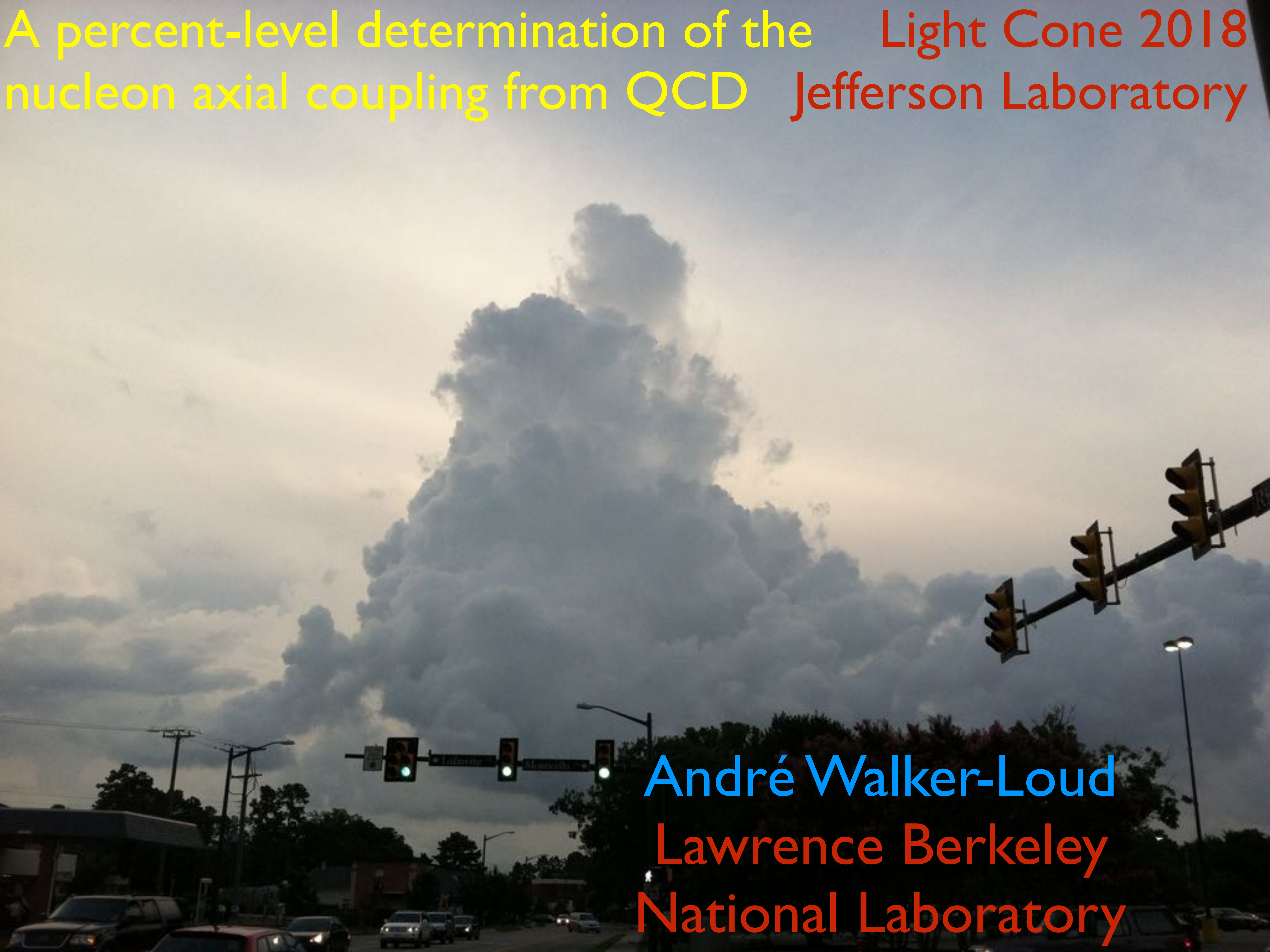


A percent-level determination of the nucleon axial coupling from QCD

Light Cone 2018
Jefferson Laboratory

André Walker-Loud
Lawrence Berkeley National Laboratory

A percent-level determination of the nucleon axial coupling from QCD

Light Cone 2018
Jefferson Laboratory

André Walker-Loud

A percent-level determination of the nucleon axial coupling from QCD (accepted for publication)

Lattice QCD Team

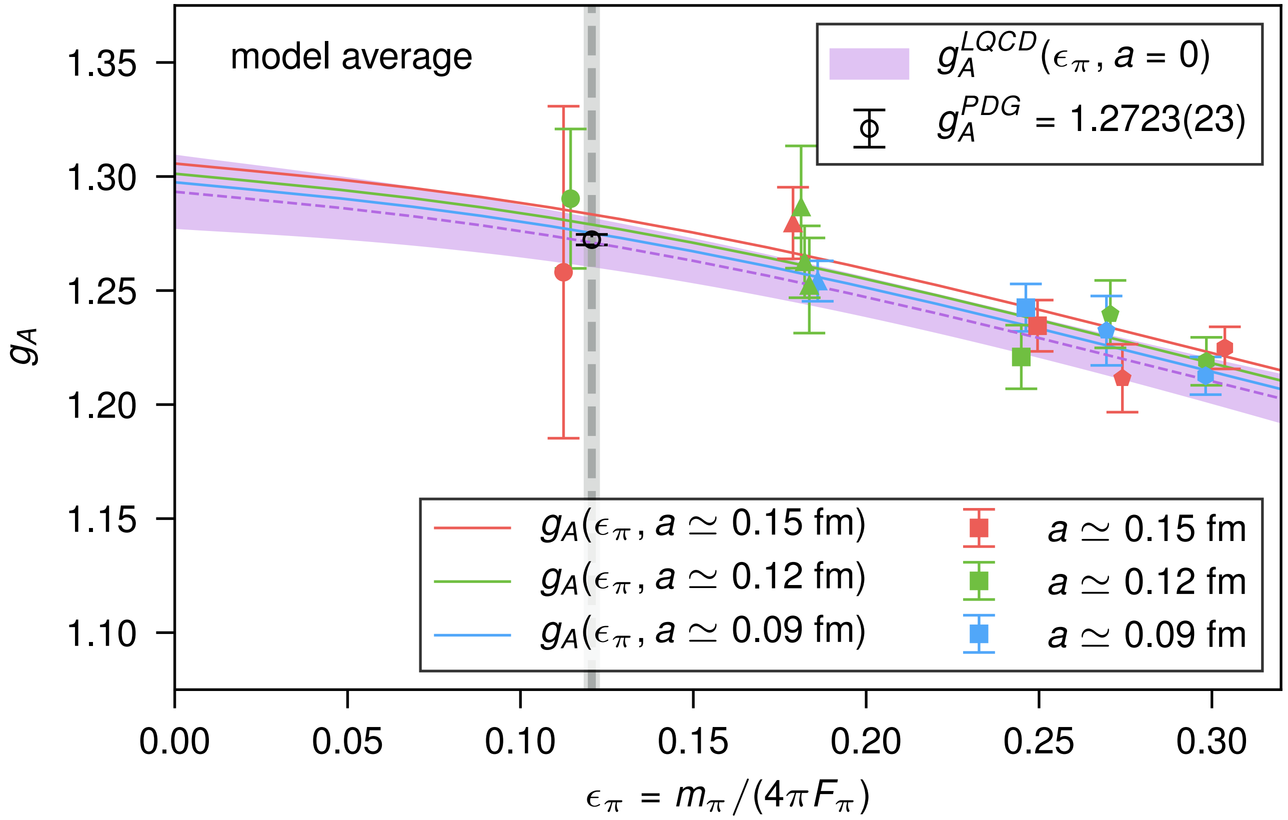
Glasgow:	Chris Bouchard
INT:	Chris Monahan
JLab:	Balint Jóo
Jülich:	Evan Berkowitz
LBL/UCB:	David Brantley, Chia Cheng (Jason) Chang, T. Kurth (NERSC), Henry Monge-Camacho, AWL
LLNL:	Pavlos Vranas
Liverpool:	Nicolas Garron
NVIDIA:	Kate Clark
RIKEN/BNL:	Enrico Rinaldi
UNC:	Amy Nicholson
William and Mary:	Kostas Orginos

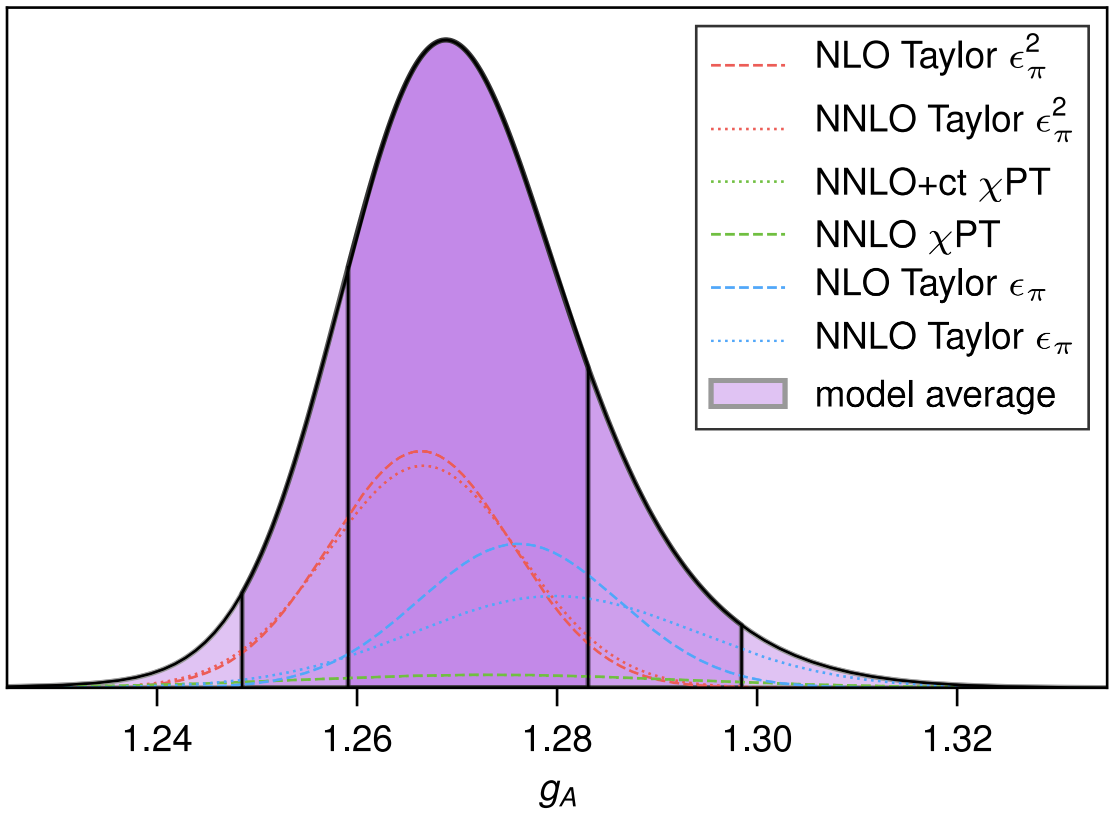
plus a few friends

DOE Topical Collaboration
Double Beta Decay

$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$
$$= 1.2711(126)$$
$$g_A^{\rm UCNA} = 1.2772(020)$$

experiment factor of 6 more precise





A percent-level determination of the nucleon axial coupling from QCD (accepted for publication)

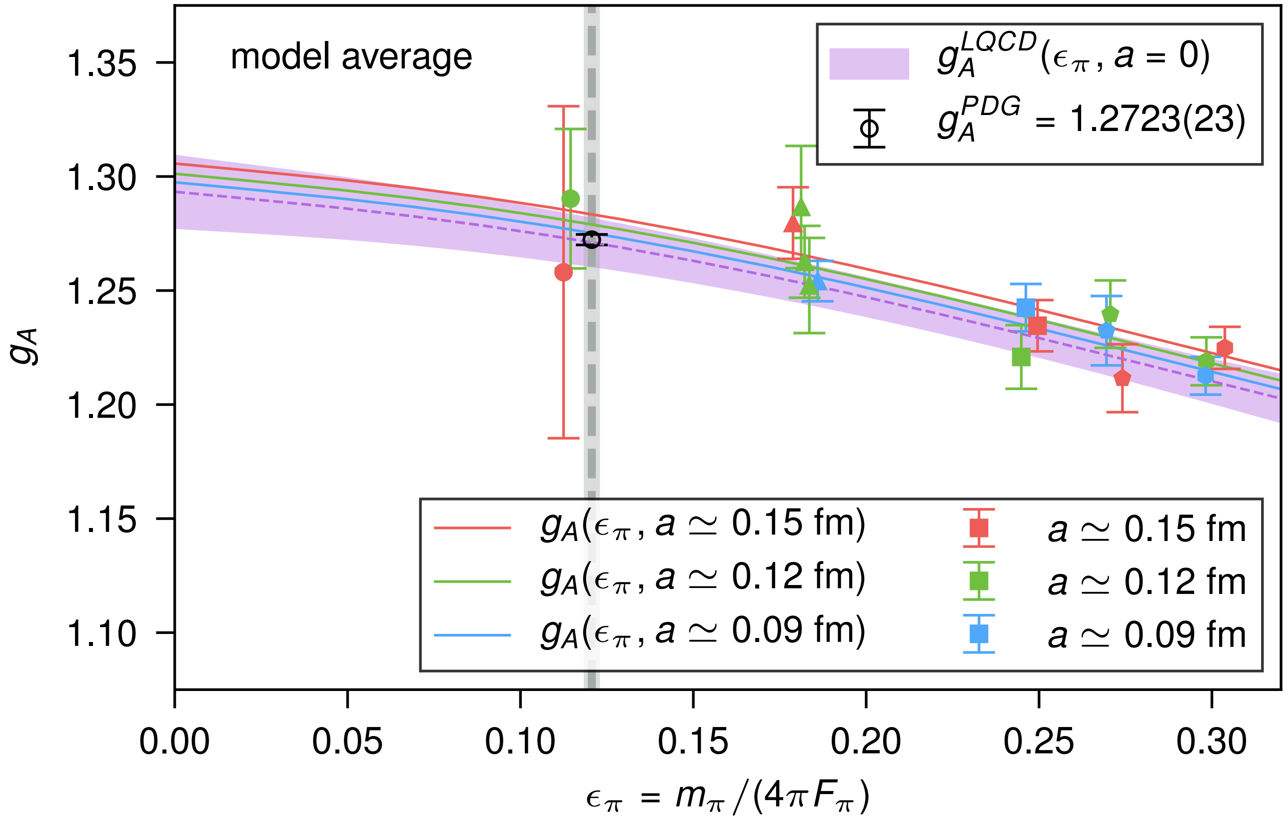
Lattice QCD Team
Glasgow: Chris Bouchard
INT: Chris Monahan
JLab: Balint Jóo
Jülich: Evan Berkowitz
LBL/UCB: David Brantley, Chia Cheng (Jason) Chang, T. Kurth (NERSC), Henry Monge-Camacho, AWL
LLNL: Pavlos Vranas
Liverpool: Nicolas Garron
NVIDIA: Kate Clark
RIKEN/BNL: Enrico Rinaldi
UNC: Amy Nicholson
William and Mary: Kostas Orginos

plus a few friends

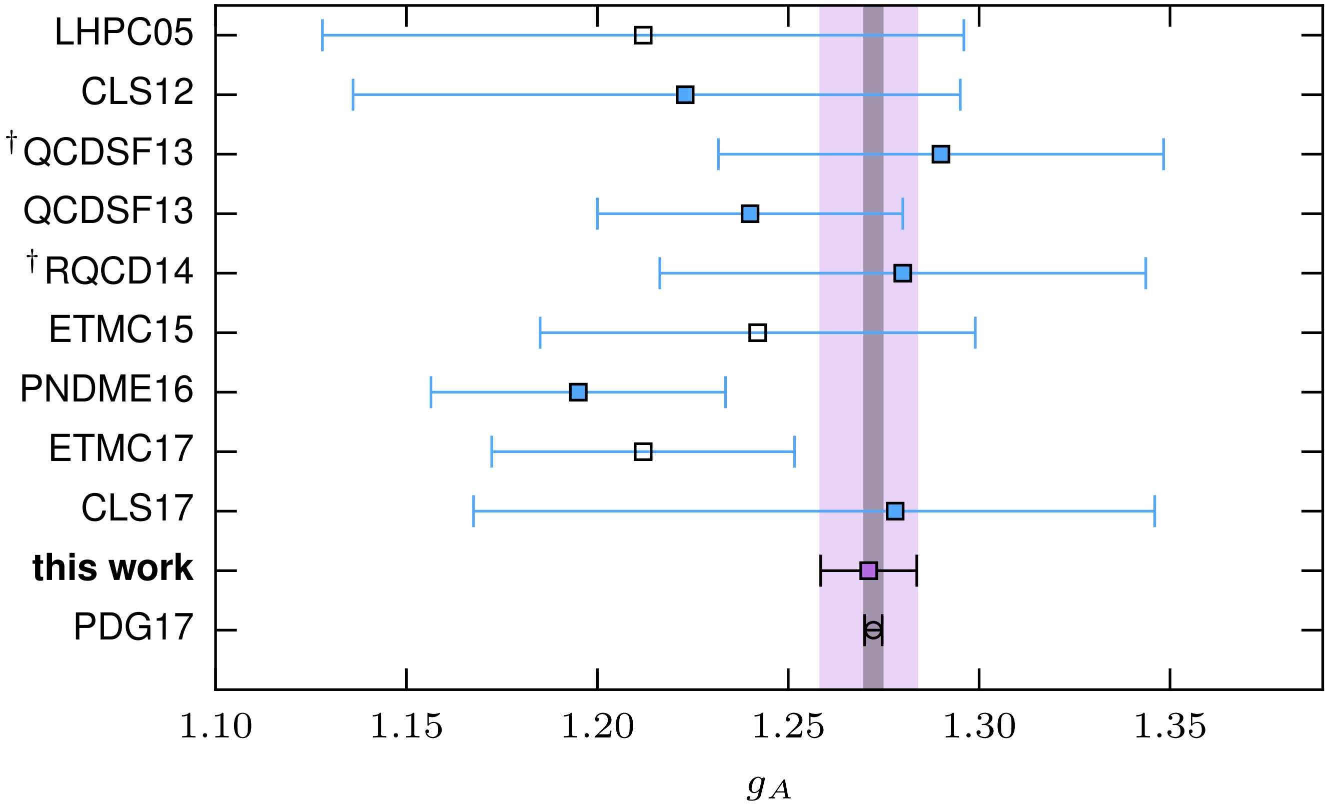
U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING
OAK RIDGE National Laboratory LEADERSHIP COMPUTING FACILITY
Lawrence Livermore National Laboratory
SciDAC Scientific Discovery through Advanced Computing
DOE Topical Collaboration Double Beta Decay
U.S. DEPARTMENT OF ENERGY Office of Science

$$
\begin{aligned}
g_A^{\rm QCD} &= 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M \\
&= 1.2711(126)
\end{aligned}
$$

previously estimated to achieve 2% by 2020



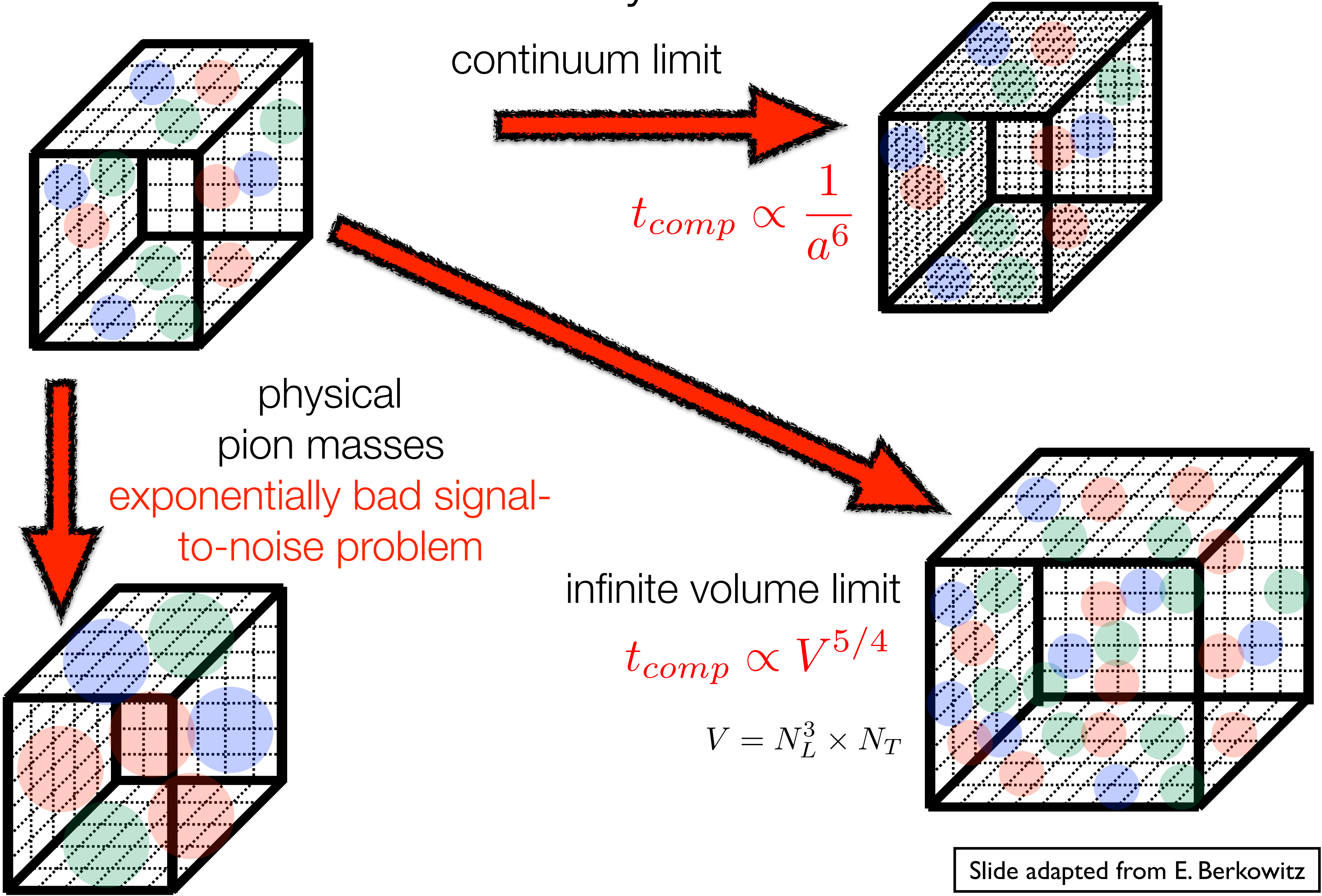
LQCD results



A percent-level determination of the nucleon axial coupling from QCD

- Challenges in applying lattice QCD to nucleon structure (and nuclear physics in general)
- An unconventional method
- g_A - details of the calculation and analysis

LQCD Systematics



continuum limit

$t_{comp} \propto \frac{1}{a^6}$

physical
pion masses

exponentially bad signal-to-noise problem

infinite volume limit

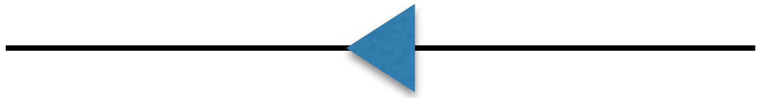
$t_{comp} \propto V^{5/4}$

$V = N_L^3 \times N_T$

Slide adapted from E. Berkowitz

LQCD Challenges for NP

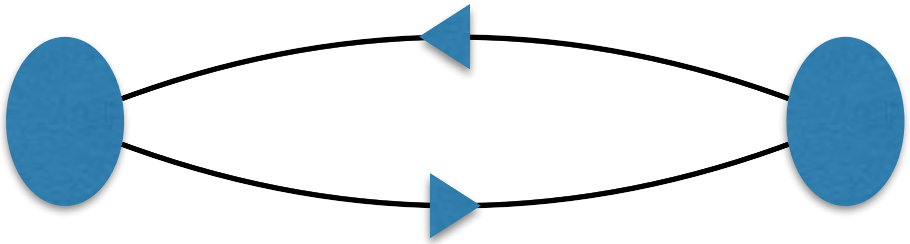
- The most difficult challenge in applying LQCD to NP is an exponentially bad signal-to-noise problem for nucleons



$$\sim e^{-\frac{1}{2}m_\pi t} + e^{-\frac{1}{3}m_N t} + \cdots$$

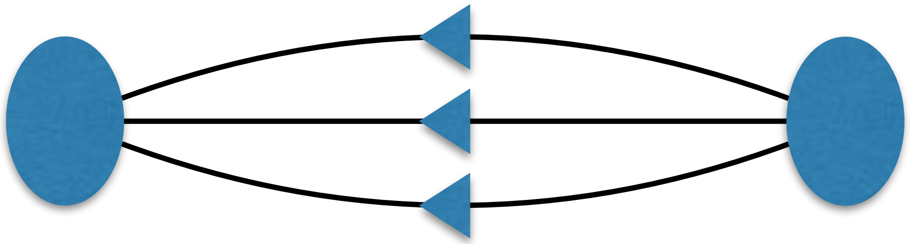
each **quark propagator** carries information about pions and nucleons
(conversations with David Kaplan)

$$\lambda_\pi(t) \gg \lambda_N(t)$$



$$\bar{d}\gamma_5 u : C(t) = A_\pi e^{-m_\pi t} + \cdots$$

For the nucleon - the large pion eigenvalues must cancel to expose the small nucleon eigenvalues



$$(u^T C\gamma_5 d)u : C(t) = A_N e^{-m_N t} + \cdots$$

$$\frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N}\exp\left[-A\left(m_N - \frac{3}{2}m_\pi\right)t\right]$$

→ **exponential noise power-law statistics**

LQCD Challenges for NP

2-point correlation function

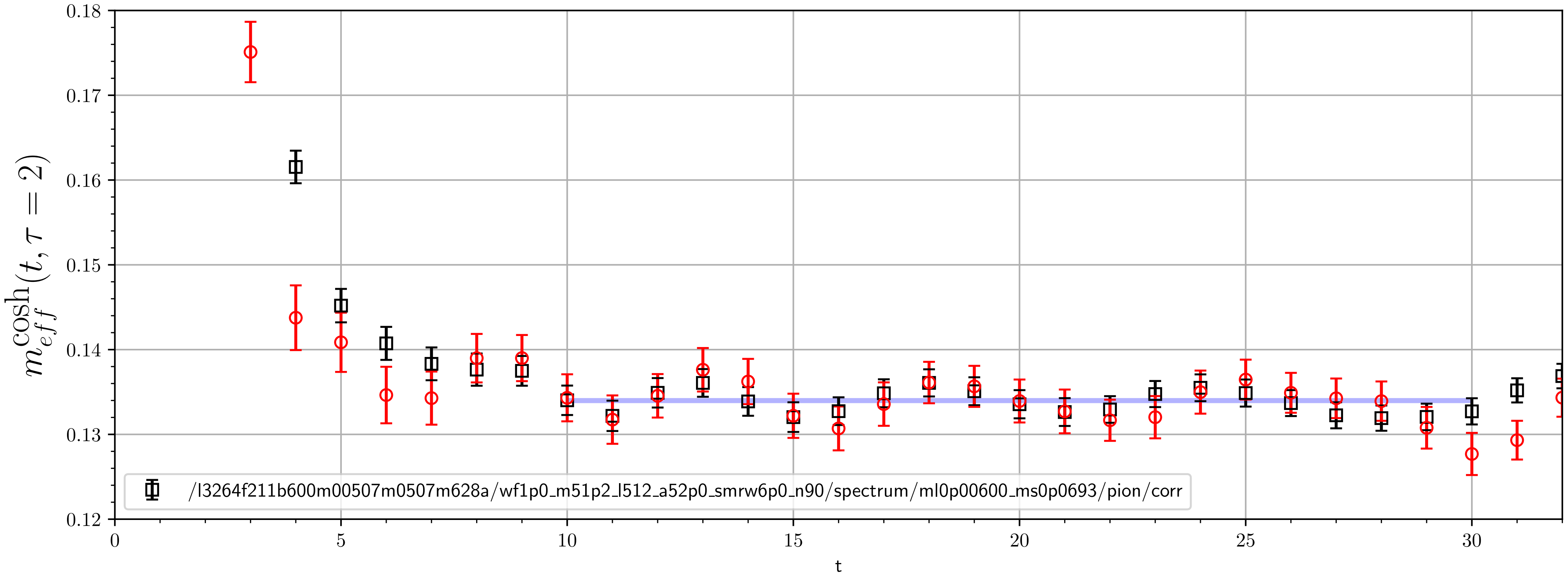
$$C(t) = \sum_n z_n z_n^\dagger e^{-E_n t}$$

$$m_{eff}(t) = \frac{1}{\tau} \ln \left(\frac{C(t)}{C(t+\tau)} \right)$$

For pions, need to consider leading finite temperature effects

$$C(t) = \sum_n z_n z_n^\dagger \left(e^{-E_n t} + e^{-E_n (T-t)} \right)$$

$$m_{eff}^{\cosh}(t,\tau) = \frac{1}{\tau} \cosh^{-1} \left(\frac{C(t+\tau) + C(t-\tau)}{2C(t)} \right)$$

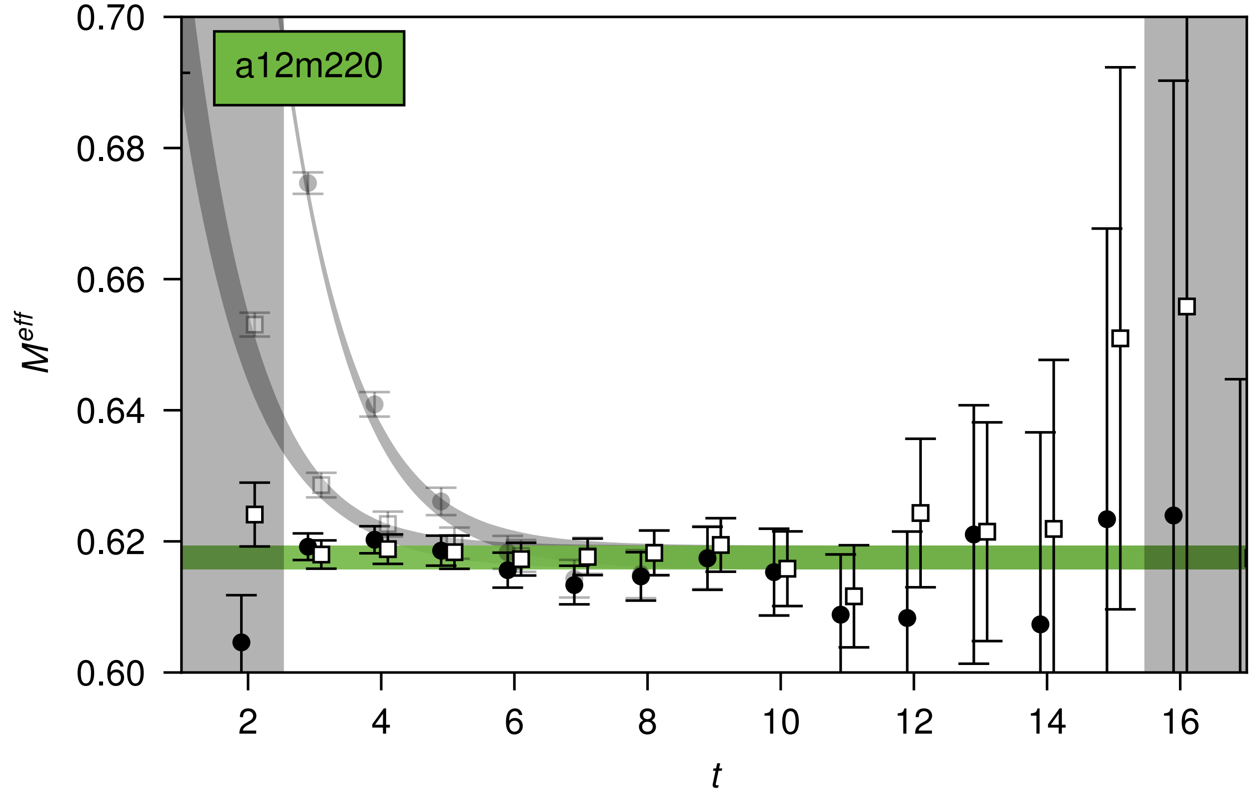


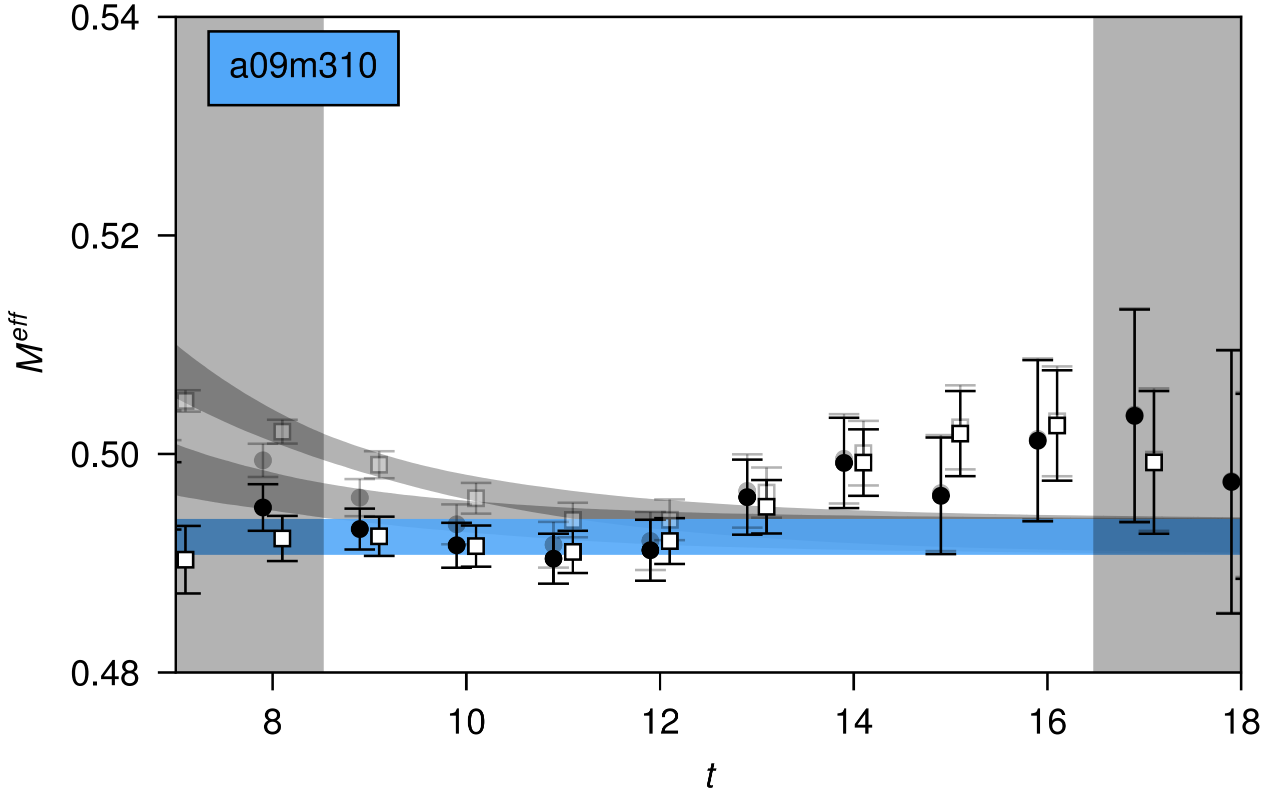
Effective mass of Pion 2-point correlation function
red and black “data” are from different choices of overlap operators

Noise is constant in time - can determine very clean ground state (blue band)

LQCD Challenges for NP

2-point correlation function





Two examples of nucleon effective mass

Noise is growing in time - can not simply go to the long-time limit without exponentially increasing the amount of statistics needed

$$\frac{Signal}{Noise} \rightarrow \sqrt{N_{stat}} e^{-(m_N - \frac{3}{2} m_\pi)t}$$

Correlated late-time fluctuations… what is the ground state?

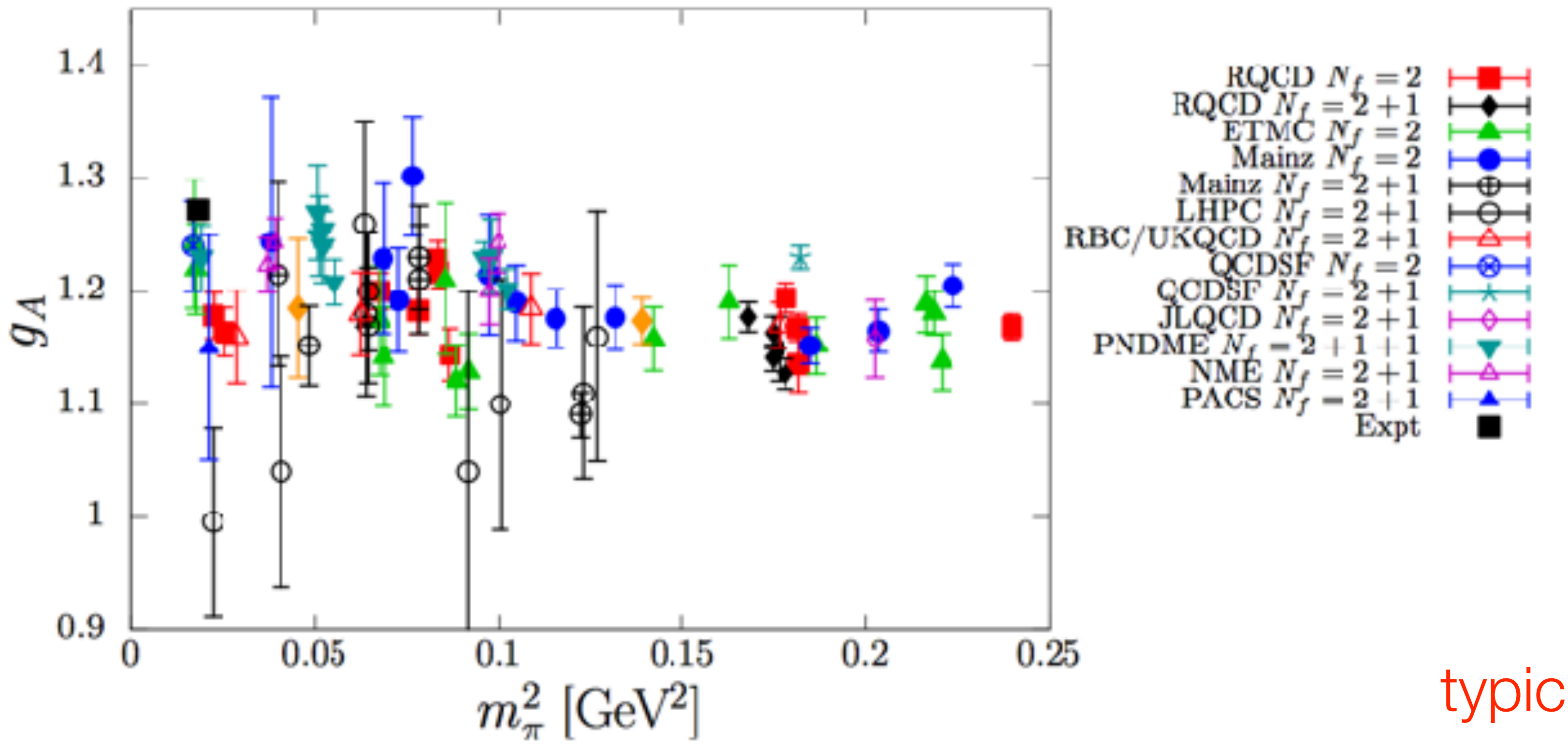
Need sophisticated analysis to ensure you are not susceptible to correlated fluctuations

This problem is exacerbated with 2+ nucleons and form-factor calculations (g_A)

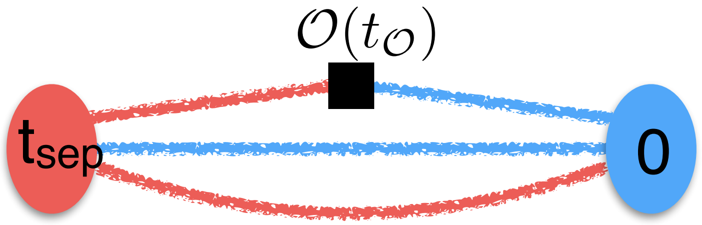
LQCD Challenges for NP

Nucleon axial charge - S. Collins - Lattice 2016 Plenary

Benchmark quantity sensitive to systematics.



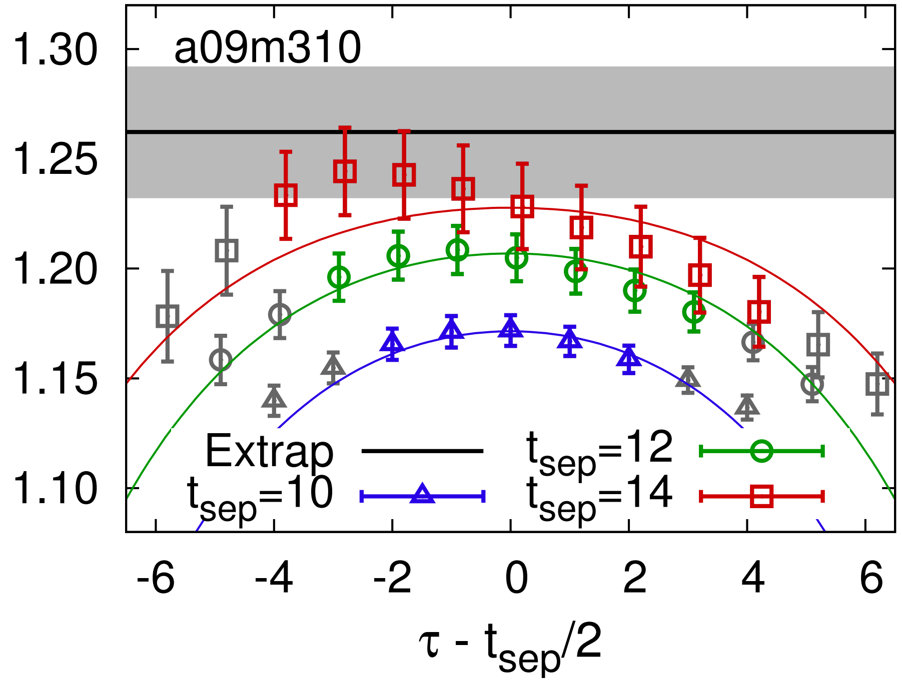
g_A is an important benchmark quantity very challenging for LQCD to get under control



Presented 2016:

PNDME, NME, Mainz, RQCD, ETMC, PACS, χQCD, QCDSF,

typical calculation



t_{sep}	fm
14	1.22
12	1.05
10	0.875

in long-time (**t_{sep}**) limit - should be flat

An unconventional method

arXiv.org > hep-lat > arXiv:1612.06963 ***Phys. Rev.* D96 (2017)**

High Energy Physics – Lattice

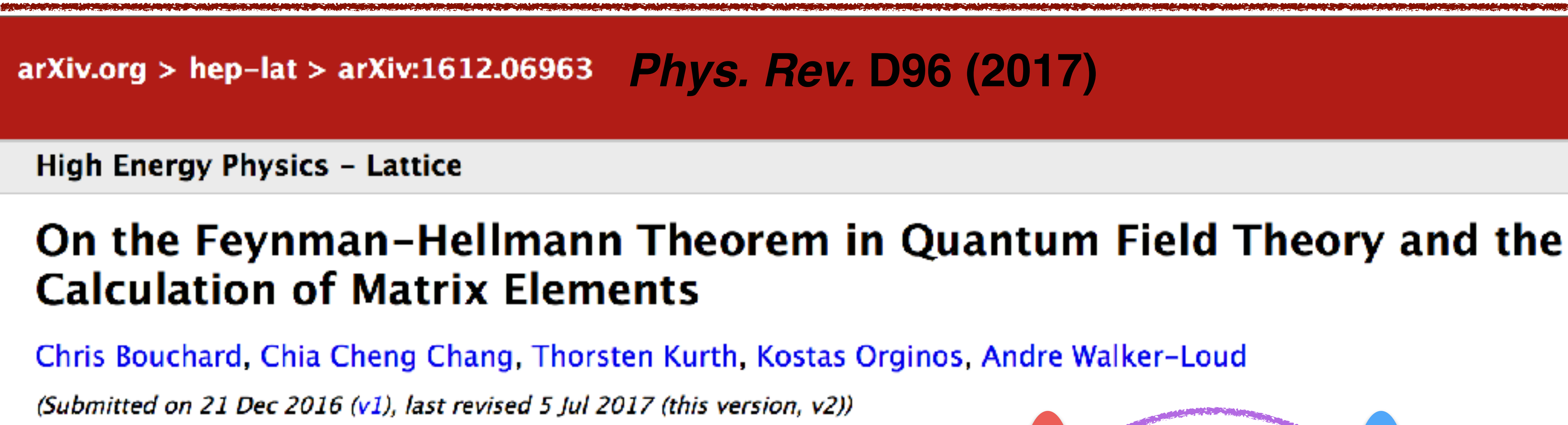
On the Feynman-Hellmann Theorem in Quantum Field Theory and the Calculation of Matrix Elements

Chris Bouchard, Chia Cheng Chang, Thorsten Kurth, Kostas Orginos, Andre Walker-Loud

(Submitted on 21 Dec 2016 (v1), last revised 5 Jul 2017 (this version, v2))

- Take the Feynman-Hellmann (FH) Theorem as a starting point: $\partial_\lambda E_n = \langle n | H_\lambda | n \rangle$
- conceptually very simple and straightforward
- applying the FH Theorem to the effective mass directly leads to the method we use
  - relates matrix elements to functional derivatives of the partition function
  - reduces the dependence on two time variables (operator insertion time, τ, and source/sink separation time, t) to a single variable, t
  - allows for demonstrable control of excited state systematics, reduced sensitivity to correlated fluctuations & the extraction of the signal early in Euclidean time (exponentially smaller relative noise)

An unconventional method



arXiv.org > hep-lat > arXiv:1612.06963 *Phys. Rev.* **D96 (2017)**

High Energy Physics – Lattice

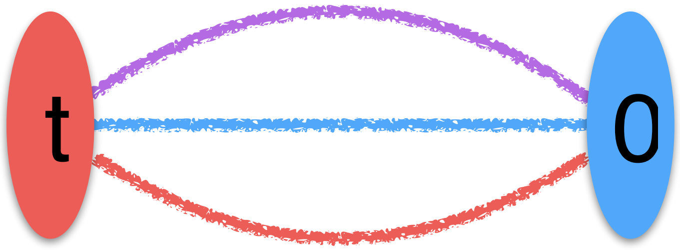
On the Feynman-Hellmann Theorem in Quantum Field Theory and the Calculation of Matrix Elements

Chris Bouchard, Chia Cheng Chang, Thorsten Kurth, Kostas Orginos, Andre Walker-Loud

(Submitted on 21 Dec 2016 (v1), last revised 5 Jul 2017 (this version, v2))

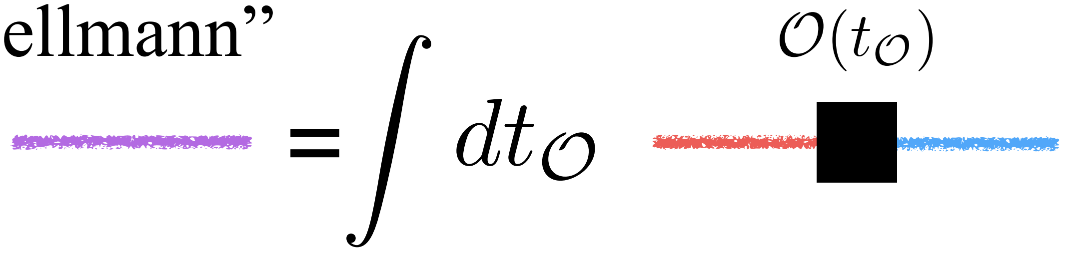
☐ No time to discuss details
cartoon explanation

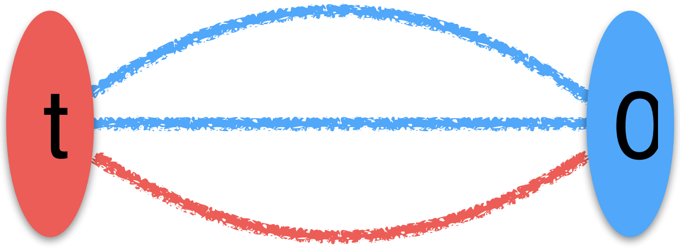
“Feynman-Hellmann”
correlation function



$$\left.\frac{\partial m^{eff}_{\lambda}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C_\lambda(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C_\lambda(t)}\right]_{\lambda=0}$$

“Feynman-Hellmann”
propagator



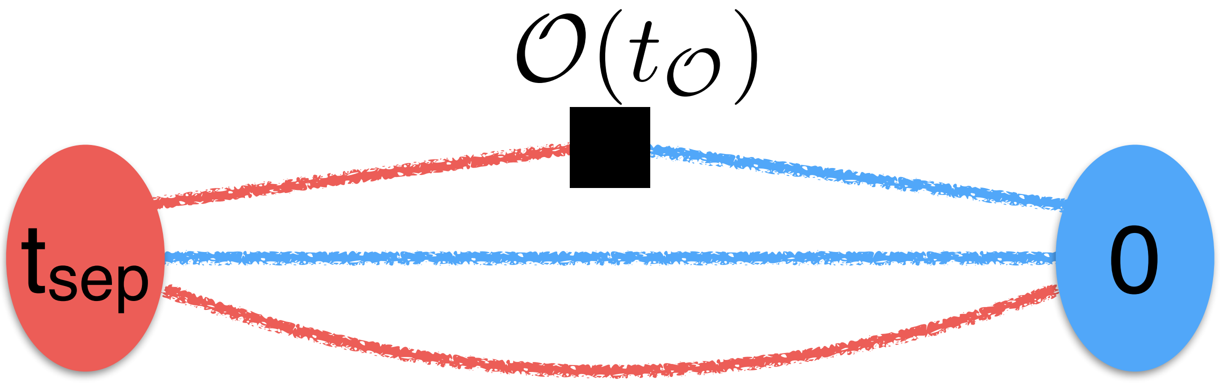


standard 2-point function

An unconventional method

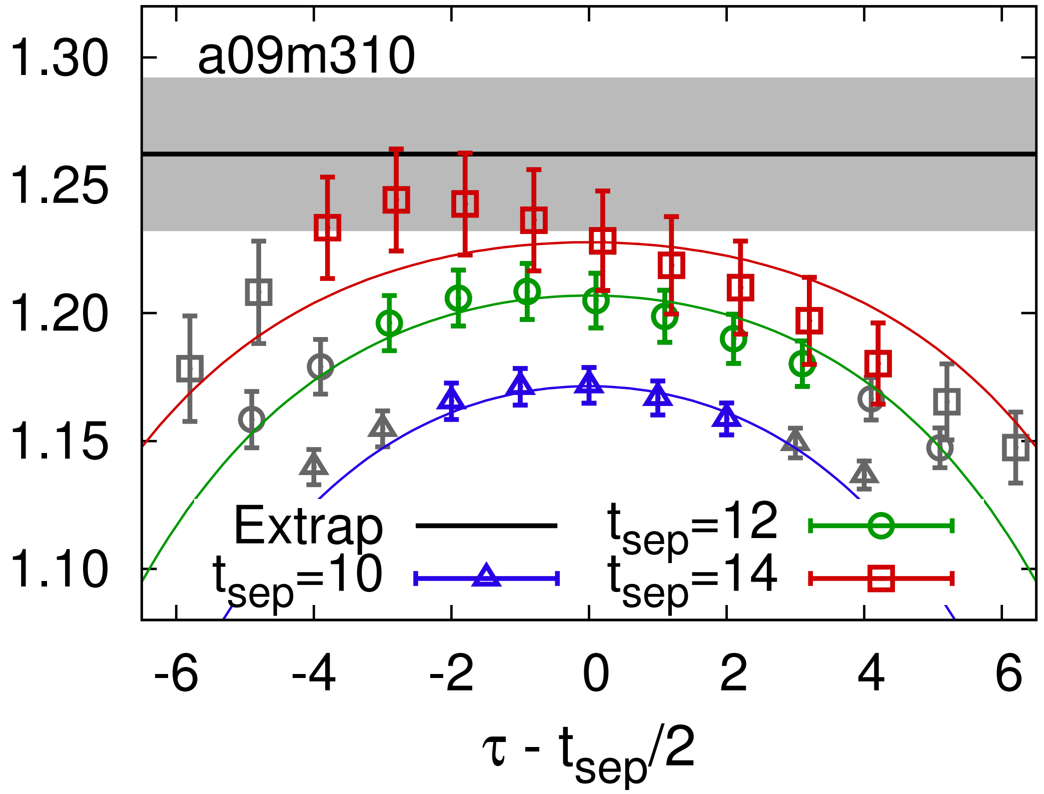
arXiv:1612.06963

standard method

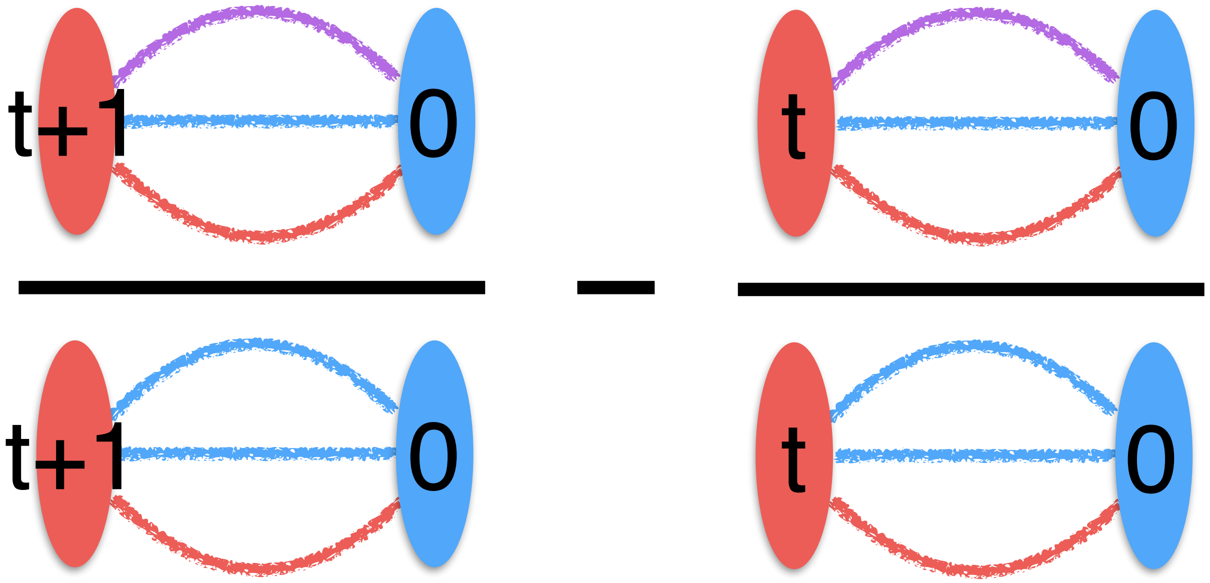


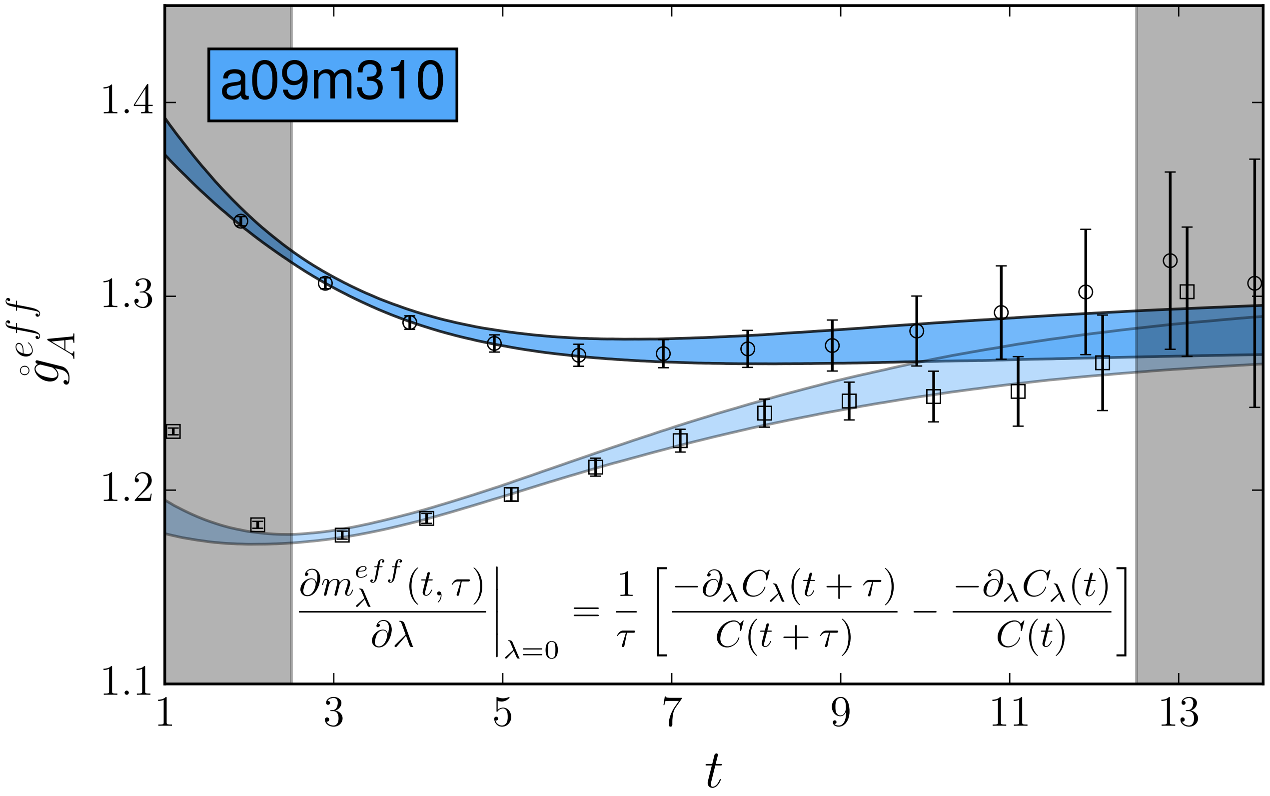
fixed source-sink separation time

PNDME arXiv:1606.07049



our unconventional method



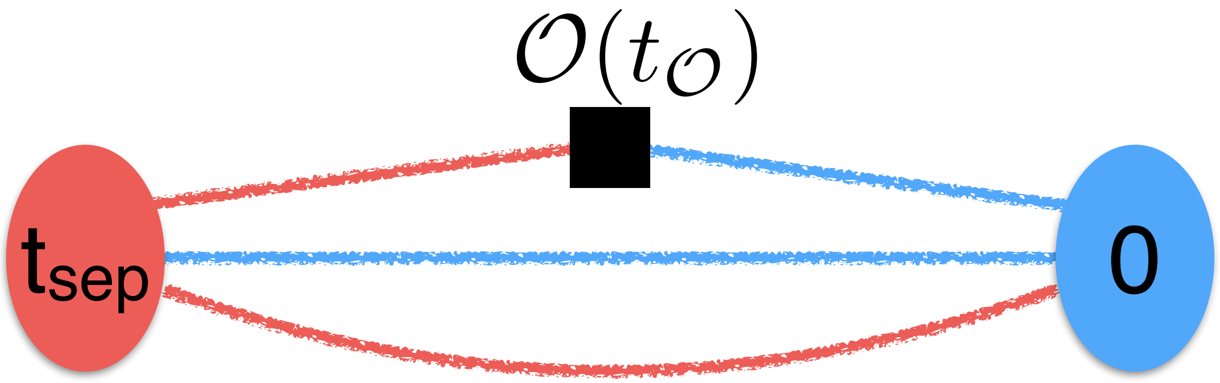


$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$

An unconventional method

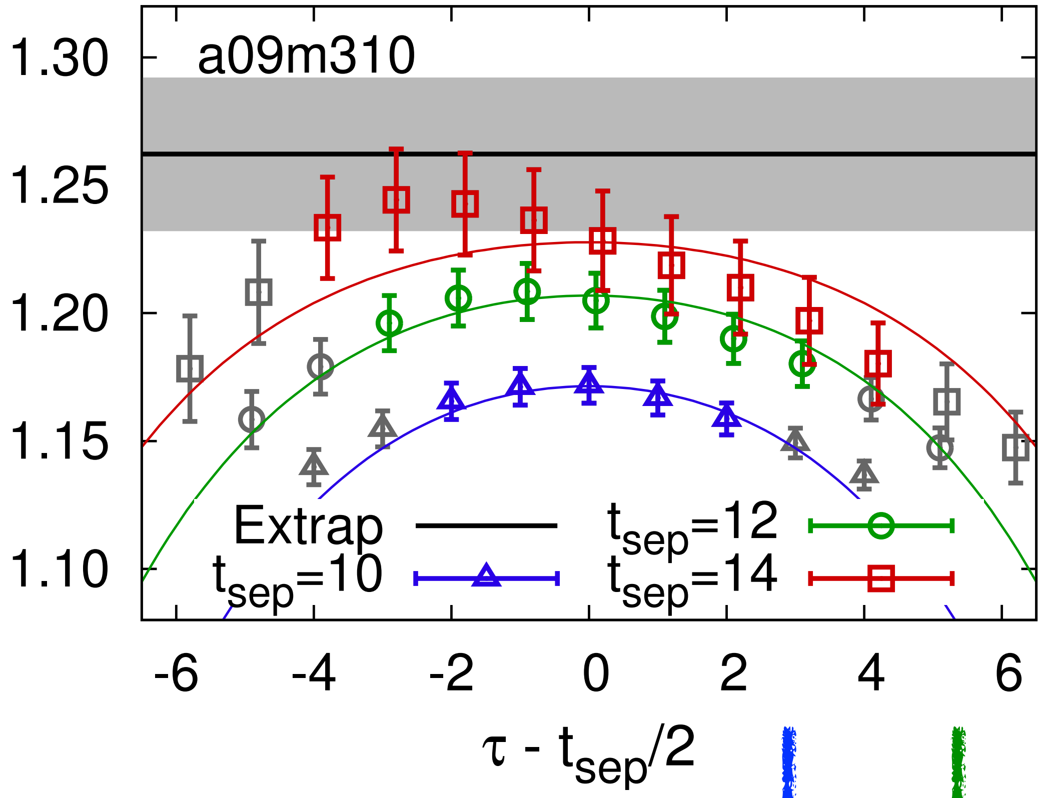
arXiv:1612.06963

standard method

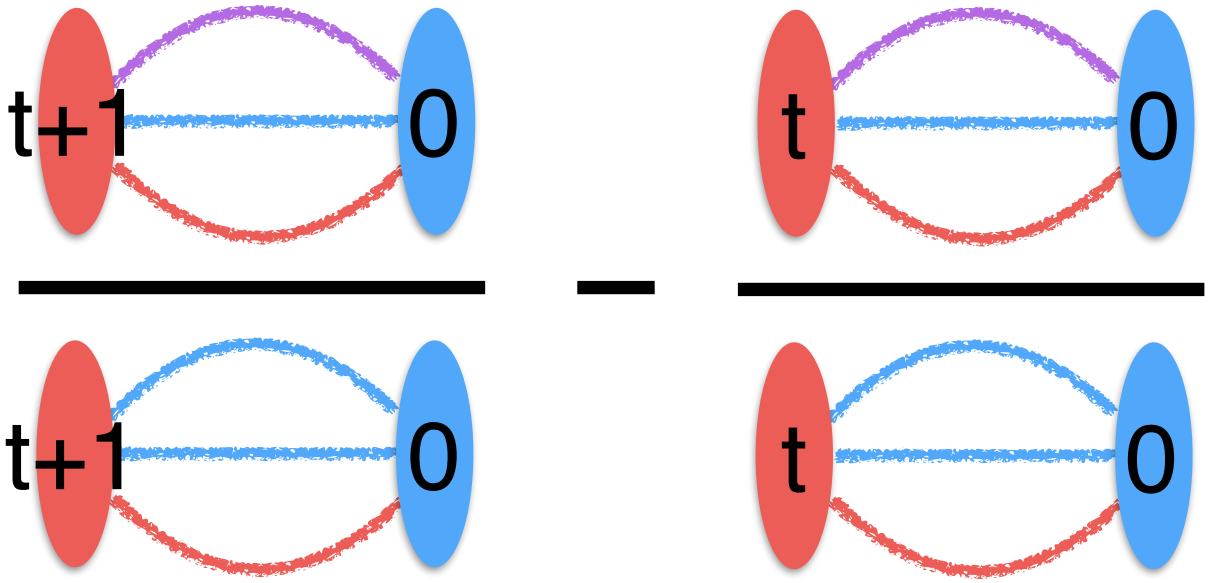


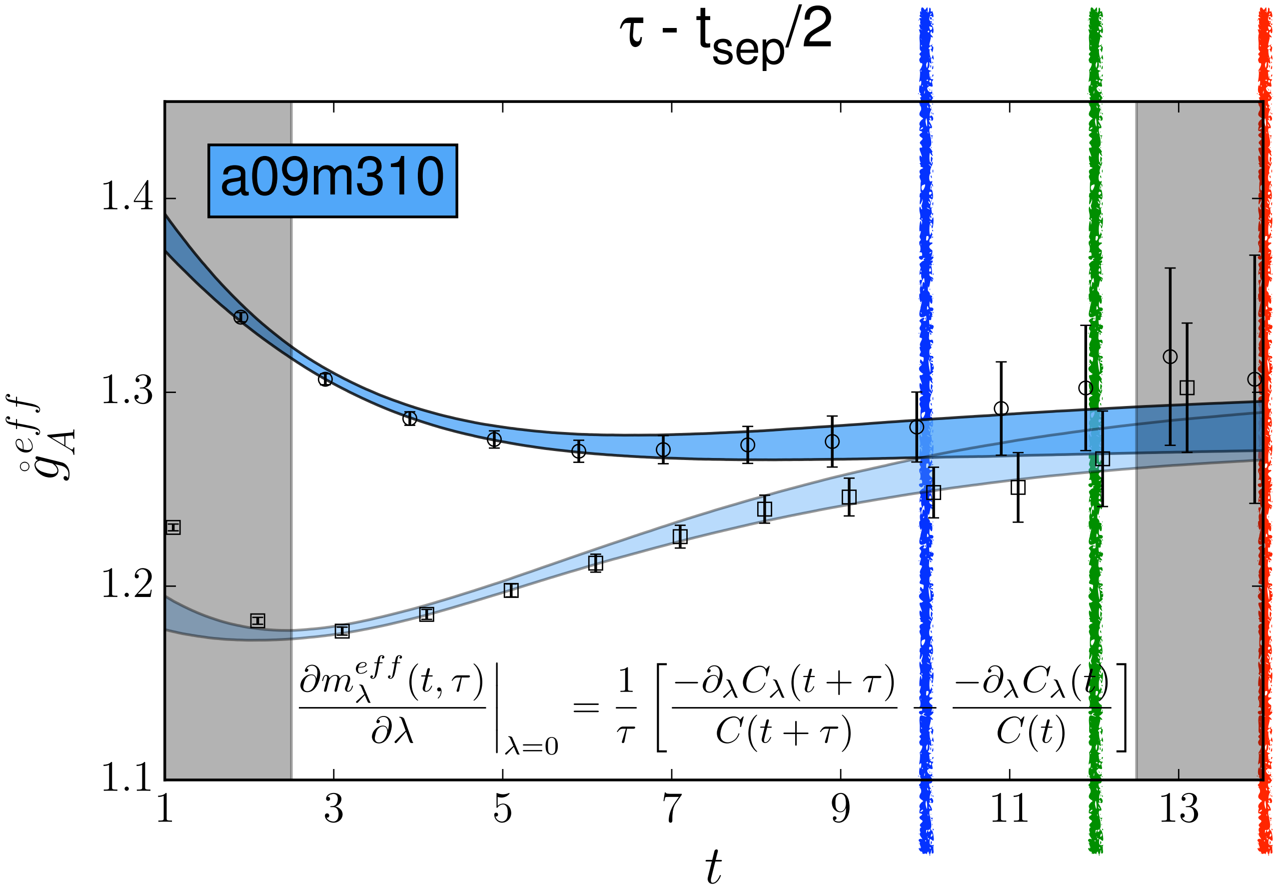
fixed source-sink separation time

PNDME arXiv:1606.07049



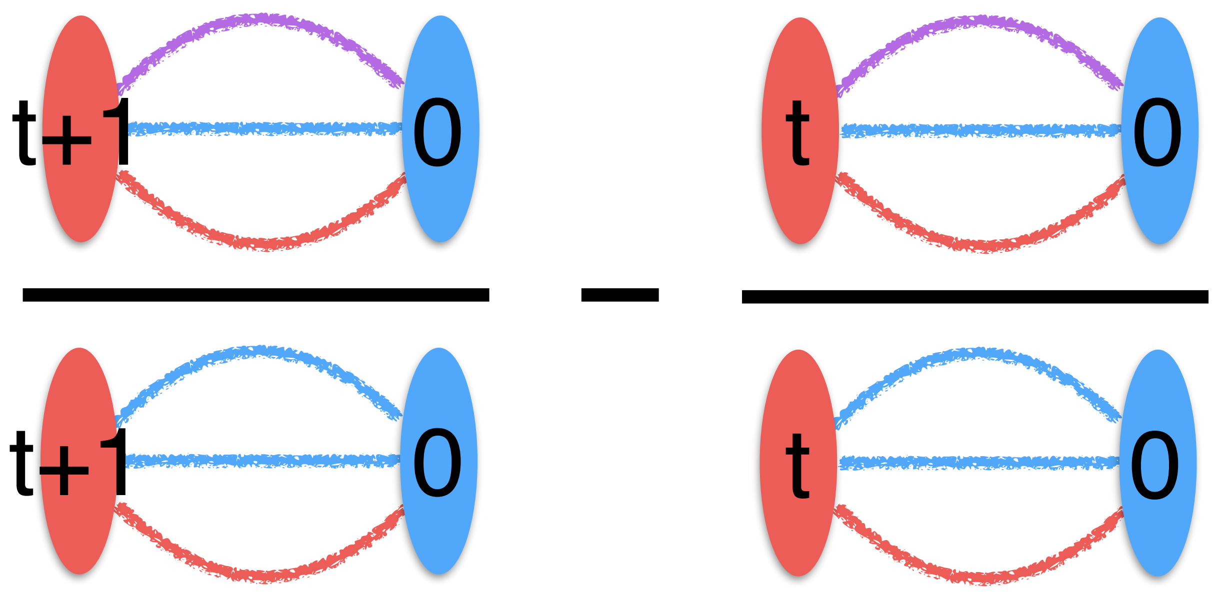
our unconventional method

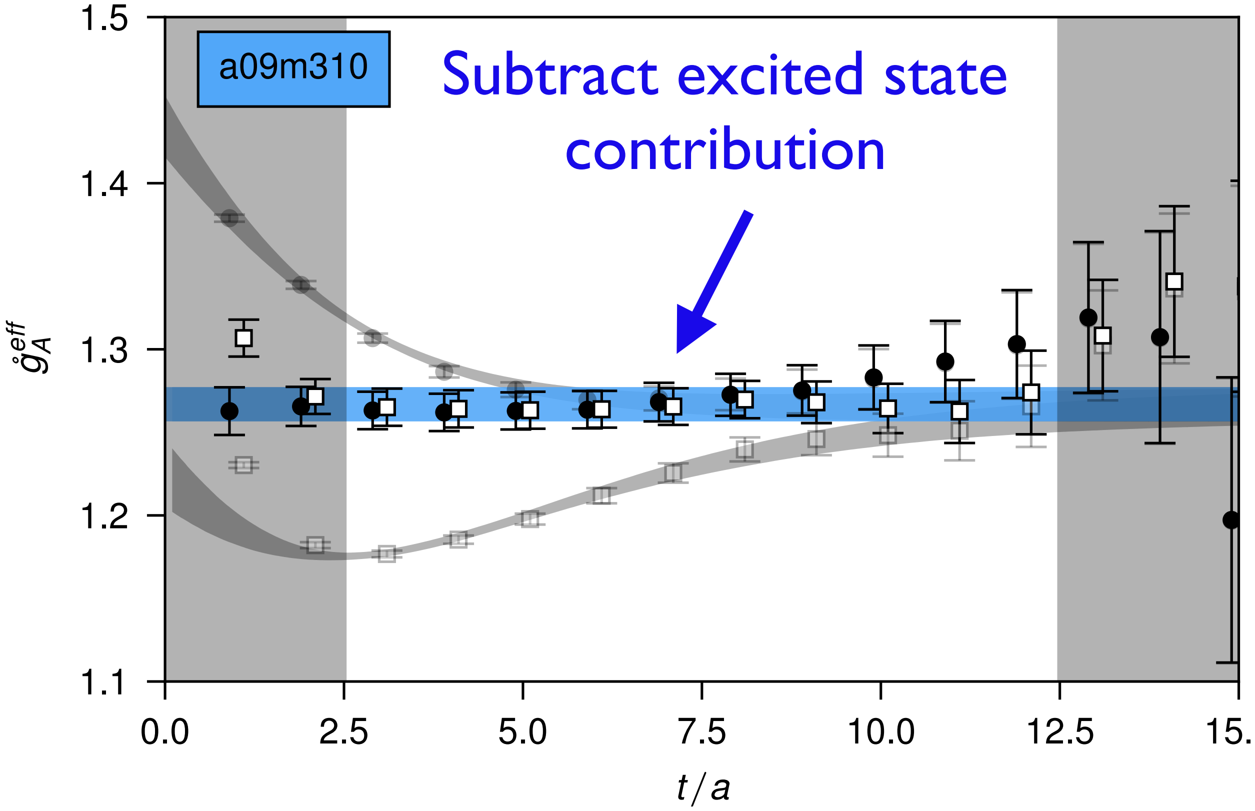




An unconventional method arXiv:1612.06963

our unconventional method





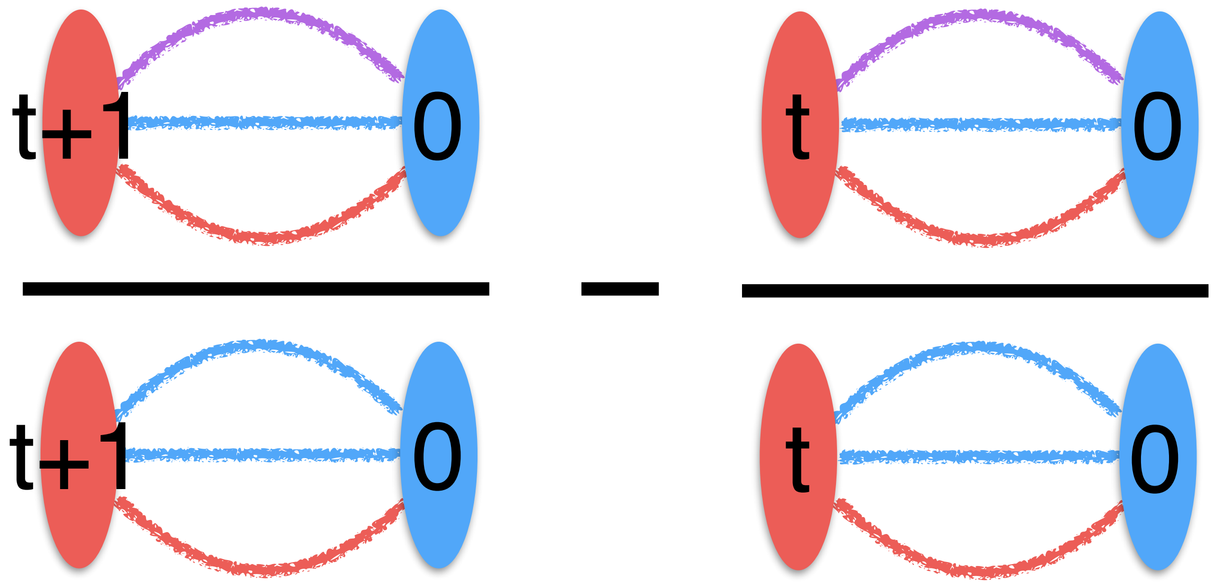
Subtract excited state contribution

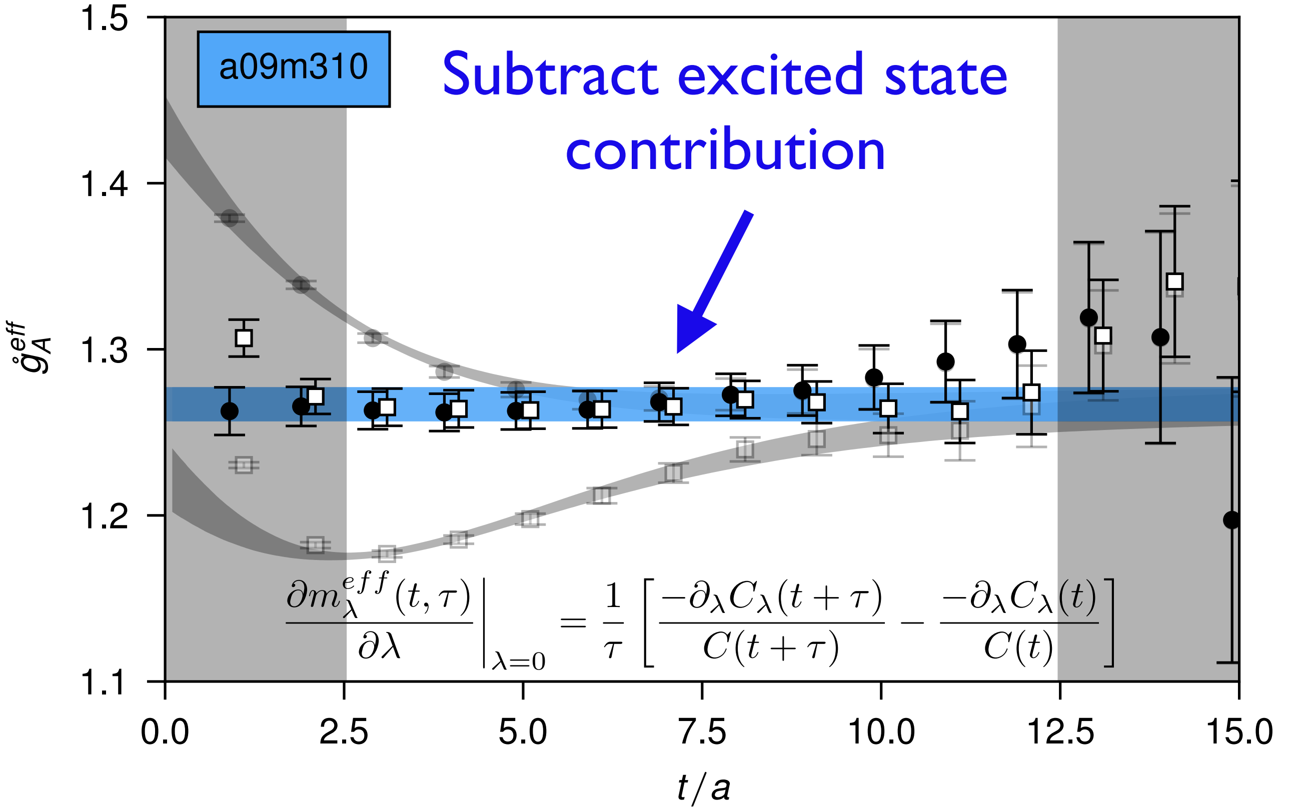
Key features of this method

- The correlation function is given by $\partial_\lambda m_\lambda^{eff}(t,\tau)\Big|_{\lambda=0} = g_{00} + z e^{-t\Delta_{e.s.}}$
- excited state contamination is demonstrably controlled
- we can access very early Euclidean time, allowing the use of exponentially more precise numerical points
- **No background field is used** - the FH-theorem is used to “derive” our “Feynman-Hellmann Correlation function” analytically

An unconventional method arXiv:1612.06963

our unconventional method





Subtract excited state contribution

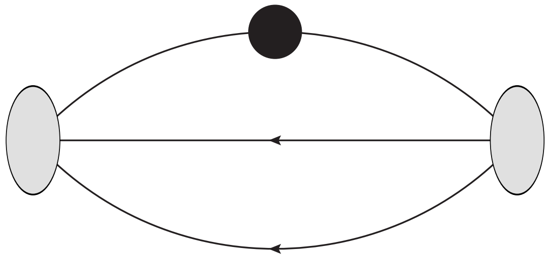
$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$

Key features of this method

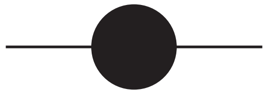
- The correlation function is given by $\left.\partial_\lambda m_\lambda^{eff}(t,\tau)\right|_{\lambda=0} = g_{00} + z e^{-t\Delta_{e.s.}}$
- excited state contamination is demonstrably controlled
- we can access very early Euclidean time, allowing the use of exponentially more precise numerical points
- **No background field is used** - the FH-theorem is used to “derive” our “Feynman-Hellmann Correlation function” analytically

An unconventional method arXiv:1612.06963

Numerical Implementation:



the “Feynman-Hellman” propagator is given by



$$= S_{FH}(y,x) = \sum_z S(y,z)\Gamma(z)S(z,x)$$

$S(z,x)$	standard quark propagator off some source at x, to all z
$\Gamma(z)$	some bi-linear operator (can be constant) e.g., γ_4 for the vector current
$\Gamma(z)S(z,x)$	treat like a source to invert off of

NOTE: this is the same equation as appears in de Divitiis, Petronzio, Tantalo, PLB718 (2012) can be traced back to Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987)

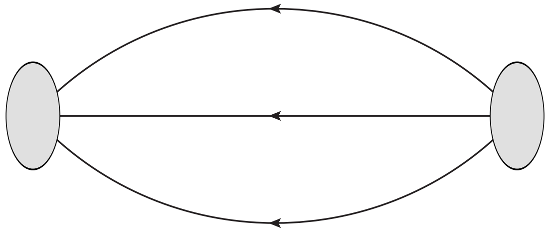
Similar ideas in literature:
Chambers et. al. Phys.Rev. D90 [arXiv:1405.3019]
Chambers et. al. Phys.Rev. D92 [arXiv:1508.06856]
Savage et. al. Phys.Rev.Lett. 119 [arXiv:1610.04545]

Already used for new processes!
Orginos, Radyushkin, Karpie, Zafeiropoulos
Phys.Rev. D96 [arXiv:1706.05373]

An unconventional method arXiv:1612.06963

Proton

$$\langle\Omega|N_\gamma(y)\bar{N}_{\gamma'}(x)|\Omega\rangle$$



creation/annihilation operators

$$\bar{N}_{\gamma'} = \epsilon_{i'j'k'}P_{\gamma'\rho'}\;\bar{u}^{i'}_{\rho'}(\bar{u}^{j'}_{\alpha'}\Gamma^{\dagger,src}_{\alpha'\beta'}\bar{d}^{k'}_{\beta'})$$

$$N_\gamma = \epsilon_{ijk}P_{\gamma\rho}\;u^{i}_{\rho}(u^{j}_{\alpha}\Gamma^{snk}_{\alpha\beta}d^{k}_{\beta})$$

quark propagators

$$U(y,x)^{ii'}_{\alpha\alpha'} = \underbracket{u^{i}_{\alpha}(y)\bar{u}^{i'}_{\alpha'}}(x)\,,$$

$$D(y,x)^{ii'}_{\alpha\alpha'} = \underbracket{d^{i}_{\alpha}(y)\bar{d}^{i'}_{\alpha'}}(x)\,,$$

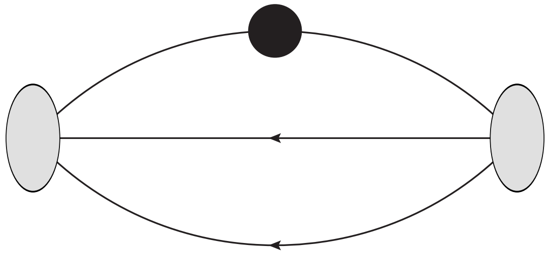
$\langle\Omega|N_\gamma(y)\bar{N}_{\gamma'}(x)|\Omega\rangle$ =

$$C_{\gamma\gamma'} = \epsilon_{ijk}\epsilon_{i'j'k'}P_{\gamma'\rho'}\Gamma^{src}_{\alpha'\beta'}\left[P_{\gamma\rho}\Gamma^{snk}_{\alpha\beta} + P_{\gamma\alpha}\Gamma^{snk}_{\rho\beta}\right]U^{ii'}_{\rho\rho'}U^{jj'}_{\alpha\alpha'}D^{kk'}_{\beta\beta'}$$

An unconventional method

arXiv:1612.06963

Proton - with FH propagator $\langle\Omega|N_\gamma(y)\bar{N}_{\gamma'}(x)|\Omega\rangle$



down-quark

$$C^{\Gamma d}_{\gamma\gamma'} = \epsilon_{ijk}\epsilon_{i'j'k'} P_{\gamma'\rho'}\Gamma^{src}_{\alpha'\beta'}\left[P_{\gamma\rho}\Gamma^{snk}_{\alpha\beta} + P_{\gamma\alpha}\Gamma^{snk}_{\rho\beta}\right] U^{ii'}_{\rho\rho'} U^{jj'}_{\alpha\alpha'} \underline{D^{\Gamma,kk'}_{\beta\beta'}}$$

up-quark

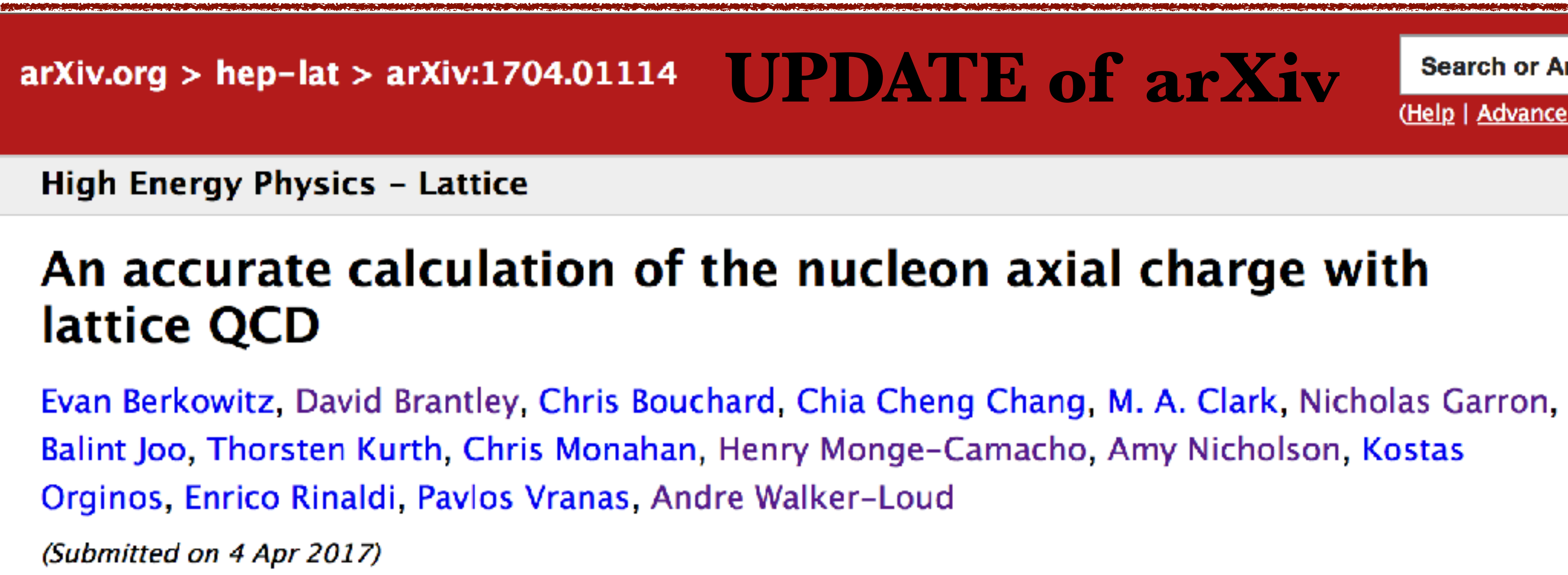
$$C^{\Gamma u}_{\gamma\gamma'} = \epsilon_{ijk}\epsilon_{i'j'k'} P_{\gamma'\rho'}\Gamma^{src}_{\alpha'\beta'}\left[P_{\gamma\rho}\Gamma^{snk}_{\alpha\beta} + P_{\gamma\alpha}\Gamma^{snk}_{\rho\beta}\right] \left[\underline{U^{\Gamma,ii'}_{\rho\rho'}}\, U^{jj'}_{\alpha\alpha'} D^{kk'}_{\beta\beta'} + U^{ii'}_{\rho\rho'} \underline{U^{\Gamma,jj'}_{\alpha\alpha'}}\, D^{kk'}_{\beta\beta'}\right]$$

up←down

$$C^{u\leftarrow d}_{\gamma\gamma'} = \epsilon_{ijk}\epsilon_{i'j'k'} \left[P_{\gamma\rho}\Gamma^{snk}_{\alpha\beta} + P_{\gamma\alpha}\Gamma^{snk}_{\rho\beta}\right]\left[P_{\gamma'\rho'}\Gamma^{src}_{\alpha'\beta'} + P_{\gamma'\beta'}\Gamma^{src}_{\alpha'\rho'}\right] U^{ii'}_{\alpha\alpha'} D^{jj'}_{\beta\beta'} \underline{(U \leftarrow D)^{kk'}_{\rho\rho'}}$$

NOTE: this method does NOT require any actual background field. Instead, we have analytically determined the linear-response correlation function

A percent-level determination of the nucleon axial coupling from QCD



arXiv.org > hep-lat > arXiv:1704.01114

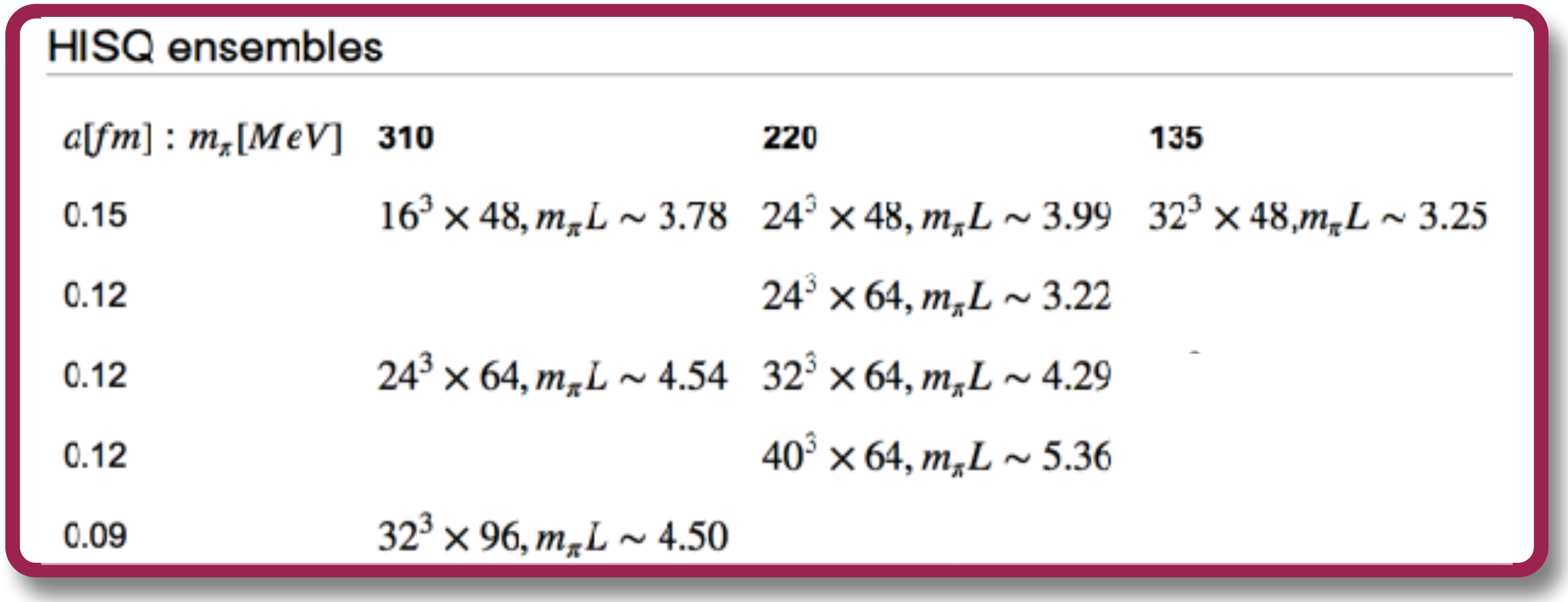
UPDATE of arXiv

High Energy Physics – Lattice

An accurate calculation of the nucleon axial charge with lattice QCD

Evan Berkowitz, David Brantley, Chris Bouchard, Chia Cheng Chang, M. A. Clark, Nicholas Garron, Balint Joo, Thorsten Kurth, Chris Monahan, Henry Monge-Camacho, Amy Nicholson, Kostas Orginos, Enrico Rinaldi, Pavlos Vranas, Andre Walker-Loud

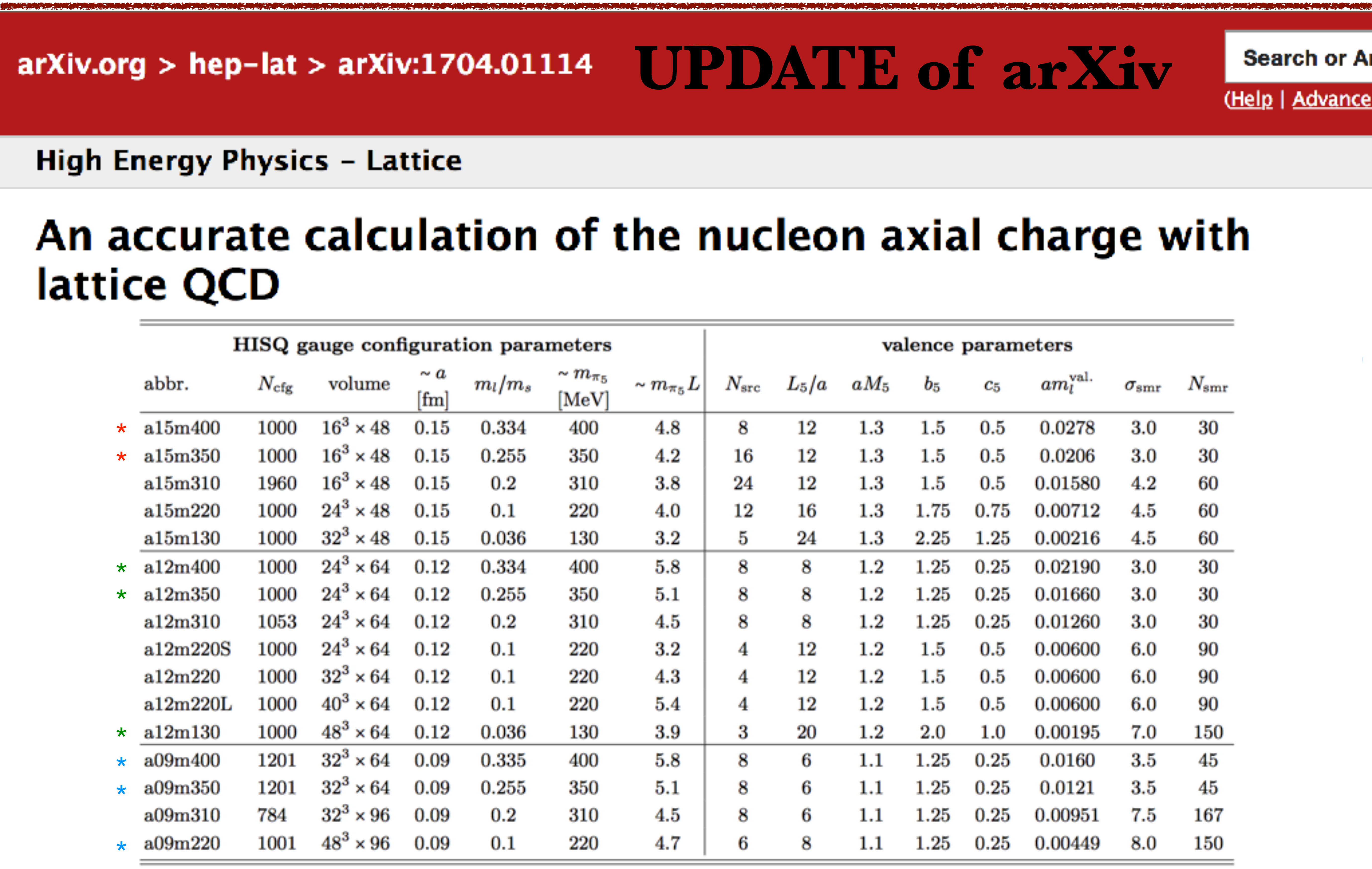
(Submitted on 4 Apr 2017)



HISQ ensembles

$a[fm] : m_\pi[MeV]$	**310**	**220**	**135**
0.15	$16^3 \times 48, m_\pi L \sim 3.78$	$24^3 \times 48, m_\pi L \sim 3.99$	$32^3 \times 48, m_\pi L \sim 3.25$
0.12		$24^3 \times 64, m_\pi L \sim 3.22$	
0.12	$24^3 \times 64, m_\pi L \sim 4.54$	$32^3 \times 64, m_\pi L \sim 4.29$	
0.12		$40^3 \times 64, m_\pi L \sim 5.36$	
0.09	$32^3 \times 96, m_\pi L \sim 4.50$		

A percent-level determination of the nucleon axial coupling from QCD



arXiv.org > hep-lat > arXiv:1704.01114

UPDATE of arXiv

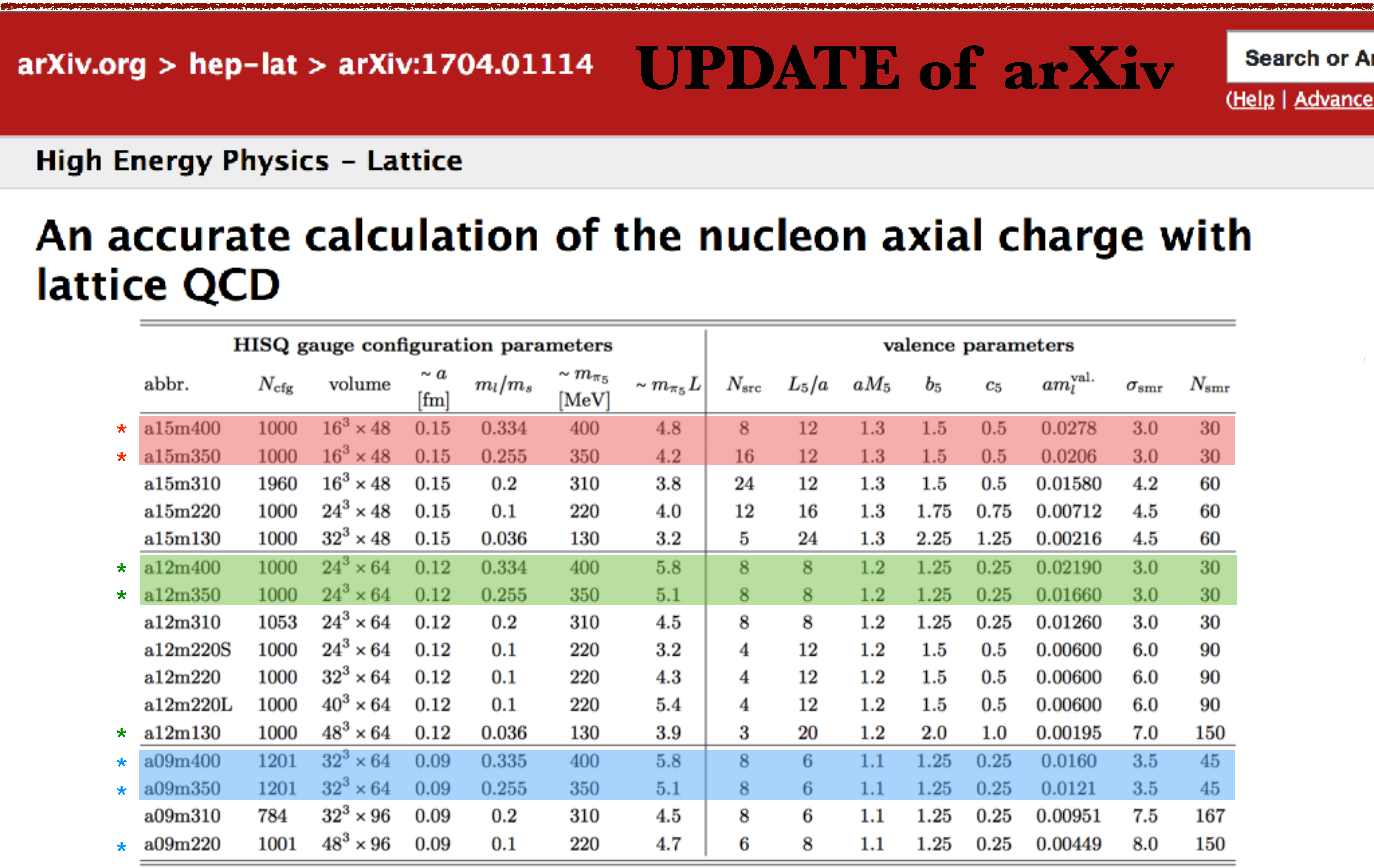
High Energy Physics – Lattice

An accurate calculation of the nucleon axial charge with lattice QCD

	HISQ gauge configuration parameters							valence parameters							
	abbr.	$N_{\rm cfg}$	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	$N_{\rm src}$	L_5/a	aM_5	b_5	c_5	$am_l^{\rm val.}$	$\sigma_{\rm smr}$	$N_{\rm smr}$
*	a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
*	a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
	a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
	a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
	a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
*	a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
*	a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
	a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
	a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
*	a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
*	a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8	8	6	1.1	1.25	0.25	0.0160	3.5	45
*	a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1	8	6	1.1	1.25	0.25	0.0121	3.5	45
	a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167
*	a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150

* New calculation

A percent-level determination of the nucleon axial coupling from QCD



UPDATE of arXiv

High Energy Physics – Lattice

An accurate calculation of the nucleon axial charge with lattice QCD

	HISQ gauge configuration parameters							valence parameters							
	abbr.	N_{cfg}	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	N_{src}	L_5/a	aM_5	b_5	c_5	$am_l^{\text{val.}}$	σ_{smr}	N_{smr}
*	a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
*	a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
	a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
	a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
	a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
*	a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
*	a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
	a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
	a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
*	a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
*	a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8	8	6	1.1	1.25	0.25	0.0160	3.5	45
*	a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1	8	6	1.1	1.25	0.25	0.0121	3.5	45
	a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167
*	a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150

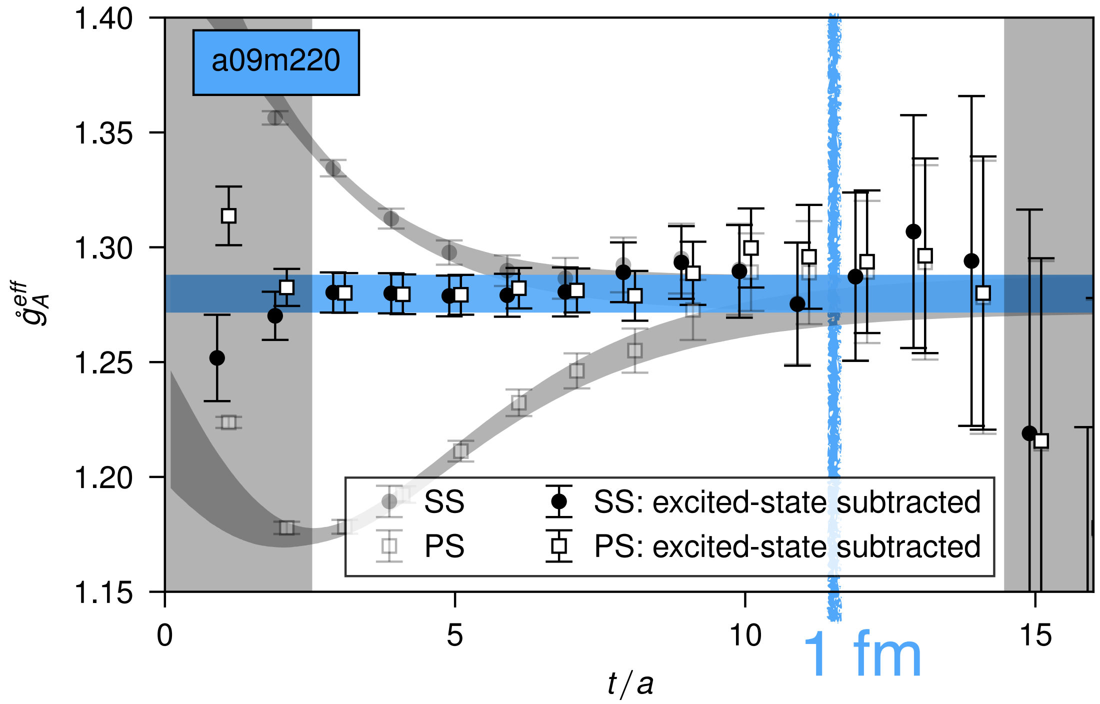
* New calculation

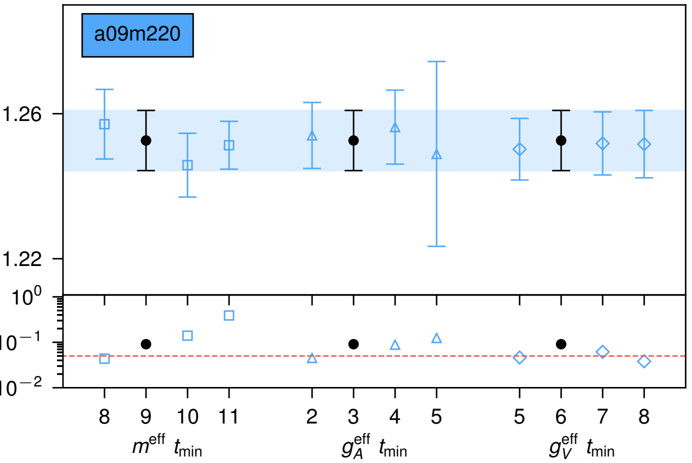
additional HISQ ensembles generated @ LLNL

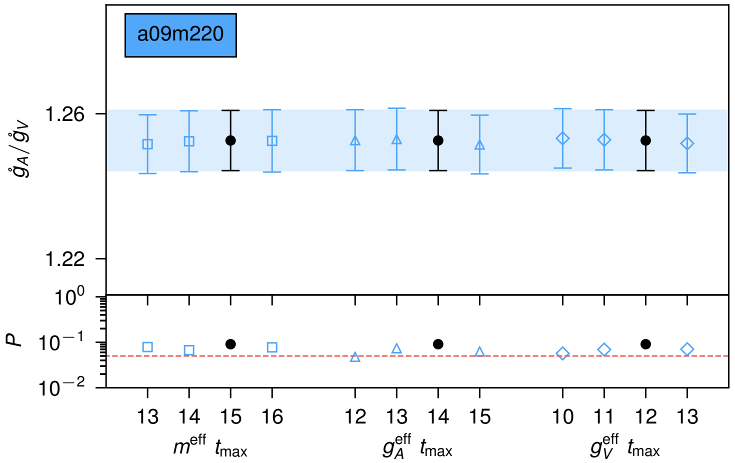
A percent-level determination of the nucleon axial coupling from QCD

Correlation function analysis

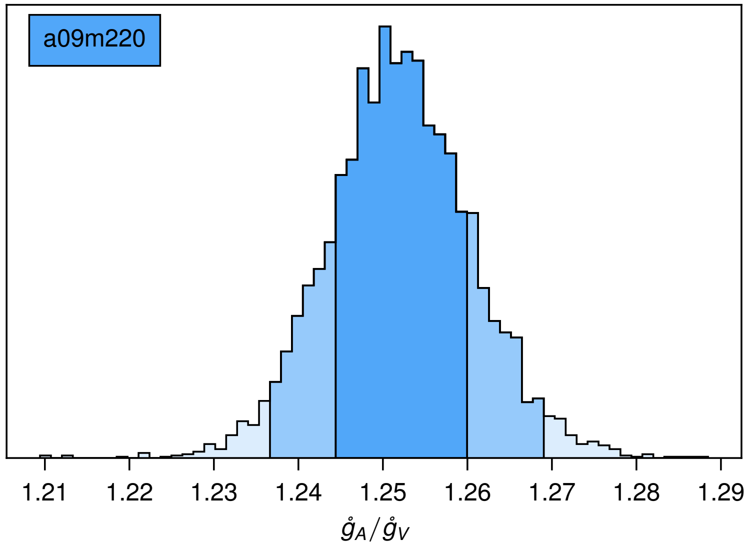
- Simultaneous fit to 6 correlation functions, 2-point, g_A, g_V (SS and PS)
- Complete 2-state fit (g.s. and 1 excited state, including transitions)
- Bayesian constrained fit as a pre-conditioner for unconstrained non-linear regression
- Stability analysis is performed varying min and max time in the correlator analysis







- early time has excited state contamination
- late time susceptible to correlated fluctuations
- excited state subtracted results are constant in fit region
- resulting bootstrap distributions (N_{bs}=5000) are Gaussian and show no outliers



A percent-level determination of the nucleon axial coupling from QCD

Renormalization

- The currents used in the calculation must be renormalized to match physical currents
- By definition

$$Z_V \mathring{g}_V = 1$$

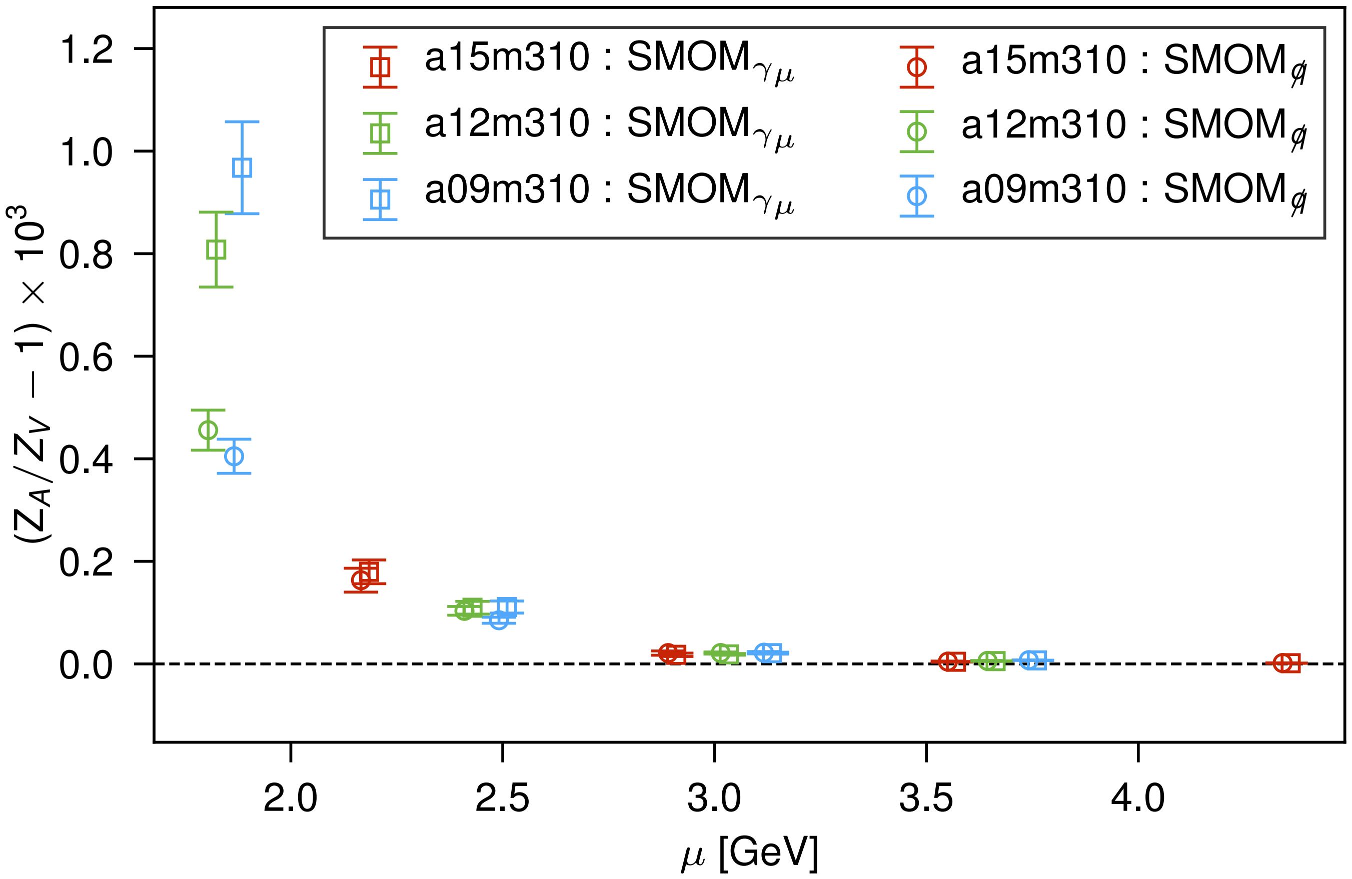
- The renormalized value of the axial coupling

$$g_A = \frac{Z_A}{Z_V} \frac{\mathring{g}_A}{\mathring{g}_V}$$

- Our action uses (Möbius) Domain-Wall fermions, which have very good chiral symmetry properties
- Using the RI/SMOM non-perturbative renormalization scheme, we find

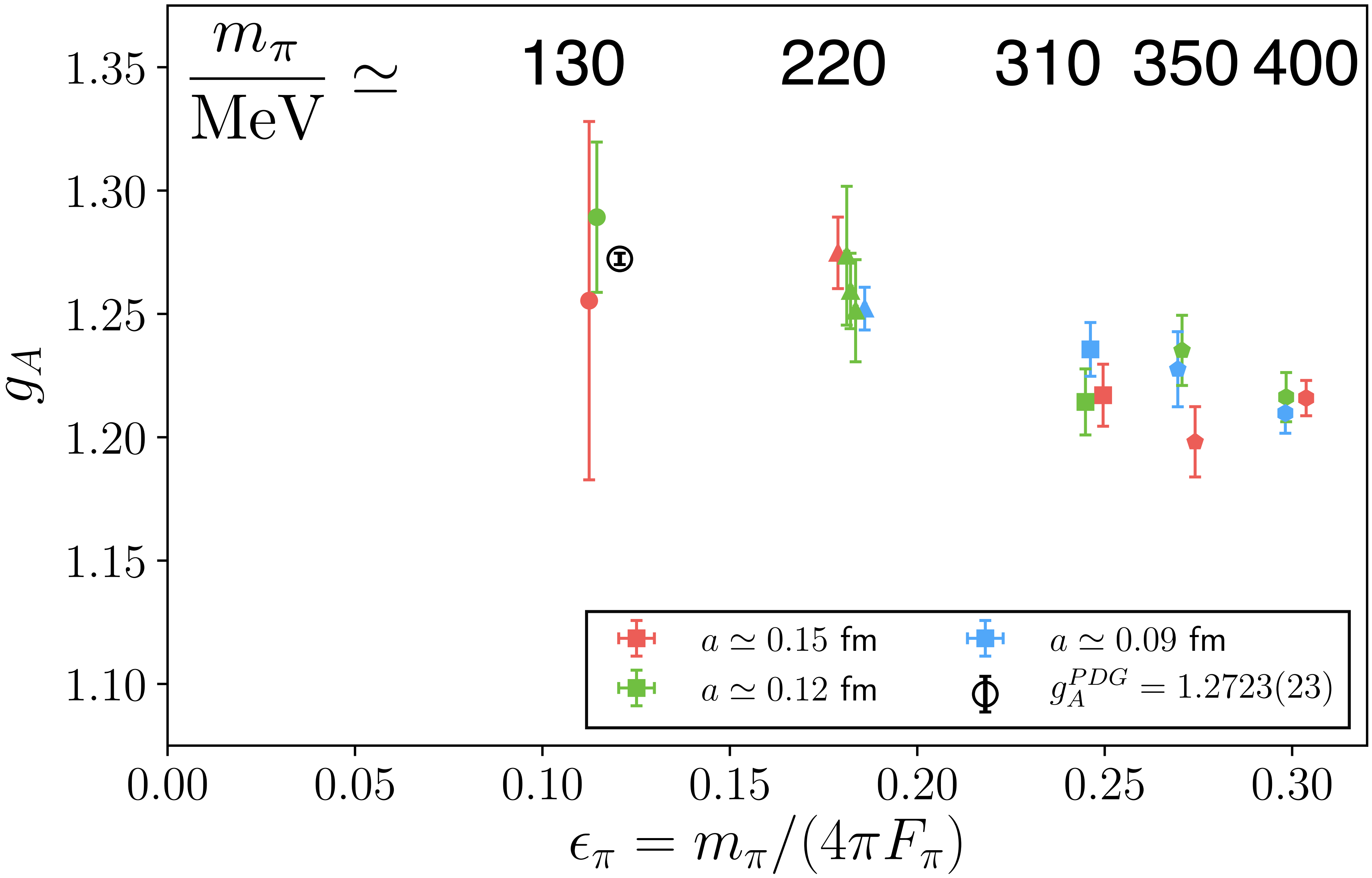
$$\frac{Z_A}{Z_V} = 1$$ **to 1 part in 10^4**

a confirmation that our action respects chiral symmetry to a good degree.



A percent-level determination of the nucleon axial coupling from QCD

Renormalized LQCD results



A percent-level determination of the nucleon axial coupling from QCD

Extrapolations

Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

- ChiPT: EFT expanding around m_π = 0
  - best hope for model-independent extrapolation
  - not guaranteed to converge around m_π = 135 MeV
- Mild m_π,a dependence
  - Taylor expansion works well for extrapolation/interpolation

Slide adapted from A. Nicholson

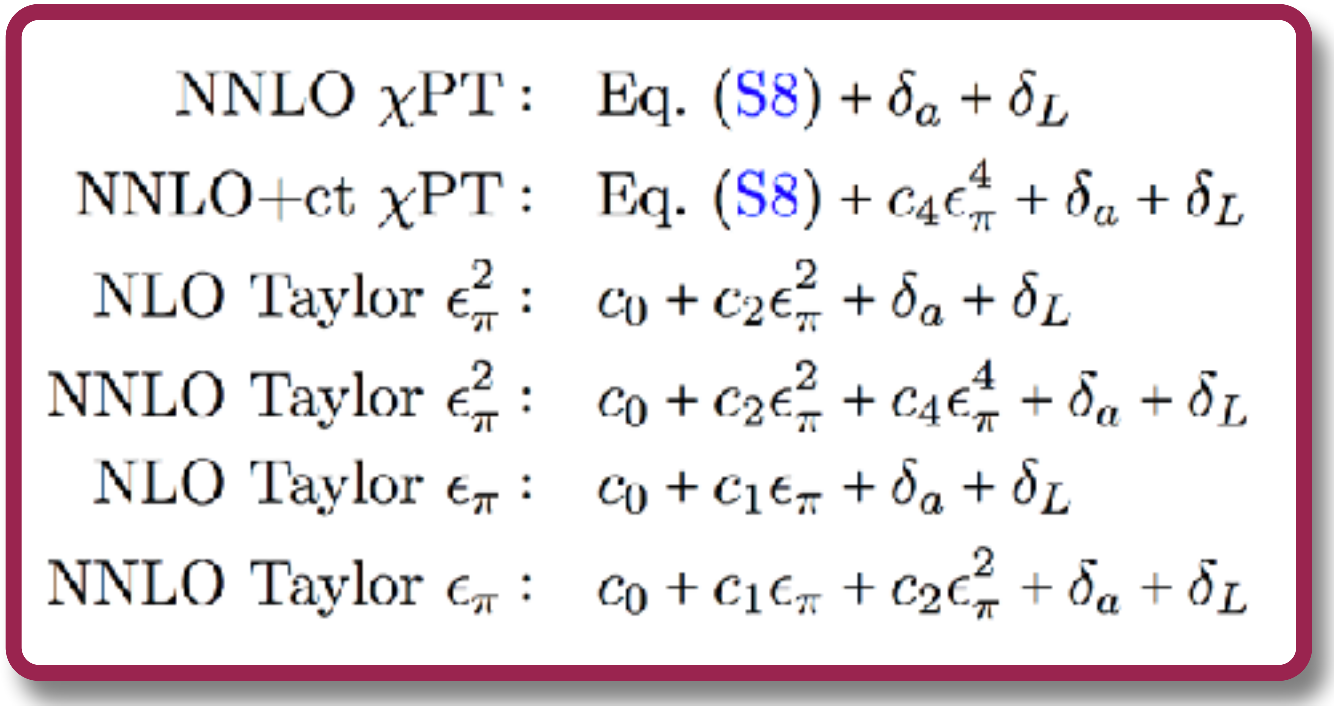
A percent-level determination of the nucleon axial coupling from QCD

Extrapolations

Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$



$$\begin{aligned}
\text{NNLO } \chi\text{PT}: &\quad \text{Eq. (S8)} + \delta_a + \delta_L \\
\text{NNLO+ct } \chi\text{PT}: &\quad \text{Eq. (S8)} + c_4\epsilon_\pi^4 + \delta_a + \delta_L \\
\text{NLO Taylor } \epsilon_\pi^2: &\quad c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L \\
\text{NNLO Taylor } \epsilon_\pi^2: &\quad c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L \\
\text{NLO Taylor } \epsilon_\pi: &\quad c_0 + c_1\epsilon_\pi + \delta_a + \delta_L \\
\text{NNLO Taylor } \epsilon_\pi: &\quad c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L
\end{aligned}$$

Slide adapted from A. Nicholson

A percent-level determination of the nucleon axial coupling from QCD

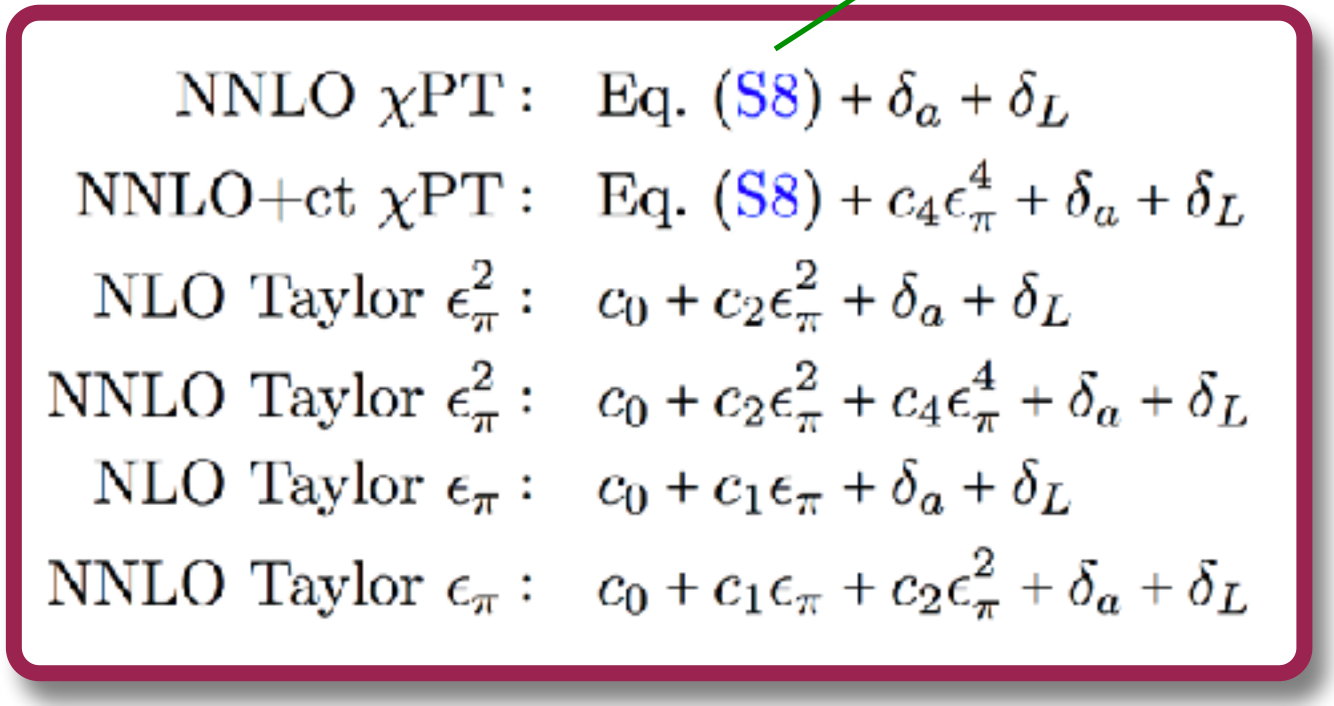
Extrapolations

Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$g_A = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln\left(\epsilon_\pi^2\right) + g_0 c_3 \epsilon_\pi^3$$



NNLO χPT : Eq. (S8) $+ \delta_a + \delta_L$

NNLO+ct χPT : Eq. (S8) $+ c_4\epsilon_\pi^4 + \delta_a + \delta_L$

NLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

NNLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$

NLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$

NNLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

Slide adapted from A. Nicholson

A percent-level determination of the nucleon axial coupling from QCD

Extrapolations

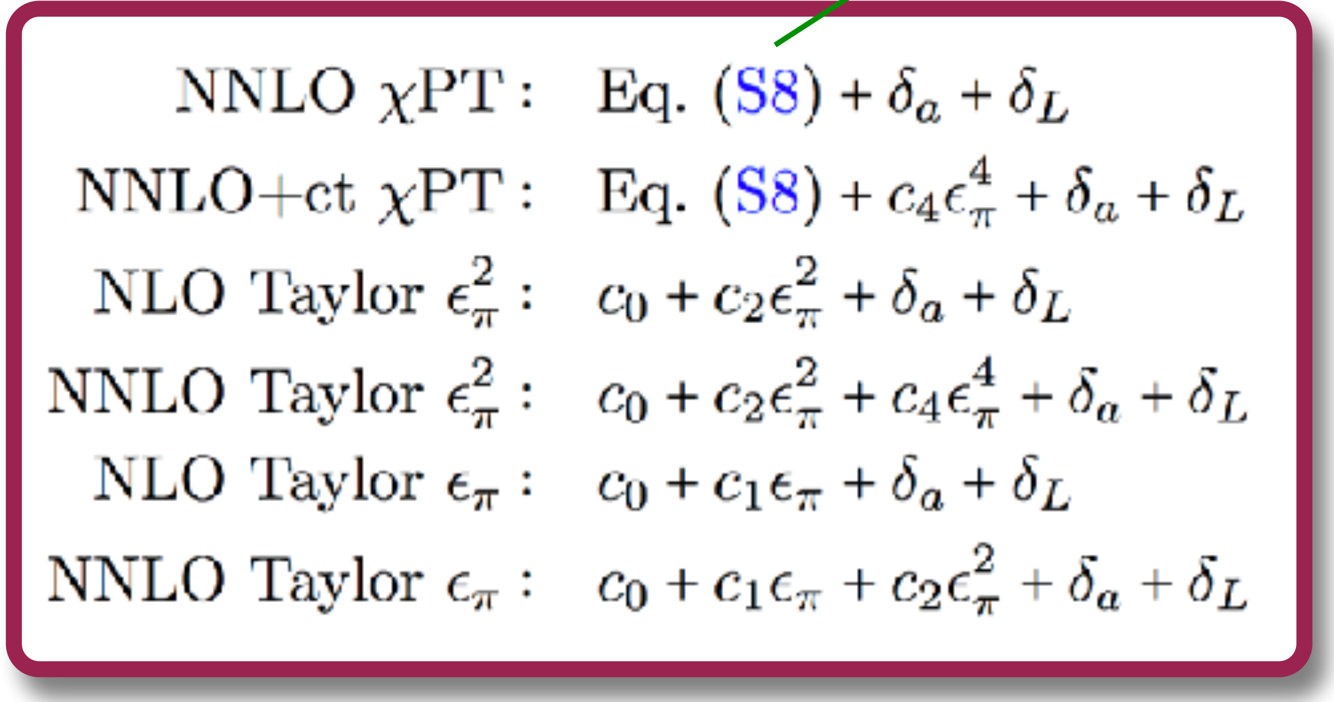
Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$\delta_a = a_2\varepsilon_a^2 + b_4\varepsilon_a^2\varepsilon_\pi^2 + a_4\varepsilon_a^4 + [a_1\sqrt{4\pi}\varepsilon_a + s_2\alpha_S\alpha_a^2]$$

$$g_A = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln\left(\epsilon_\pi^2\right) + g_0c_3\epsilon_\pi^3$$



NNLO χPT : Eq. (S8) $+ \delta_a + \delta_L$

NNLO+ct χPT : Eq. (S8) $+ c_4\epsilon_\pi^4 + \delta_a + \delta_L$

NLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

NNLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$

NLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$

NNLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

Slide adapted from A. Nicholson

A percent-level determination of the nucleon axial coupling from QCD

Extrapolations

Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$\delta_a = a_2\varepsilon_a^2 + b_4\varepsilon_a^2\varepsilon_\pi^2 + a_4\varepsilon_a^4 + [a_1\sqrt{4\pi}\varepsilon_a + s_2\alpha_S\alpha_a^2]$$

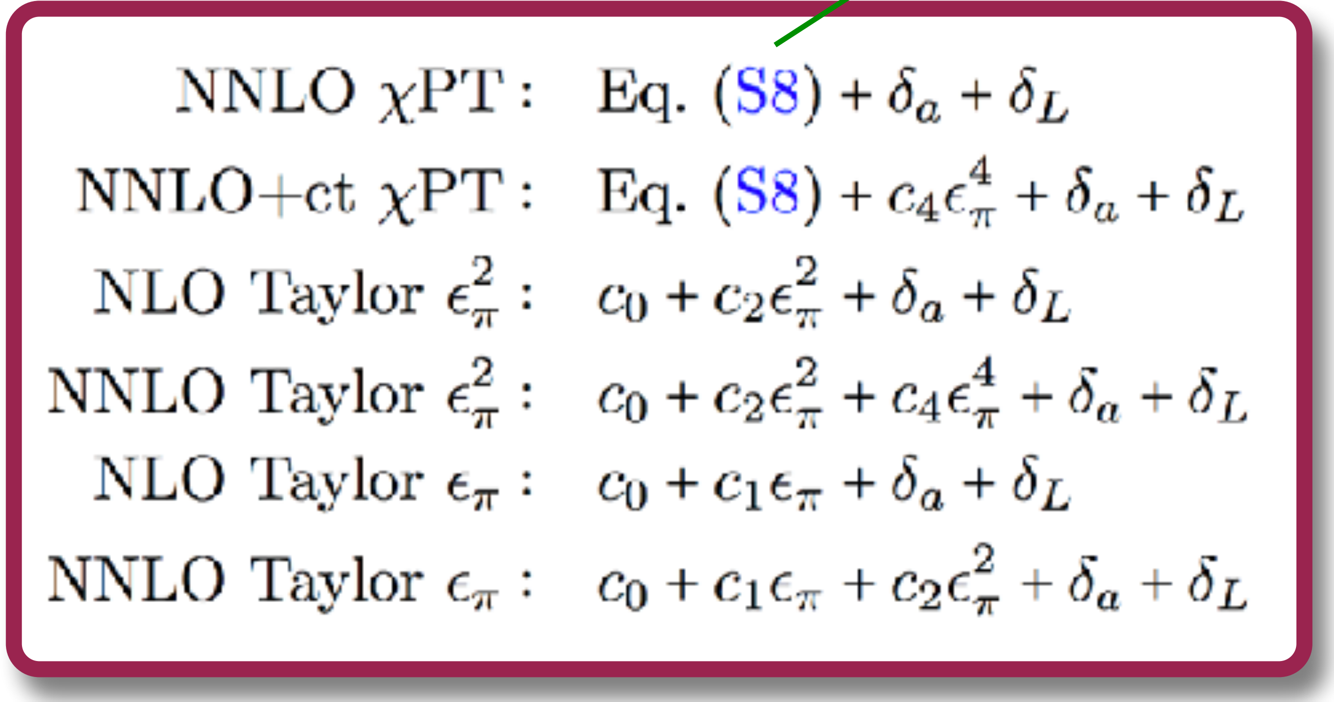
$$F_1(x) = \sum_{\mathbf{n}\neq 0}\left[K_0(x|\mathbf{n}|) - \frac{K_1(x|\mathbf{n}|)}{x|\mathbf{n}|}\right]$$

$$\delta_L = \underline{\frac{8}{3}\varepsilon_\pi^2\left[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)\right]} + f_3\varepsilon_\pi^3 F_1(m_\pi L)$$

$$F_3(x) = -\frac{3}{2}\sum_{\mathbf{n}\neq 0}\frac{K_1(x|\mathbf{n}|)}{x|\mathbf{n}|}$$

Beane and Savage
Phys.Rev.D70 [hep-ph/0404131]

$$g_A = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln\left(\epsilon_\pi^2\right) + g_0 c_3\epsilon_\pi^3$$

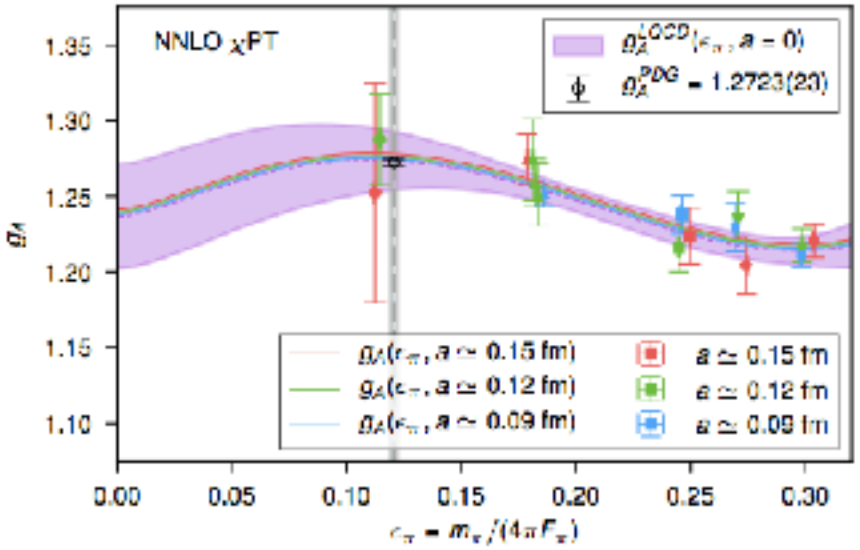


NNLO χPT : Eq. (S8) $+ \delta_a + \delta_L$
NNLO+ct χPT : Eq. (S8) $+ c_4\epsilon_\pi^4 + \delta_a + \delta_L$
NLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$
NNLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$
NLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$
NNLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

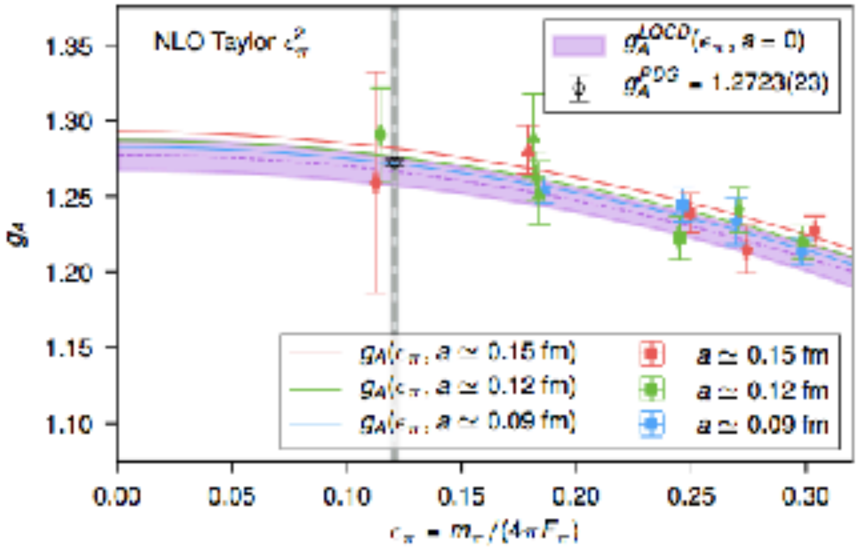
Slide adapted from A. Nicholson

A percent-level determination of the nucleon axial coupling from QCD

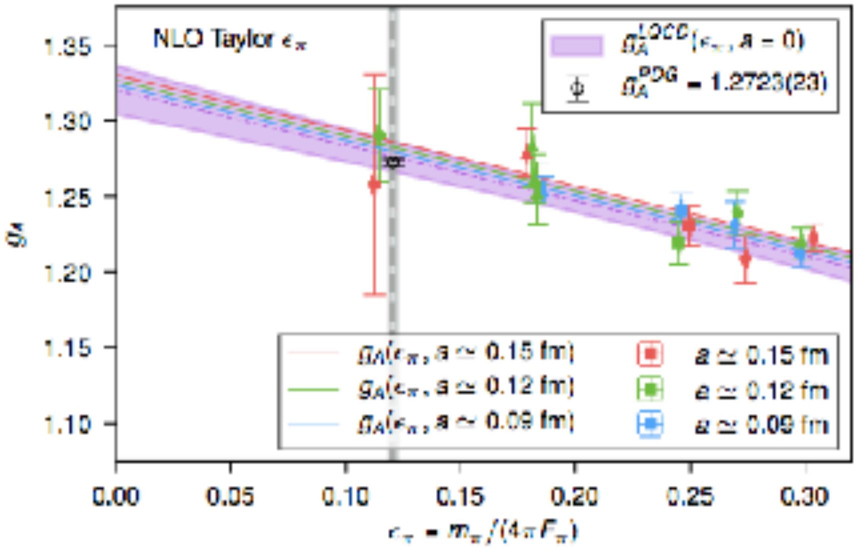
Extrapolations



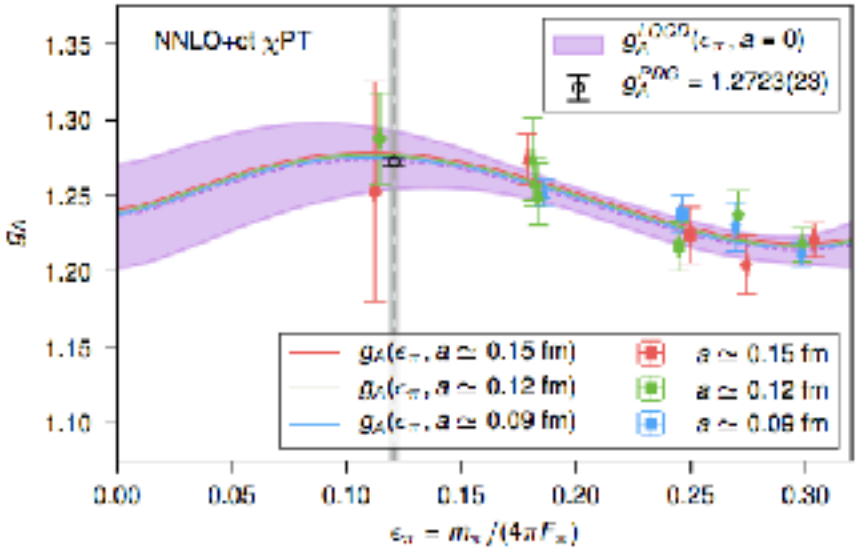
c



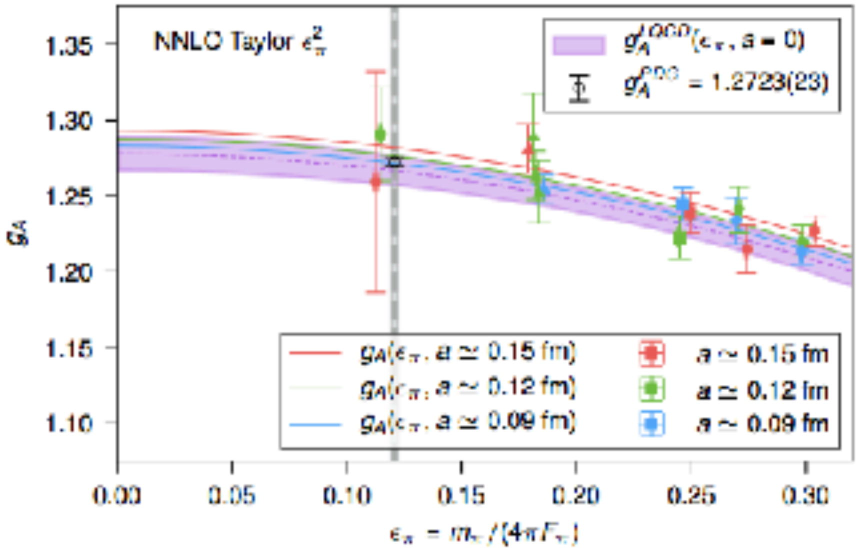
d



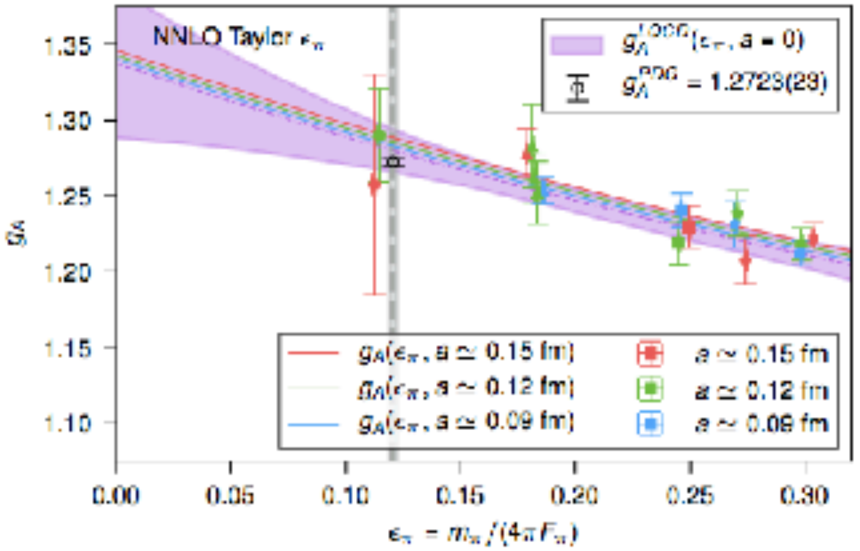
e



f



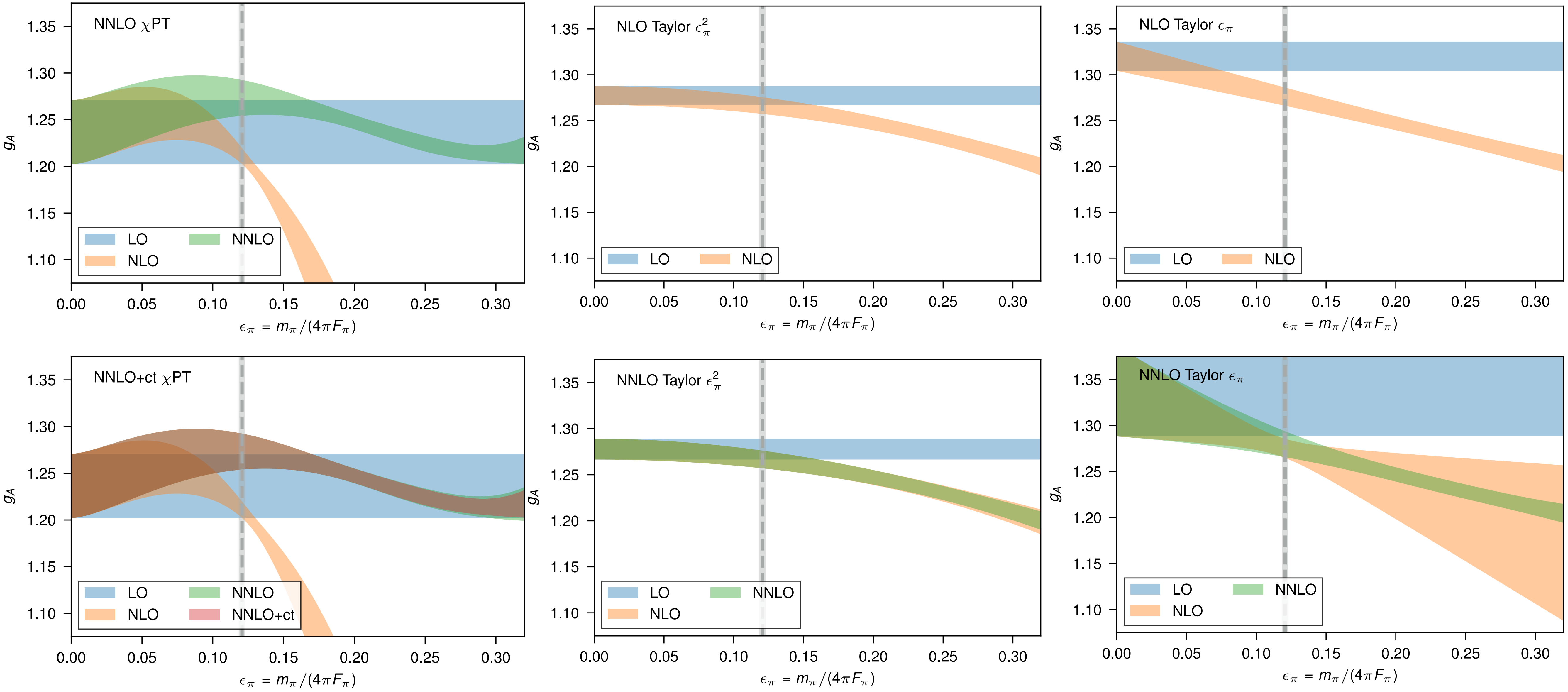
g



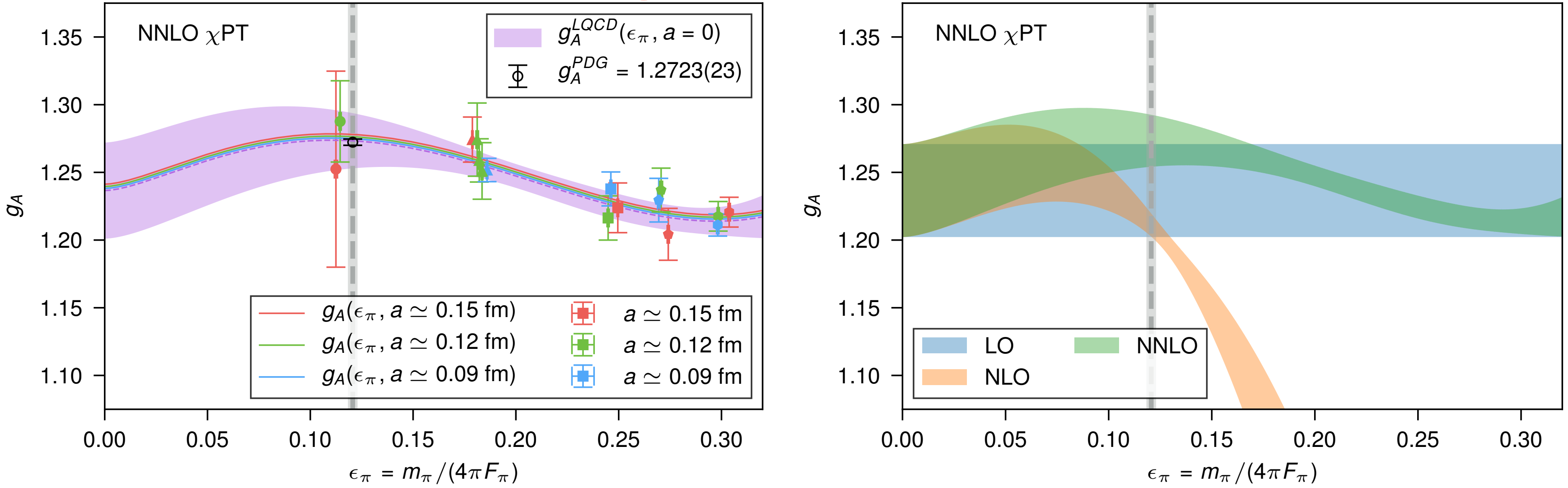
h

A percent-level determination of the nucleon axial coupling from QCD

Convergence of chiral extrapolation

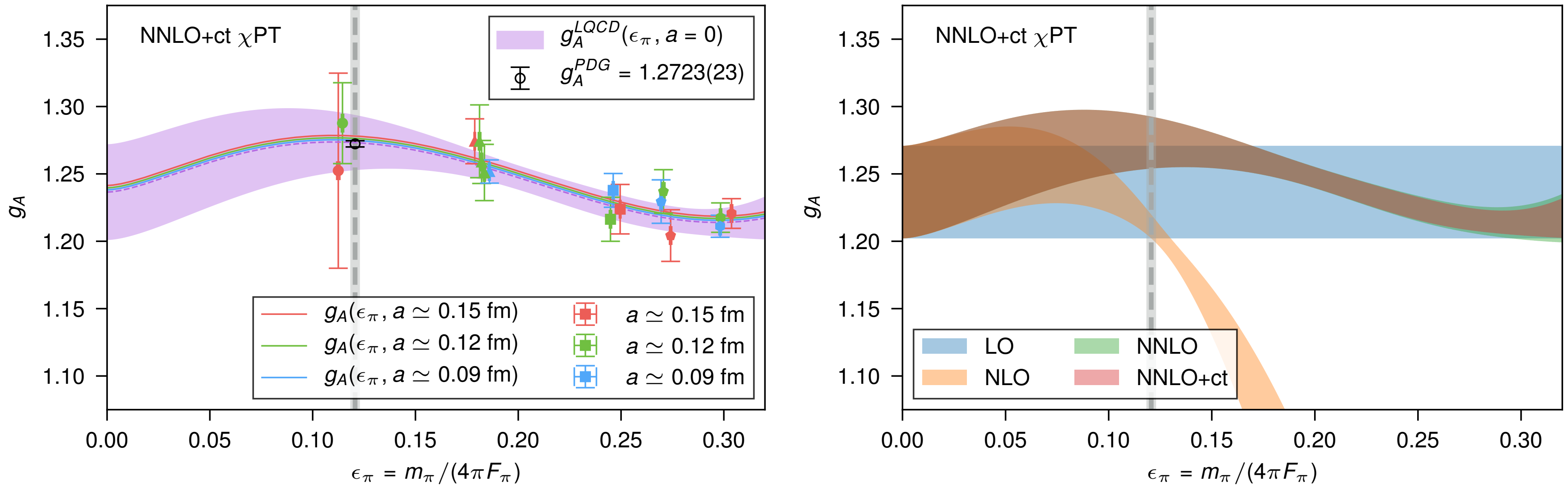


convergence of the chiral expansion…

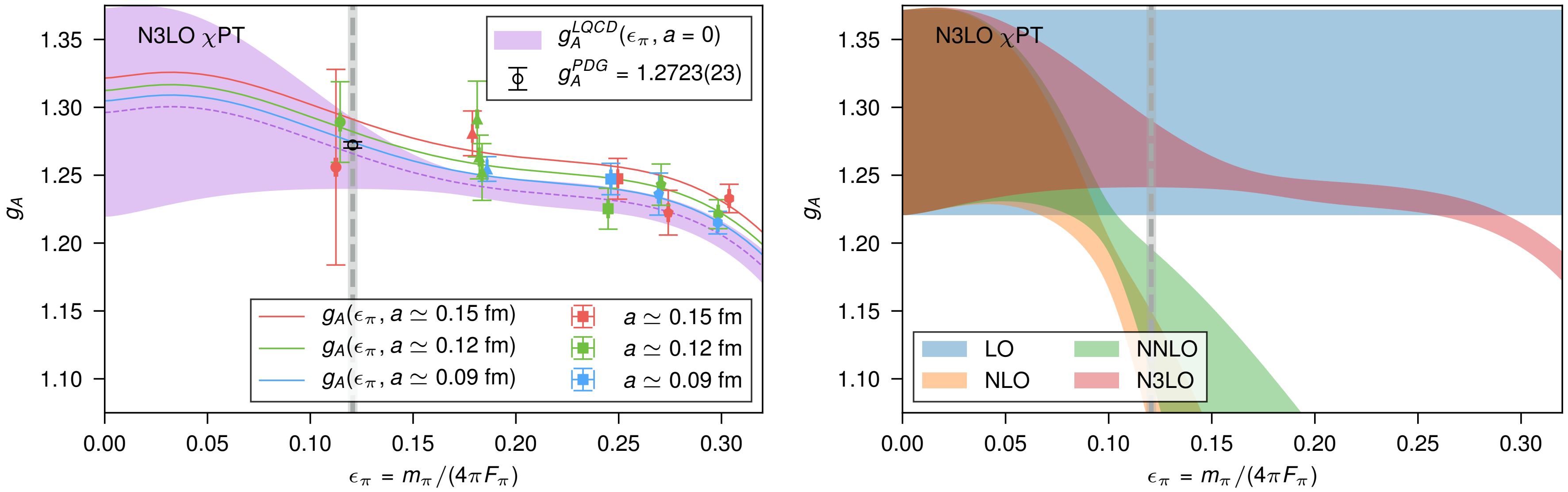


$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3\epsilon_\pi^3$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

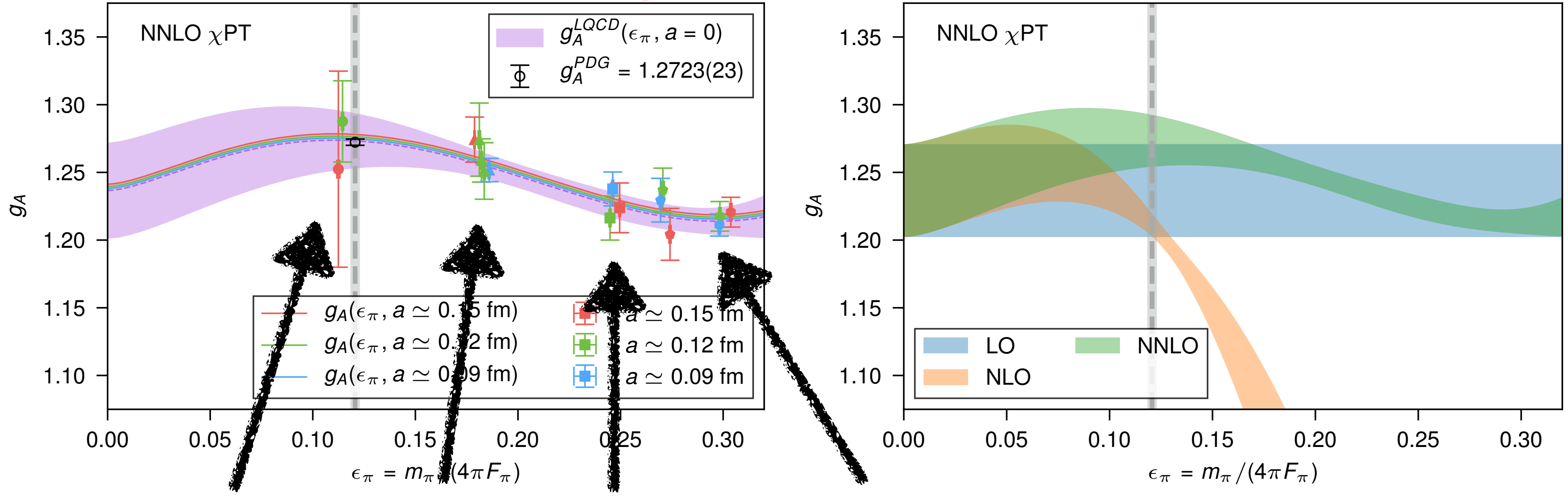


$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3\epsilon_\pi^3 + c_4\epsilon_\pi^4$$



$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3\epsilon_\pi^3 + \epsilon_\pi^4\Big[c_4 + \tilde{\gamma}_4\ln(\epsilon_\pi^2) + \left(\frac{2}{3}g_0 + \frac{37}{12}g_0^3 + 4g_0^5\right)\ln^2(\epsilon_\pi^2)\Big]$$

convergence of the chiral expansion…



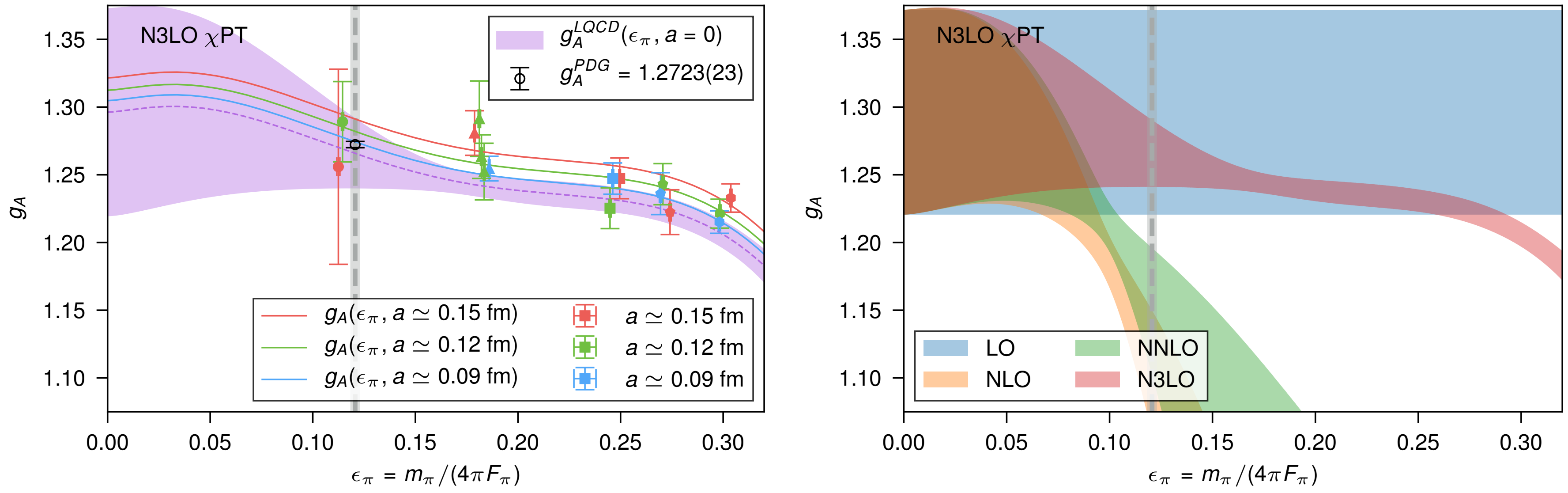
$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$m_\pi \sim 130$ MeV 220 310 400

can we trust extrapolation of quantities with chiraly-enhanced behavior?

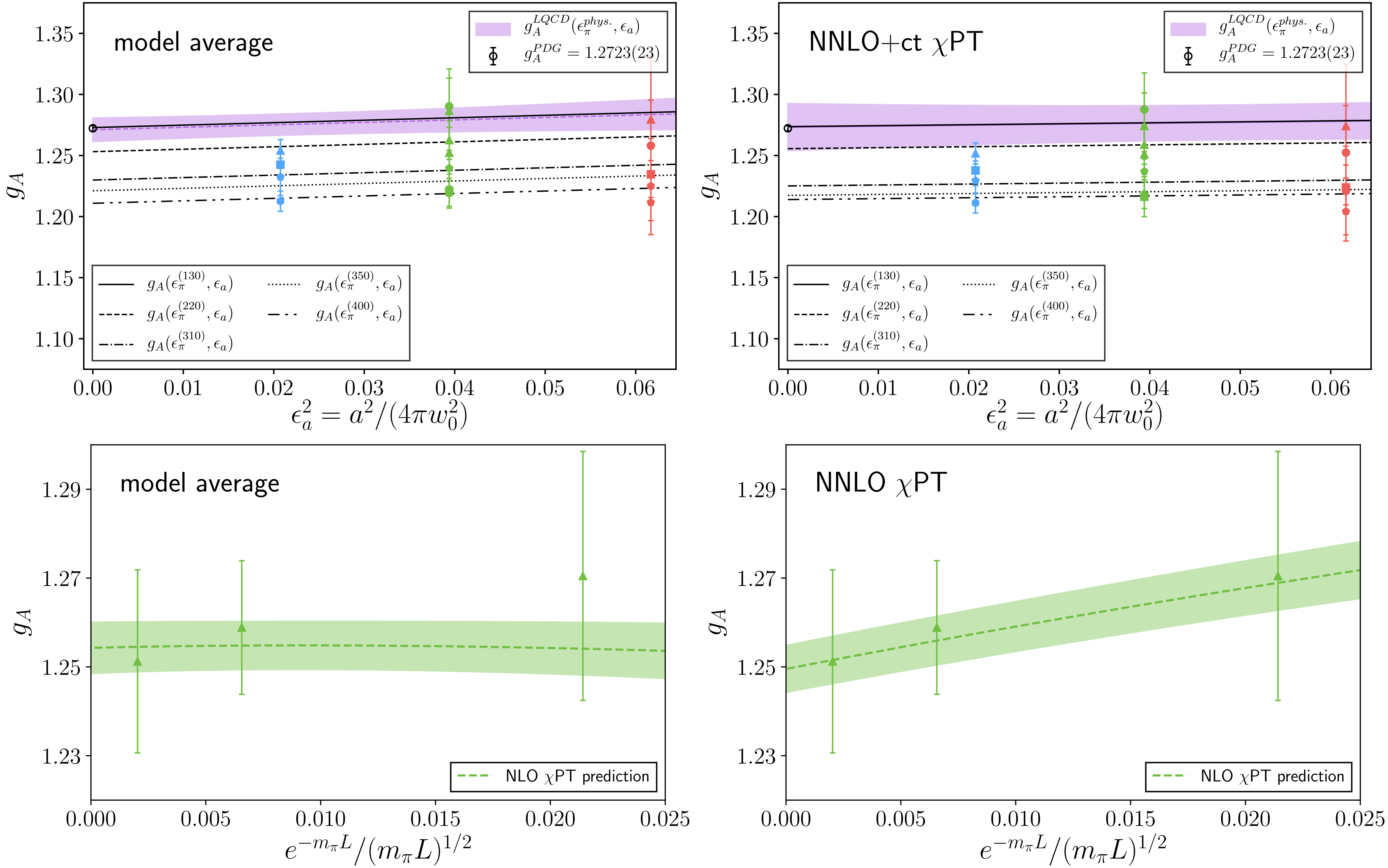
if the single nucleon is not converging, would you trust chiral extrapolations of two or more nucleons?



$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4\left[c_4 + \tilde{\gamma}_4\ln(\epsilon_\pi^2) + \left(\frac{2}{3}g_0 + \frac{37}{12}g_0^3 + 4g_0^5\right)\ln^2(\epsilon_\pi^2)\right]$$

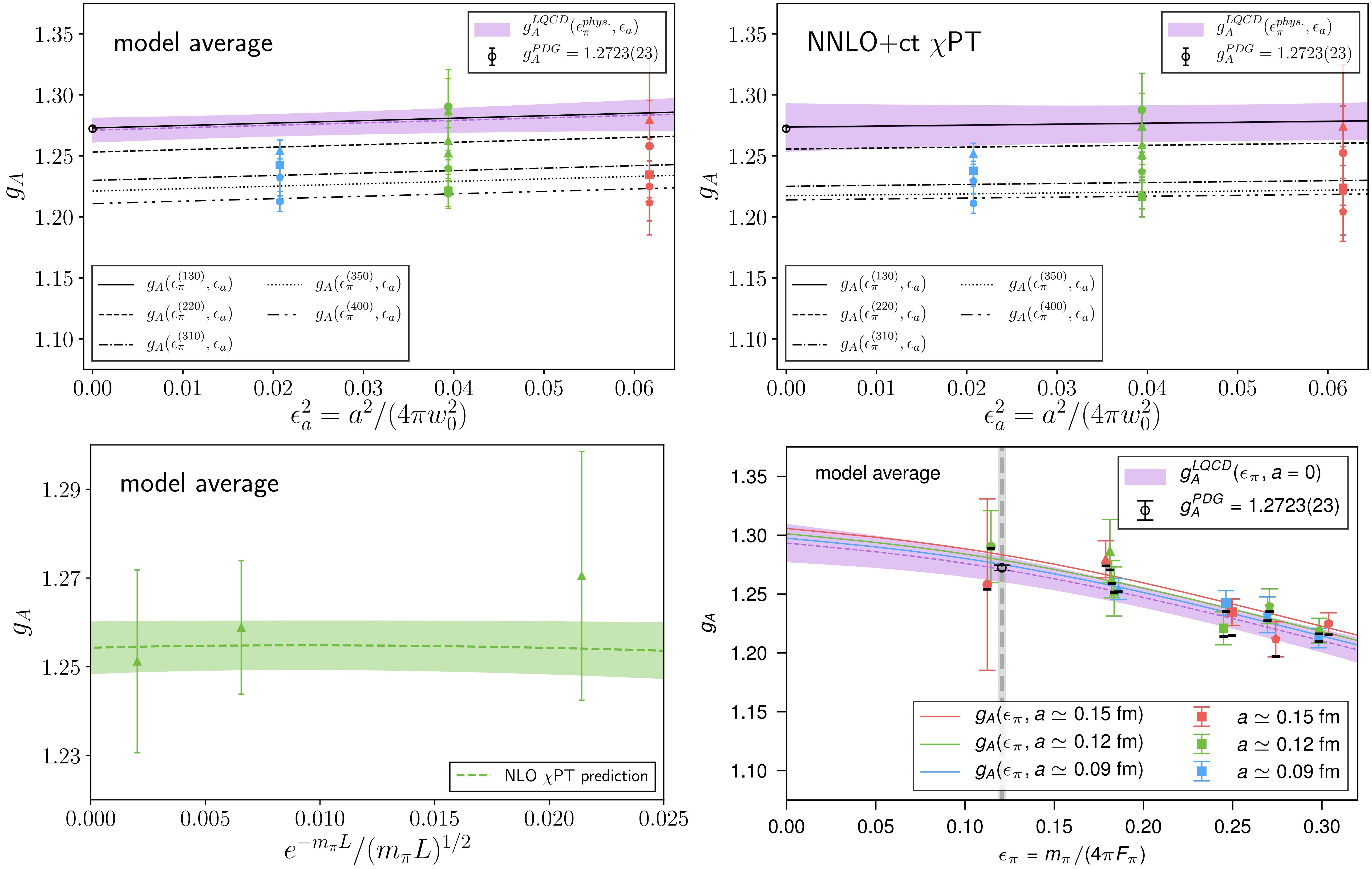
A percent-level determination of the nucleon axial coupling from QCD

Continuum and infinite volume extrapolation



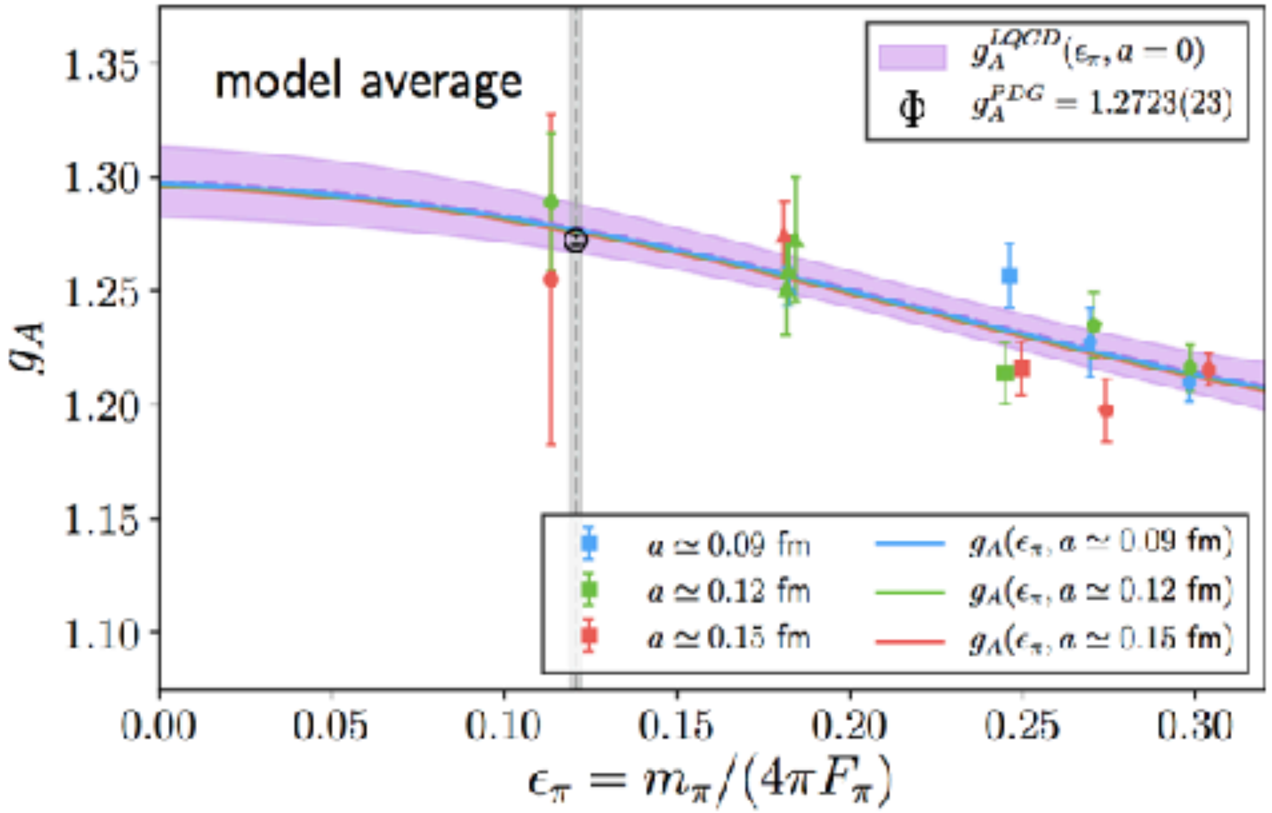
A percent-level determination of the nucleon axial coupling from QCD

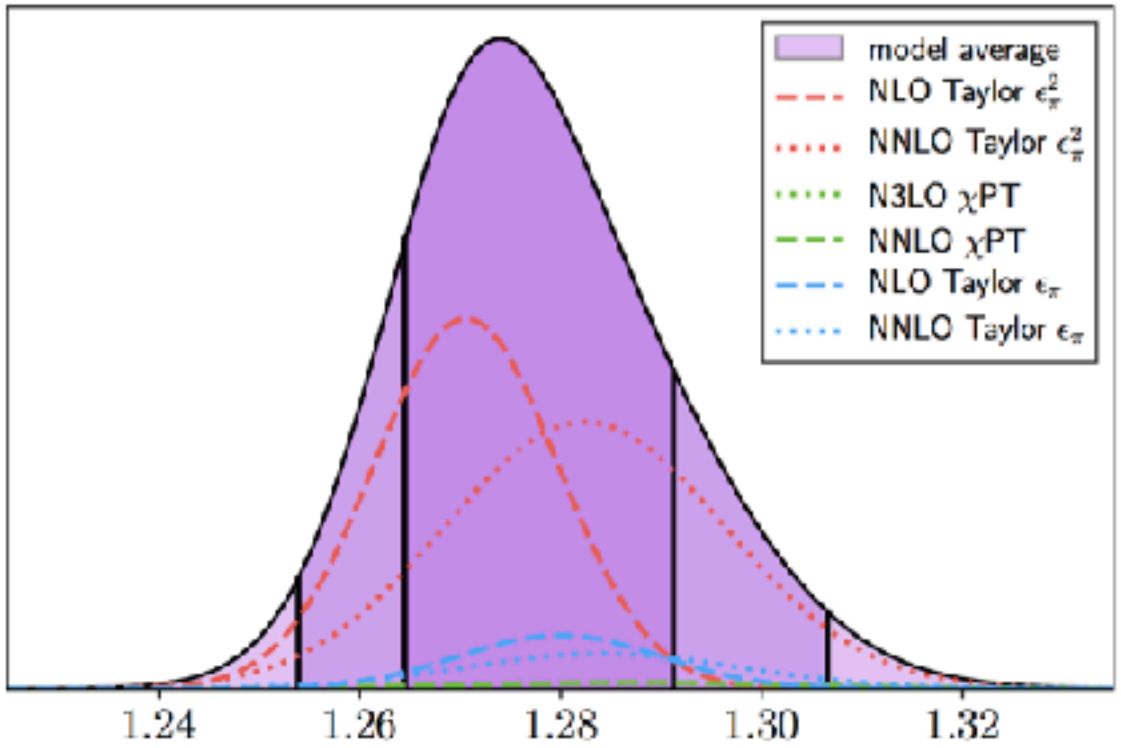
Continuum and infinite volume extrapolation



A percent-level determination of the nucleon axial coupling from QCD

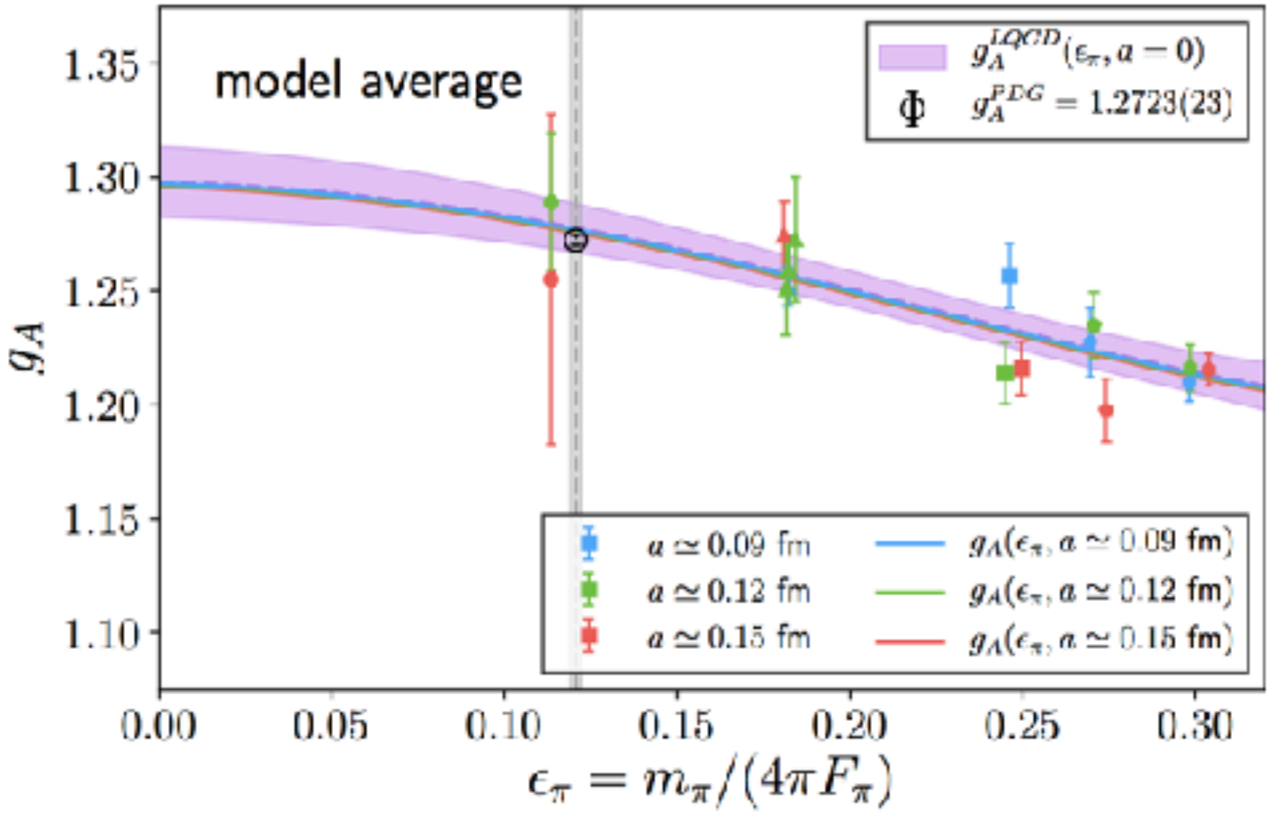
Model average extrapolation

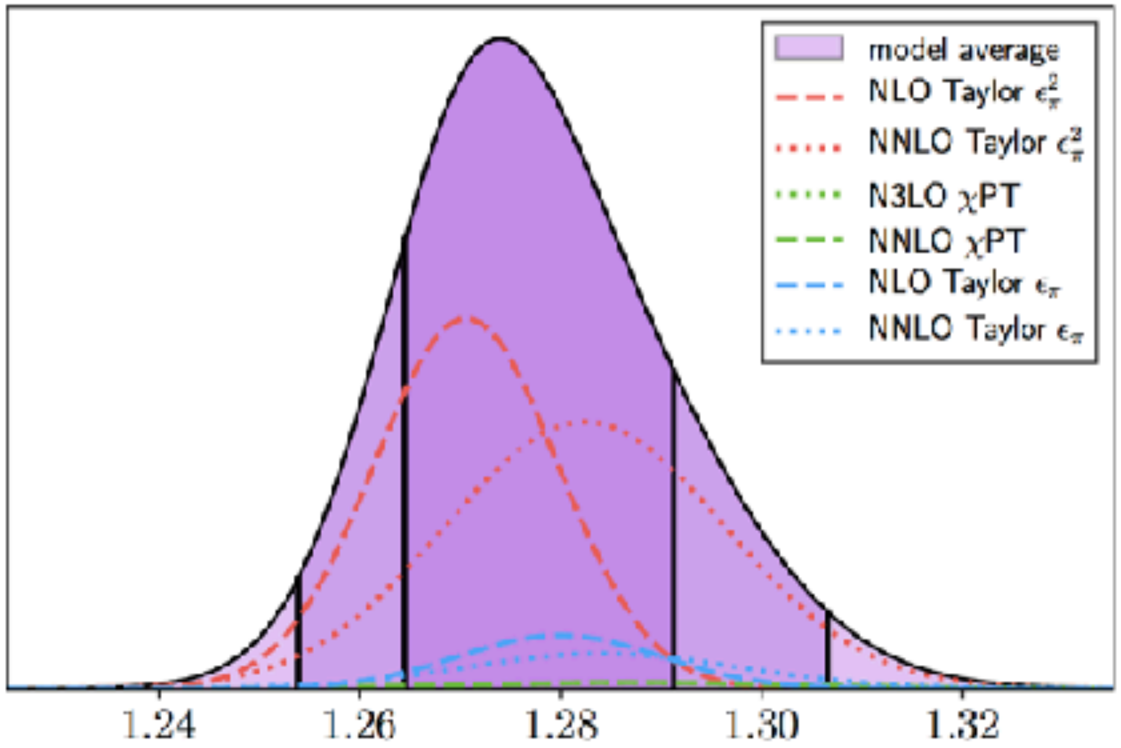




A percent-level determination of the nucleon axial coupling from QCD

Model average extrapolation





Fit	χ^2/dof	$\mathcal{L}(D\mid M_k)$	$P(M_k\mid D)$	$P(g_A\mid M_k)$
NNLO χPT	0.727	22.734	0.033	1.273(19)
NNLO+ct χPT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
NNLO Taylor ϵ_π^2	0.787	24.897	0.284	1.267(10)
NLO Taylor ϵ_π	0.700	24.855	0.191	1.276(10)
NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

NNLO χPT : Eq. (S8) $+ \delta_a + \delta_L$

NNLO+ct χPT : Eq. (S8) $+ c_4\epsilon_\pi^4 + \delta_a + \delta_L$

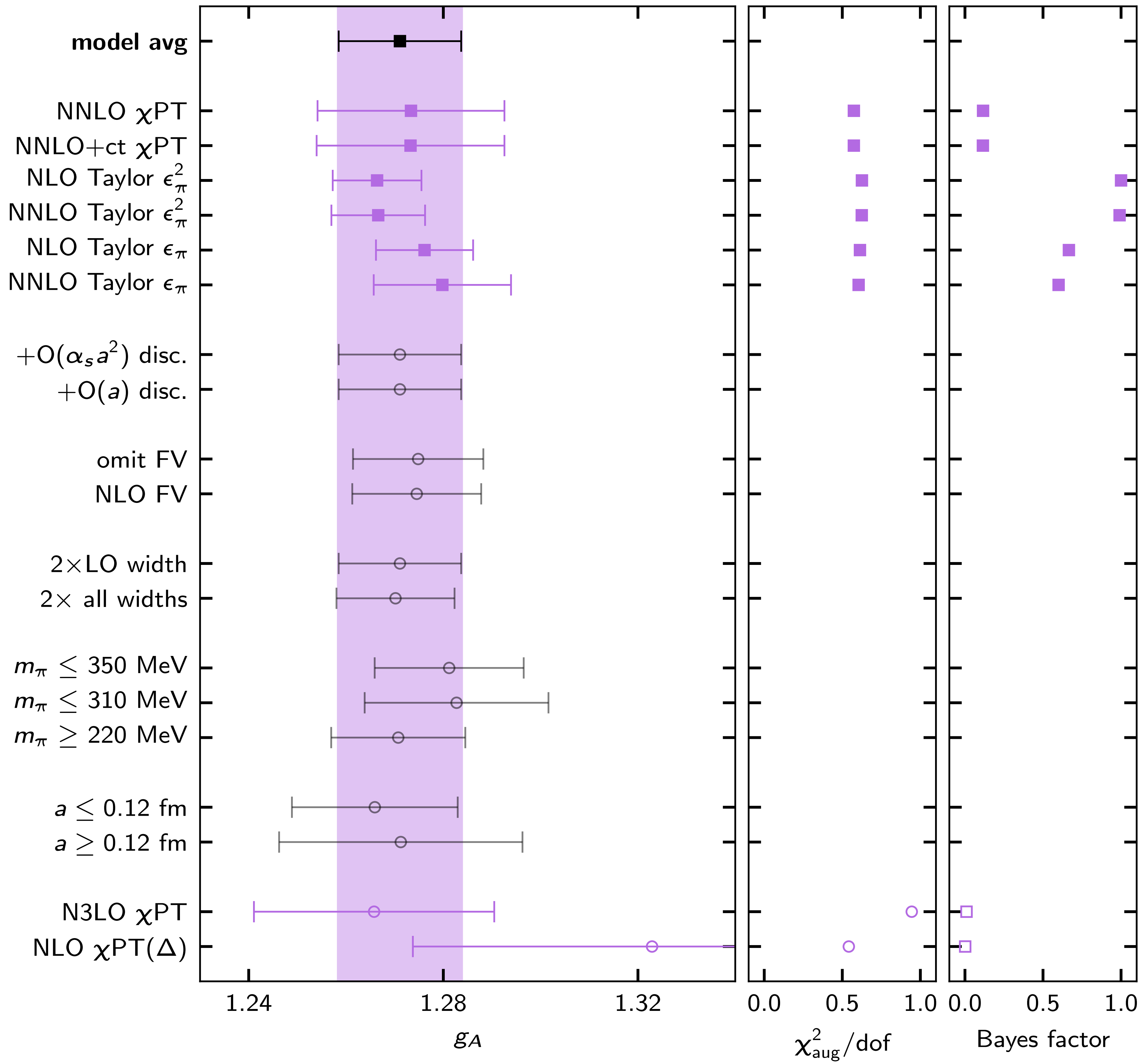
NLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

NNLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$

NLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$

NNLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

Stability of Extrapolation Analysis



Final result

6 fits included in the model average and their relative weights

discretization corrections

finite volume corrections

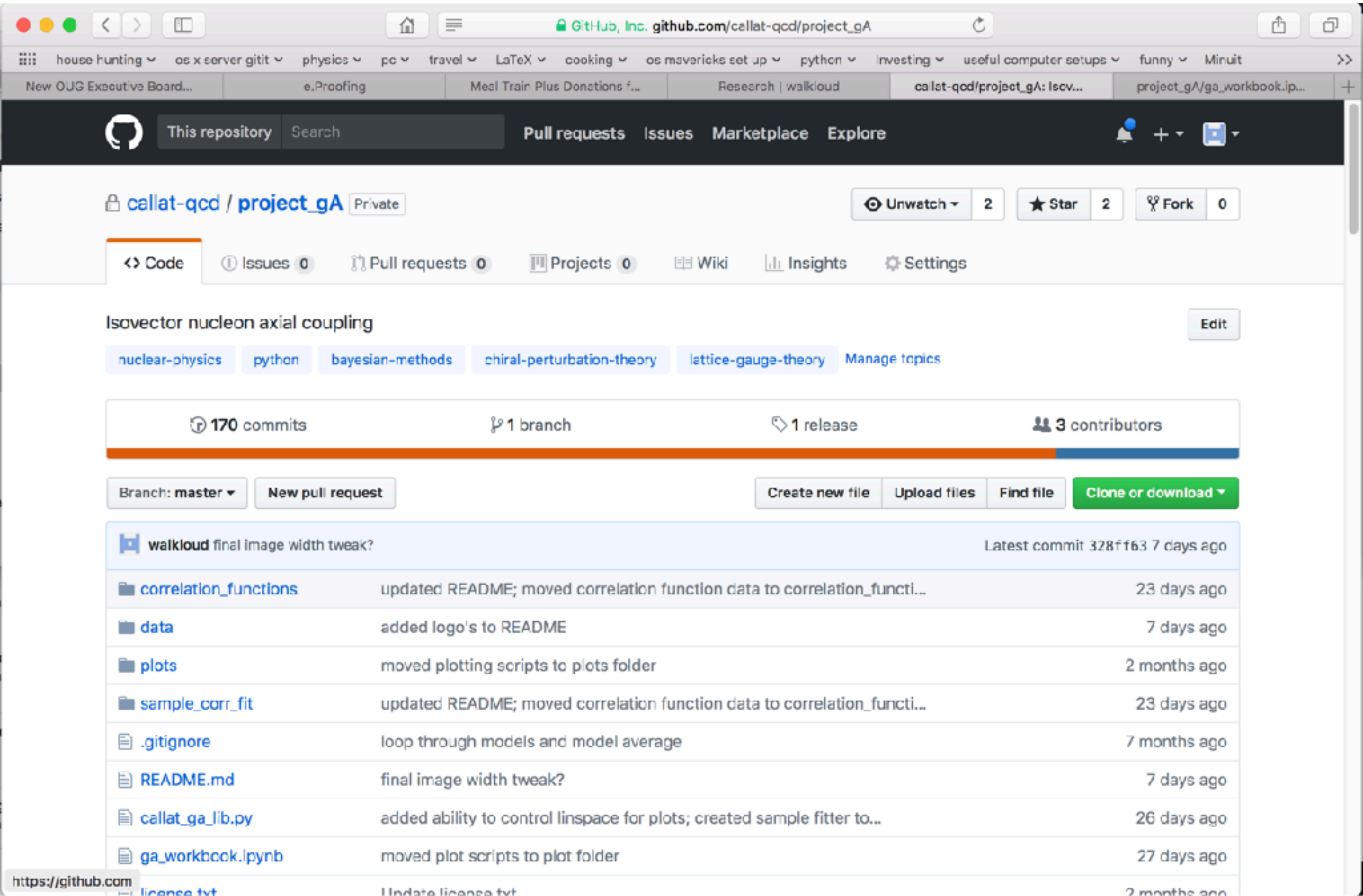
sensitivity to prior width

sensitivity to pion mass cuts

sensitivity to lattice spacing cuts

additional XPT analysis

Stability of Extrapolation Analysis



callat-qcd / **project_gA** Private

Isovector nucleon axial coupling

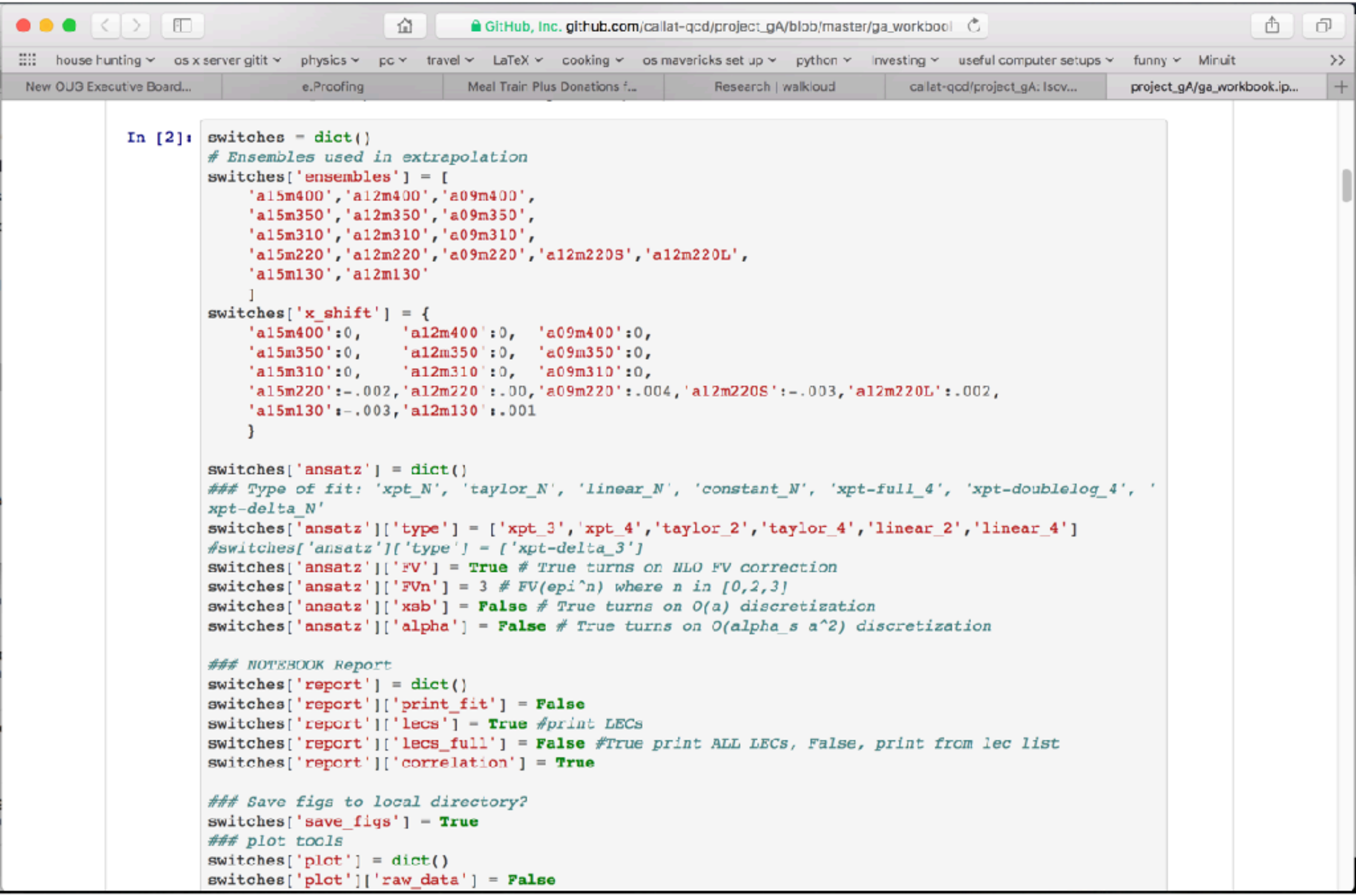
nuclear-physics · python · bayesian-methods · chiral-perturbation-theory · lattice-gauge-theory · Manage topics

170 commits · 1 branch · 1 release · 3 contributors

walkloud final image width tweak? — Latest commit 328ff63 7 days ago

correlation_functions	updated README; moved correlation function data to correlation_functi...	23 days ago
data	added logo's to README	7 days ago
plots	moved plotting scripts to plots folder	2 months ago
sample_corr_fit	updated README; moved correlation function data to correlation_functi...	23 days ago
.gitignore	loop through models and model average	7 months ago
README.md	final image width tweak?	7 days ago
callat_ga_lib.py	added ability to control linspace for plots; created sample fitter to...	26 days ago
ga_workbook.ipynb	moved plot scripts to plot folder	27 days ago
license.txt	Update license.txt	2 months ago

Stability of Extrapolation Analysis

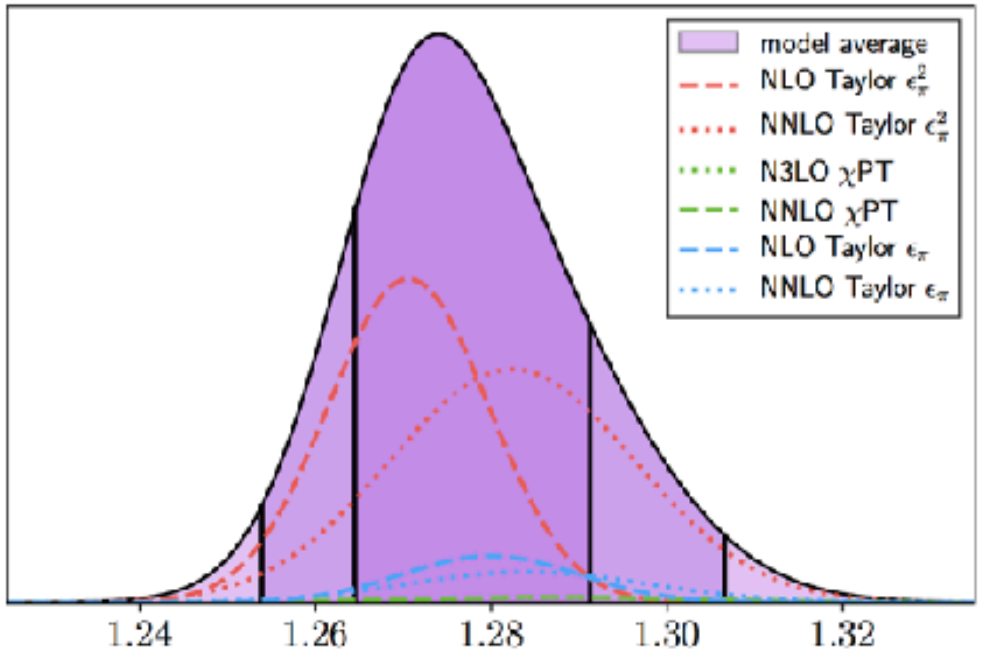


```
switches = dict()
# Ensembles used in extrapolation
switches['ensembles'] = [
    'a15m400','a12m400','a09m400',
    'a15m350','a12m350','a09m350',
    'a15m310','a12m310','a09m310',
    'a15m220','a12m220','a09m220','a12m220S','a12m220L',
    'a15m130','a12m130'
    ]
switches['x_shift'] = {
    'a15m400':0,    'a12m400':0,  'a09m400':0,
    'a15m350':0,    'a12m350':0,  'a09m350':0,
    'a15m310':0,    'a12m310':0,  'a09m310':0,
    'a15m220':-.002,'a12m220':.00,'a09m220':.004,'a12m220S':-.003,'a12m220L':.002,
    'a15m130':-.003,'a12m130':.001
    }

switches['ansatz'] = dict()
### Type of fit: 'xpt_N', 'taylor_N', 'linear_N', 'constant_N', 'xpt-full_4', 'xpt-doublelog_4', '
xpt-delta_N'
switches['ansatz']['type'] = ['xpt_3','xpt_4','taylor_2','taylor_4','linear_2','linear_4']
#switches['ansatz']['type'] = ['xpt-delta_3']
switches['ansatz']['FV'] = True # True turns on NLO FV correction
switches['ansatz']['FVn'] = 3 # FV(epi^n) where n in [0,2,3]
switches['ansatz']['xsb'] = False # True turns on O(a) discretization
switches['ansatz']['alpha'] = False # True turns on O(alpha_s a^2) discretization

### NOTEBOOK Report
switches['report'] = dict()
switches['report']['print_fit'] = False
switches['report']['lecs'] = True #print LECs
switches['report']['lecs_full'] = False #True print ALL LECs, False, print from lec list
switches['report']['correlation'] = True

### Save figs to local directory?
switches['save_figs'] = True
### plot tools
switches['plot'] = dict()
switches['plot']['raw_data'] = False
```

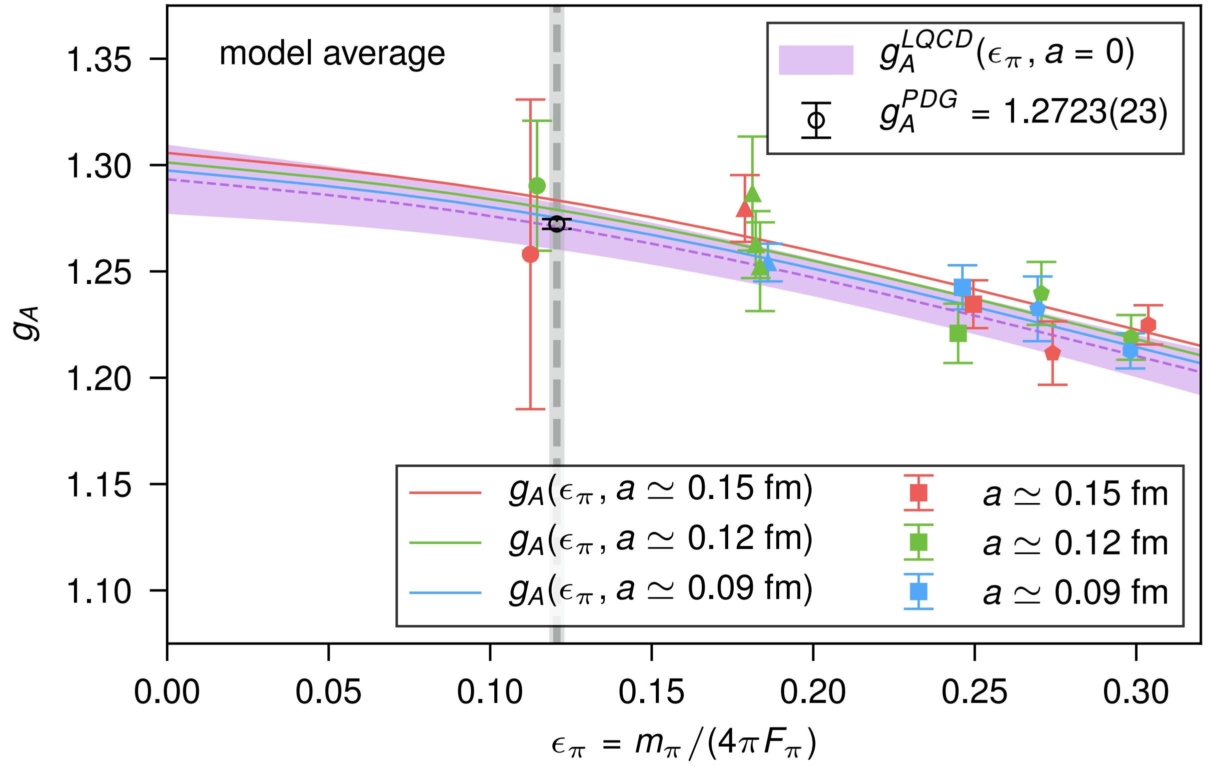
A percent-level determination of the nucleon axial coupling from QCD

Final result

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



$$g_A^{\mathrm{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$



A percent-level determination of the nucleon axial coupling from QCD

Lattice QCD Team
Glasgow: Chris Bouchard
INT: Chris Monahan
JLab: Balint Jóo
Jülich: Evan Berkowitz
LBL/UCB: David Brantley, Chia Cheng (Jason) Chang, T. Kurth (NERSC), Henry Monge-Camacho, AWL
LLNL: Pavlos Vranas
Liverpool: Nicolas Garron
NVIDIA: Kate Clark
RIKEN/BNL: Enrico Rinaldi
UNC: Amy Nicholson
William and Mary: Kostas Orginos

plus a few friends

DOE Topical Collaboration
Double Beta Decay

$$\begin{aligned} g_A^{\rm QCD} &= 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M \\ &= 1.2711(126) \\ g_A^{\rm UCNA} &= 1.2772(020) \end{aligned}$$

experiment factor of 6 more precise

- ☐ The success of this result was enabled through a few features of the calculation:
  - ☐ an unconventional strategy that can exploit **exponentially more precise data** at early time and has **demonstrable control of excited state** contributions
  - ☐ an action with **improved stochastic behavior**, a very **mild continuum** extrapolation, **highly suppressed chiral symmetry breaking**
  - ☐ access to a set of ensembles (**MILC**) that allowed for control over all standard lattice systematics (physical pion mass, continuum and infinite volume limits)
  - ☐ ***ludicrously fast*** GPU code

A percent-level determination of the nucleon axial coupling from QCD

Lattice QCD Team
Glasgow: Chris Bouchard
INT: Chris Monahan
JLab: Balint Jóo
Jülich: Evan Berkowitz
LBL/UCB: David Brantley, Chia Cheng (Jason) Chang, T. Kurth (NERSC), Henry Monge-Camacho, AWL
LLNL: Pavlos Vranas
Liverpool: Nicolas Garron
NVIDIA: Kate Clark
RIKEN/BNL: Enrico Rinaldi
UNC: Amy Nicholson
William and Mary: Kostas Orginos

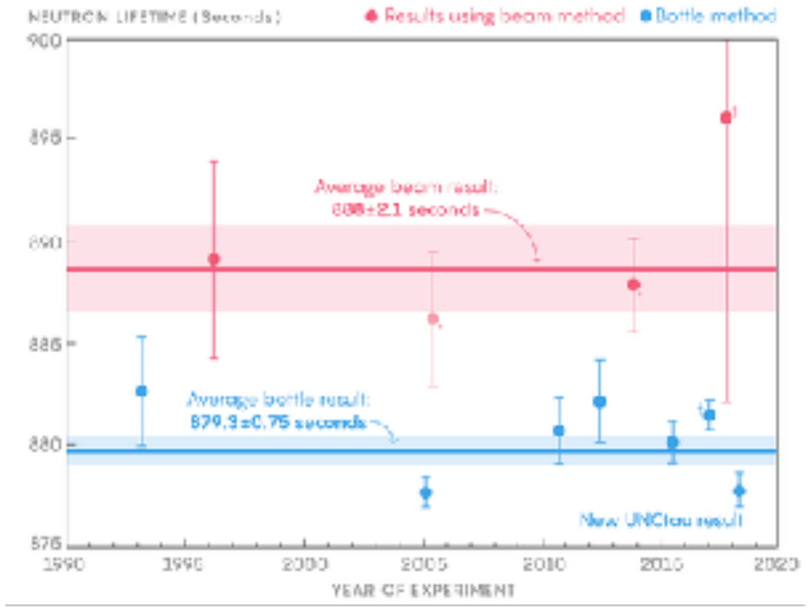
plus a few friends

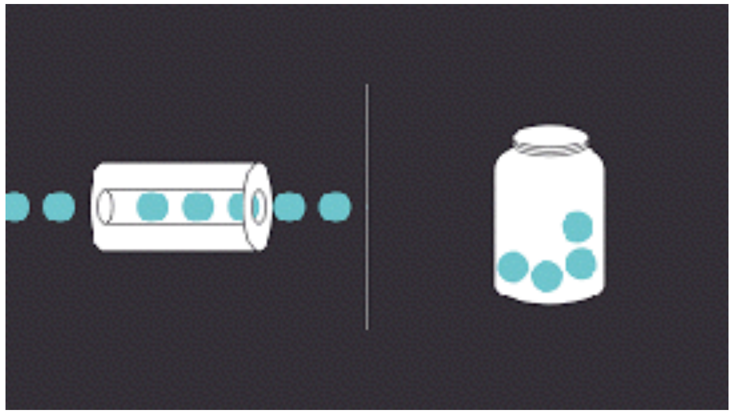
DOE Topical Collaboration
Double Beta Decay

$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$
$$= 1.2711(126)$$
$$g_A^{\text{UCNA}} = 1.2772(020)$$

experiment factor of 6 more precise

- ☐ there is a 4-sigma discrepancy: beam $\tau_n^{\text{beam}} = 888.0(2.0)s$ and bottle $\tau_n^{\text{bottle}} = 879.4(0.6)s$ measurements of the neutron lifetime, new physics (dark matter) or unknown systematic?





Czarnecki, Marciano, Sirlin
arXiv:1802.01804

$$\tau_n = \frac{5172.0(1.0)\ s}{1 + 3g_A^2}$$

A percent-level determination of the nucleon axial coupling from QCD

Lattice QCD Team
Glasgow: Chris Bouchard
INT: Chris Monahan
JLab: Balint Jóo
Jülich: Evan Berkowitz
LBL/UCB: David Brantley, Chia Cheng (Jason) Chang, T. Kurth (NERSC), Henry Monge-Camacho, AWL
LLNL: Pavlos Vranas
Liverpool: Nicolas Garron
NVIDIA: Kate Clark
RIKEN/BNL: Enrico Rinaldi
UNC: Amy Nicholson
William and Mary: Kostas Orginos

plus a few friends

DOE Topical Collaboration
Double Beta Decay

$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

☐ The method is readily extended to
  ☐ flavor changing currents
  ☐ non-zero momentum transfer
  ☐ multiple current insertions
  ☐ multi-nucleon systems

A percent-level determination of the nucleon axial coupling from QCD

Lattice QCD Team

Glasgow:	Chris Bouchard
INT:	Chris Monahan
JLab:	Balint Jóo
Jülich:	Evan Berkowitz
LBL/UCB:	David Brantley, Chia Cheng (Jason) Chang, T. Kurth (NERSC), Henry Monge-Camacho, AWL
LLNL:	Pavlos Vranas
Liverpool:	Nicolas Garron
NVIDIA:	Kate Clark
RIKEN/BNL:	Enrico Rinaldi
UNC:	Amy Nicholson
William and Mary:	Kostas Orginos

CalLat

plus a few friends

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

OAK RIDGE National Laboratory LEADERSHIP COMPUTING FACILITY

Lawrence Livermore National Laboratory

SciDAC

DOE Topical Collaboration
Double Beta Decay

U.S. DEPARTMENT OF ENERGY Office of Science

$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

The era of precision applications of LQCD to Nuclear Physics is now upon us! (at least for single nucleons)

- ☐ The method is readily extended to
  - ☐ flavor changing currents
  - ☐ non-zero momentum transfer
  - ☐ multiple current insertions
  - ☐ multi-nucleon systems

M. Constantinou	Mon 9:45
D. Richards	Tues 17:30
C. Egerer	Thur 15:00

New Complex Quantities

C. Monahan	Mon 10:40
F. Steffens	Mon 11:05
G. Schierholz	Mon 11:55
P. Bedaque	Mon 15:15
B. Hoerz	Mon 16:35
R. Sufian	Tues 14:00
J. Guerrero	Tues 16:50
J. Karpie	Tues 17:10
A. Baroni	Thur 15:20
W. Detmold	Fri 14:50
D. Lee	Fri 15:15
D. Robaina	Fri 16:35

Thank You