GPDs and LFWFs: Crossing the $x = |\xi|$ line.

Cédric Mezrag

INFN Roma1

May 18^{th}, 2018

Chapter 1:
Definitions and Properties

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  - are defined according to a non-local matrix element,

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0}$$
$$=\frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u+E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0}$$
$$=\frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5u+\tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

- Generalised Parton Distributions (GPDs):
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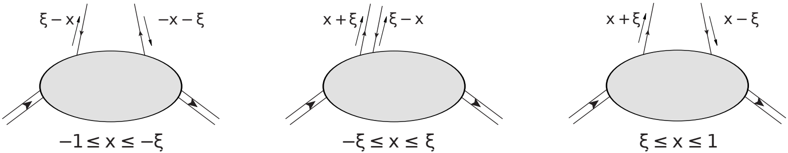
  - depend on three variables (x, ξ, t),



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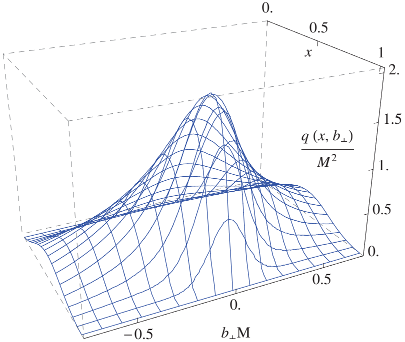
M. Burkardt, Phys. Rev. **D62**, 071503 (2000)



Pion GPD in Impact parameter space from: CM *et al.*, Phys. Lett. **B741**, 190-196 (2015)

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  - depend on three variables (x, ξ, t),
  - can split in terms of quark flavour and gluon contributions,
  - can be related to the 2+1D parton number density when $\xi \to 0$.
  - are univeral, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions:

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x\ C(x, \xi) H(x, \xi, t)$$

Properties

- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x \; x^m H^q(x,\xi,t) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C_{2j}^q(t) + mod(m,2)\xi^{m+1} C_{m+1}^q(t)$$

Lorentz Covariance

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left|H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2}E^q(x,\xi,t)\right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right)q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

- Polynomiality Property:

Lorentz Covariance

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Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Soft pion theorem (pion GPDs only)

M.V. Polyakov, Nucl. Phys. **B555**, 231 (1999)
CM *et al.*, Phys. Lett. **B741**, 190 (2015)

Axial-Vector WTI

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Relativistic quantum mechanics

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Axial-Vector WTI

No model (so far) fulfils all the constraints a priori

People emphasise either:

- Polynomiality through Double Distribution modeling,
- Positivity through LFWFs approaches

- Definition in terms of matrix element for $z^2 = 0$:

$$\langle P + \frac{\Delta}{2} | \bar{q}\left(-\frac{z}{2}\right)\gamma_\mu q\left(\frac{z}{2}\right)|P - \frac{\Delta}{2}\rangle = 2P_\mu \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\, e^{-i\beta(P\cdot z)+i\alpha\frac{(\Delta\cdot z)}{2}} F^q(\beta,\alpha,t) - \Delta_\mu \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\, e^{-i\beta(P\cdot z)+i\alpha\frac{(\Delta\cdot z)}{2}} G^q(\beta,\alpha,t) + \text{higher twist terms.}$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
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- Simple relation to GPDs:

$$H(x,\xi,t) = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha \, \delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right]$$

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- Automatically fulfil the polynomiality property

Double Distributions for spin 0

- Definition in terms of matrix element for $z^2 = 0$:

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- Automatically fulfil the polynomiality property
- But positivity is not fulfilled a priori

- Lightfront quantization allows to expand hadrons on a Fock basis

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

LFWFs

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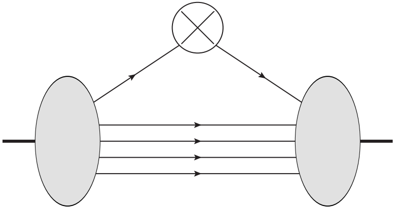
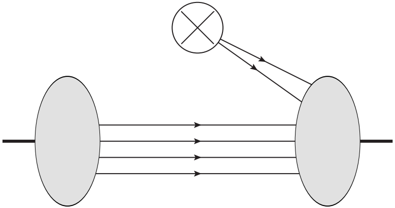
DGLAP: $|x| > |\xi|$



- Same N LFWFs
- Truncation unambiguous

ERBL: $|x| < |\xi|$



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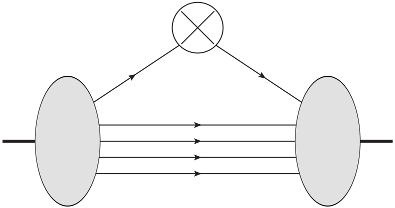

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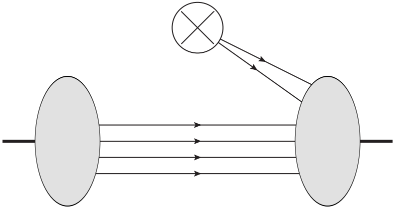

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LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

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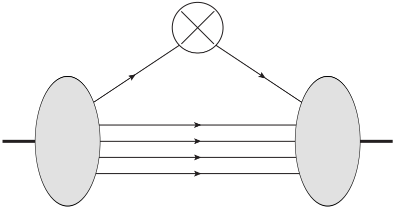

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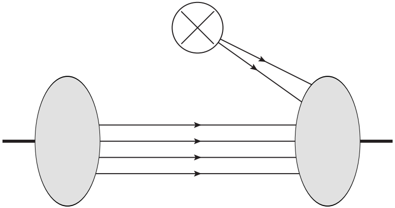

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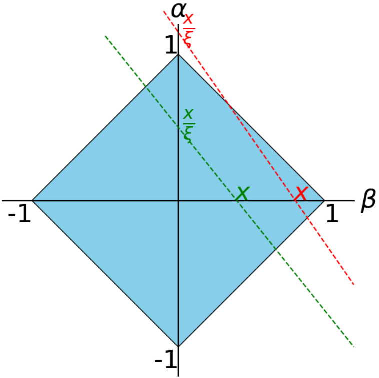
Is there a solution to get all the good properties?

Chapter 2:
The Inverse Radon Transform

N.Chouika, CM, H. Moutarde, J. Rodriguez-Quintero,
EPJC 77 (2017) no.12, 906

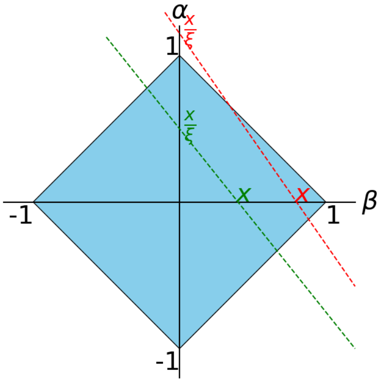
$$H(x,\xi) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha)+\xi G(\beta,\alpha)\right]$$



- DGLAP (red) and ERBL (green) lines cut $\beta = 0$ outside or inside the square
- Every point $(\beta \neq 0, \alpha)$ contributes **both** to DGLAP and ERBL regions
- For every point $(\beta \neq 0, \alpha)$ we can draw an infinite number of DGLAP lines.

Intuitive picture

$$H(x,\xi) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha) + \xi G(\beta,\alpha)\right]$$



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Is it possible to recover the DDs from the DGLAP region only?

- We can define a D-term such that:

$$\int_{-1}^{1} \mathrm{d}x\, x^m \left(H(x,\xi) - D(x/\xi)\right) = \sum_{i \text{ even}}^{m} (2\xi)^i C_{m,i},$$

yielding the **Ludwig-Helgason** consistency conditions.

- From **Hertle theorem** (1983), we know that $H - D$ is in the range of the Radon transform and that:

$$H(x,\xi) = D(x/\xi) + \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x - \beta - \alpha\xi) F_D(\beta,\alpha)$$

This allows us to identify the DD F_D with the Radon transform of $H - D$. This has been first noticed by O. Teryaev (PLB510 2001 125).

- It should be possible to use the **limited** Radon inverse transform to obtain the DD and thus the ERBL part.

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NB: This is equivalent to fixing the DD to the Polyakov-Weiss scheme. The same argument can be done in other schemes, but the D-term remains unknown.

Uniqueness of the Extension

- Since DD are compactly supported, we can use the **Boman and Todd-Quinto theorem** which tells us

$$H(x,\xi) = 0 \quad \text{for} \quad (x,\xi) \in \mathrm{DGLAP} \Rightarrow F_D(\beta,\alpha) = 0 \quad \text{for all} \quad (\beta \neq 0, \alpha) \in \Omega$$

Boman and Todd-Quinto, Duke Math. J. 55, 943 (1987)

insuring the uniqueness of the extension up to D-term like terms.

- The DGLAP region almost completely characterises the entire GPD.

New modeling strategy

- Compute the DGLAP region through overlap of LFWFs
  $\Rightarrow$ **fulfilment of the positivity property**
- Extension to the ERBL region using the Radon inverse transform
  $\Rightarrow$ **fulfilment of the polynomiality property**

Chapter 3:
An example on the pion

N.Chouika, CM, H. Moutarde, J. Rodriguez-Quintero,
PLB 780 (2018) 287-293

An algebraic model for the Pion BSWF

- Consider the Euclidean Bethe-Salpeter Wave Function based on the Nakanishi representation:

$$\Psi(k,P) = S(k - P/2)\Gamma(k,P)S(k + P/2)$$

$$S(k) = \frac{i\gamma \cdot k + M}{k^2 + M^2} \qquad \Gamma(k,P) = i\mathcal{N}\gamma_5 \int_{-1}^{1} \frac{\mathrm{d}z(1-z^2)M^2}{\left[\left(k - \frac{1-z}{2}P\right)^2 + M^2\right]}$$

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- We can compute from it the 2-body LFWFs:

$$\Phi_{\uparrow\downarrow}(x,\mathbf{k}_\perp) = 8\sqrt{15}\,\pi \frac{M^3}{(\mathbf{k}_\perp^2+M^2)^2}(1-x)\,x$$

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Nakanishi Representation

The present model is very simple, but the Nakanishi formalism is completely general, and can be straigthforwardly apply to more complicated models.

Algebraic Results

$$H^u_{\pi^+}(x,\xi,t)|_{\xi\leq x} = \frac{15}{2}\frac{(1-x)^2(x^2-\xi^2)}{(1-\xi^2)^2}\frac{1}{(1+\zeta)^2}\left(3+\frac{1-2\zeta}{1+\zeta}\frac{\text{arctanh}\left(\sqrt{\frac{\zeta}{1+\zeta}}\right)}{\sqrt{\frac{\zeta}{1+\zeta}}}\right),$$

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Few comments

- Simple LFWFs yield quite complicated GPDs in the DGLAP region;
- Yet algebraic results can be obtained both for the DD and the GPD in the ERBL region;
- Provide us with a benchmarck for numerical approaches

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Another model have been inverted by Müller and Hwang (PLB660 (2008) 350-359)

Numerical Inversion

- For more complicated LFWFs, algebraic inversion is not possible, we need to develop a **systematic** numerical method to handle it.

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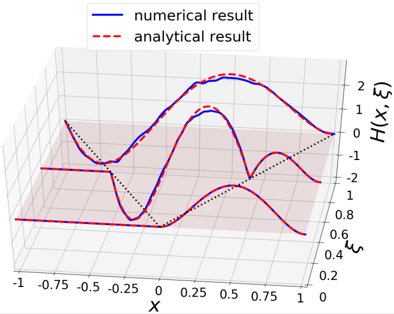
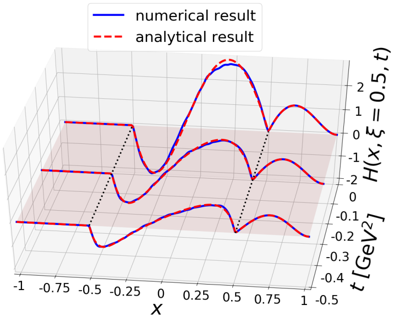
- Difficulty: The limited inverse Radon transform is a severely ill-posed problem in the sens of Hadamard.
- Using finite element analysis with LSMR regularisation we obtained:

$H(x, \xi, 0)$



$H(x, 0.5, t)$



Chapter 4:
Toward a Nucleon Wave Function

CM, J. Segovia, L. Chang, C.D. Roberts,
arXiv:1711.09101

Faddeev Framework

- The Faddeev equation provides a covariant framework to describe the nucleon as a bound state of three dressed quarks.

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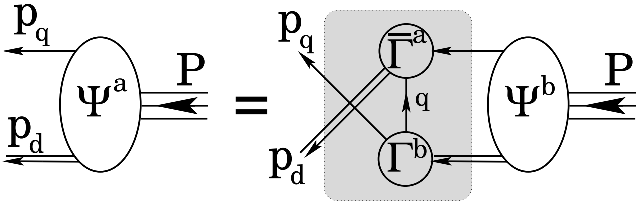

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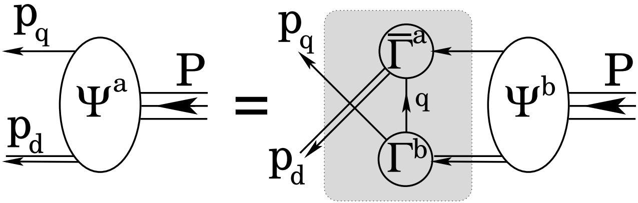

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks, whose mass is roughly 2/3 of the nucleon mass,
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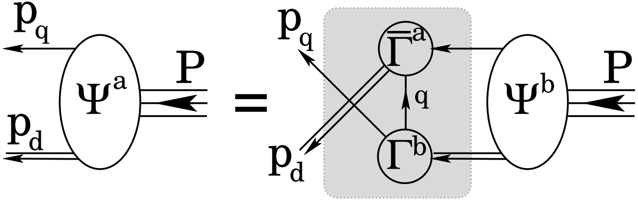

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  - Axial-Vector (AV) diquarks, whose mass is around 3/4 of the nucleon one.
- Can we understand the nucleon wave function in terms of quark-diquarks correlations?

Wave Function Model

- Algebraic parametrisation inspired by the results obtained from DSEs and Faddeev equations.
- It is based on Nakanishi representation, which is completely general.
- This is an exploratory work: we want to know what we can or cannot do.
- We also assume the dynamical diquark correlations, both scalar and AV, and compare in the end with Lattice QCD one.

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_{\uparrow}^{i}(z_1)C\not{n}u_{\downarrow}^{j}(z_2)\right)\not{n}d_{\uparrow}^{k}(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

Nucleon Distribution Amplitude

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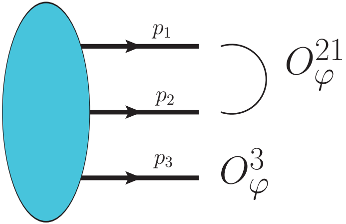
$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

- We can apply it on the wave function:



- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

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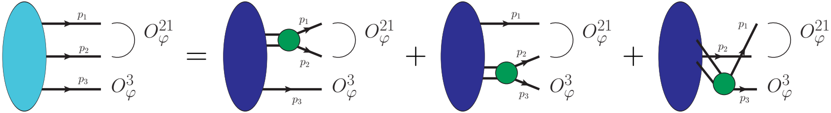


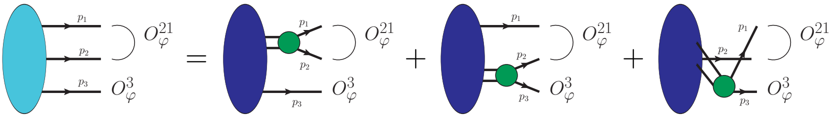
Nucleon Distribution Amplitude

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

- We can apply it on the wave function:



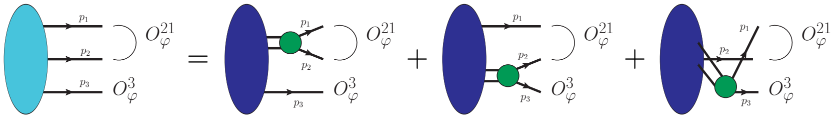
- The operator then selects the relevant component of the wave function.

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

- We can apply it on the wave function:

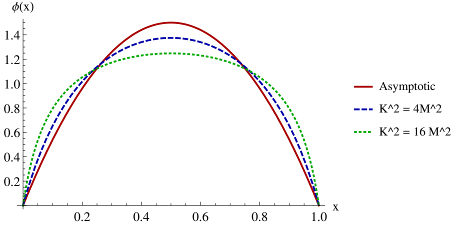


- The operator then selects the relevant component of the wave function.
- Our ingredients are:
  - Perturbative-like quark and diquark propagator
  - Nakanishi based diquark Bethe-Salpeter-like amplitude (green disks)
  - Nakanishi based quark-diquark amplitude (dark blue ellipses)

Diquark DA

$$\phi(x) \propto 1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar diquark



Diquark DA

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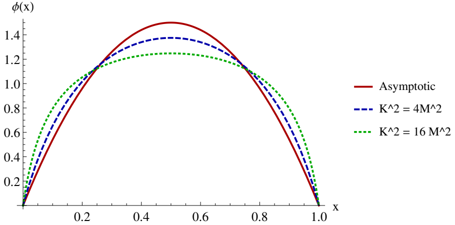
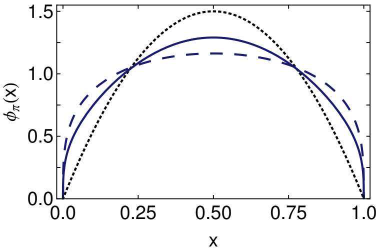
Scalar diquark



Pion

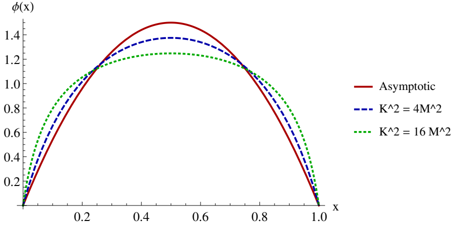


Pion figure from L. Chang *et al.*, PRL 110 (2013)

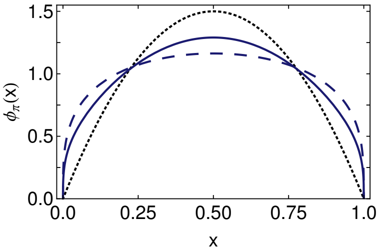
Diquark DA

$$\phi(x) \propto 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2} x(1-x)\right]}{x(1-x)}$$

Scalar diquark

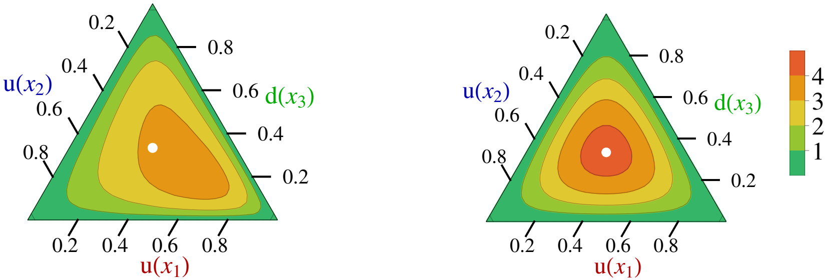


Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

- This results provide a broad and concave meson DA parametrisation
- The endpoint behaviour remains linear

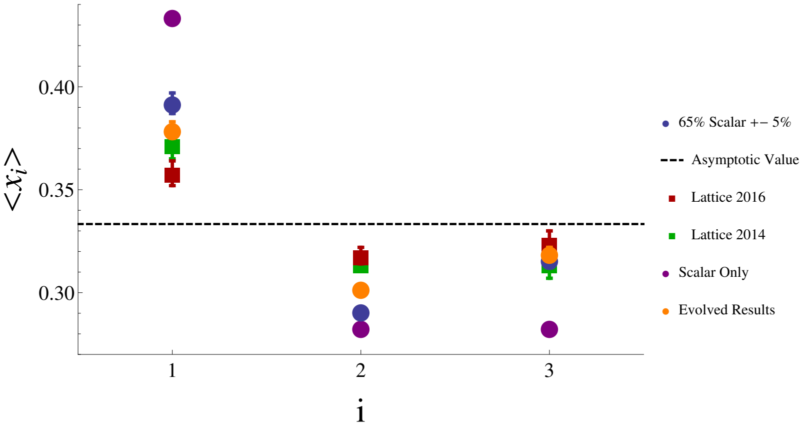


Our Result — Asymptotic DA

- Results evolved from 0.51 to 2 GeV with both scalar and AV diquark
- Nucleon DA is skewed compared to the asymptotic one
- It is also broader than the asymptotic results
- These properties are consequences of our quark-diquark picture
- Can be extended to the radial excitations (Roper)

Comparison with lattice

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

Conclusion

Conclusion

GPDs Theory

- We can now fulfil **positivity** and **polynomiality** a priori.
- We have a **systematic** way to do it.

Nakanishi Parametrisation

- Simple algebraic Nakanishi-like models for the pion and nucleon.
- Algebraic models have their **successes** and their **limitations**.
- Aim : numerical solution from **Dyson-Schwinger Equations**.

Phenomenology

- Final goal: DVCS/TCS/DVMP **cross sections**.
- Use **PARTONS** to achieve it.

Thank you for your attention

Back up slides

http://partons.cea.fr

PARTONS

PARtonic Tomography Of Nucleon Software

Main Page   Reference documentation +   Search

Main Page

What is PARTONS?

PARTONS is a C++ software framework dedicated to the phenomenology of Generalized Parton Distributions (GPDs). GPDs provide a comprehensive description of the partonic structure of the nucleon and contain a wealth of new information. In particular, GPDs provide a description of the nucleon as an extended object, referred to as 3-dimensional nucleon tomography, and give an access to the orbital angular momentum of quarks.

PARTONS provides a necessary bridge between models of GPDs and experimental data measured in various exclusive channels, like Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP). The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron Ion Collider (EIC).

PARTONS is useful to theorists to develop new models, phenomenologists to interpret existing measurements and to experimentalists to design new experiments. A detailed description of the project can be found here.

Table of Contents

- What is PARTONS?
- Get PARTONS
- Configure PARTONS
- How to use PARTONS
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- License
- Contact and newsletter

Get PARTONS

Here you can learn how to get your own version of PARTONS. We offer two ways.

You can use our provided virtual machine with an out-of-the-box PARTONS runtime and development environment. This is the easiest way to start your experience with PARTONS.

Using PARTONS with our provided Virtual Machine

You can also build PARTONS by your own on either GNU/Linux or Mac OS X. This is useful if you want to have PARTONS on your computer without using the virtualization technology or if you want to use PARTONS on computing farms.

Using PARTONS on GNU/Linux

Using PARTONS on Mac OS X

Configure PARTONS

If you are using **our virtual machine**, you will find all configuration files set up and ready to be used. However, if you want to tune the configuration or if you have installed PARTONS by your own, this tutorial will be helpful for

$$\begin{aligned}
H(x,\xi,t) &= (1-x)\int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi) h_P(\beta,\alpha,t) \\
h_P(\beta,\alpha,t) &= \frac{15}{2}\theta(\beta)\left[1+\frac{-t}{4M^2}\left((1-\beta)^2-\alpha^2\right)\right]^{-3} \\
&\quad \times \left[1-3(\alpha^2-\beta^2)-2\beta+\frac{-t}{4M^2}\left(1-(\alpha^2-\beta^2)^2-4\beta(1-\beta)\right)\right],
\end{aligned}$$

From the algebraic DD we can deduce the GPD in ERBL region

$$H(x,\xi,0)|_{|x|\leq\xi} = \frac{15}{2}\frac{(1-x)(\xi^2-x^2)}{\xi^3(1+\xi)^2}\left(x+2x\xi+\xi^2\right),$$

- Use of a P_1 (planar by pieces) basis
- We have to trade of precision and noise:
  In ill-posed inverse problem, small errors coming from our discretisations can trigger significant increases in the numerical noise.