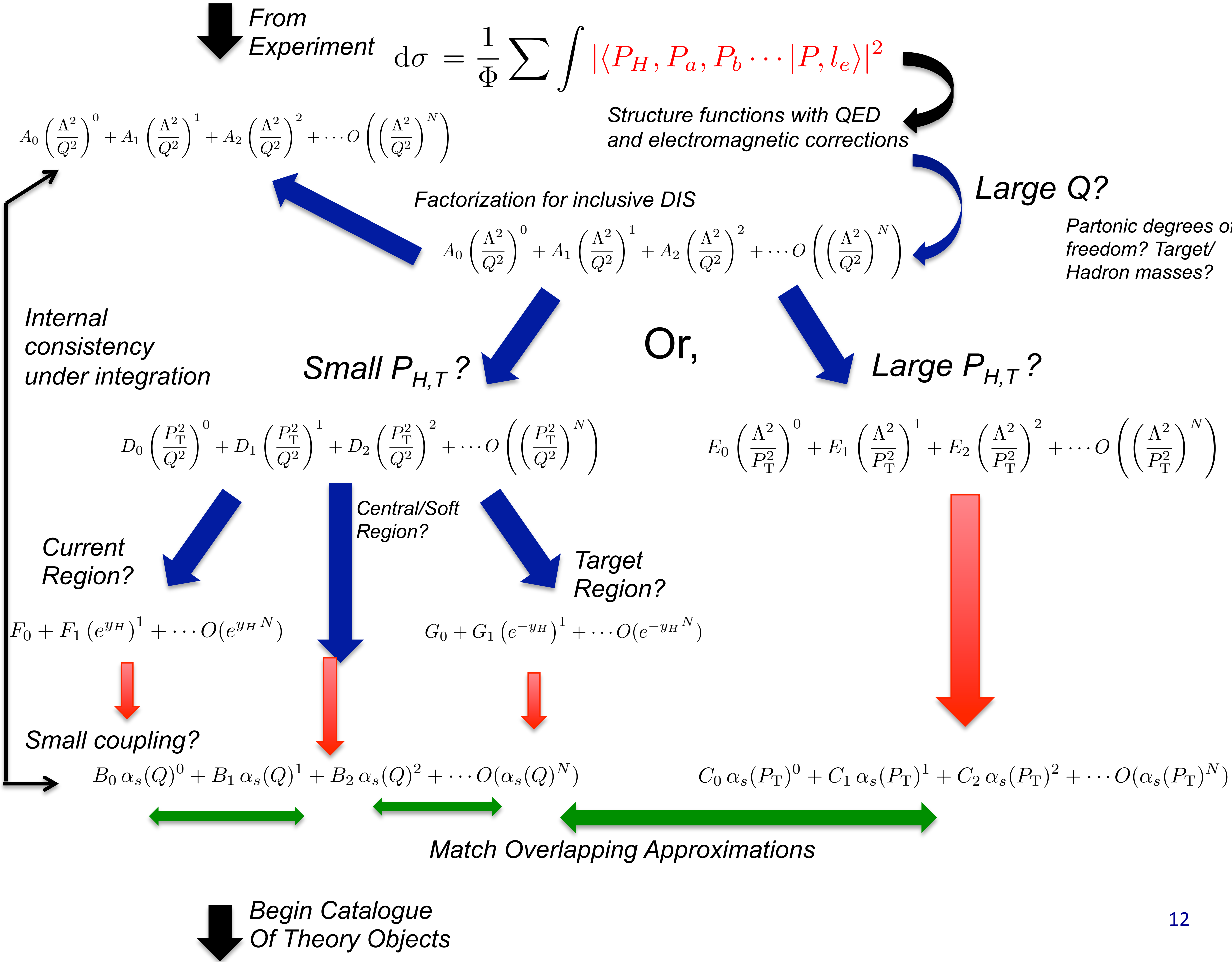


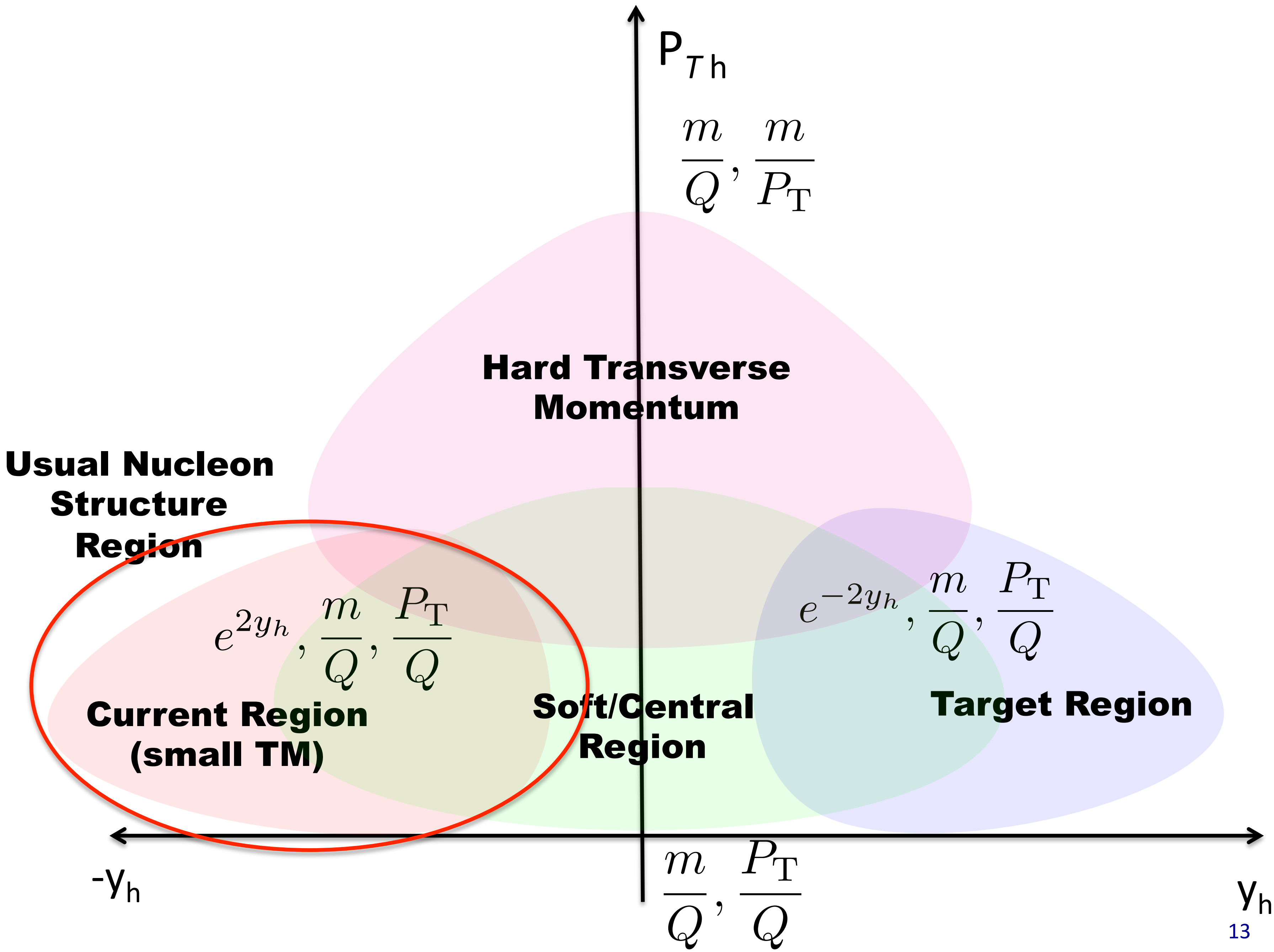
From Experiment
$\mathrm{d}\sigma = \frac{1}{\Phi}\sum\int |\langle P_H, P_a, P_b \cdots | P, l_e\rangle|^2$
Structure functions with QED and electromagnetic corrections
Large Q?
Partonic degrees of freedom? Target/ Hadron masses?
Factorization for inclusive DIS
$A_0\left(\frac{\Lambda^2}{Q^2}\right)^0 + A_1\left(\frac{\Lambda^2}{Q^2}\right)^1 + A_2\left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$
$\bar{A}_0\left(\frac{\Lambda^2}{Q^2}\right)^0 + \bar{A}_1\left(\frac{\Lambda^2}{Q^2}\right)^1 + \bar{A}_2\left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$
Small $P_{H,T}$?
Or,
Large $P_{H,T}$?
$D_0\left(\frac{P_T^2}{Q^2}\right)^0 + D_1\left(\frac{P_T^2}{Q^2}\right)^1 + D_2\left(\frac{P_T^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{P_T^2}{Q^2}\right)^N\right)$
$E_0\left(\frac{\Lambda^2}{P_T^2}\right)^0 + E_1\left(\frac{\Lambda^2}{P_T^2}\right)^1 + E_2\left(\frac{\Lambda^2}{P_T^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{P_T^2}\right)^N\right)$
Current Region?
Central/Soft Region?
Target Region?
$F_0 + F_1\left(e^{y_H}\right)^1 + \cdots O(e^{y_H N})$
$G_0 + G_1\left(e^{-y_H}\right)^1 + \cdots O(e^{-y_H N})$
Small coupling?
$B_0\,\alpha_s(Q)^0 + B_1\,\alpha_s(Q)^1 + B_2\,\alpha_s(Q)^2 + \cdots O(\alpha_s(Q)^N)$
$C_0\,\alpha_s(P_T)^0 + C_1\,\alpha_s(P_T)^1 + C_2\,\alpha_s(P_T)^2 + \cdots O(\alpha_s(P_T)^N)$
Begin Catalogue Of Theory Objects



P_{Th}
$\frac{m}{Q}, \frac{m}{P_T}$
Hard Transverse Momentum
$e^{2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Current Region
Soft/Central Region
$e^{-2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Target Region
$-y_h$
$\frac{m}{Q}, \frac{P_T}{Q}$
y_h


From Experiment
$d\sigma = \frac{1}{\Phi} \sum \int |\langle P_H, P_a, P_b \cdots | P, l_e \rangle|^2$
Structure functions with QED and electromagnetic corrections
$\bar{A}_0 \left(\frac{\Lambda^2}{Q^2}\right)^0 + \bar{A}_1 \left(\frac{\Lambda^2}{Q^2}\right)^1 + \bar{A}_2 \left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$
Large Q?
Partonic degrees of freedom? Target/ Hadron masses?
Factorization for inclusive DIS
$A_0 \left(\frac{\Lambda^2}{Q^2}\right)^0 + A_1 \left(\frac{\Lambda^2}{Q^2}\right)^1 + A_2 \left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$
Small $P_{H,T}$?
Or,
Large $P_{H,T}$?
$D_0 \left(\frac{P_T^2}{Q^2}\right)^0 + D_1 \left(\frac{P_T^2}{Q^2}\right)^1 + D_2 \left(\frac{P_T^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{P_T^2}{Q^2}\right)^N\right)$
$E_0 \left(\frac{\Lambda^2}{P_T^2}\right)^0 + E_1 \left(\frac{\Lambda^2}{P_T^2}\right)^1 + E_2 \left(\frac{\Lambda^2}{P_T^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{P_T^2}\right)^N\right)$
Current Region?
Central/Soft Region?
Target Region?
$F_0 + F_1 \left(e^{y_H}\right)^1 + \cdots O(e^{y_H N})$
$G_0 + G_1 \left(e^{-y_H}\right)^1 + \cdots O(e^{-y_H N})$
Small coupling?
$B_0\, \alpha_s(Q)^0 + B_1\, \alpha_s(Q)^1 + B_2\, \alpha_s(Q)^2 + \cdots O(\alpha_s(Q)^N)$
$C_0\, \alpha_s(P_T)^0 + C_1\, \alpha_s(P_T)^1 + C_2\, \alpha_s(P_T)^2 + \cdots O(\alpha_s(P_T)^N)$
Match Overlapping Approximations
Begin Catalogue Of Theory Objects


From Experiment
$d\sigma = \frac{1}{\Phi}\sum\int |\langle P_H, P_a, P_b \cdots | P, l_e\rangle|^2$
Structure functions with QED and electromagnetic corrections
$\bar{A}_0\left(\frac{\Lambda^2}{Q^2}\right)^0 + \bar{A}_1\left(\frac{\Lambda^2}{Q^2}\right)^1 + \bar{A}_2\left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$
Large Q?
Partonic degrees of freedom? Target/ Hadron masses?
Factorization for inclusive DIS
$A_0\left(\frac{\Lambda^2}{Q^2}\right)^0 + A_1\left(\frac{\Lambda^2}{Q^2}\right)^1 + A_2\left(\frac{\Lambda^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{Q^2}\right)^N\right)$
Internal consistency under integration
Small $P_{H,T}$?
Or,
Large $P_{H,T}$?
$D_0\left(\frac{P_T^2}{Q^2}\right)^0 + D_1\left(\frac{P_T^2}{Q^2}\right)^1 + D_2\left(\frac{P_T^2}{Q^2}\right)^2 + \cdots O\left(\left(\frac{P_T^2}{Q^2}\right)^N\right)$
$E_0\left(\frac{\Lambda^2}{P_T^2}\right)^0 + E_1\left(\frac{\Lambda^2}{P_T^2}\right)^1 + E_2\left(\frac{\Lambda^2}{P_T^2}\right)^2 + \cdots O\left(\left(\frac{\Lambda^2}{P_T^2}\right)^N\right)$
Current Region?
Central/Soft Region?
Target Region?
$F_0 + F_1\left(e^{y_H}\right)^1 + \cdots O(e^{y_H N})$
$G_0 + G_1\left(e^{-y_H}\right)^1 + \cdots O(e^{-y_H N})$
Small coupling?
$B_0\,\alpha_s(Q)^0 + B_1\,\alpha_s(Q)^1 + B_2\,\alpha_s(Q)^2 + \cdots O(\alpha_s(Q)^N)$
$C_0\,\alpha_s(P_T)^0 + C_1\,\alpha_s(P_T)^1 + C_2\,\alpha_s(P_T)^2 + \cdots O(\alpha_s(P_T)^N)$
Match Overlapping Approximations
Begin Catalogue Of Theory Objects



$P_{T h}$
$\frac{m}{Q}, \frac{m}{P_T}$
Hard Transverse Momentum
Usual Nucleon Structure Region
$e^{2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
$e^{-2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Current Region (small TM)
Soft/Central Region
Target Region
$-y_h$
$\frac{m}{Q}, \frac{P_T}{Q}$
y_h

Combining Results in TMD Factorization, order ≈ α_s^3 (Drell-Yan)

See L. Gamberg talk

Sudakov Form Factor: (Moch,Vermaseren (2005), Vogt, Gehrmann et al (2014))

α_s^2 Wilson Coefficients from Collinear Factorization: (Catani et al, (2012)), and SCET (Echevarria, Scimemi, Vladimirov (2016))

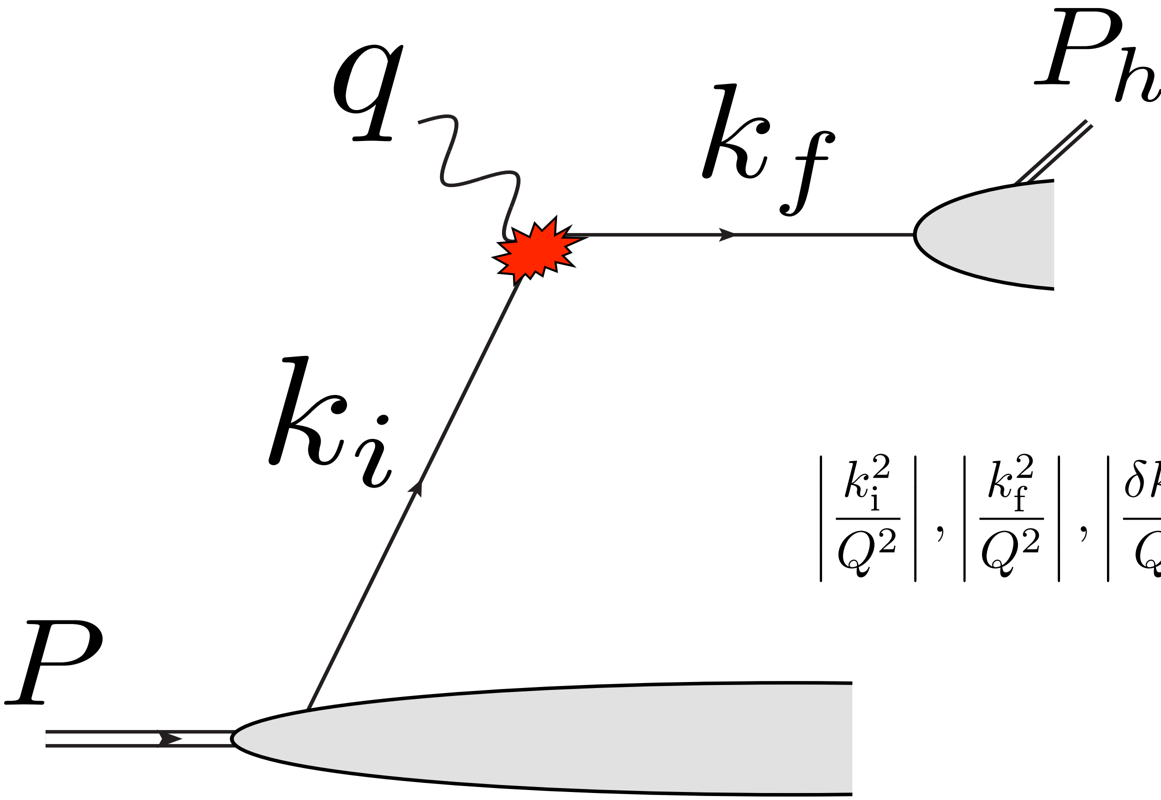
$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} &= \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} H^{\mathrm{DY}}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \\
&\times e^{-g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right) \\
&\times e^{-g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})} \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right) \\
&\times \exp\left\{-g_K(b_{\mathrm{T}};b_{\max})\ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\} \\
&+ \text{suppressed corrections.}
\end{aligned}
$$

Ex: Konychev, Nadolsky (2006)
ResBos extractions (and others)

Li, Zhu (2017)
Vladimirov (2017)

From
Sudakov Form Factor: (Moch,Vermaseren (2005), Vogt, Gehrmann et al (2014))

Kinematical Regions of Small P_T



$$\left|\frac{k_{\rm i}^2}{Q^2}\right|, \left|\frac{k_{\rm f}^2}{Q^2}\right|, \left|\frac{\delta k_{\rm T}^2}{Q^2}\right| << 1$$

M. Boglione , J. Collins, L. Gamberg , J. O. Gonzalez-Hernandez , TCR , N. Sato (2017)

Quantify proximity to current fragmentation

$$R(y_{\rm h}, z_{\rm h}, x_{\rm bj}, Q) \equiv \frac{P_h \cdot k_{\rm f}}{P_h \cdot k_{\rm i}}\,, \qquad \stackrel{\frac{m}{Q} \to 0}{=} \quad e^{2y_h}$$

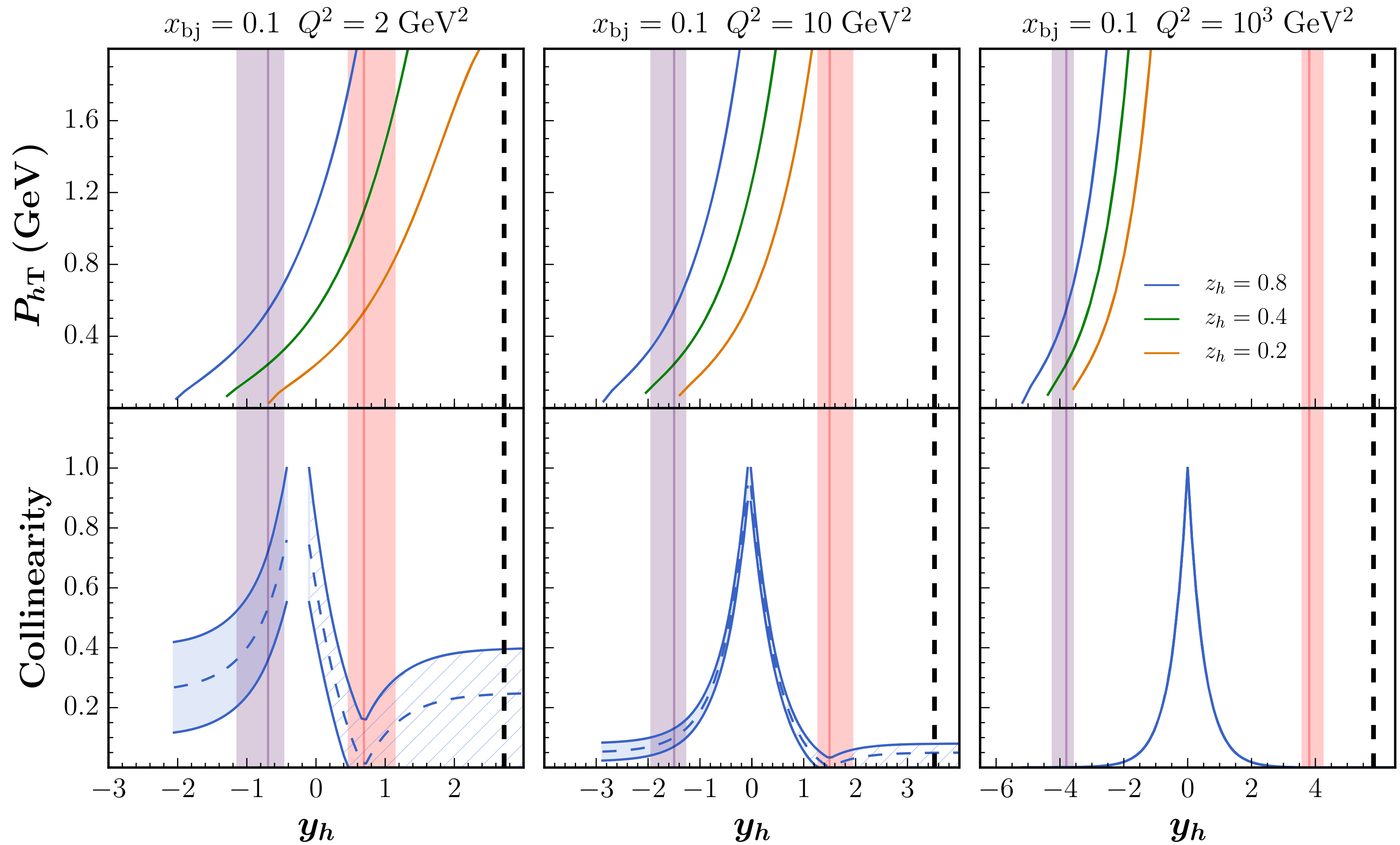
- Need estimates of non-perturbative scales:

$$y_i = \ln \frac{Q}{M_{i,\rm T}}\,; \qquad y_f = -\ln \frac{Q}{M_{f,\rm T}}$$

$$M_{i,\rm T} \approx M_{f,\rm T} \approx 0.5 \pm 0.3 \text{ GeV}$$

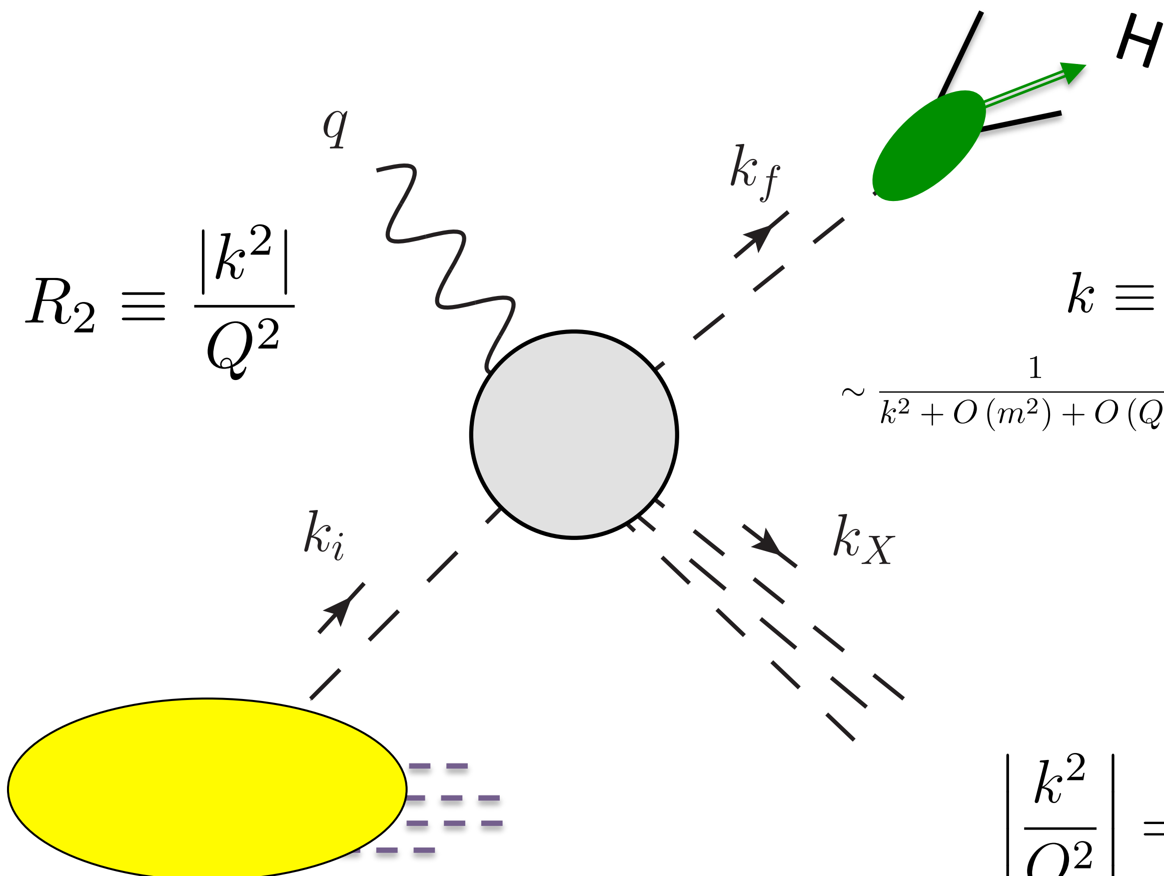
M. Boglione , J. Collins, L. Gamberg , J. O. Gonzalez-Hernandez , TCR , N. Sato (2017)

Rapidity Regions



Parton Region Mapping

$$R_2 \equiv \frac{|k^2|}{Q^2}$$

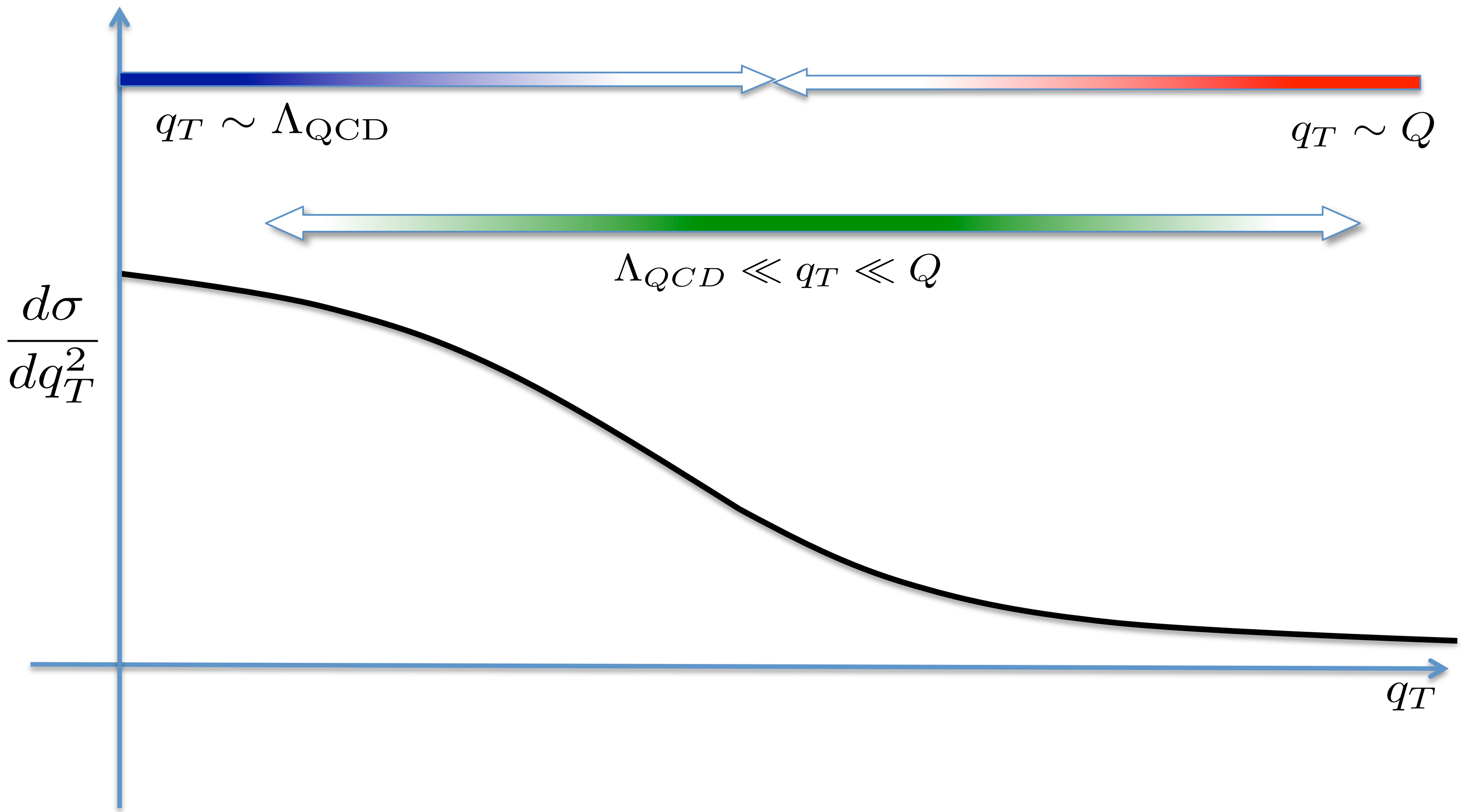


$$k \equiv k_{\mathrm{f}} - q \, .$$

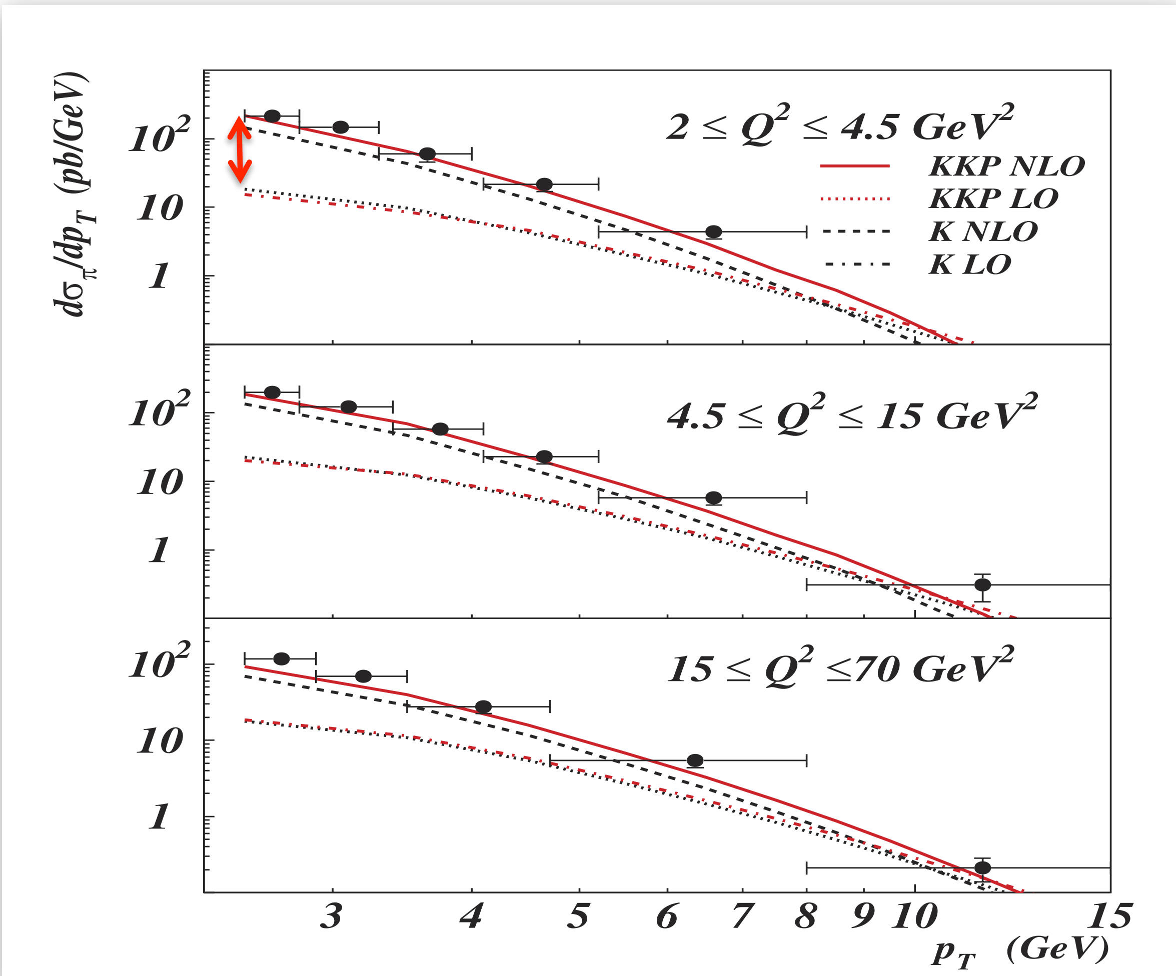
$$\sim \frac{1}{k^2 + O\left(m^2\right) + O\left(Q^2\right)} = \frac{1}{Q^2}\left(1 + O\left(\frac{k^2}{Q^2}\right) + O\left(\frac{m^2}{Q^2}\right)\right)$$

$$\left|\frac{k^2}{Q^2}\right| = (1 - \hat{z}) + \hat{z}\frac{q_{\mathrm{T}}^2}{Q^2}$$

- Merging large and small transverse momenta



Very Large Transverse Momentum



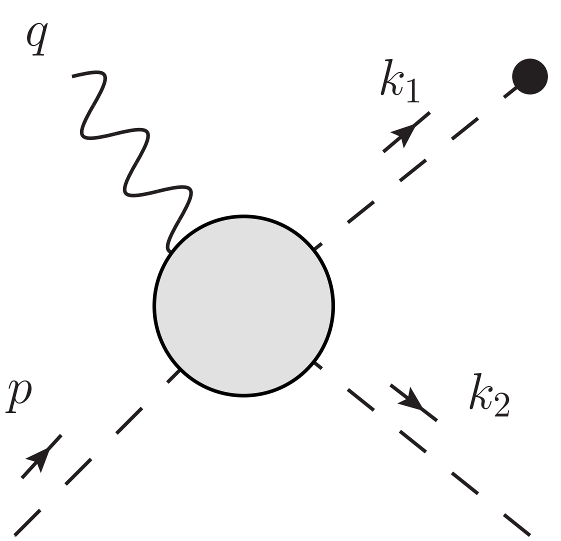
Daleo, de Florian, Sassot (2005)
Phys.Rev. D71 (2005) 034013

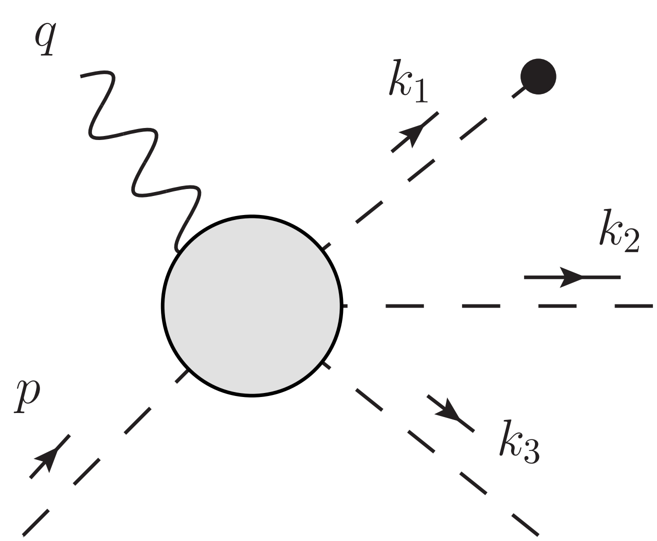
Data: H1 (2004)
Eur.Phys.J.C36:441-452,2004

Large Transverse Momentum

B. Wang, J. O. Gonzalez-Hernandez , TCR , N. Sato (In Preparation)

See N. Sato talk





Kinematical Corrections

- Old (1970s) methods of DIS mass corrections not based on factorization logic.

- Lightcone fractions versus x and z:

$$x_{\mathrm{N}} = -\frac{q^+}{P^+} = \frac{2x_{\mathrm{Bj}}}{1+\sqrt{1+\frac{4x_{\mathrm{Bj}}^2 M^2}{Q^2}}} \qquad x_{\mathrm{Bj}} = \frac{Q^2}{2P\cdot q}$$

$$z_{\mathrm{N}} = \frac{P_{\mathrm{B}}^-}{q^-} \qquad z_{\mathrm{h}} = \frac{P\cdot P_{\mathrm{B}}}{P\cdot q} = 2x_{\mathrm{Bj}}\frac{P\cdot P_{\mathrm{B}}}{Q^2}$$

- Final state hadron mass corrections

$$z_{\mathrm{N}} = \frac{x_{\mathrm{N}} z_{\mathrm{h}}}{2x_{\mathrm{Bj}}}\left(1+\sqrt{1-\frac{4M^2 M_{\mathrm{B,T}}^2 x_{\mathrm{Bj}}^2}{Q^4 z_{\mathrm{h}}^2}}\right)$$

$$= z_{\mathrm{h}}\left(1-\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2}\left(1+\frac{P_{\mathrm{B,T}}^2}{z_{\mathrm{h}}^2 Q^2}\right)+\left(\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2}\right)^2\left(\frac{P_{\mathrm{B,T}}^2}{z_{\mathrm{h}}^2 Q^2}-\frac{P_{\mathrm{B,T}}^4}{z_{\mathrm{h}}^4 Q^4}+2-\frac{M_{\mathrm{B}}^2}{z_{\mathrm{h}}^2 M^2 x_{\mathrm{Bj}}^2}\right)+O\left(\left(\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2}\right)^3\right)\right)$$

Other Challenges

- Describing non-perturbative TMD behavior

- Transition from current region to non-current region

- Target and final state hadron masses

Other Challenges

Thank you!