Progress on Calculation of Pion Valence Distribution form Hadronic Lattice Cross Sections

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in Collaboration with

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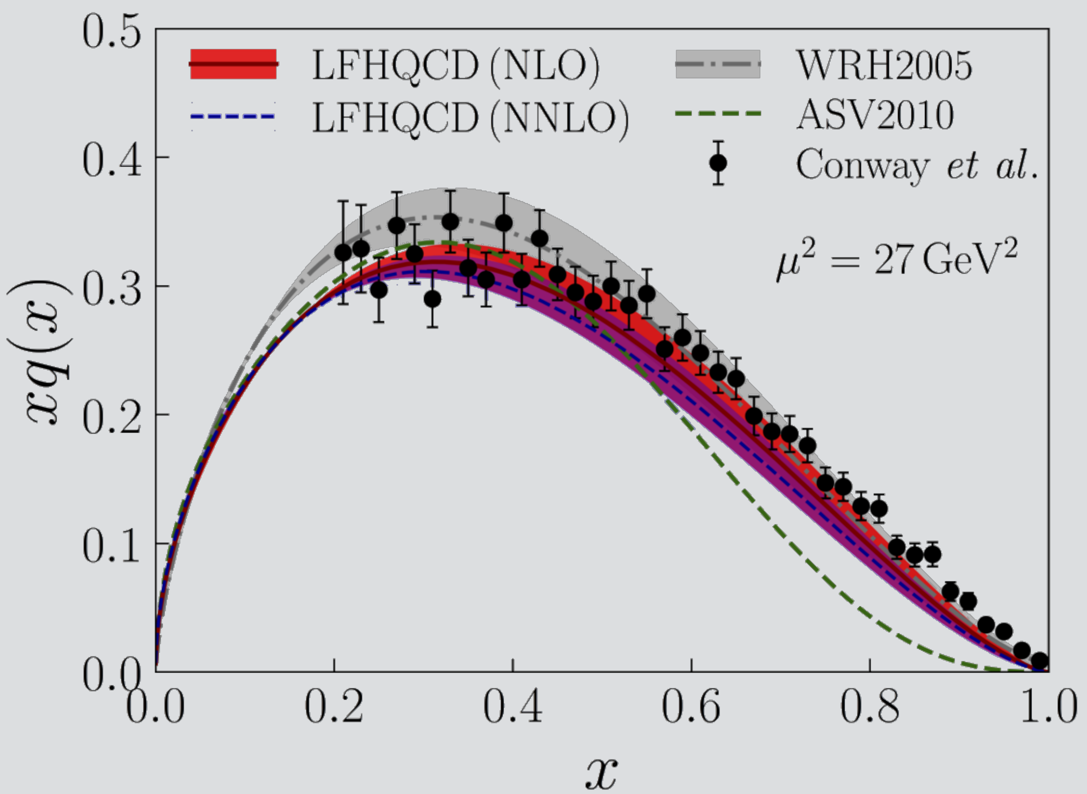
Why Pion Valence Distribution

Large-x behavior of pion valence distribution an unresolved problem

★ Perturbative QCD, Dyson-Schwinger model $(1-x)^2$ fall-off

★ Nambu-Jona-Lasino (NJL) model, Duality arguments $(1-x)^1$ fall-off



de Téramond, Liu, ***RSS***, Dosch, Brodsky, Deur PRL (2018)

See talk by Tianbo Liu (HLFHS Collaboration)

★ C12-15-006 experiment (Tagged DIS) at JLab to explore large-x behavior

★ *Lattice QCD can play vital role in understanding large x-behavior*

Calculations of Parton Distributions on the Lattice

★ Quasi PDFs (X. Ji, PRL (2013))

$$\tilde{q}(x,\mu^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-ixP_z\xi_z} \langle P|\bar{\psi}(\xi_z)\gamma_z \exp\left\{ -ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

Proposed Matching

$$\tilde{q}(x,\Lambda,P_z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right)_{\mu^2=Q^2} q(y,Q^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Power-law UV divergence from Wilson line in the non-local operator

★ Pseudo-PDFs (A. Radyushkin, PLB (2017))

$$M(\xi,P_Z) \rightarrow \mathcal{M}(\omega,\xi^2) \quad \text{Lorentz invariant Ioffe time } \omega = \xi\cdot P$$

$$\mathcal{P}(x,\xi^2) \equiv \int \frac{d\omega}{2\pi} e^{-ix\omega}\mathcal{M}(\omega,\xi^2)$$

Beautiful feature of canceling UV divergence as $\xi^2 \rightarrow 0$

See talk by Joseph Karpie (Pseudo PDFs)

....

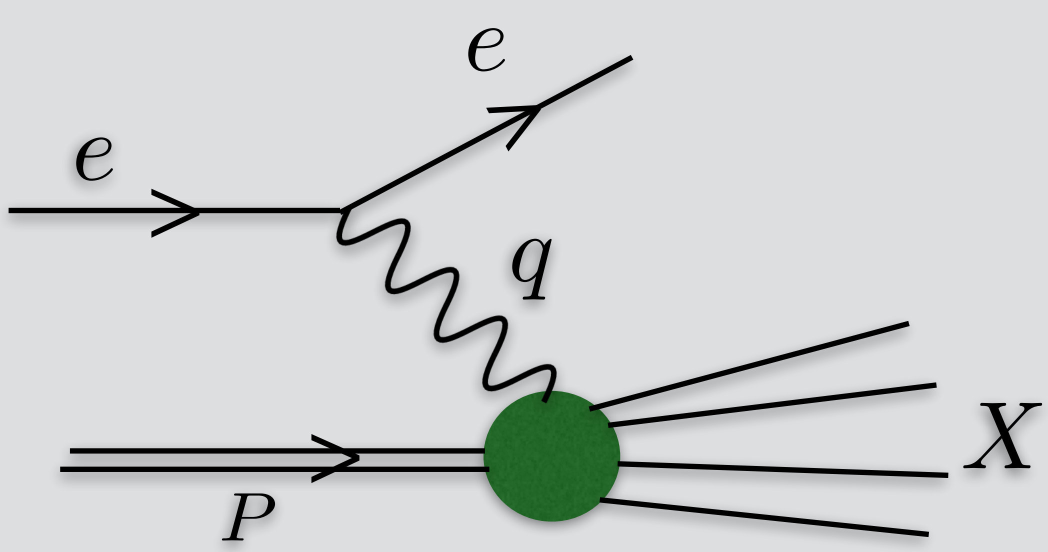
- Hadronic tensor method (K. F. Liu, PRL 1994, PRD 200))
- Compton amplitude / OPE without OPE
  (A. Chambers, et al PRL (2017))
- Position-space correlators (V. M. Braun & D. Müller (2008))
- **Good Lattice Cross Sections**
  (Y. Q. Ma, J.-W. Qiu, PRL 2018)

→ Requires four-point correlation function calculation on the lattice

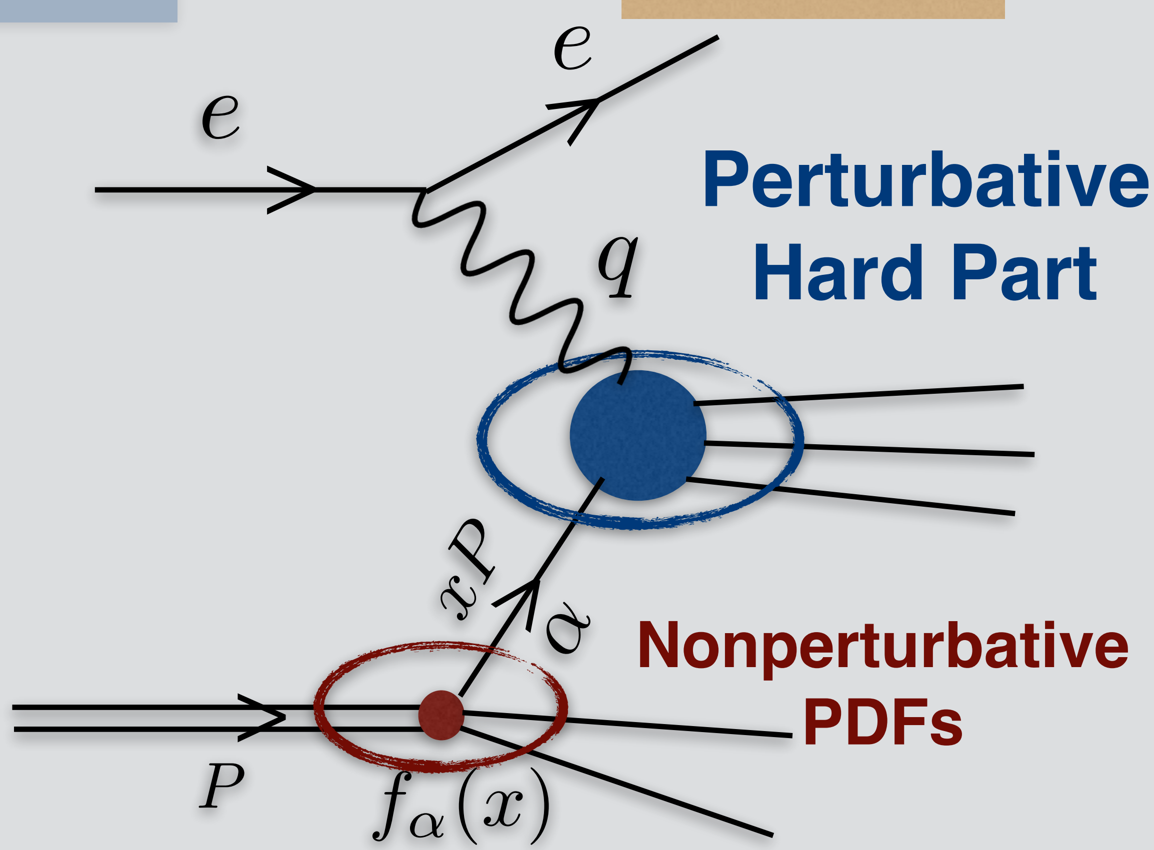
→ Collaboration between lattice QCD and perturbative QCD

Parton Distribution Functions (PDFs)

$$\sigma^{DIS}(x,Q^2,\sqrt{s}) = \sum_{\alpha=q,\bar{q},g} C_\alpha\left(x,\frac{Q^2}{\mu^2},\sqrt{s}\right)\otimes f_\alpha(x,\mu^2)+\mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$



DIS



Parton Picture

x :Bjorken-x

$\sqrt{s}$:Collision Energy

Q :Momentum Transfer

μ :Factorization Scale

What are Good Lattice "Cross Sections" (LCSs)

Single hadron matrix elements:

Ma & Qiu
PRL (2018)

1. Calculable using lattice QCD with Euclidean time

2. Well defined continuum limit ($a \to 0$), UV finite

3. Share the same perturbative collinear divergences with PDFs

4. Factorizable to PDFs with IR-safe hard coefficients
   with controllable power corrections

Lattice Calculable + Renormalizable + Factorizable

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) \times K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{QCD}^2)$$

Nonperturbative PDFs of flavor $a = q, g$

Perturbatively Calculable Hard Coefficients

$$f_{\bar{a}}(x, \mu^2) = -f_a(-x, \mu^2)$$

P and ξ → Collision Kinematics

$\mathcal{O}_n$ → Dynamical Features of LCSs

Factorization holds for any finite ω and $P^2\xi^2$
if ξ is short distance

★ Hadron matrix elements: $\sigma_n(\omega, \xi^2, P^2) = \langle P | T\{\mathcal{O}_n(\xi)\} | P \rangle$

$$\omega \equiv P \cdot \xi, \; \xi^2 \neq 0, \; \xi_0 = 0$$

★ Current-current correlators

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1} + d_{j_2} - 2} \, Z_{j_1}^{-1} \, Z_{j_2}^{-1} j_1(\xi) \, j_2(0)$$

d_j : Dimension of the current

Z_j : Renormalization constant of the current

Z_j already known for the lattice ensembles being used

★ Different choices of currents

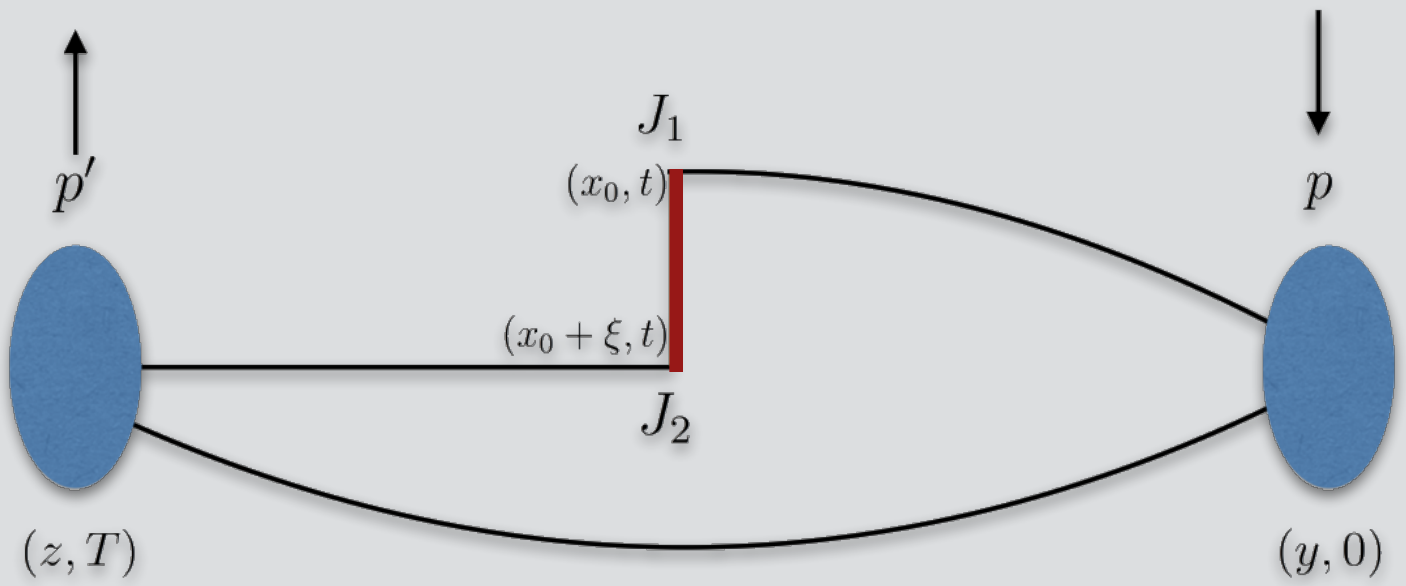
$$j_S(\xi) = \xi^2 Z_S^{-1} [\overline{\psi}_q \psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1} [\overline{\psi}_q \gamma \cdot \xi \psi_q](\xi),$$

$$j_{V'}(\xi) = \xi Z_{V'}^{-1} [\overline{\psi}_q \gamma \cdot \xi \psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1} [-\frac{1}{4} F_{\mu\nu}^c F_{\mu\nu}^c](\xi), \ldots$$

flavor changing current

gluon distribution

Lattice Calculation Setup



possible ξ
on/off axis

$$
\begin{aligned}
&\langle \Pi(-p')|\mathcal{O}_{J_1}(x_0)\mathcal{O}_{J_2}(\xi)|\Pi(-p')\rangle = \\
&\quad = \sum_{y,z} e^{i(p'.z-p.y)}\langle \bar{q}\,\Gamma_\Pi\, q(z,T)\,\bar{q}\,J_2\,q(x_0+\xi,t)\,\bar{q}\,J_1\,q(x_0,t)\,\bar{q}\,\Gamma_\Pi\,q(y,0)\rangle \\
&\quad = \sum_{y,z} e^{i(p'.z-p.y)}\mathrm{tr}[J_2\,D^{-1}(x_0+\xi,t;x_0,t)\,J_1\,D^{-1}(x_0,t;y,0)\,\Gamma_\Pi \\
&\qquad \times D^{-1}(y,0;z,T)\Gamma_\Pi\,D^{-1}(z,T;x_0+\xi,t)],
\end{aligned}
$$

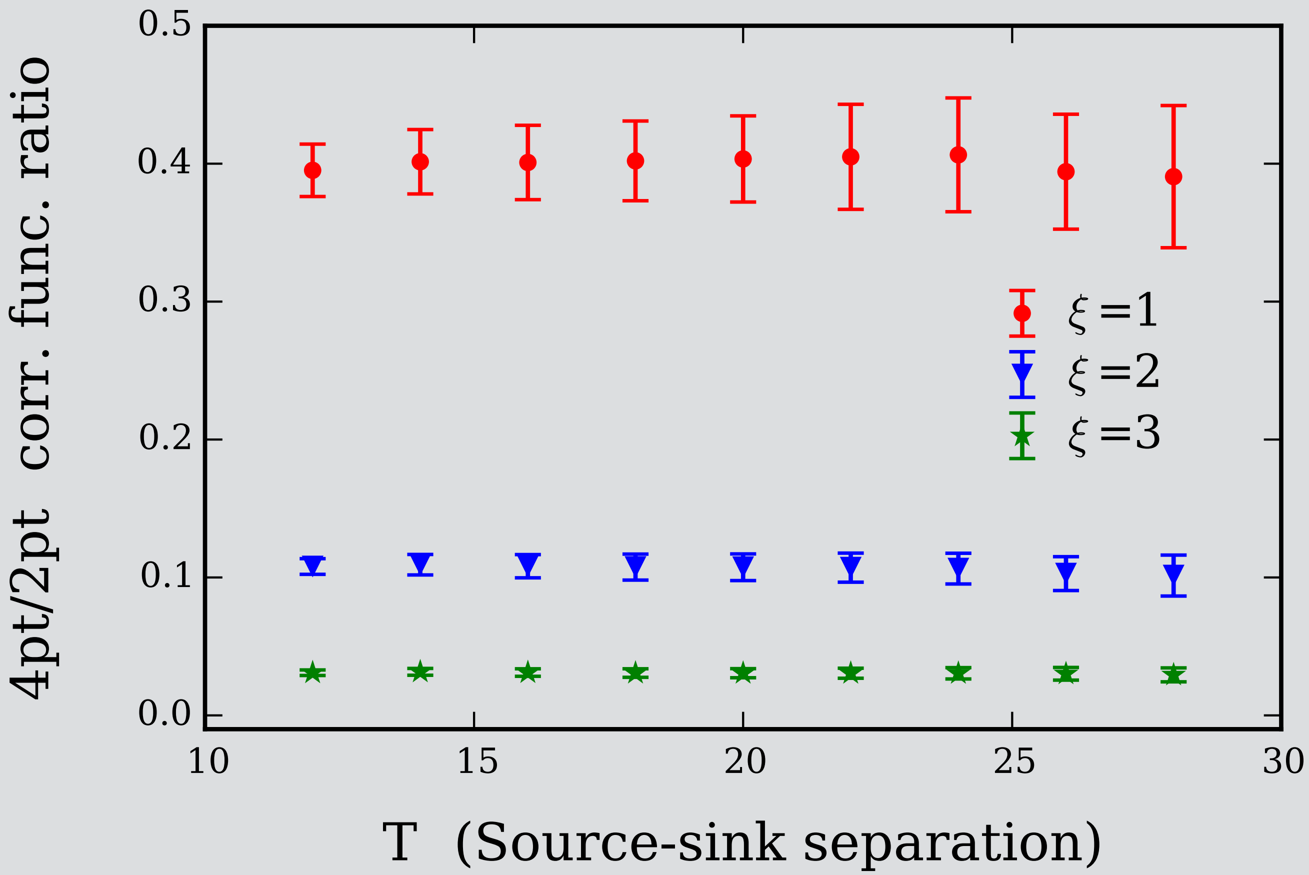
- ★ Analysis shown here on isoClover with 110 Configurations
- ★ Lattice spacing ~ 0.127 fm, 0.09 fm $(32^3 \times 96,\ 32^3 \times 64)$

  $m_\pi \approx 440, 400\ \mathrm{MeV}$
- ★ Projected calculation with $m_\pi \approx 170\ \mathrm{MeV}$ on $64^3 \times 128$ ensemble

(Very) Preliminary Lattice Results

- Matrix element with currents: J_1 =Vector, J_2 =Axial
- Only 110 configurations and 1 random source used
- Momentum smearing to be used for higher momentum

Gunnar S. Bali, et al (PRD 2016))



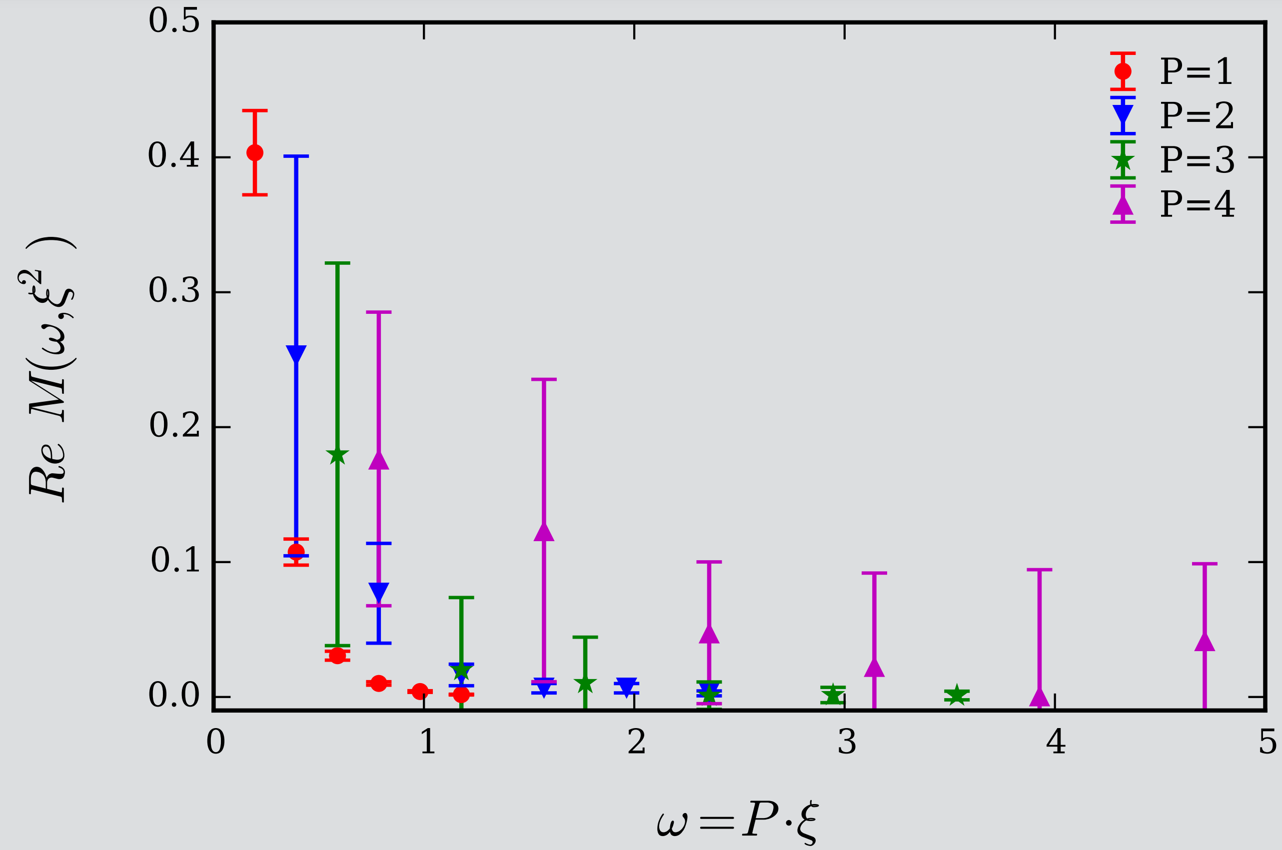
T ➡ Source-sink separation

P_z= 1 (~0.3 GeV)

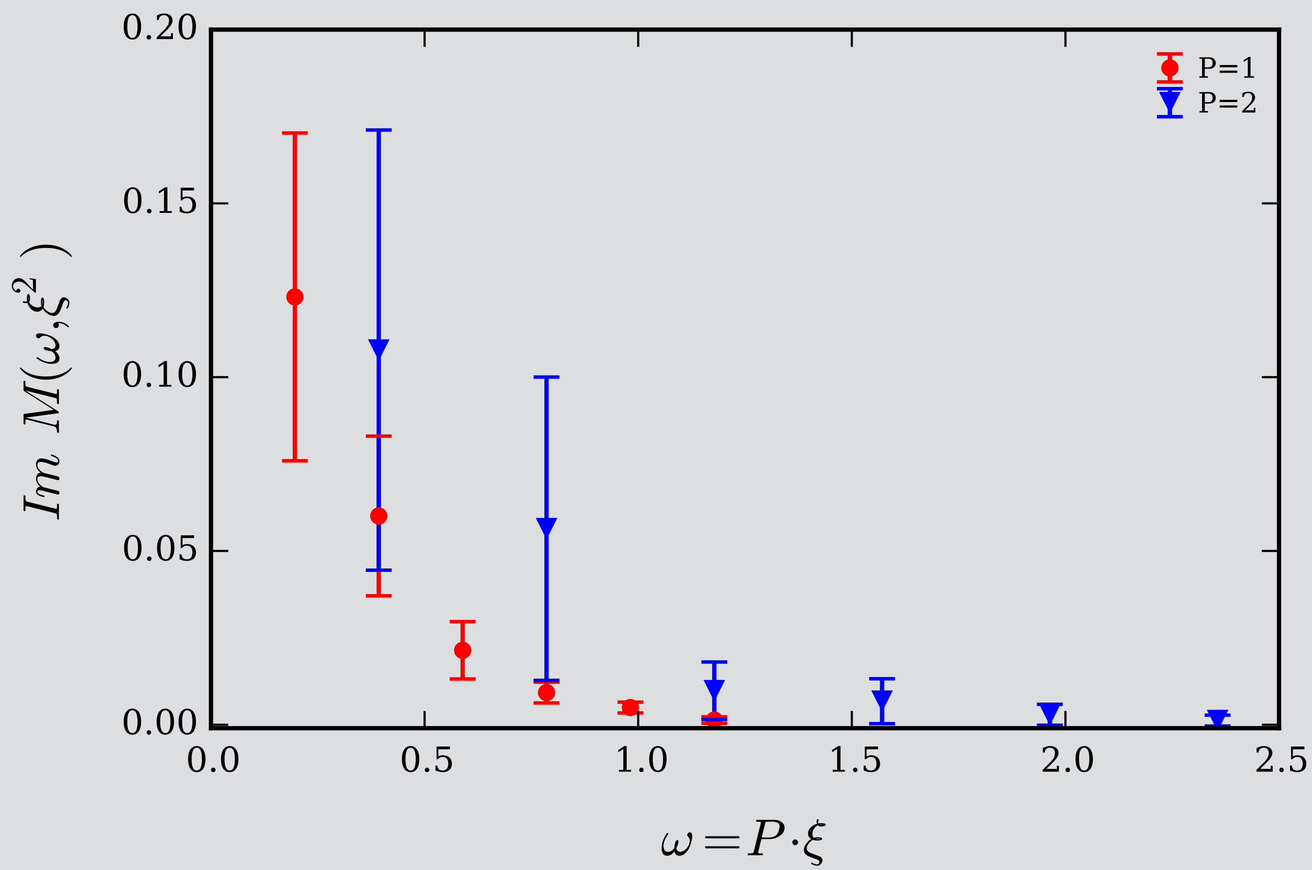
$m_\pi \approx 440$ MeV

$a \approx 0.127$ fm

(Very) Preliminary Lattice Results



We have ~ 30 times more statistics being produced



No signal for imaginary part with P=3, 4 from ~110 configs

Outlook

- Many different LSCs with different currents are being analyzed

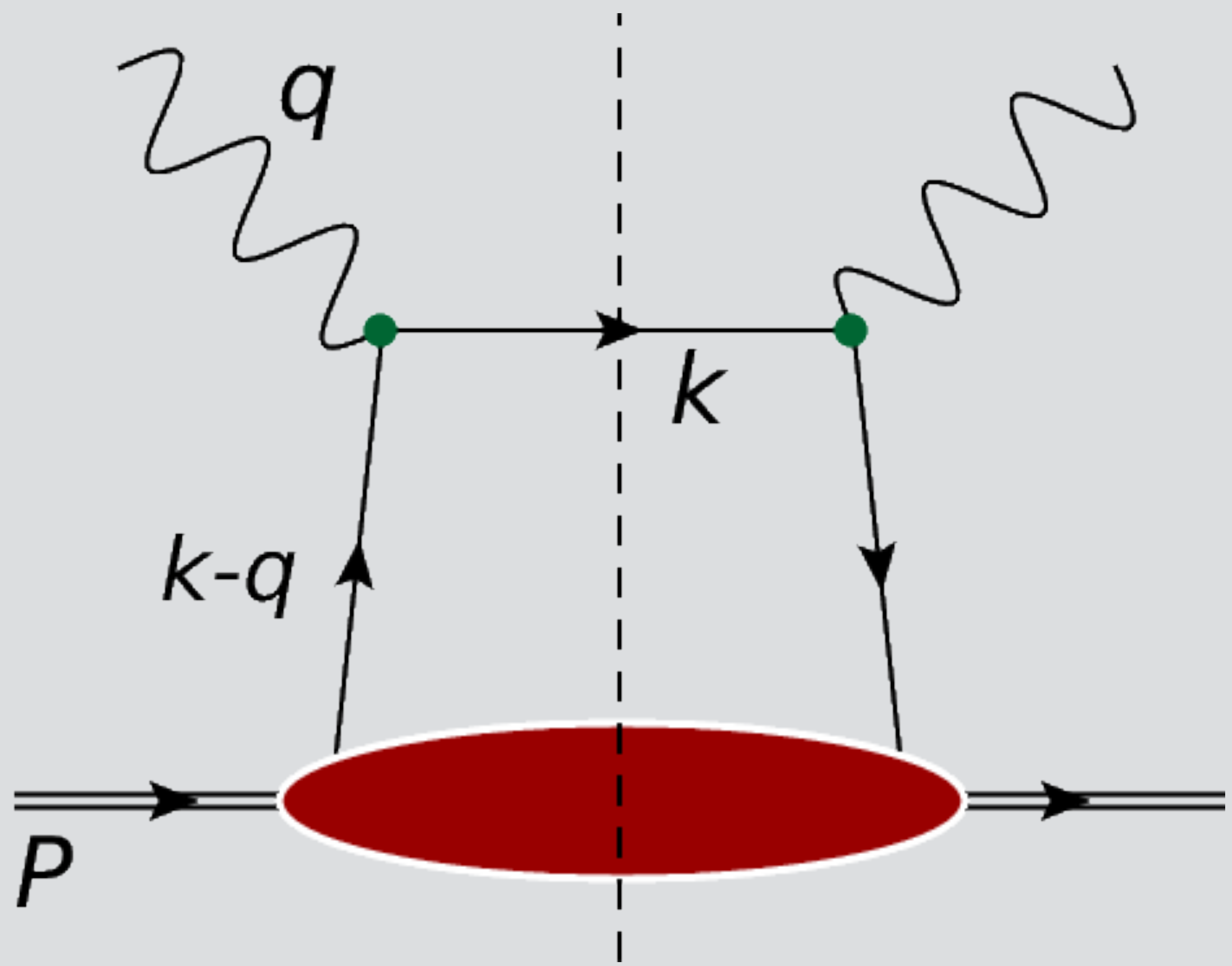
- Momentum space matrix elements

$$\tilde{\sigma}_n(\tilde{\omega}, q^2, P^2) \equiv \int \frac{d\xi^4}{\xi^4} e^{iq.\xi} \sigma_n(\omega, \xi^2, P^2)$$

$$\tilde{\sigma}_n = \sum_a f_a \otimes \tilde{K}_n^a + \mathcal{O}(\Lambda_{QCD}^2/q^2)$$

- x-dependence of pion valence distribution can be obtained from $\tilde{\omega} = 1/x$

Thank You



q
k
k-q
P

ξ^2 be small but not vanishing

Apply OPE to non-local op $\mathcal{O}_n(\xi)$

$$\sigma_n(\omega, \xi^2, P^2) = \sum_{J=0} \sum_{a} W_n^{(J,a)}(\xi^2, \mu^2)\, \xi^{\nu_1} \cdots \xi^{\nu_J} \times \langle P | \mathcal{O}^{(J,a)}_{\nu_1 \cdots \nu_J}(\mu^2) | P \rangle \,,$$

$\mathcal{O}^{(J,a)}_{\nu_1 \cdots \nu_J}(\mu^2)$ Local, symmetric , traceless op