Basis Light-Front Quantization Approach to Proton

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Contents

☐ Introduction: overview about some nucleon properties

☐ Basis Light-Front Quantization (BLFQ) approach to nucleon

✓ Form factors

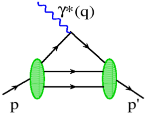
✓ Generalized parton distributions (GPDs)

☐ Light-front quark-diquark model (LFQDM)

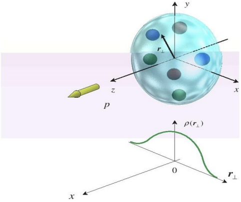
☐ Conclusions

Form factors Vs PDFs Vs GPDs

Elastic Scattering

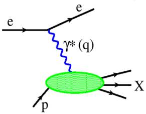


Established extended nature of nucleon

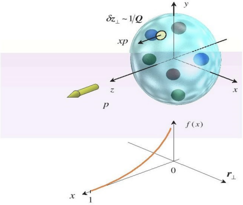


charge and magnetization distribution

Deep Inelastic Scattering

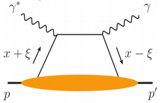


discovered the existence (quarks) inside the nucleon

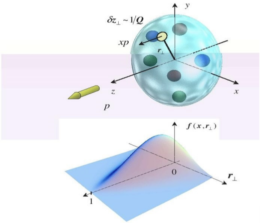


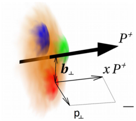
longitudinal momentum distribution

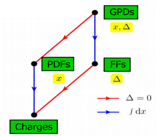
Deeply virtual Compton Scattering



provides 3D spatial structure of the nucleon







Basis Light-Front Quantization (BLFQ)

J. Vary *et. al.*, PRC 81 (2010)

BLFQ: approach for solving quantum field theory

- **Nonperturbative:**
  for systems with strong interaction
- **First-principles:**
  effective Hamiltonian as input/ direct access to wavefunction of bound states
- **Light-front dynamics:**
  spectrum and light-front Fock-state wavefunctions are obtained from

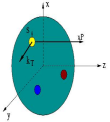
$$H_{LF}|\psi\rangle = M^2|\psi\rangle$$

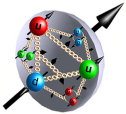
$$H_{LF} \equiv P_\mu P^\mu = P^+P^- - \mathbf{P}_\perp^2$$

$$P^\pm = P^0 \pm P^3$$

LF wavefunctions

Proton 3D imaging | **Proton spin**





FF | GPD | TMD...

General Procedure for BLFQ discussed by Vary, Chen & Zhao

✓ Construct the basis state: $|\alpha\rangle$

✓ Derive/write the Light-Front Hamiltonian: P^-

✓ Calculate Hamiltonian matrix elements: $\langle\alpha'|P^-|\alpha\rangle$

✓ Diagonalize the Hamiltonian: $P^-|\beta\rangle = P^-_\beta|\beta\rangle$

✓ Evalute the observables $\mathcal{O} \equiv \langle\beta|\hat{\mathcal{O}}|\beta\rangle$

Previous application (QCD)

- In heavy quarkonium: decay constant, elastic form factor, radii, radiative transitions, distribution amplitude, GPDs

—Y Li, G Chen, X Zhao, P Maris, J Vary, L Adhikari, M Li, A El-Hady (2016 - 2018)

Previous application (QED)

- electron anomalous magnetic moments
- wave function, spectroscopy of positronium system
- GPDs of the electron and positronium

—X. Zhao, P. Wiecki, H. Honkanen, D. Chakrabarti, P. Maris, J. P. Vary, S. J. Brodsky (2013 - 2018)

Basis construction

☐ Example: the basis state of proton

- Fock's space expansion

$$|\,N\,\rangle_{\text{proton}} = a|\,qqq\,\rangle + b|\,qqqg\,\rangle + c|\,qqqq\bar{q}\,\rangle + \cdots\,.$$

- For each Fock particle

  ✓ For each quark: n_q, m_q, k_q, $\lambda_q = (\frac{1}{2}, -\frac{1}{2})$
  ✓ For each gluon: n_g, m_g, k_g, $\lambda_g = (1, -1)$

- For the first Fock sector:

$$|\,qqq\,\rangle = |\,n_{q_1}, m_{q_1}, k_{q_1}, \lambda_{q_1}\,\rangle \otimes |\,n_{q_2}, m_{q_2}, k_{q_2}, \lambda_{q_2}\,\rangle \otimes |\,n_{q_3}, m_{q_3}, k_{q_3}, \lambda_{q_3}\,\rangle.$$

☐ Truncation of the basis

- Fock sector truncation

- For each Fock sector:

  ✓ "K_{max}" truncation in the longitudinal direction: $\sum_i k_i = K_{max}$
  ✓ "N_{max}" in the transverse direction: $\sum_i (2n_i + |m_i| + 1) = N_{max}$

Basis construction: quantum numbers

- Longitudinal direction: plane-wave basis
  ✓ discrete longitudinal momentum (labeled by k): $p^+ = \frac{2\pi}{L} k$
- Transverse: ✓ 2D harmonic oscillator basis (labeled by $n,\ m$)

$$\phi^b_{n,m}(p_\perp) = \frac{1}{b\sqrt{\pi}}\sqrt{\frac{n!}{(n+|m|)!}}\, e^{-\frac{p^2}{2b^2}}\, e^{-im\phi}\left(\frac{p}{b}\right)^{|m|} L_n^{|m|}\left(\frac{p^2}{b^2}\right) \begin{cases} b \equiv \sqrt{M\Omega} \\ p = \sqrt{p_1^2 + p_2^2} \end{cases}$$

- For the leading Fock sector:

$$|\, qqq\,\rangle = |\, n_{q_1}, m_{q_1}, k_{q_1}, \lambda_{q_1}\,\rangle \otimes |\, n_{q_2}, m_{q_2}, k_{q_2}, \lambda_{q_2}\,\rangle \otimes |\, n_{q_3}, m_{q_3}, k_{q_3}, \lambda_{q_3}\,\rangle.$$

☐ Truncation of the basis

- Fock sector truncation
- For each Fock sector:
  ✓ "K_{max}" truncation in the longitudinal direction: $\sum_i k_i = K_{max}$
  ✓ "N_{max}" in the transverse direction: $\sum_i (2n_i + |m_i| + 1) = N_{max}$

Effective Hamiltonian

$$H_{eff} = \underbrace{\sum_a \frac{k_{a\perp}^2 + m_a^2}{x_a}}_{\text{LF Kinetic energy}} + \underbrace{\frac{1}{2}\sum_{a,b} V_{ab}^{(CON)}}_{\text{Confinement}} + \underbrace{\frac{1}{2}\sum_{a,b} V_{ab}^{(OGE)}}_{\text{One gluon exchange}}$$

- Light-Front kinetic energy
- Confinement in transverse direction $\Rightarrow$
  $V_{ab}^{(SW)} = \kappa_T^4 x_a x_b (r_{a\perp} - r_{b\perp})^2$ inspired by Light-Front holography

—Brodsky, Teramond (2006)

- Longitudinal confinement $\Rightarrow V_{ab}^{(L)} = \frac{\kappa_L^4}{(m_a+m_b)^2}\partial_{x_a}(x_a x_b \partial_{x_b})$
  ✓ reduce to harmonic oscillator potential at non-relativistic limit

—Y Li, X Zhao, P Maris, J Vary (2016)

- $V_{ab}^{(OGE)} = f\frac{4\pi\alpha_s(Q_{ab}^2)}{Q_{ab}^2}\bar{u}_{s'_a}(k'_a)\gamma^\mu u_{s_a}(k_a)\bar{u}_{s'_b}(k'_b)\gamma^\nu u_{s_b}(k_b)d_{\mu\nu}$
  ✓ introduce short distance physics with spin structure
  ✓ provides the P-wave WFs, essential to generate the Pauli-FF

Wave-function production

- Calculate the Hamiltonian matrix elements:

$$H_{eff}^{\alpha'\alpha} = \langle\alpha'|H|\alpha\rangle$$

$\alpha'|$ $\&$ $|\alpha\rangle$ are the basis state of BLFQ, such as $|qqq\rangle$.

- Diagonalize H_{eff} and obtain its eigen spectrum

$$H_{eff}|\beta\rangle = H_{eff}^{\beta}|\beta\rangle$$

✓ $|\beta\rangle$ is the physical state and eigenstate of Hamiltonian.
In case of proton $|\beta\rangle = |P_{proton}\rangle$.

- Evaluate observables:

$$\mathcal{O} = \langle\beta|\hat{\mathcal{O}}|\beta\rangle$$

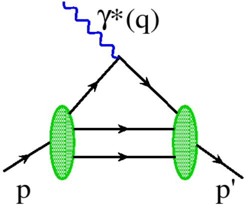
Form factor in BLFQ

work in progress

- EM form factors in light-front (with $q^+ = 0$),

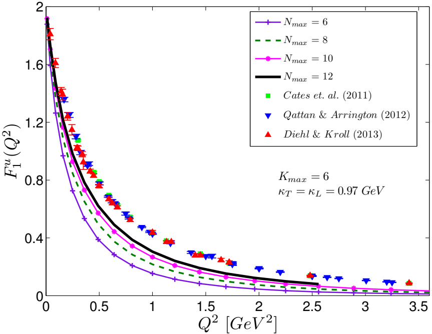
$$\checkmark\ \langle P+q;\Lambda|\frac{J^+(0)}{2P^+}|P;\Lambda\rangle = F_1(q^2)$$

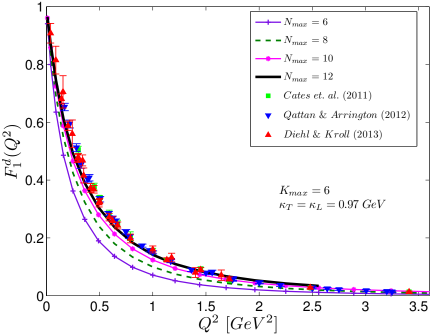
$$\langle P+q;\Lambda|\frac{J^+(0)}{2P^+}|P;-\Lambda\rangle = -(q^1 - iq^2)\frac{F_2(q^2)}{2M}$$



very preliminary results

$(Q^2 = -q^2)$





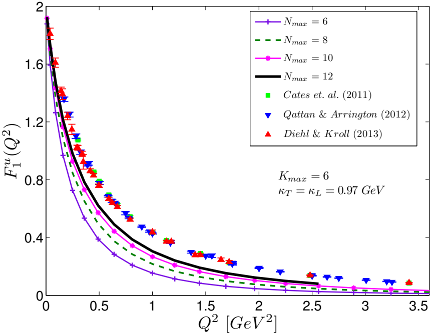
Form factor in BLFQ

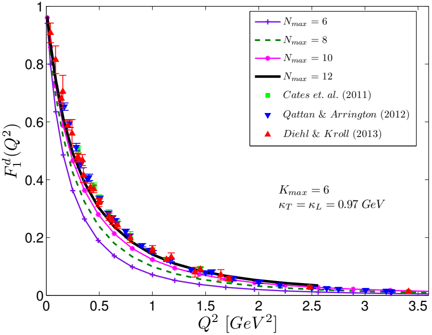
work in progress

- In terms of overlap of light-front WFs,

$$F_1(q^2) = \sum_{\lambda_i} \int \prod_{i=1}^{3} dx_i \; d^2\mathbf{k}_{\perp i} \delta(1 - \sum_{j=1}^{3} x_j) \delta^2(\sum_{j=1}^{3} \mathbf{k}_{\perp j}) \Psi^{\Lambda *}_{\lambda_i}(x_i, \mathbf{k}'_{\perp i}) \Psi^{\Lambda}_{\lambda_i}(x_i, \mathbf{k}_{\perp i})$$

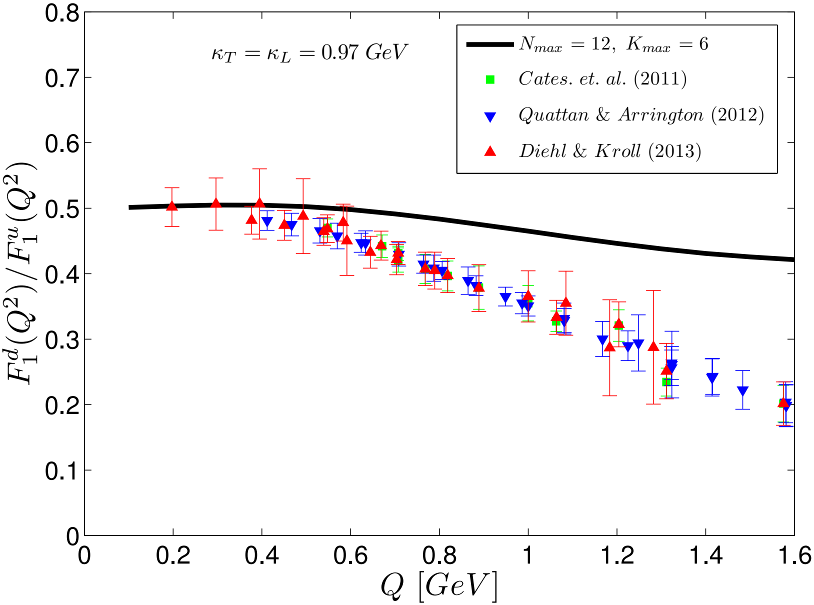
very preliminary results $(Q^2 = -q^2)$





Ratio of Dirac FFs for flavor

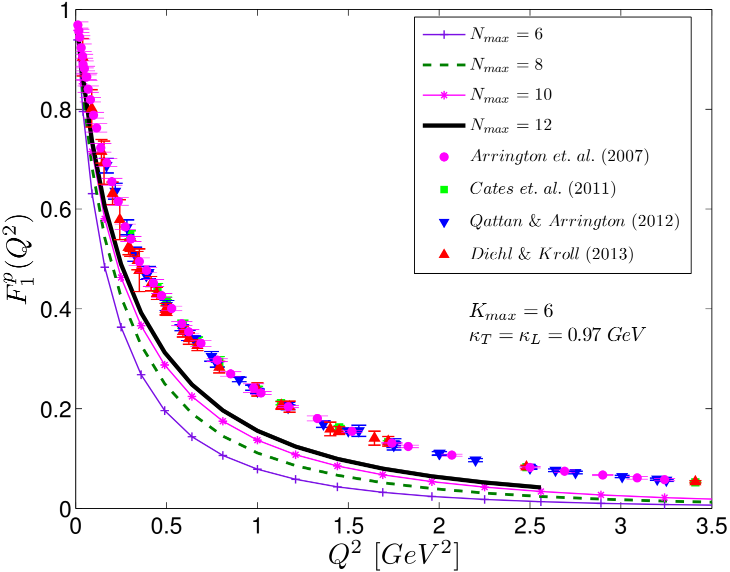
work in progress



Dirac form factor for proton in BLFQ

work in progress

Flavor decomposition: $F_1^p = e_u F_1^u + e_d F_1^d$ — Cates $et.\ al.$ PRL 106 (2011)



GPDs in BLFQ

work in progress

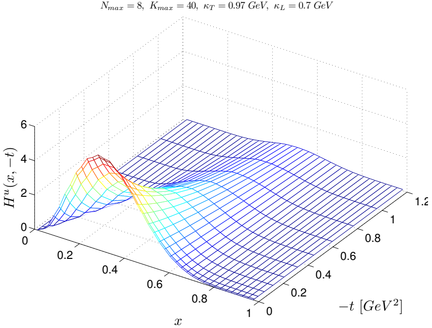
☐ Dirac form factor in light-front [with $q^+ = 0$],

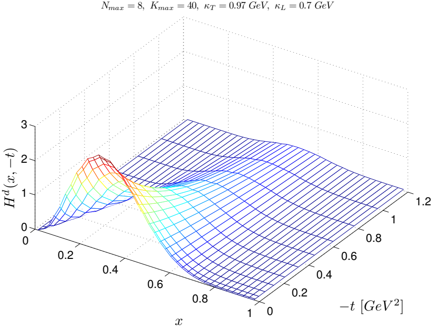
$$F_1(-q^2) = \langle P+q;\Lambda|\frac{J^+(0)}{2P^+}|P;\Lambda\rangle; \qquad F_1^q(-q^2) = \int dx\, H^q(x,-q^2).$$

✓ In terms of overlap of light-front WFs:, $H^q(x,-q^2)$:

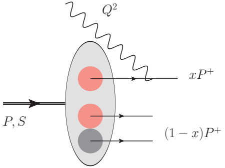
$$\sum_{\lambda_i}\int \prod_{i=1}^{3} dx_i\, d^2\mathbf{k}_{\perp i}\delta(1-\sum x_j)\delta(x-x_1)\delta^2(\sum \mathbf{k}_{\perp j})\Psi^{\Lambda *}_{\lambda_i}(x_i,\mathbf{k}'_{\perp i})\Psi^{\Lambda}_{\lambda_i}(x_i,\mathbf{k}_{\perp i})$$

with $\mathbf{k}'_{\perp 1} = \mathbf{k}_{\perp 1} - (1-x_1)\mathbf{q}_\perp$ & $\mathbf{k}'_{\perp a} = \mathbf{k}_{\perp a} - x_a\mathbf{q}_\perp \quad (a \neq 1)$





Light-front quark-diquark Model



In the quark-diquark picture, nucleon ($p = |uud\rangle$, $n = |udd\rangle$) is considered to be a bound state of *a single quark* and *a scalar diquark state*.

The two particle Fock-state expansion for $J^z = \pm 1/2$

$$|P;\pm\rangle = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_\lambda \psi^\pm_{\lambda q}(x,\mathbf{p}_\perp)|\lambda\ \Lambda_S; xP^+, \mathbf{p}_\perp\rangle\Big|_{\Lambda_S=0}$$

Light front wave functions are constructed from the *AdS/QCD soft wall model WFs*.

The light-front wave functions:

$$\psi^\pm_{\lambda q}(x,\mathbf{p}_\perp) = N_q f(x,\mathbf{p}_\perp,\lambda,\Lambda_s=0)\ \varphi^i_q(x,\mathbf{p}_\perp)\Big|_{i=1,2}$$

Normalized by quark counting rules.

Light-front wavefunctions

The light-front wave functions:

$$\psi^{\pm}_{\lambda q}(x,\mathbf{p}_\perp) = \quad N_q f(x,\mathbf{p}_\perp,\lambda,\Lambda_s=0)\ \varphi^i_q(x,\mathbf{p}_\perp)\Big|_{i=1,2}$$

$$\begin{aligned}
\psi^{+}_{+q}(x,\mathbf{p}_\perp) &= \varphi^{(1)}_q(x,\mathbf{p}_\perp), \\
\psi^{+}_{-q}(x,\mathbf{p}_\perp) &= -\frac{p^1+ip^2}{xM}\varphi^{(2)}_q(x,\mathbf{p}_\perp), \\
\psi^{-}_{+q}(x,\mathbf{p}_\perp) &= \frac{p^1-ip^2}{xM}\varphi^{(2)}_q(x,\mathbf{p}_\perp), \\
\psi^{-}_{-q}(x,\mathbf{p}_\perp) &= \varphi^{(1)}_q(x,\mathbf{p}_\perp),
\end{aligned}$$

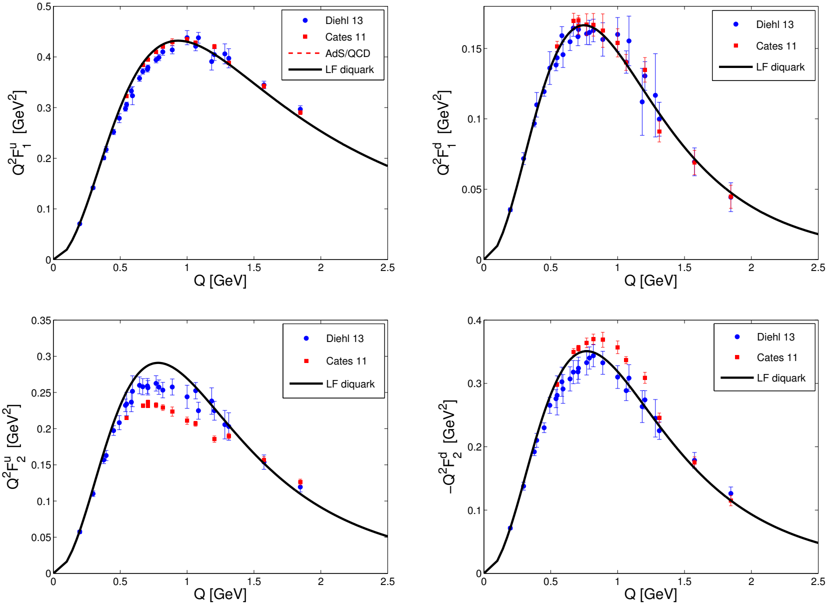
—Gutsche et. al. PRD 89 (2014)

Modified soft-wall AdS/QCD wave function for two particle bound state:

$$\varphi^{(\nu)}_i(x,\mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\,x^{a^\nu_i}(1-x)^{b^\nu_i}\exp\left[-\frac{\mathbf{p}^2_\perp}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

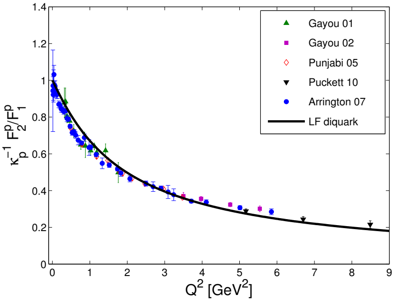
—G. F. de Teramond and S. J. Brodsky, arXiv:1203.4025 [hep-ph].

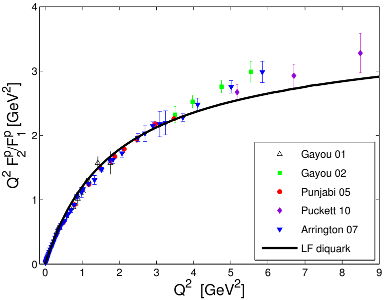
LF quark-diquark model: Flavor form factor

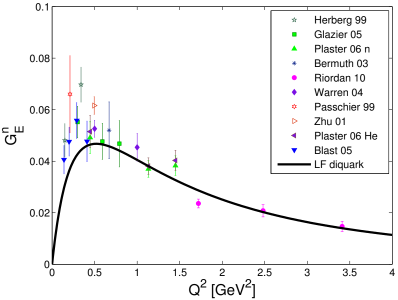


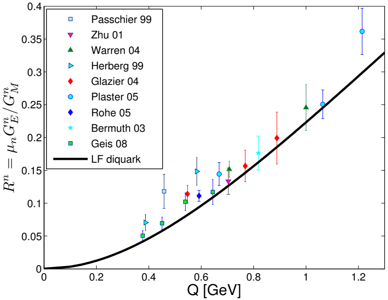
Nucleon form factor

CM, D. Chakrabarti: EPJC 75 (2015)









Electromagnetic radii

$$\langle r_E^2 \rangle^N = -6 \frac{dG_E^N(Q^2)}{dQ^2}\Big|_{Q^2=0},$$

$$\langle r_M^2 \rangle^N = -\frac{6}{G_M^N(0)} \frac{dG_M^N(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

The Sachs form factors are defined as

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4M_N^2} F_2^N(Q^2),$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2).$$

Quantity	quark-diquark	Data from PDG
r_E^p (fm)	0.786	0.877 ± 0.005
r_M^p (fm)	0.772	0.777 ± 0.016
$\langle r_E^2 \rangle^n$ (fm^2)	-0.085	-0.1161 ± 0.0022
r_M^n (fm)	0.760	$0.862^{+0.009}_{-0.008}$

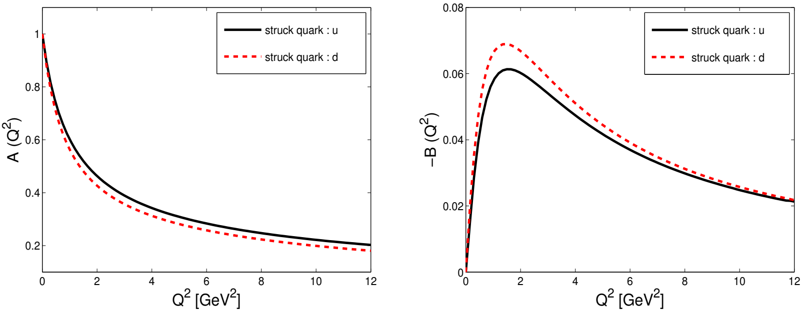
—CM, D. Chakrabarti, EPJC 75 (2015)

Gravitational form factors

Chakrabarti, CM, Mukherjee, PRD 92 (2015)

In the light-front:

$$\langle P+q,\uparrow | \frac{T_i^{++}(0)}{2(P^+)^2} | P,\uparrow\rangle = A_i(q^2),$$
$$\langle P+q,\uparrow | \frac{T_i^{++}(0)}{2(P^+)^2} | P,\downarrow\rangle = -(q^1 - iq^2)\frac{B_i(q^2)}{2M}.$$



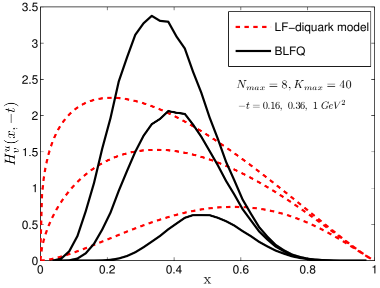
Nucleon GFFs= Quark GFFs + diquark GFFs⇒ $[\mathbf{A(0) = 1 \;\&\; B = 0}]$

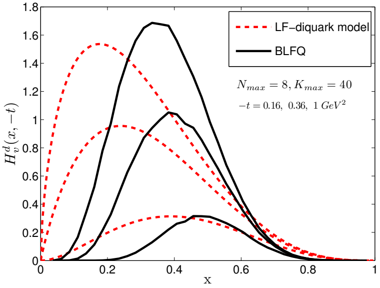
GPDs: BLFQ vs LF quark-diquark

✓ In terms of overlap of light-front WFs:, $H^q(x, -q^2)$:

$$H^q(x,-t) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[\psi^{+*}_{+q}(x,\mathbf{k}'_\perp)\psi^{+}_{+q}(x,\mathbf{k}_\perp) + \psi^{+*}_{-q}(x,\mathbf{k}'_\perp)\psi^{+}_{-q}(x,\mathbf{k}_\perp)\right]$$

where $\mathbf{k}'_\perp = \mathbf{k}_{\perp 1} - (1-x)\mathbf{q}_\perp; \quad t = -\mathbf{q}^2_\perp$





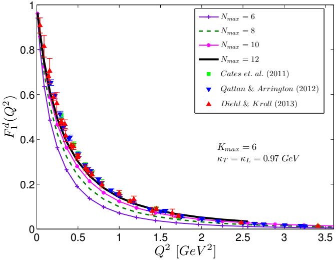
- This is not the complete picture !!
  We need to include the one gluon exchange interaction in the H_{eff}

Conclusions & outlook

- We have discussed the very preliminary results of proton form factors & GPDs in BLFQ approach and LF quark-diquark model.
- In the effective Hamiltonian, we have the kinetic energy & the confining potential in both the transverse and longitudinal direction. Here, we consider only the leading Fock sector.
- As basis size increases, we obtain converging results.

Outlook:

- Increase basis size
- Include the higher Fock component $|qqqg\rangle$.
- Investigate other nucleon properties..
- Investigate the structure of other baryons.



Thank You

Effect of longitudinal confinement