Pseudo Distributions on the Lattice

WILLIAM & MARY
CHARTERED 1693

Jefferson Lab

Joe Karpie
William & Mary / Jefferson Lab

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In Collaboration with

Kostas Orginos (W&M / JLab)
Anatoly Radyushkin (Old Dominion U / JLab)
Savvas Zafeiropoulos (Heidelberg U)

Introduction

- Lattice calculations moving from Hadron bulk properties (masses, charges,...) to Hadron distributions (Form Factors, Structure Functions, Distribution functions, …)
- Project Goals
  - Long Term: Study various methods of calculating parton distributions from ab initio Lattice QCD
  - Short Term: Understand systematic effects in the simple case of iso-vector unpolarized quark PDF
- Mellin moments and OPE
  - Restricted to low moments by reduced rotational symmetry
- Hadronic Tensor Methods A Chambers et.al (2017) 1703.01153
  - "Light-like" separated Hadronic Tensor K-F Liu et al Phys. Rev. Lett. 72 1790 (1994) , Phys. Rev. D62 (2000) 074501
  - Good lattice cross sections Y.-Q. Ma J.-W. Qiu (2014) 1404.6860 Y.-Q. Ma, J.-W. Qiu (2017) 1709.03018
- Ioffe Time Pseudo Distribution Methods J.-W. Chen et.al. (2018) 1803.04393 C Alexandrou et.al. (2018) 1803.02685
  - Quasi PDF X. Ji, Phys.Rev.Lett. 110, (2013)
  - Pseudo PDF A. Radyushkin Phys.Lett. B767 (2017) K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

Ioffe Time distribution

B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

$$\nu = p \cdot \xi$$

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

- $\mathcal{I}(\nu,\mu^2) = \int_{-1}^{1} dx e^{i\nu x} f(x,\mu^2) \quad = \frac{1}{2p^+}\langle h(p)|\bar{\psi}_q(0,\xi^-,0_T)\gamma^+ W((0,\xi^-,0_T);0)\psi_q(0)|h(p)\rangle_{\mu^2}$

- CP Even/Odd combinations
  - Even: $q_-(x) = f(x) + f(-x) = q(x) - \bar{q}(x) \equiv q_V(x)$
  - Odd: $q_+(x) = f(x) - f(-x) = q(x) + \bar{q}(x) = q_V(x) + 2\bar{q}(x)$

$$\mathfrak{Re}\,[\mathcal{I}(\nu)] = \int_0^1 dx \cos(\nu x) q_V(x) \equiv \mathcal{I}_V(\nu)$$

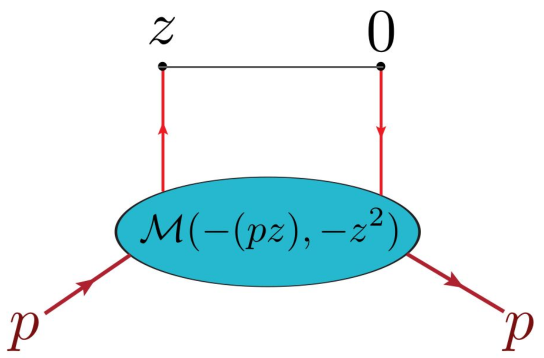
$$\mathfrak{Im}\,[\mathcal{I}(\nu)] = \int_0^1 dx \sin(\nu x)(q(x) + \bar{q}(x))$$

- Perturbative DGLAP evolution

I.I. Balitsky and V.M. Braun, Nucl. Phys. B311, 541 (1988)

$$\mathcal{I}_v(\nu,\mu_2^2) = \mathcal{I}_v(\nu,\mu_1^2) - \frac{C_F\alpha_s}{2\pi}\log\frac{\mu_2^2}{\mu_1^2}\int_0^1 du\left[\frac{1}{2}\delta(1-u) - (1-u) - 2[\frac{u}{1-u}]_+\right]\mathcal{I}_v(u\nu,\mu_1^2)$$

Ioffe Time Pseudo Distributions



- A general matrix element of interest

$$M^{\alpha}(z,p) = \langle h(p)|\bar{\psi}_q(z)\gamma^{\alpha}W(z;0)\psi_q(0)|h(p)\rangle$$

- Lorentz decomposition
  - Use of symmetry
  - Choice of p, z, and α can remove higher twist term

$$M^{\alpha}(z,p) = 2p^{\alpha}\mathcal{M}_p(\nu, z^2) + z^{\alpha}\mathcal{M}_z(\nu, z^2)$$

- Factorization Relation to ITDF
  - Perturbatively calculable Wilson coefficients for each parton

$$\mathcal{M}(\nu, -z^2) = \sum_i C_i(z^2\mu^2, \alpha_s) \otimes \mathcal{I}_i(\nu, \mu^2) + H.T.$$

A. Radyushkin (2017) 1710.08813
J.-H. Zhang (2018) 1801.03023
T. Izubuchi (2018) 1801.03917

$$M^{\alpha}(z,p) = \langle h(p)|\bar{\psi}_q(z)\gamma^{\alpha}W(z;0)\psi_q(0)|h(p)\rangle$$

$$M^{\alpha}(z,p) = 2p^{\alpha}\mathcal{M}_p(\nu, z^2) + z^{\alpha}\mathcal{M}_z(\nu, z^2)$$

Special Cases

- Light cone PDF

$$p = (p^+, \frac{m^2}{2p^+}, 0_T) \qquad z = (0, z^-, 0_T) \qquad \alpha = +$$

$$\mathcal{M}_p((p^+z^-), 0) = \int_{-1}^{1} dx e^{ix(p^+z^-)} f(x)$$

A. Radyushkin (2017) 1612.05170

- Straight Link "Primordial" TMD

$$p = (p^+, \frac{m^2}{2p^+}, 0_T) \qquad z = (0, z^-, z_T) \qquad \alpha = +$$

$$\mathcal{M}_p((p^+z^-), -z_T^2) = \int_{-1}^{1} dx e^{ix(p^+z^-)} \int d^2k_T e^{ik_T\cdot z_T} F(x, k_T^2)$$

- Pseudo PDF

$$p = (E, 0, 0, p_3) \qquad z = (0, 0, 0, z_3) \qquad \alpha = 0$$

$$\mathcal{M}_p((-z_3 * p_3), -z_3^2) = \int_{-1}^{1} dx e^{ix(-z_3 * p_3)} P(x, -z_3^2)$$

Pseudo PDF vs Quasi PDF

$0.2 GeV \approx 1 fm^{-1}$

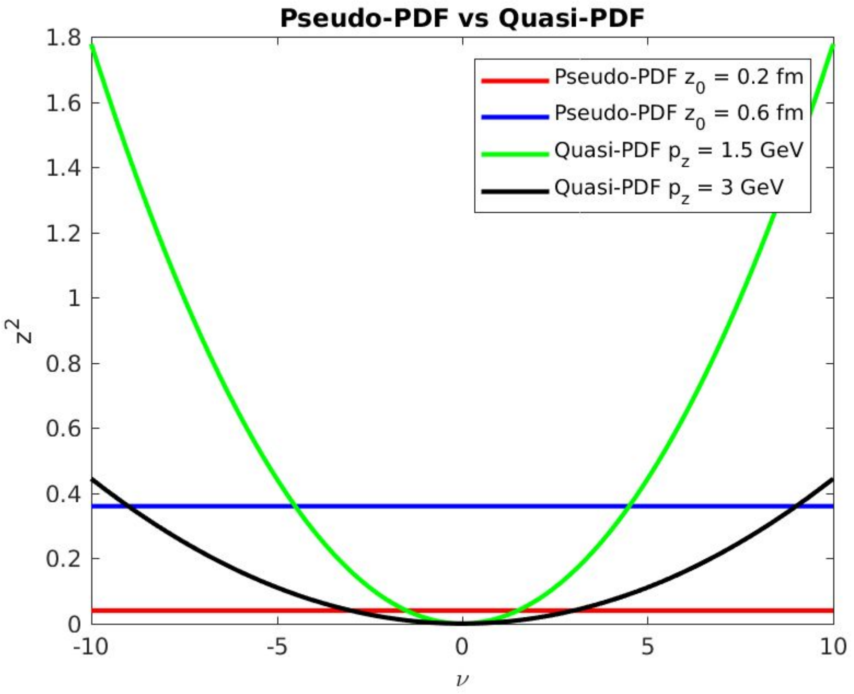
- Both are integrals of pseudo ITDF
  - Pseudo PDF has fixed invariant scale dependence

$$P(x, z_0^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, z_0^2)$$

  - Quasi PDF mixes invariant scales until p_z is effectively large enough

$$Q(x, p_z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, \frac{\nu^2}{p_z^2})$$

- Expedite desired limit of $z^2 \longrightarrow 0$
  - Pseudo-PDFs use reduced distributions
  - Quasi-PDFs use LaMET



Numerical Lattice Field Theory

- Importance sampling of path integral

$$\langle O(\bar{\psi}, \psi, A_\mu)\rangle = \frac{1}{Z}\int D[\bar{\psi}]D[\psi]D[A_\mu]O(\bar{\psi}, \psi, A_\mu)e^{-S(\bar{\psi},\psi,A_\mu)}$$

$$\approx \frac{1}{N}\sum_i^N F_O(U_\mu^{(i)})$$

- Correlation functions

$$C_2(\vec{p}, T) = \langle O_N(-\vec{p}, T)\bar{O}_N(\vec{p}, 0)\rangle$$

$$C_{op}(O_{op}; \vec{p}, T) = \sum_t \sum_{\vec{x}} \langle O_N(-\vec{p}, T) O_{op}(\vec{x}, t)\bar{O}_N(\vec{p}, 0)\rangle$$

- Feynman-Hellman matrix element extraction C. Bouchard et.al Phys. Rev. D 96, no. 1, 014504 (2017)

$$\frac{\langle N(p)|O_{op}|N(p)\rangle}{2E_{N(p)}} = \lim_{T\to\infty}\frac{1}{\tau}(R(T+\tau) - R(T)) \qquad R(T) = \frac{C_{op}(O_{op}; \vec{p}, T)}{C_2(\vec{p}, T)}$$

Renormalization and the Reduced distribution

- Effective Bare matrix element

$$M^{eff}(T) = (R(T+1) - R(T)) + O(e^{-\Delta T})$$

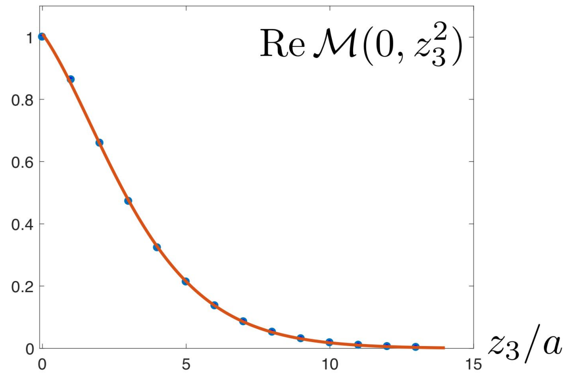
- Vector current to cancel quark field renormalization
  - Forces matrix elements to give unit nucleon charge

$$Z_p^{-1} = M^4(0,p)$$

- Reduced distribution
  - TMD "Factorization" and suppression of polynomial corrections

$$F(x,k_T^2) = f(x)g(k_T^2) \quad \mathcal{M}(\nu,z^2) = \mathcal{M}(\nu,0)\mathcal{M}(0,z^2)$$

  - BONUS: UV corrections from Wilson line exactly cancels. No complicated renormalizations.



$$\mathfrak{M}(\nu,z^2) = \frac{\mathcal{M}(\nu,z^2)}{\mathcal{M}(0,z^2)}$$

Numerical Study

$$O_q^\alpha(z;T) = \sum_{\vec{x}} \bar{\psi}_q(\vec{x}+\vec{z},T)\lambda^3\gamma^\alpha W((\vec{x}+\vec{z},T);(\vec{x},T))\psi_q(\vec{x},T)$$

Quenched

K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

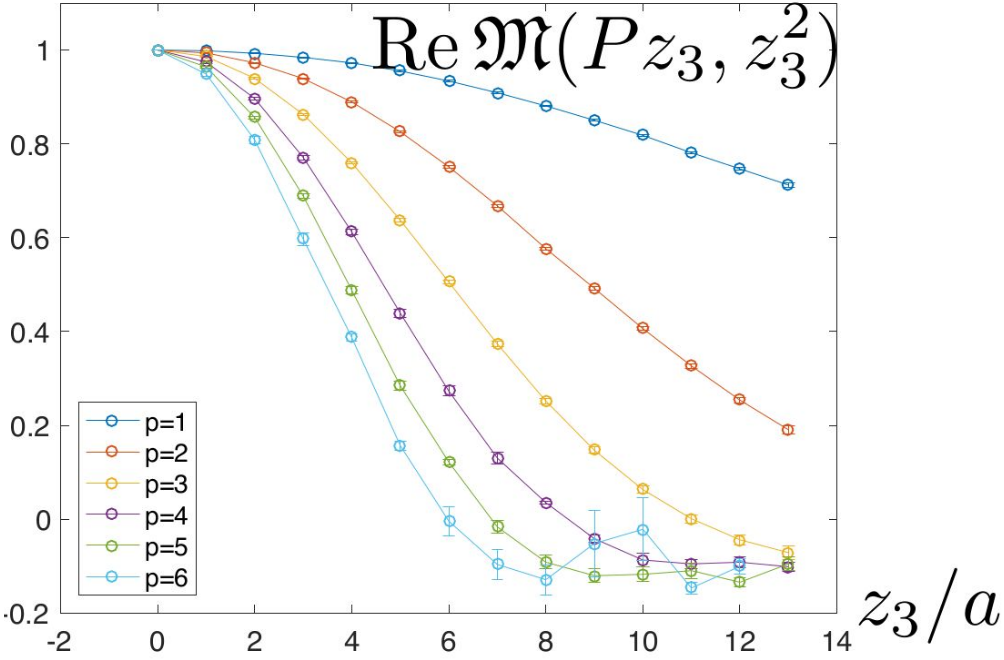
- $\beta = 6.0$ $m_\pi = 600$ MeV $32^3 \times 64$ $a = 0.1$ fm

Dynamical (Preliminary)

Unpublished

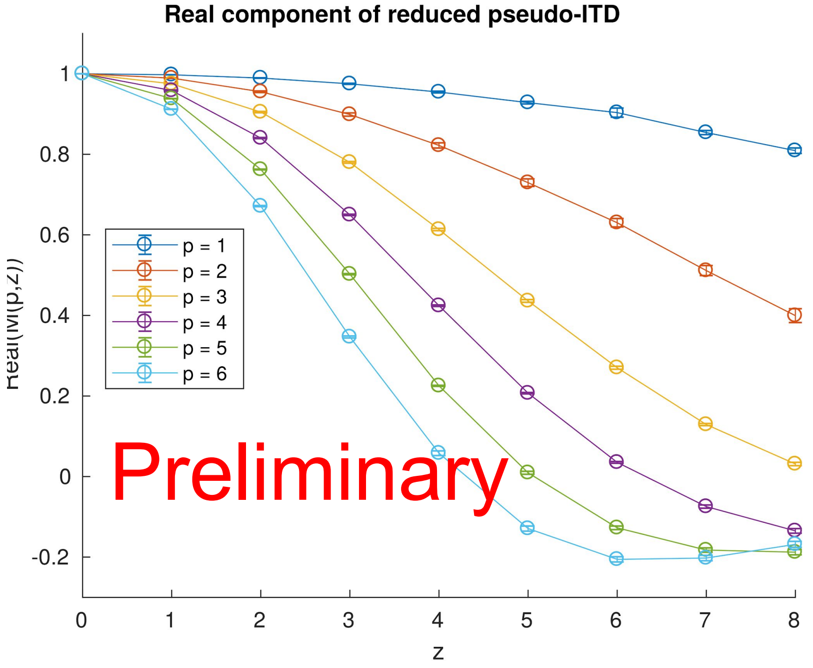
- $a127m440:$ $\beta = 6.1$ $m_\pi = 440$ MeV $24^3 \times 64$ $a = 0.127$ fm
- $a127m440L:$ $\beta = 6.1$ $m_\pi = 440$ MeV $32^3 \times 96$ $a = 0.127$ fm
- $a094m400:$ $\beta = 6.3$ $m_\pi = 400$ MeV $32^3 \times 64$ $a = 0.094$ fm

Quenched Results



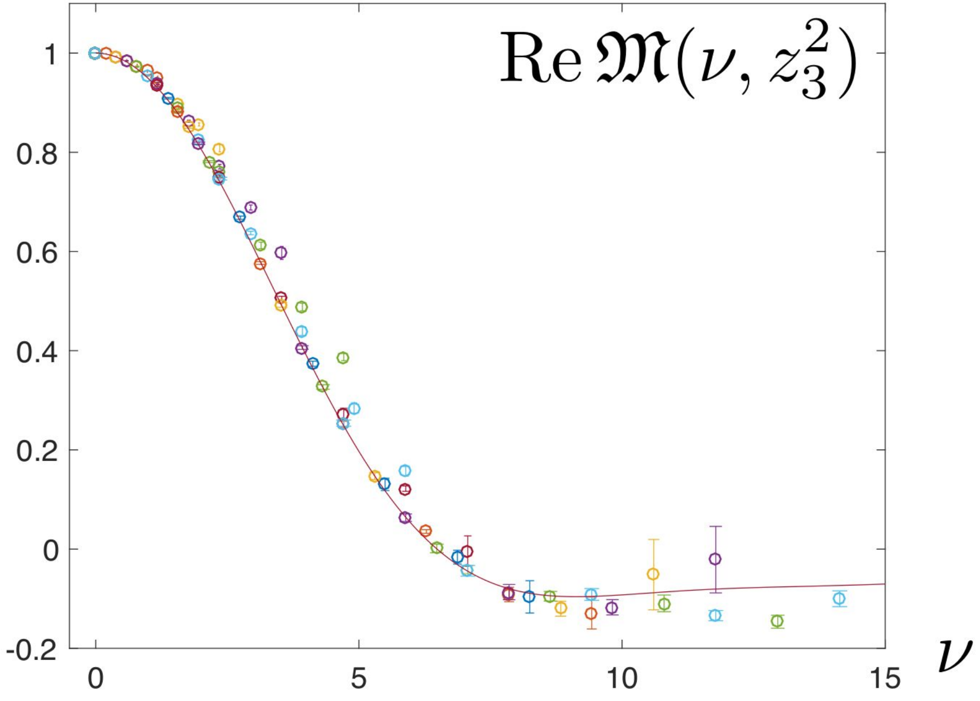
Dynamical Results

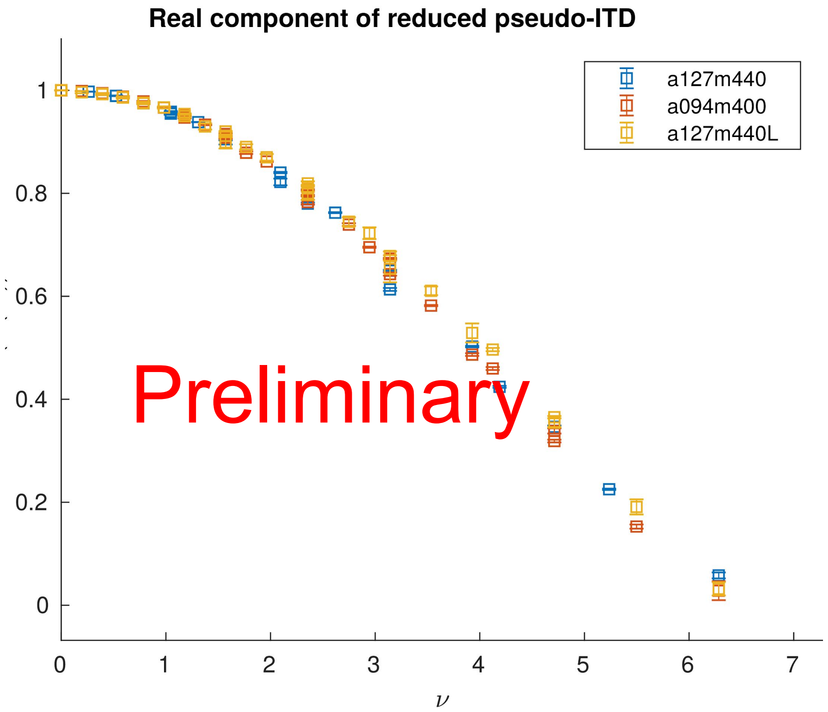
$a127m440$



Quenched Results

Dynamical Results





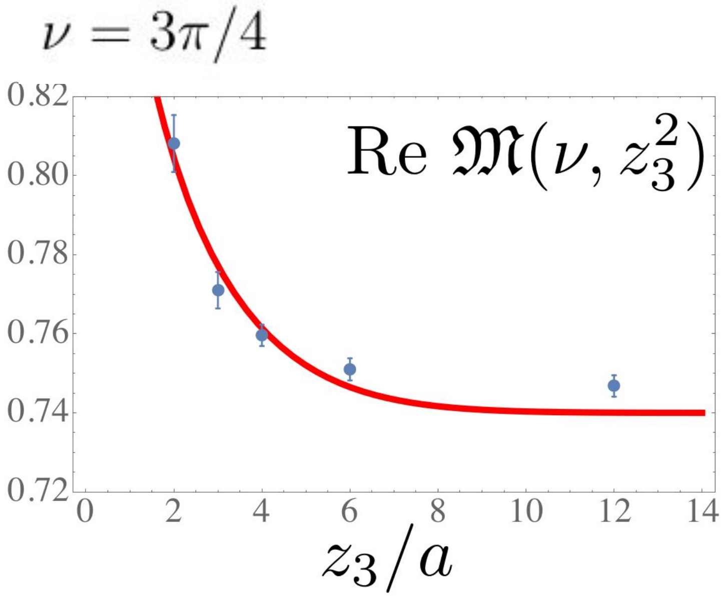
Perturbative Evolution of Lattice data

$$B \otimes \mathfrak{M}(\nu, z^2) = \int_0^1 du B(u) M(u\nu, z^2)$$

$$\mathfrak{M}(\nu, z_0^2) = \mathfrak{M}(\nu, z^2) + \frac{C_F \alpha_s}{2\pi} \log(\frac{z^2}{z_0^2}) B \otimes \mathfrak{M}(\nu, z^2)$$

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

- Position space DGLAP evolution
- Improvement of Almost Universal curve
- Separation of Regimes
  - Small separation matrix elements follow log behavior expected from perturbation theory
  - Large separation matrix elements seem z_3 independent expected from cancellation of polynomial effects



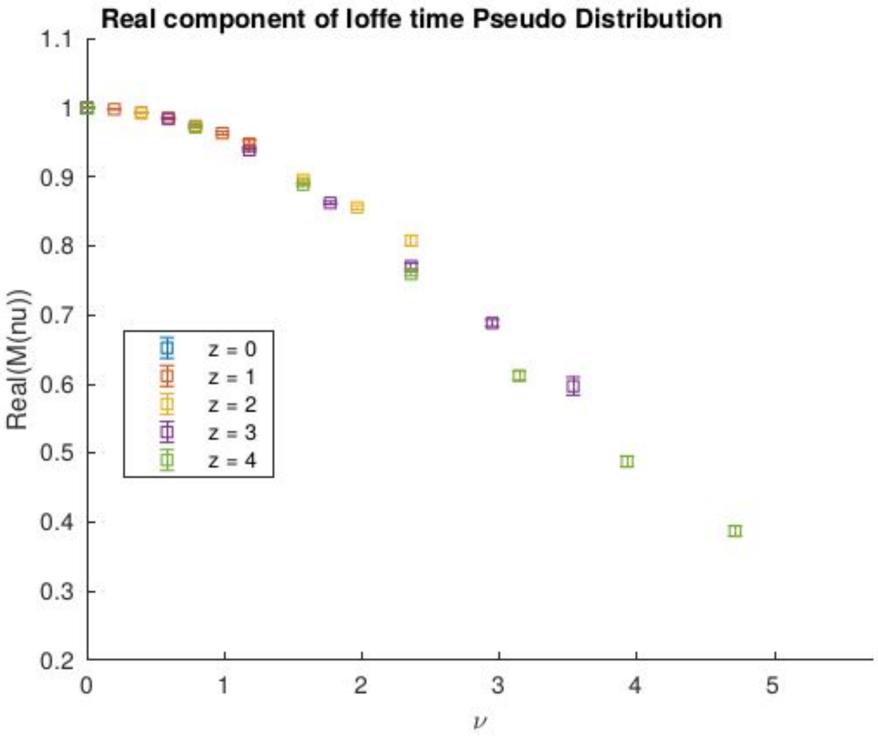
Matching Lattice data to Ioffe Time distribution

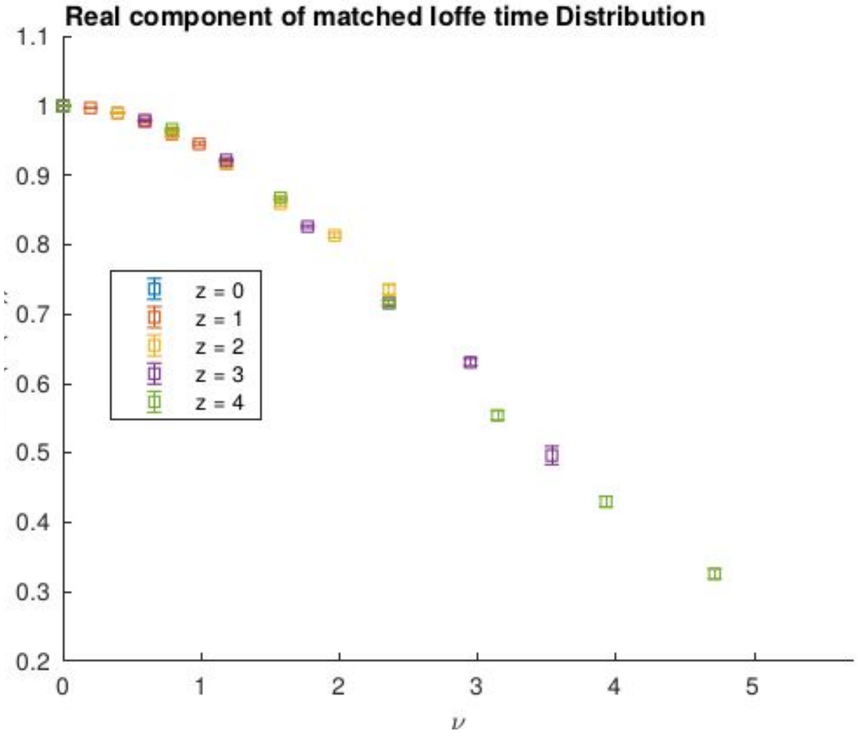
- Without proper matching, the results are only comparable to global fits up to α_s corrections
- Yet another convolution of reduced pseudo-ITD
- At 1-loop, scale evolution and matching can be simultaneous
- Results are matched to μ = 1 GeV MS bar ITD

$$\mathcal{I}(\nu,\mu^2) = \mathfrak{M}(\nu,z^2) + \frac{C_F\alpha_S}{2\pi}\int_0^1 du\Bigg(B(u)\left(\log(z^2\mu^2\frac{e^{2\gamma_E}}{4})+1\right) + \left[4\frac{\log(1-u)}{1-u} - 2(1-u)\right]_+\Bigg)M(u*\nu,z^2)$$

$$= \mathfrak{M}(\nu,z^2) + \frac{C_F\alpha_S}{2\pi}\left[(\log(z^2\mu^2\frac{e^{2\gamma_E}}{4})+1)B\otimes M(\nu,z^2) + L\otimes M(\nu,z^2)\right]$$

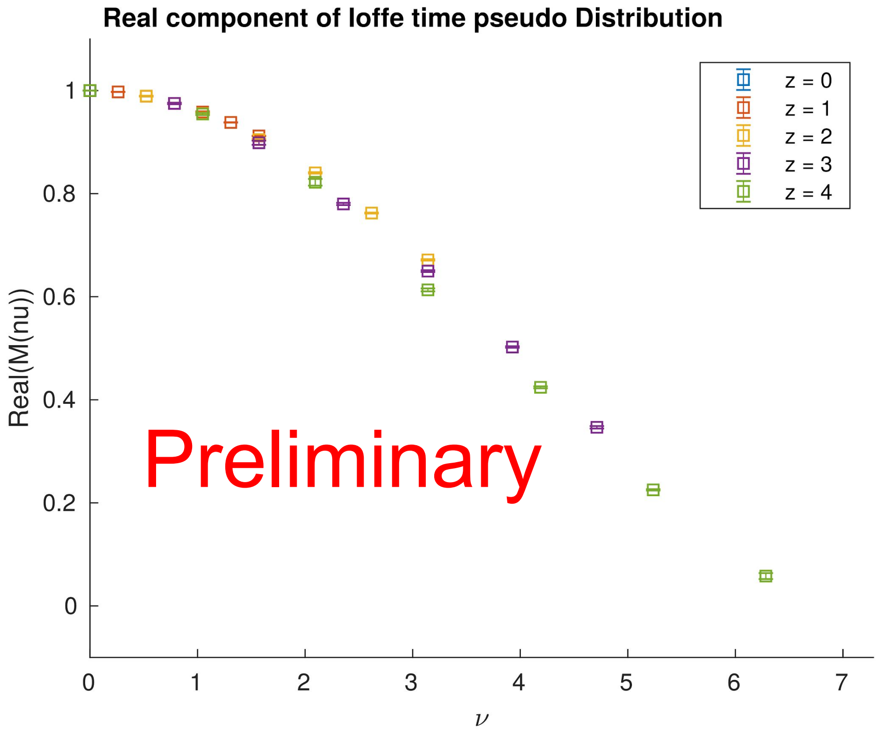
Perturbative Matching of Quenched Lattice data

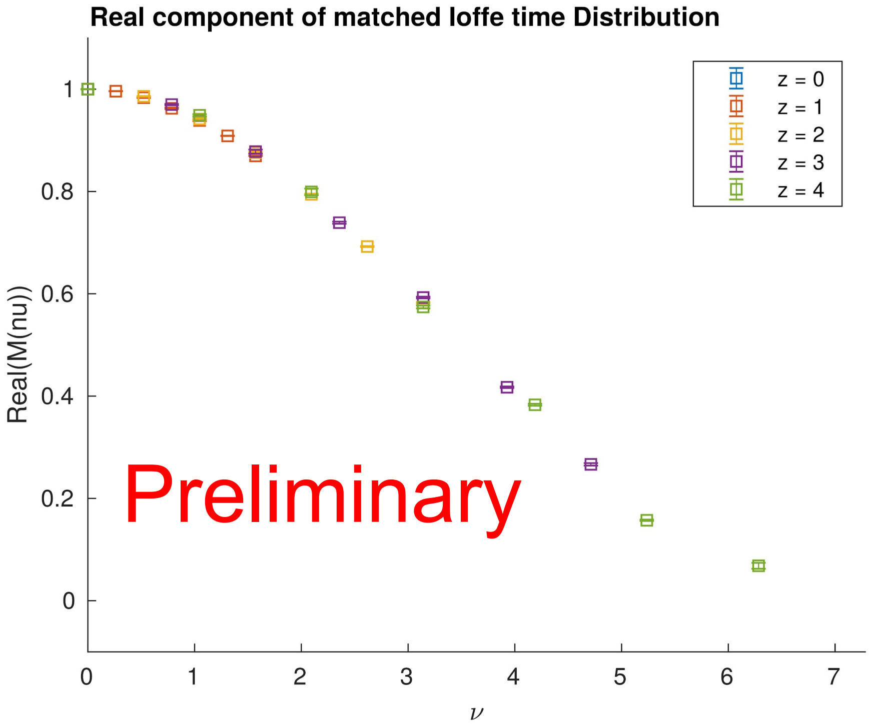




Perturbative Matching of Dynamical Lattice data

$a127m440$





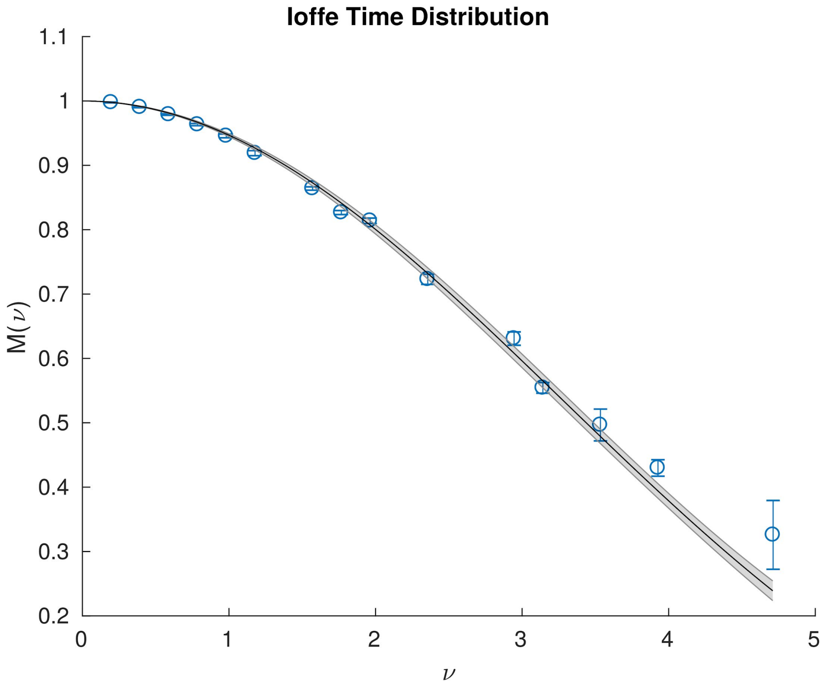
Real component and the Valence Quark distribution

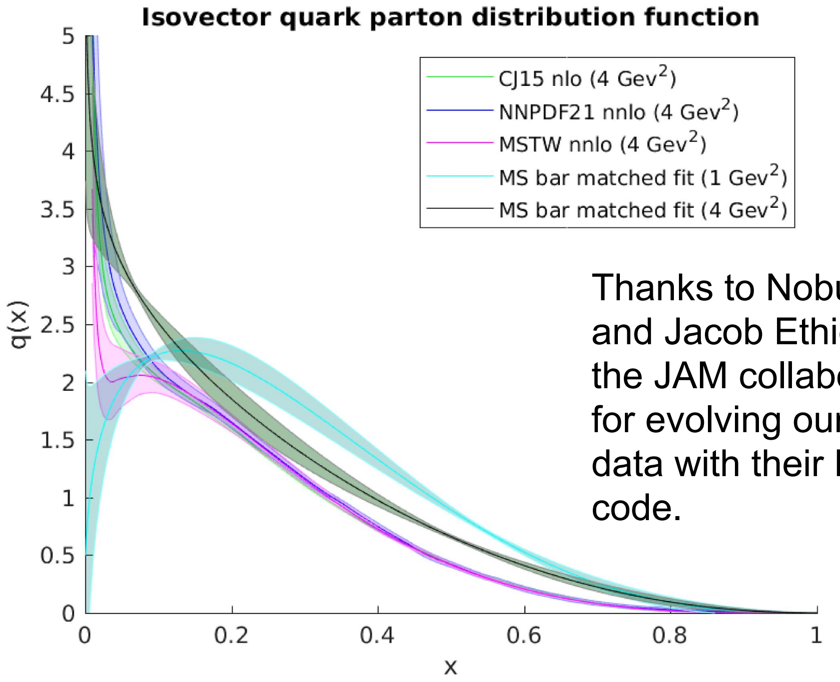
- A more general model PDF

$$f_{abcd}(x) = N_{abcd}\, x^a (1-x)^b (1 + c\sqrt{x} + dx)$$

- Limiting behaviors
  - Regge $a = -0.5$
  - Quark counting $b = 3$
  - Small Corrections $c \sim 0 \quad d \sim 0$

Quenched Pseudo PDF Matched to MS bar Compared to Global fit PDFs

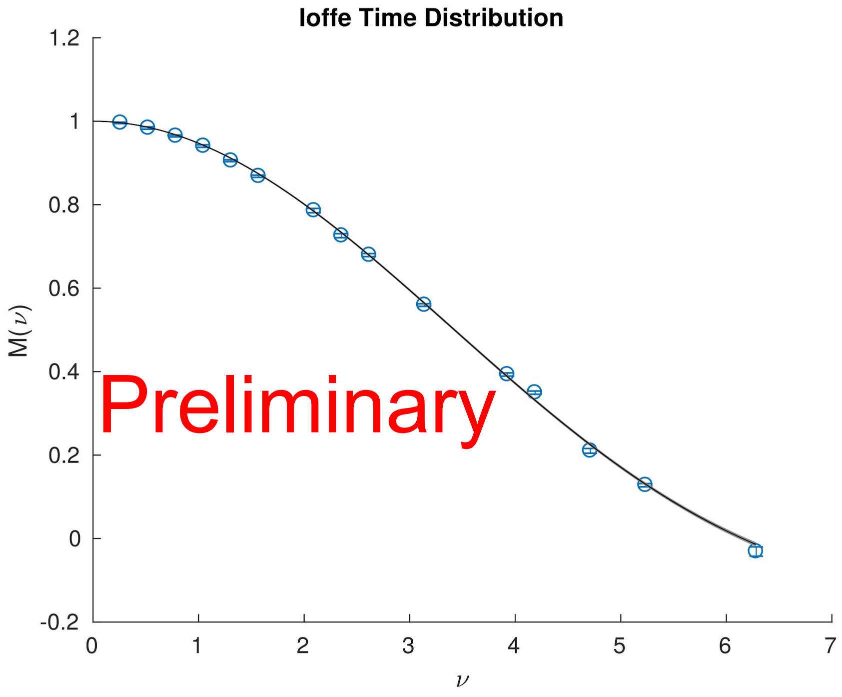


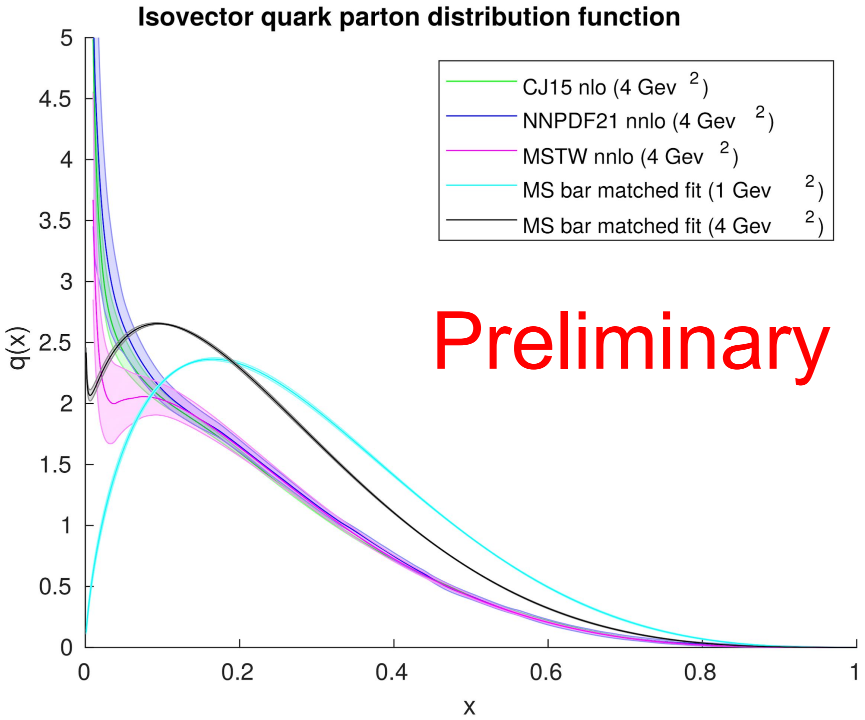


Thanks to Nobuo Sato and Jacob Ethier of the JAM collaboration for evolving our lattice data with their NLO code.

Dynamical Pseudo PDF Matched to MS bar Compared to Global fit PDFs

$a127m440$





Summary

- First study of pseudo ITDF analyzed as reduced pseudo PDFs
- Quenched and Dynamical Results are in agreement with PDF fits at large x
- Treatment of z^2 dependence guided by data
- Application of proper matching to perturbative scheme
- Missing divergent behavior improves after scale evolution to 4 GeV^2
- Systematics left to thoroughly study
  - Continuum limit
  - Control of Excited states
  - Finite Volume
  - Physical Pion mass limit

Thank you for listening