Leading twist GTMDs and Wigner distributions in the LFQDM

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Ref.: TM, D. Chakrabarti, C. Mondal and A. Mukherjee, Phys. Rev. D 95, 074028(2017)

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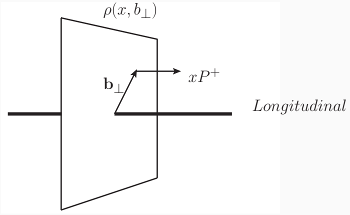
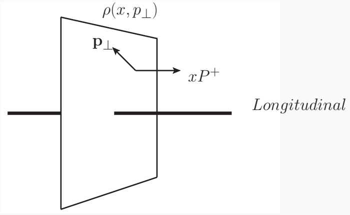
Introduction: non-perturbative structure





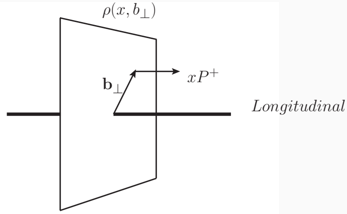
GPD$(x, \mathbf{b}_\perp)$

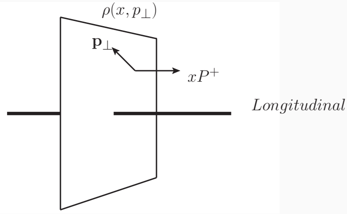
[Talk by S. Liuti, C. Mondal]

TMD$(x, \mathbf{p}_\perp)$

[Talk by A. Accardi, I. Balitsky]

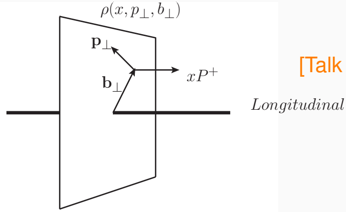
Introduction: non-perturbative structure





GPD$(x, \mathbf{b}_\perp)$

[Talk by S. Liuti, C. Mond



TMD$(x, \mathbf{p}_\perp)$

[Talk by A. Accardi, I. Balitsky]

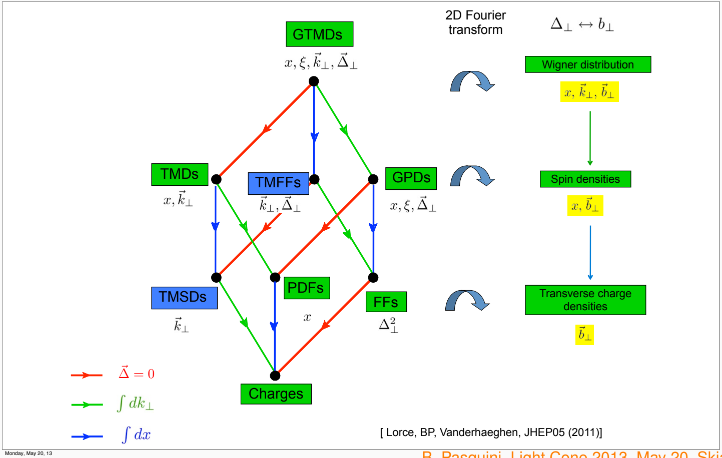
Wigner Distribution $(x, \mathbf{p}_\perp, \mathbf{b}_\perp)$

$\Updownarrow$ F.T. $\mathbf{b}_\perp \leftrightarrow \Delta_\perp$

GTMDs $(x, \mathbf{p}_\perp, \Delta_\perp)$

Generalised Transverse Momentum Dependent parton Distributions

GTMDs $\rightleftharpoons$ WD



— B. Pasquini, Light Cone 2013, May 20, Skiathos, Greece

- Parton distributions in the phase space; quasi-probabilistic int.
- Correlation of quark momentum and position in the trans. mom. plane.

- Not directly measurable in experiments.
- Proposal to extract GTMDs from the linear combination of asymmetries in double Drell Yan process.
  [–S. Bhattacharya, A. Metz, J. Zhou,PLB771(2017)]
- Important to investigate phenomenologically.
  [LCCQM: C. Lorce, B. Pasquini,PRD84(2011)]
  [Dressed quark: J. More, A. Mukherjee, S. Nair, PRD95(2017); PRD91(2015)]

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We present the leading twist GTMDs and Wigner distributions (WDs) in the light-front quark diquark model (LFQDM) and compare with other model results.

Light-front quark-diquark Model(LFQDM)

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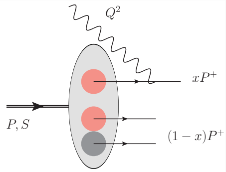


- In this model proton is considered as a bound state of a quark and a diquark with an effective mass. The diquark can have spin-0 singlet(scalar diquark) or spin-1 triplet(axial-vector diquark). The proton state is written in the spin-flavor SU(4) structure as[1]

$$|P;\pm\rangle = C_S|u\ S^0\rangle^{\pm} + C_V|u\ A^0\rangle^{\pm} + C_{VV}|d\ A^1\rangle^{\pm}$$

— R. Jakob, P. J. Mulders, j. Rodrigues NPA626(1997)937

- The two particle Fock-state expansion for $J^z = \pm 1/2$

$$|u\ S\rangle^{\pm} = \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_{\lambda} \psi_{\lambda}^{\pm(u)}(x,\mathbf{p}_\perp)|\lambda\ \Lambda_S; xP^+, \mathbf{p}_\perp\rangle\Big|_{\Lambda_S=0}$$

$$|\nu\ A\rangle^{\pm} = \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_{\lambda} \sum_{\Lambda_A} \psi_{\lambda\Lambda_A}^{\pm(\nu)}(x,\mathbf{p}_\perp)|\lambda\ \Lambda_A; xP^+, \mathbf{p}_\perp\rangle\Big|_{\Lambda_A=1,0,-1}$$

[1] TM, D.Chakrabarti, PRD94,094020(2016)

Wave function

- The light-front wave functions:

$$\psi_{\lambda\Lambda}^{\pm(\nu)}(x, \mathbf{p}_\perp) = N^\nu f(x, \mathbf{p}_\perp, \lambda, \Lambda)\, \varphi_i^{(\nu)}(x, \mathbf{p}_\perp)\Big|_{i=1,2}$$

- Normalized by quark counting rules.
- Modified soft-wall AdS/QCD wave function[2] for two particle bound state:

$$\varphi_i^{(\nu)}(x, \mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\, x^{a_i^\nu}(1-x)^{b_i^\nu} \exp\left[-\delta^\nu \frac{\mathbf{p}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

with the AdS/QCD scale parameter $\kappa = 0.4\ GeV$

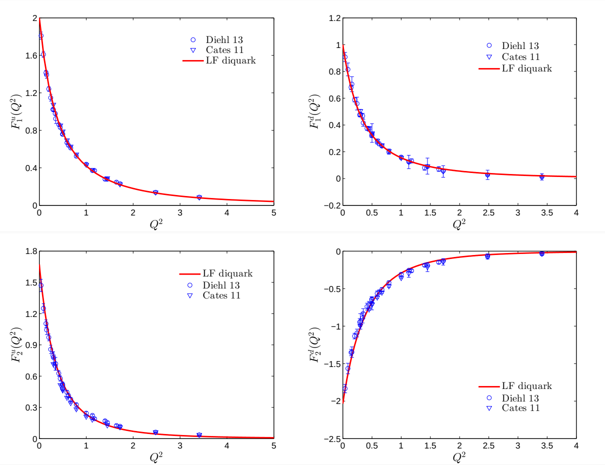
- We determine the parameters a_i^ν, b_i^ν, δ^ν by fitting the experimental data of the Dirac $F_1(Q^2)$ and Pauli $F_2(Q^2)$ form factors.

—-*T.Maji, D.Chakrabarti, PRD94,094020(2016)*

[2] G. F. de Teramond and S. J. Brodsky, arXiv:1203.4025 [hep-ph].
D. Chakrabarti and C. Mondal, Eur. Phys. J. C **73**, 2671 (2013).

Parameter fitting

$$\langle P+q;+|\frac{J^+(0)}{2P^+}|P;+\rangle = F_1(q^2); \qquad \langle P+q;+|\frac{J^+(0)}{2P^+}|P;-\rangle = -(q^1 - iq^2)\frac{F_2(q^2)}{2M}$$



ν	a_1^ν	b_1^ν	a_2^ν	b_2^ν	δ^ν
u	0.280 ± 0.001	0.1716 ± 0.0051	0.84 ± 0.02	0.2284 ± 0.0035	1.0
d	0.5850 ± 0.0003	0.7000 ± 0.0002	$0.9434^{+0.0017}_{-0.0013}$	$0.64^{+0.0082}_{-0.0022}$	1.0

—-T.Maji, D.Chakrabarti, PRD94,094020(2016)

Check

- CHECK 1: The electric and magnetic charge radius for nucleons are found to be in good agreement with the experimental measurement.

Quantity	Our result	Measured Data
$r_E^p\ (fm)$	0.830 ± 0.025	0.877 ± 0.005
$r_M^p\ (fm)$	0.779 ± 0.007	0.777 ± 0.016
$\langle r_E^2 \rangle^n\ (fm^2)$	-0.064 ± 0.018	-0.1161 ± 0.0022
$r_M^n\ (fm)$	0.758 ± 0.005	$0.862^{+0.009}_{-0.008}$

- CHECK 2: helicity and transversity PDFs are compared with phenomenological fits: shows qualitatively good agreement.
  - Model result for axial charge has good agreement with the measured data.

	g_A^u	g_A^d	g_A
LFQDM	0.71 ± 0.02	$-0.50^{+0.09}_{-0.12}$	$1.22^{+0.11}_{-0.13}$
Measured Data	0.82 ± 0.07	-0.45 ± 0.07	1.27 ± 0.14

[—E. Leader, A. V. Sidorov, and D. B. Stamenov,PRD82(2010)

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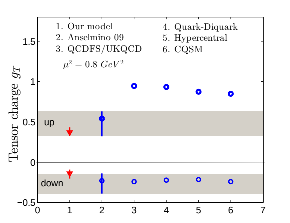


- Tensor charge falls in the region which is proposed by the experimental findings.

LFQDM

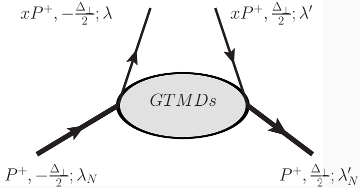
- A simple model.
- Quark mass $m_q = 0$.
- No antiparticles are considered.
- All the gluon interactions are included in the diquark with an effective mass.
- Parameters $\Leftarrow$ fitting of F1 and F2 data.

- Check 1: Electromagnetic radii of nucleons.
- Check 2: Unpolarised PFDs

- So far, in LFQDM:
  - Polarised PDFs —TM, D. Chakrabarti, PRD94(2016)
  - GPDs —TM, C. Mondal, D. Chakrabarti; PRD96(2017)
  - TMDs —TM, D. Chakrabarti; PRD95(2017)
- Wigner Distributions & GTMDs — PRD95(2017)

GTMDs and Wigner Distributions

GTMDs



—Meissner $et.al.$ JHEP08(2009)056

$$W^{\nu[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S) = \frac{1}{2}\int \frac{dz^-}{(2\pi)} \frac{d^2 z_T}{(2\pi)^2} e^{ip.z} \times \langle P''; S|\bar{\psi}_i^\nu(-z/2)\Gamma \mathcal{W}_{[-z/2,z/2]}\psi_j^\nu(z/2)|P'; S\rangle\Big|_{z^+=0}.$$

Leading twist: $\Gamma = \gamma^+, \gamma^+\gamma^5, \sigma^{j+}\gamma^5$

- Bilinear decomposition satisfying hermiticity, parity and time reversal:

$$W^{\nu[\gamma^+]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S) \sim F_{1,1}, F_{1,2}, F_{1,3}, F_{1,4}$$
$$W^{\nu[\gamma^+\gamma^5]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S) \sim G_{1,1}, G_{1,2}, G_{1,3}, G_{1,4}$$
$$W^{\nu[\sigma^{j+}\gamma^5]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S) \sim H_{1,1}, H_{1,2}, H_{1,3}, H_{1,4}, H_{1,5}, H_{1,6}, H_{1,7}, H_{1,8}$$

Wigner Distributions

- 5-dimensional quark Wigner distributions

$$\rho^{\nu[\Gamma]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; S) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp . b_\perp} W^{\nu[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S).$$

Leading twist: $\Gamma = \gamma^+, \gamma^+\gamma^5, \sigma^{j+}\gamma^5$

Quark pol. $\rightarrow$ U L T

$$\rho^{\nu}_{XY} = \frac{1}{2}\Big[\rho^{\nu[\Gamma]}(\mathbf{b}_\perp, \mathbf{p}_\perp; +\hat{S}) \pm \rho^{\nu[\Gamma]}(\mathbf{b}_\perp, \mathbf{p}_\perp; -\hat{S})\Big]$$

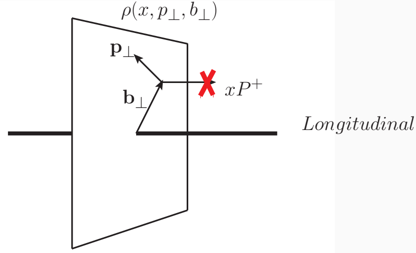
X : proton Pol., Y : quark Pol.

—C. Lorce, B. Pasquini, PRD84(2011)

$\rho^\nu_{UU}, \rho^\nu_{UL}, \rho^\nu_{UT}$, $\rho^\nu_{LU}, \rho^\nu_{LL}, \rho^\nu_{LT}$, $\rho^\nu_{TU}, \rho^\nu_{TL}, \rho^\nu_{TT}$; $\nu = \mathrm{u}, \mathrm{d}$

Unpol. Longi. pol. Trans. pol. $\Leftarrow$ Proton pol.

Wigner Distributions



$$\rho^{\nu[\Gamma]}(\mathbf{b}_\perp, \mathbf{p}_\perp) = \int dx \rho^{\nu[\Gamma]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; S)$$

—C. Lorce, B. Pasquini, PRD84(2011)

- Distribution due to correlation between $\mathbf{p}_\perp$ and $\mathbf{b}_\perp$ (absent in GPDs, TMDs).

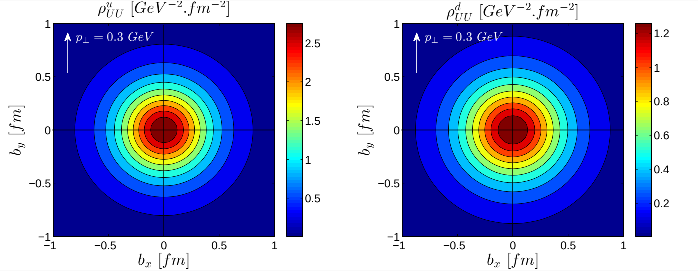
  ◇ At fixed $\mathbf{p}_\perp \Rightarrow$ 2D distributions in impact parameter space

  ◇ At fixed $\mathbf{b}_\perp \Rightarrow$ 2D distributions in transverse momentum space

Results in LFQDM

WD at fixed $\mathrm{p}_\perp$: unpol. quark in unpol. proton

$\rho^{\nu}_{UU}(\mathbf{p}_\perp, \mathbf{b}_\perp)$: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV



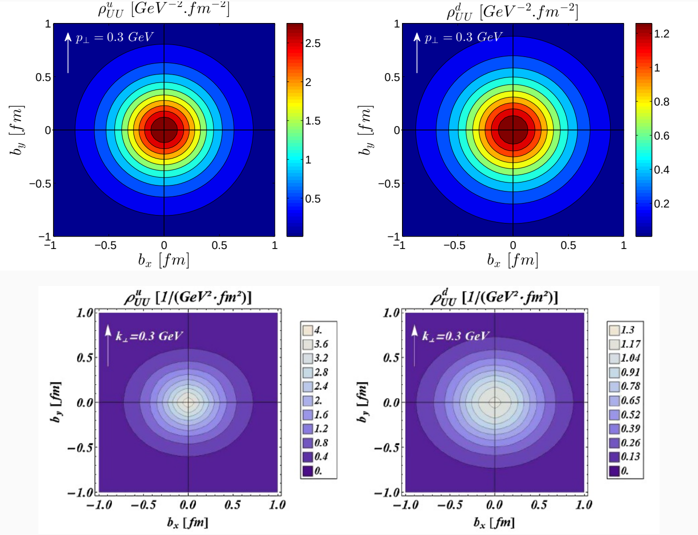
Up quark Down quark

- Monopole distributions.
- Axially symmetric $\Longrightarrow$ no total orbital angular momentum (OAM)
- As found in other models: LCCQM [Lorce, Pasquini, PRD84(2011)].

WD at fixed $\mathrm{p}_\perp$: unpol. quark in unpol. proton

$\rho^{\nu}_{UU}(\mathbf{p}_\perp, \mathbf{b}_\perp)$:

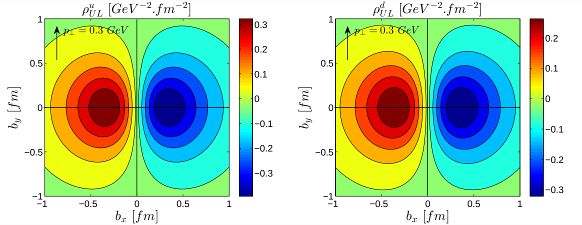
Comparison with LCCQM



[—Lorce, Pasquini, PRD84(2011)]

WD at fixed $\mathrm{p}_\perp$: Longi. pol. quark in Unpol. proton

$\rho^{\nu}_{UL}(\mathbf{p}_\perp, \mathbf{b}_\perp)$: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV



Up quark Down quark

- Correlation between quark spin and quark OAM

$$C^{\nu}_z = \int dx d^2\mathbf{p}_\perp d^2\mathbf{b}_\perp (\mathbf{b}_\perp \times \mathbf{p}_\perp)_z \rho^{\nu}_{UL}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)$$

- $C^{\nu}_z > 0 (< 0)$: q-spin and OAM tend to be aligned(anti-aligned).
- Our model: $C^u_z = 0.55$ $C^d_z = 0.75$ LCCQM: $C^u_z = 0.23$ $C^d_z = 0.19$

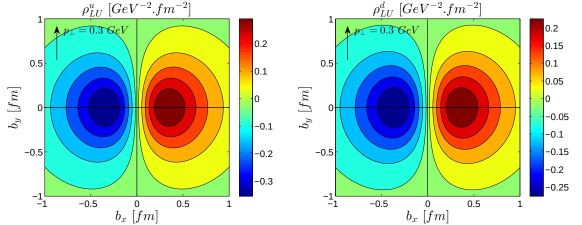
$\Longrightarrow$ q-spin

$\rightarrow$ u-OAM

$\longrightarrow$ d-OAM

WD at fixed $\mathrm{p}_\perp$: Unpol. quark in Longi. pol. proton

$\rho^{\nu}_{LU}(\mathbf{p}_\perp, \mathbf{b}_\perp)$: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV



Up quark   Down quark

- Correlation between proton spin and quark OAM

$$\ell^{\nu}_z = \int dx d^2\mathbf{p}_\perp d^2\mathbf{b}_\perp (\mathbf{b}_\perp \times \mathbf{p}_\perp)_z \rho^{\nu}_{LU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)$$

- $\ell^{\nu}_z > 0 (< 0)$: Proton-spin and q-OAM tend to be aligned(anti-aligned).
- Our model: $\ell^u_z = 0.49$ $\ell^d_z = 0.58$ LCCQM: $\ell^u_z = 0.131$ $\ell^d_z = -0.005$

Our model		LCCQM
$\Longrightarrow$	$q - spin$	$\Longrightarrow$
$\rightarrow$	$u - OAM$	$\longrightarrow$
$\longrightarrow$	$d - OAM$	$\leftarrow$

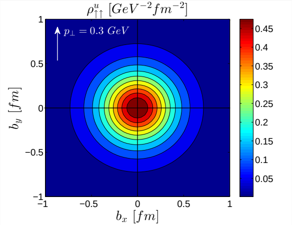
—Lorce,Pasquini,PRD85(2012); Y. Hatta,PLB708(2012)

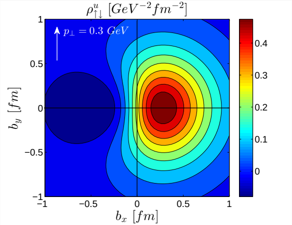
Spin-spin correlations: Longitudinal

- Quarks with longitudinal polarization λ in a longitudinally polarised proton Λ is defined for $\Gamma = \gamma^+ \frac{\mathbb{1}+\lambda\gamma^5}{2}$ and $\vec{S} = \Lambda \hat{S}_z$ as: [—Lorce, Pasquini, PRD84(2011)]

$$\begin{aligned}\rho^{\nu}_{\Lambda\lambda}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) \quad &= \quad \frac{1}{2}[\rho^{\nu}_{UU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda\rho^{\nu}_{LU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) \\ &\quad +\lambda\rho^{\nu}_{UL}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda\lambda\rho^{\nu}_{LL}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)],\end{aligned}$$

- u quark: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV ;





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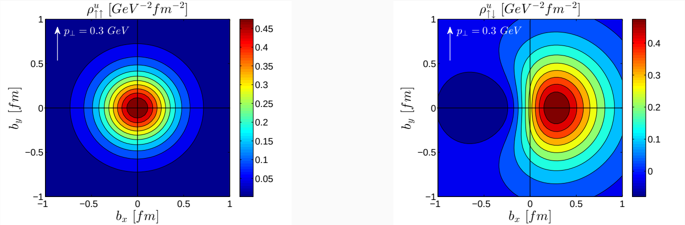
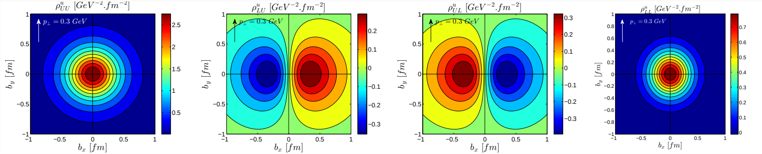
$$\rho^{\nu}_{\Lambda\lambda}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2}[\rho^{\nu}_{UU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda\rho^{\nu}_{LU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \lambda\rho^{\nu}_{UL}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda\lambda\rho^{\nu}_{LL}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)],$$

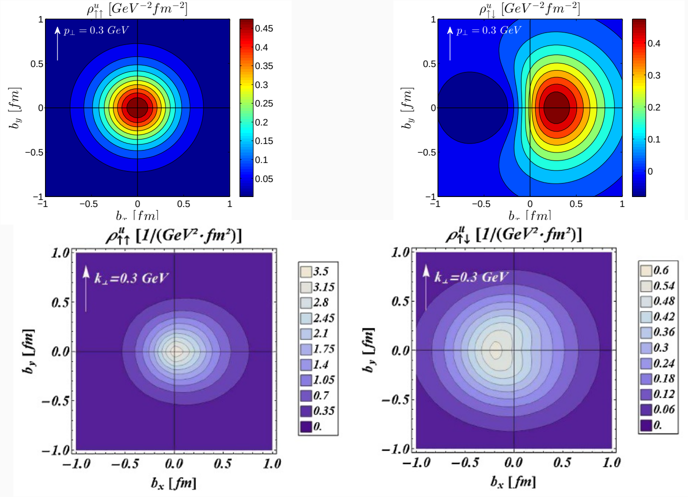
- u quark: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV ;





Spin-spin correlations: Longitudinal

- u quark: Comparison with LCCQM: ;



[—Lorce, Pasquini, PRD84(2011)]

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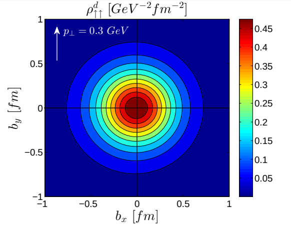
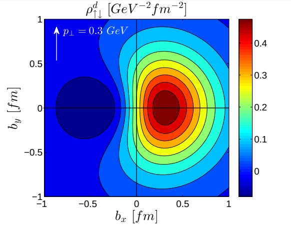
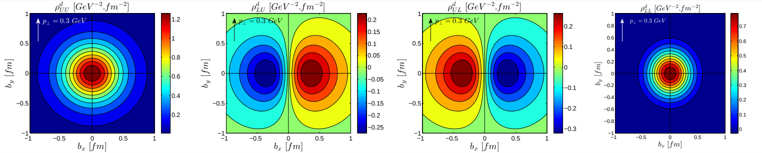
- d quark: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV ;

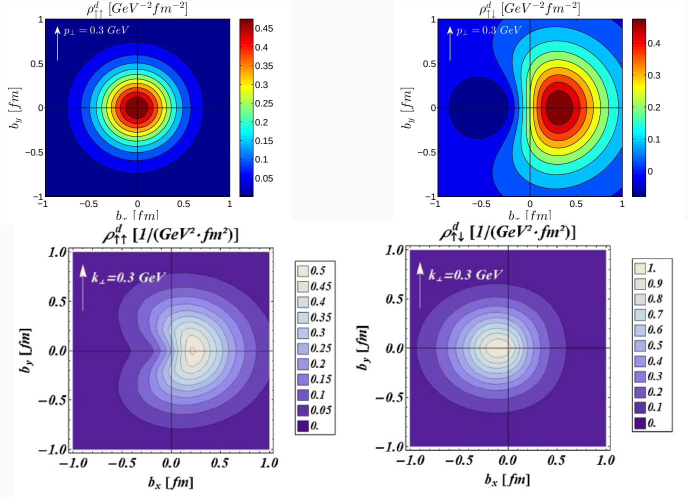






Spin-spin correlations: Longitudinal

- d quark: Comparison with LCCQM ;

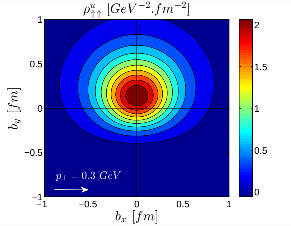


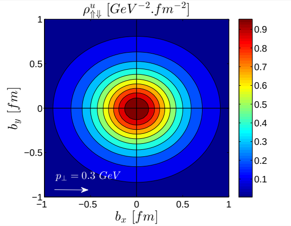
[—Lorce, Pasquini, PRD84(2011)]

Spin-spin correlations: Transverse

$$\rho^{i\nu}_{\Lambda_T\lambda_T}(\mathbf{b}_\perp,\mathbf{p}_\perp,x) = \frac{1}{2}[\rho^{\nu}_{UU}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)+\Lambda_T\rho^{i\nu}_{TU}(\mathbf{b}_\perp,\mathbf{p}_\perp,x) + \lambda_T\rho^{\nu i}_{UT}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)+\Lambda_T\lambda_T\rho^{q}_{TT}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)].$$

- u quark: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV ;

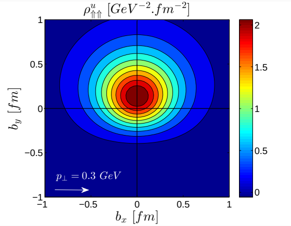


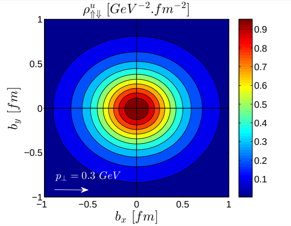


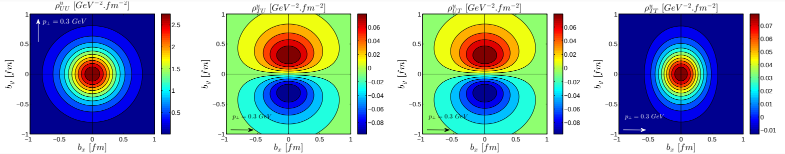
Spin-spin correlations: Transverse

$$\rho^{i\nu}_{\Lambda_T \lambda_T}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2}[\rho^{\nu}_{UU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda_T \rho^{i\nu}_{TU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \lambda_T \rho^{\nu i}_{UT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda_T \lambda_T \rho^{q}_{TT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)].$$

- u quark: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV ;





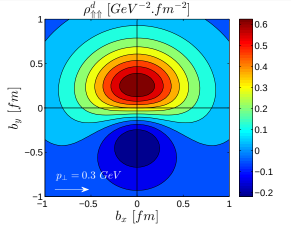


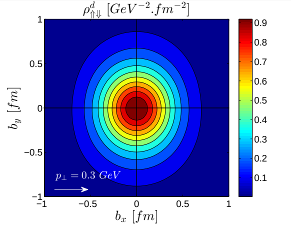
Up quark Down quark

Spin-spin correlations: Transverse

$$\rho^{i\nu}_{\Lambda_T\lambda_T}(\mathbf{b}_\perp,\mathbf{p}_\perp,x) = \frac{1}{2}[\rho^{\nu}_{UU}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)+\Lambda_T\rho^{i\nu}_{TU}(\mathbf{b}_\perp,\mathbf{p}_\perp,x) + \lambda_T\rho^{\nu i}_{UT}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)+\Lambda_T\lambda_T\rho^{q}_{TT}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)].$$

- d quark: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV ;

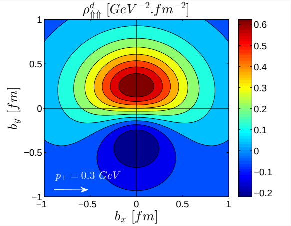


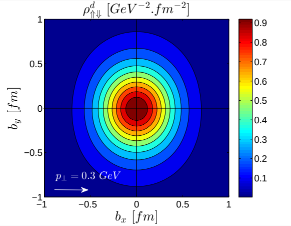


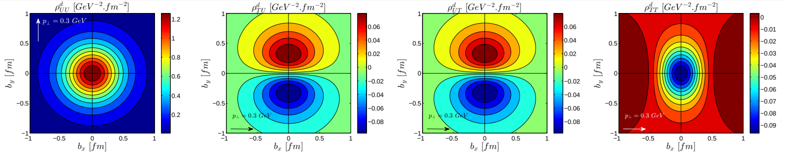
Spin-spin correlations: Transverse

$$\rho^{i\nu}_{\Lambda_T \lambda_T}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2}[\rho^{\nu}_{UU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda_T \rho^{i\nu}_{TU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)$$
$$+\lambda_T \rho^{\nu i}_{UT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda_T \lambda_T \rho^{q}_{TT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)].$$

- d quark: $\mathbf{p}_\perp$ along $\hat{y}$ and $p_y = 0.3$ GeV ;





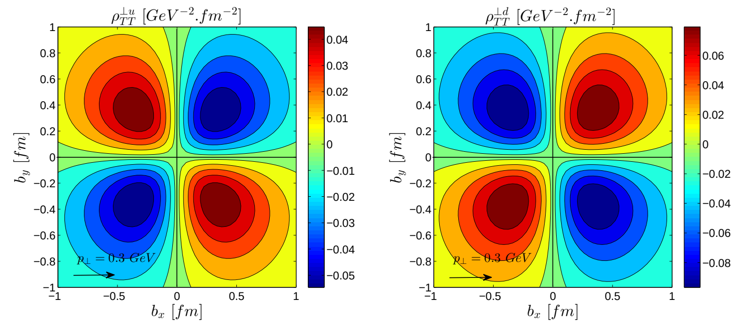


Up quark

Down quark

WD at fixed $\mathrm{p}_\perp$: quark & proton both Trans. pol. (mutually $\perp$)

$\rho^{\perp\nu}_{TT}(\mathbf{p}_\perp, \mathbf{b}_\perp)$: $\mathbf{p}_\perp$ along $\hat{x}$ and $p_x = 0.3$ GeV; Proton pol. $\hat{y}$



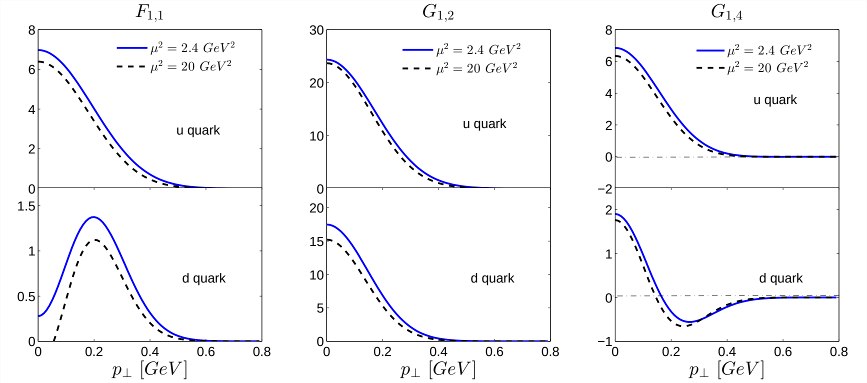
Up quark Down quark

- Quadrupole distributions
- polarity of the distributions flip for the $u \leftrightarrow d$

GTMDs

- $\Gamma = \gamma^+$: $F_{1,1} \leftrightarrow f_1$, $F_{1,2} \leftrightarrow f_{1T}^{\perp}$; T-even: $F_{1,2}^e = 0$
  $F_{1,3}$ & $F_{1,4}$ do not have TMD partners. [Meissner, JHEP'09]

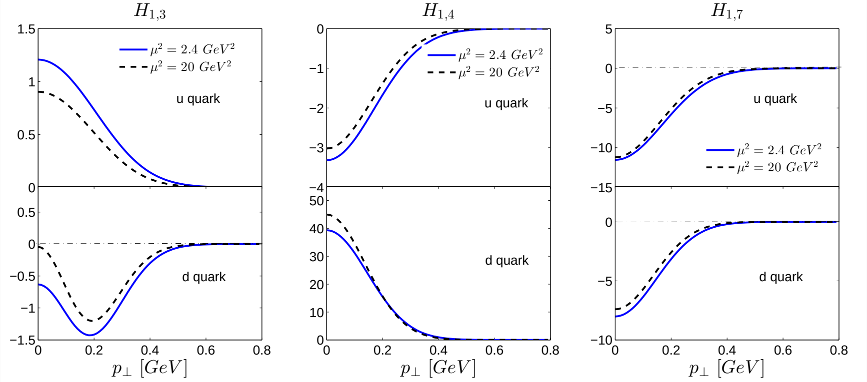
$$x = 0.3 \;\&\; \Delta_\perp^2 = 0.1\ GeV^2$$



- $\Gamma = \gamma^+\gamma^5$: $G_{1,2} \leftrightarrow g_{1T}$, $G_{1,4} \leftrightarrow g_{1L}$
  $G_{1,1}$ & $G_{1,3}$ do not have TMD partners.

GTMDs

- $\Gamma = \sigma^{j+}\gamma^5$: $H_{1,3} \leftrightarrow h_{1T}$, $H_{1,4} \leftrightarrow h_{1T}^{\perp}$,
  $H_{1,1}^{o} \leftrightarrow h_1^{\perp}$; LFQDM $H_{1,1}^{e} = 0$
  $H_{1,2}, H_{1,5}, H_{1,6}$ & $H_{1,8}$ do not have TMD partners.
  $x = 0.3$ & $\Delta_{\perp}^2 = 0.1\ GeV^2$



Inequalities

- Relations among GTMDs for $\mathbf{p}_\perp^2 > \frac{\mathbf{\Delta}_\perp^2}{4}(1-x)^2$ found in LFQDM

$$|H_{1,3}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp^2}{2M^2}H_{1,4}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2)| \quad < \quad \frac{1}{2}|F_{1,1}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2) + G_{1,4}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2)|,$$

Soffer bound for GTMDs in LFQDM.

$$\begin{aligned}
|F_{1,1}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2)| &> |H_{1,3}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp^2}{2M^2}H_{1,4}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2)|,\\
|F_{1,1}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2)| &> |H_{1,3}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2)|,\\
F_{1,1}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2) &> 0,\\
F_{1,1}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2) &> G_{1,4}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2),\\
\frac{\mathbf{p}_\perp^2}{2M^2}|H_{1,4}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2)| &< \frac{1}{2}|F_{1,1}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2) + G_{1,4}^{\nu}(x,\mathbf{\Delta}_\perp^2,\mathbf{p}_\perp^2)|.
\end{aligned}$$

- Other model results are needed to check the universality of the inequalities.

Conclusion

- We present Wigner distributions for unpolarized, longitudinally polarized, and transversely polarized quarks and protons using light-front quark-diquark model. The light-front wave functions are modelled using ADS/QCD predictions.
- The model result is compared with other model predictions.
- Spin-spin and spin-OAM correlations are discussed.
- We have noted a few inequalities among GTMDs. It will be interesting to check if such inequalities are present in other models.

THANK YOU

Form factor fitting and the parameters

- In the light-front formalism, for a spin-$\frac{1}{2}$ composite particle system the Dirac and Pauli form factors are defined as

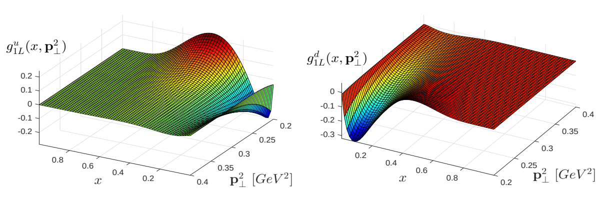
$$
\begin{aligned}
\langle P+q;+|\frac{J^+(0)}{2P^+}|P;+\rangle &= F_1(q^2) \\
\langle P+q;+|\frac{J^+(0)}{2P^+}|P;-\rangle &= -(q^1-iq^2)\frac{F_2(q^2)}{2M}
\end{aligned}
$$

  Where the q^2 is square of the momentum transferred to the nucleon of mass M.

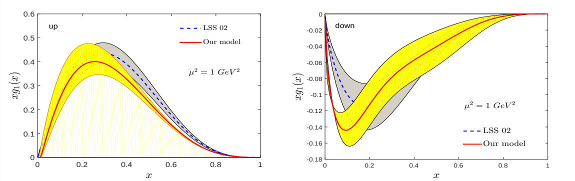
- The normalization of form factors for proton and neutron are given as $F_1^p(0)=1,\ F_2^p(0)=\kappa^p=1.793$ and $F_1^n(0)=0,\ F_2^n(0)=\kappa^n=-1.913$
- Considering the charge and isospin symmetry the nucleon form factors are decomposed into flavour form factors as $F_i^{p(n)}=e_uF_i^{u(d)}+e_dF_i^{d(u)}$.

Helicity TMD: $g_{1L}^{\nu}(x, \mathbf{p}_\perp^2)$

$$\int d^2\mathbf{p}_\perp \ldots$$



$$\int dx \ldots$$

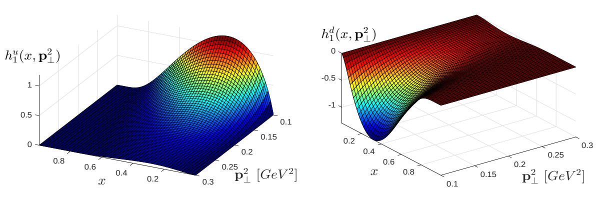


$\mu^2 = 1\ GeV^2$	g_A^u	g_A^d	$g_A = g_A^u - g_A^d$
Our result:	0.71 ± 0.09	$-0.54^{+0.19}_{-0.13}$	$1.25^{+0.28}_{-0.22}$
Measured Data:	0.82 ± 0.07	-0.45 ± 0.07	1.27 ± 0.14

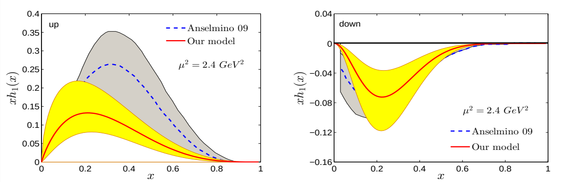
—E. Leader $et.al.$,Phys. Rev. D 67 (2003) 037503

Transversity TMD: $h_1^{\nu}(x, \mathbf{p}_\perp^2)$

$\int d^2\mathbf{p}_\perp \ldots$



$\int dx \ldots$



$\mu^2 = 0.8\ GeV^2$	g_T^u	g_T^d	$g_T = g_T^u - g_T^d$
Our result:	$0.37^{+0.06}_{-0.05}$	$-0.14^{+0.05}_{-0.06}$	$0.51^{+0.12}_{-0.11}$
Measured Data:	$0.59^{+0.14}_{-0.13}$	$-0.20^{+0.05}_{-0.07}$	$0.79^{+0.19}_{-0.20}$

—M. Anselmino $et.al.$ Nucl.Phys.Proc.Suppl.191(2009)98

Scale evolution modelling

- The scale evolution of the PDFs are simulated by making parameters scale dependent and fit the unpolarized PDFs; Consistent with DGLAP for the range upto $150\ GeV^2$.
- The explicit scale evolution of each parameter are given, so that any distribution which involved these parameters, can be calculated at any arbitrary scale.
- The model reproduces the scale evolution of unpolarized PDF for a wide range of energy scale $(10^4\ GeV^2)$ and matches accurately with the HERAPDF15(nnlo), MSTW2008(nnlo), NNPDF21(nnlo).

— TM, D.Chakrabarti, PRD94,094020(2016)

Parameret evolution approach

- In this model:

$$f_1^\nu(x,\mathbf{p}_\perp^2) = N_{f_1}^\nu \frac{\ln(1/x)}{\pi\kappa^2}\left[x^{2a_1^\nu}(1-x)^{2b_1^\nu-1} + \frac{\mathbf{p}_\perp^2}{M^2}x^{2a_2^\nu-2}(1-x)^{2b_2^\nu-1}\right]e^{-\mathbf{p}_\perp^2\frac{\delta^\nu\ln(1/x)}{\kappa^2(1-x)^2}}$$

$$a_i^\nu(\mu) \quad b_i^\nu(\mu) \quad \delta^\nu(\mu)$$

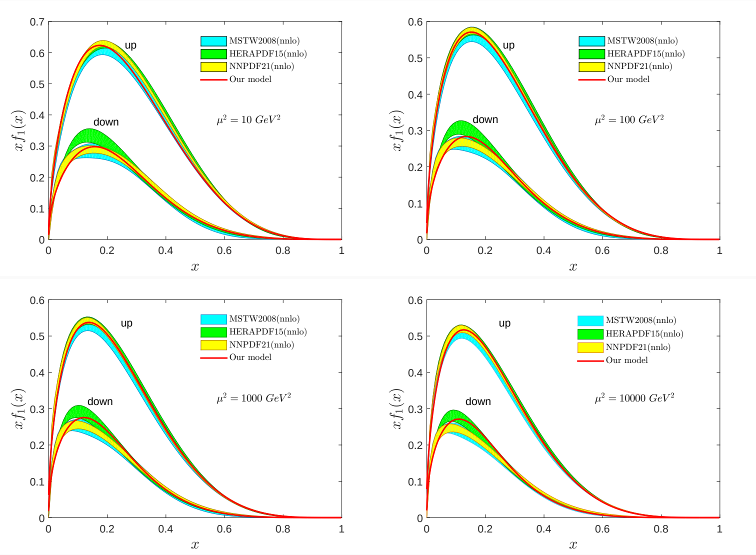
- Scale variation of the parameters is determined by fitting the DGLAP evolution for unpolarised PDFs. .

$$\begin{aligned} a_i^\nu(\mu) &= a_i^\nu(\mu_0) + A_i^\nu(\mu), \\ b_i^\nu(\mu) &= b_i^\nu(\mu_0) - B_i^\nu(\mu)\frac{4C_F}{\beta_0}\ln\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right), \\ \delta^\nu(\mu) &= \exp\left[\delta_1^\nu\left(\ln(\mu^2/\mu_0^2)\right)^{\delta_2^\nu}\right], \end{aligned}$$

$$P_i^\nu(\mu) = \alpha_{P,i}^\nu\,\mu^{2\beta_{P,i}^\nu}\left[\ln\left(\frac{\mu^2}{\mu_0^2}\right)\right]^{\gamma_{P,i}^\nu}\Bigg|_{P=A,B\ at\ i=1,2},$$

TM, D.Chakrabarti, Phys. Rev. D 94,094020(2016)

Unpol. PDFs evolution upto $\mu^2 = 10^4\ GeV^2$



- This model evolution predicts the unpolarised PDFs accurately for a wide range of scale.

QCD evolution for $f_1(x, \mathrm{p}_\perp^2)$ and $D_1^{h/\nu}(z, \mathrm{k}_\perp^2)$

$$\tilde{F}(x, \mathbf{b}_\perp; \mu) = \tilde{F}(x, \mathbf{b}_\perp; \mu_0) \exp\left(\ln \frac{\mu}{\mu_0} \tilde{K}(b_\perp; \mu) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\big(\mu', \frac{\mu^2}{\mu'^2}\big) \right),$$

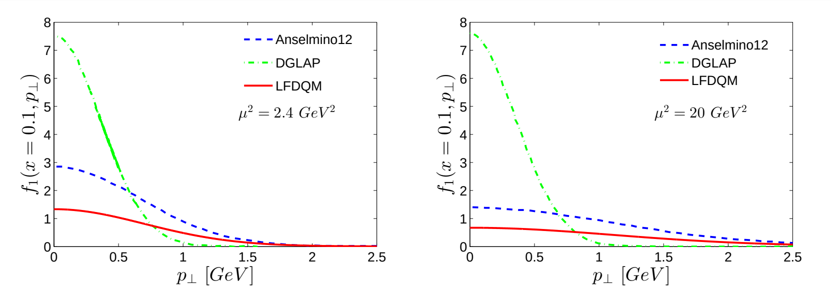
where, at $\mathcal{O}(\alpha_s)$

$$\begin{aligned}
\tilde{K}(b_\perp; \mu) &= -\frac{\alpha_s C_F}{\pi}[\ln(b_*^2 \mu_b^2) - \ln(4) + 2\gamma_E] + \left[\int_{\mu}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') \right] - g_K(b_T) \\
b_*(b_T) &= \left. \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{max}^2}}} \right|_{b_{max}=0.5\ GeV^{-1}} \quad ; \quad \mu_b = \left. \frac{C_1}{b_*(b_T)} \right|_{C_1 = 2e^{-\gamma_E}} \\
& g_K = \left. \frac{1}{2} g_2 b_T^2 \right|_{b_2 = 0.68\ GeV^2}
\end{aligned}$$

TMDs evolution

- A reduced kernel approach$(C_1/b_{max} = \mu_b(b_\perp \to \infty)$ for TMDs evolution is presented by Anselmino, PRD86,014028(2012) and extended to spin dependent TMDs e.g., $f_{1T}^{\perp}(x, \mathbf{p}_\perp^2)$

$$f(x, \mathbf{p}_\perp^2, \mu) = \int d^2\mathbf{b}_\perp e^{i\mathbf{b}_\perp \cdot \mathbf{p}_\perp} \tilde{F}(x, \mathbf{b}_\perp^2, \mu)$$



—M. Anselmino $et.al.$ PRD 86, 014028 (2012)

TMDs evolution from DGLAP : $\quad \mathrm{TMD}^\nu(x, \mathbf{p}_\perp^2; \mu) = \mathrm{PDF}^\nu(x; \mu)\, \dfrac{e^{-\mathbf{p}_\perp^2/\langle p^2{}_\perp \rangle}}{\pi \langle p^2{}_\perp \rangle}$