Baryon spectroscopy and structure with functional methods

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Light Cone 2018
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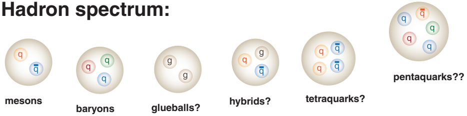
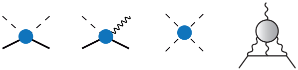
Motivation

Hadron spectrum:

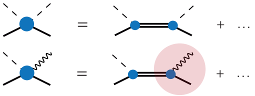


Form factors: resonance transition FFs, spacelike vs. timelike properties

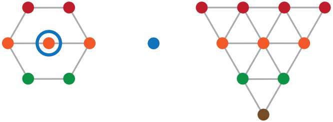
Hadron structure & scattering amplitudes



Extraction of resonances?



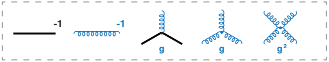
$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1520)$ $N(1700)$ $N(1875)$	$N(1680)$ $N(1860)$ $N(2000)$	$N(1675)$	$N(1990)$
$\Delta(1910)$	$\Delta(1620)$ $\Delta(1900)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Delta(1700)$ $\Delta(1940)$	$\Delta(1905)$ $\Delta(2000)$	$\Delta(1930)$	$\Delta(1950)$
$\Lambda(1116)$ $\Lambda(1600)$ $\Lambda(1810)$	$\Lambda(1405)$ $\Lambda(1670)$ $\Lambda(1800)$	$\Lambda(1890)$	$\Lambda(1520)$ $\Lambda(1690)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Sigma(1189)$ $\Sigma(1660)$ $\Sigma(1880)$	$\Sigma(1750)$	$\Sigma(1385)$	$\Sigma(1670)$ $\Sigma(1940)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Xi(1315)$		$\Xi(1530)$	$\Xi(1820)$			
		$\Omega(1672)$				



DSEs & BSEs

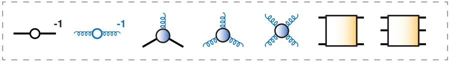
QCD's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + igA\!\!\!/ + m \right) \psi + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \right]$$

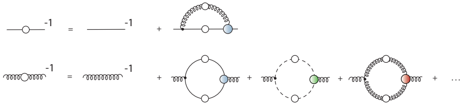
$=$ 

Quantum "effective action":

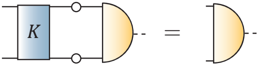
$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} = e^{-\Gamma}$$



DSEs = quantum equations of motion:
derived from path integral, relate n-point functions



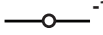
Bethe-Salpeter equations for hadronic bound states:

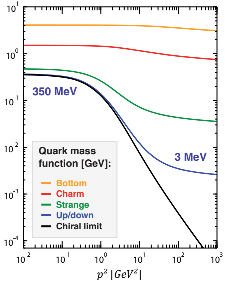


- Poincaré covariance
- Chiral symmetry
- EM gauge invariance
- Only quark & gluon d.o.f., hadronic poles generated dynamically
- multiscale problems feasible
- gauge-fixed
- **truncations:** neglect higher n-point functions to obtain closed system

QCD's n-point functions

- **Quark propagator**

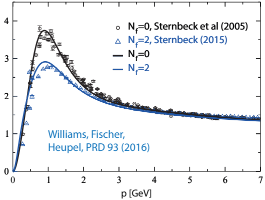
$A(p^2)\,(i\not{p} + M(p^2))$  $^{-1}$



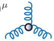
Dynamical chiral symmetry breaking generates 'constituent-quark masses'

- **Gluon propagator**

$\frac{D(p^2)}{p^2}\left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$ 



- **Three-gluon vertex**

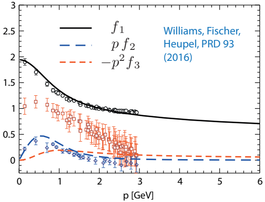
$F_1\,[\,\delta^{\mu\nu}(p_1-p_2)^\rho + \delta^{\nu\rho}(p_2-p_3)^\mu + \delta^{\rho\mu}(p_3-p_1)^\nu] + \dots$ 

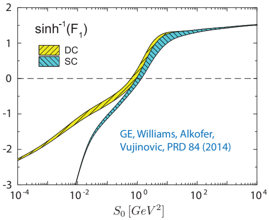
Agreement between lattice, DSE & FRG within reach

Huber, EPJ C77 (2017),
Cyrol, Mitter, Pawlowski, PRD 97 (2018), ...

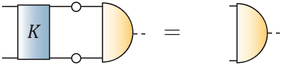
- **Quark-gluon vertex**

$f_1\,\gamma^\mu + f_2\,ip^\mu + f_3\,p^\mu\not{p} + \dots$ 

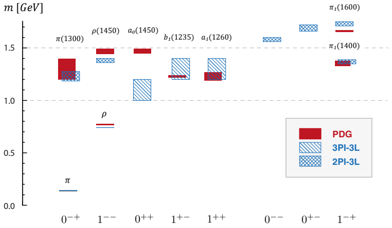




Truncations

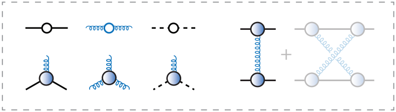


Light meson spectrum beyond rainbow-ladder:

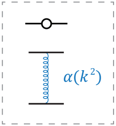


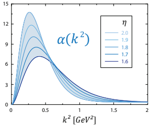
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **3PI system:** all 2 & 3-point functions calculated
  Williams, Fischer, Heupel, PRD 93 (2016)



- **Rainbow-ladder:** only quark propagator calculated, kernel = effective gluon exchange





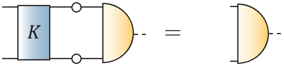
$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(\frac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

adjust scale Λ to observable, keep width η as parameter

Maris, Tandy, PRC 60 (1999),
Qin et al., PRC 84 (2011)

- Beyond rainbow-ladder constructions with **axial WTI**
  Chang, Roberts, PRL 103 (2009), PRC 85 (2012), . . .

Truncations



$K(M)\ G(M)\ \phi_i(M) = \lambda_i(M)\ \phi_i(M)$

- **3PI system:** all 2 & 3-point functions calculated
  Williams, Fischer, Heupel, PRD 93 (2016)

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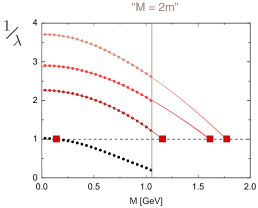
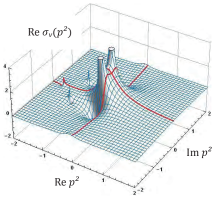
adjust scale Λ to observable, keep width η as parameter

Maris, Tandy, PRC 60 (1999), Qin et al., PRC 84 (2011)

Eigenvalues in **pion** channel:



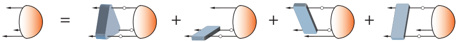
Quark propagator has **complex singularities:** no physical threshold



Baryons

Covariant Faddeev equation for baryons:

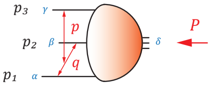
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)



- 3-gluon diagram vanishes ⇒ **3-body effects small?**

  Sanchis-Alepuz, Williams, PLB 749 (2015)

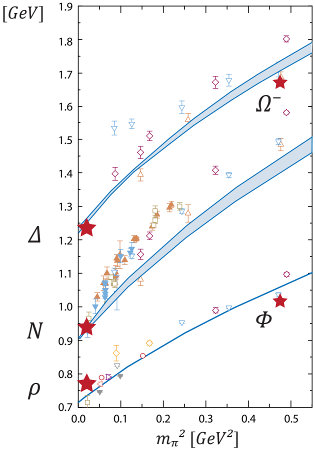
- 2-body kernels same as for mesons, no further approximations:



$$\Psi_{\alpha\beta\gamma\delta}(p,q,P) = \sum_i f_i(p^2, q^2, p\cdot q, p\cdot P, q\cdot P)\; \tau_i(p,q,P)_{\alpha\beta\gamma\delta}$$

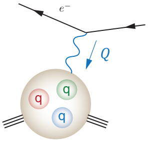
Lorentz-invariant dressing functions

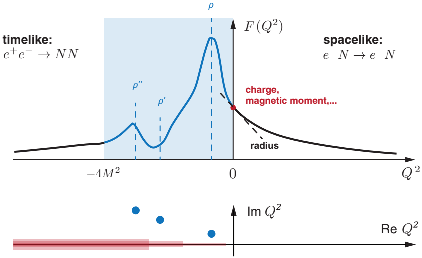
Dirac-Lorentz tensors carry OAM: s, p, d,...



Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

Form factors

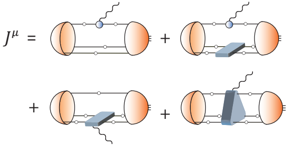




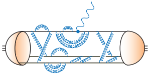
$$J^\mu = e\,\bar{u}(p_f)\left(F_1(Q^2)\,\gamma^\mu + F_2(Q^2)\,\frac{i}{4m}\,[\gamma^\mu, \not{Q}]\right)u(p_i)$$

Consistent derivation of **current matrix elements & scattering amplitudes**

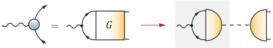
Kvinikhidze, Blankleider, PRC 60 (1999),
GE, Fischer, PRD 85 (2012) & PRD 87 (2013)



- **rainbow-ladder** topologies (1st line):



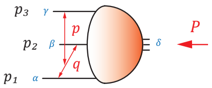
- **quark-photon vertex** preserves em. gauge invariance, dynamically generates **VM poles:**



GE, Sanchis-Alepuz, Williams, Alkofer, Fischer,
PPNP 91 (2016), 1606.09602

The role of diquarks

Three-body equation knows nothing of **diquarks**,
but dynamically generates them in iteration



Group Lorentz invariants into
multiplets of permutation group S3:

GE, Fischer, Heupel, PRD 92 (2015), GE, Sanchis-Alepuz, in preparation

- **Singlet:**
  symmetric variable,
  carries overall scale:

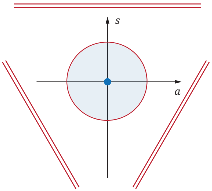
$$\mathcal{S}_0 \sim p_1^2 + p_2^2 + p_3^2 + \frac{M^2}{3}$$

- **Doublet:**

$$\mathcal{D}_0 \sim \frac{1}{\mathcal{S}_0} \begin{bmatrix} -\sqrt{3}\,(\delta x + 2\delta\omega) \\ x + 2\omega \end{bmatrix}$$

- Second **doublet:**

$$\mathcal{D}_1 \sim \frac{1}{\sqrt{\mathcal{S}_0}} \begin{bmatrix} -\sqrt{3}\,(\delta x - \delta\omega) \\ x - \omega \end{bmatrix}$$



Mandelstam plane,
outside: **diquark poles!**

The role of diquarks

Three-body equation knows nothing of **diquarks**,
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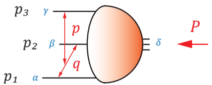

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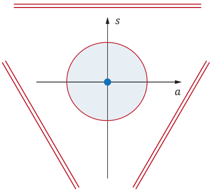

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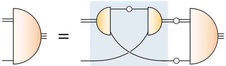
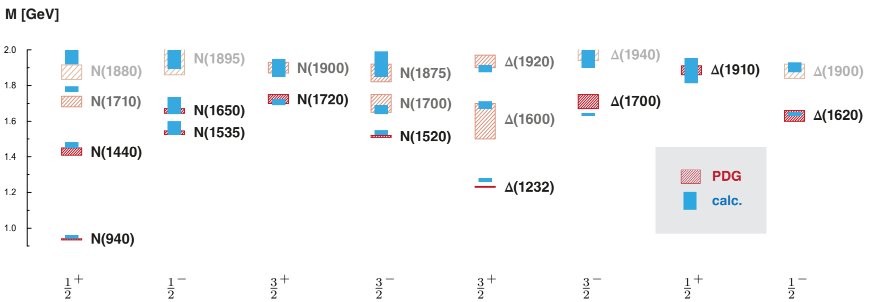
$\Rightarrow$ Simplify 3-body equation to
quark-diquark BSE

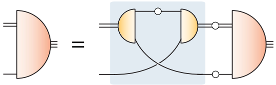


Oettel, Alkofer, Hellstern Reinhardt, PRC 58 (1998),
Cloet, GE, El-Bennich, Klähn, Roberts, FBS 46 (2009)
GE, Krassnigg, Schwinzerl, Alkofer,
Ann. Phys. 323 (2008)
...

Baryon spectrum

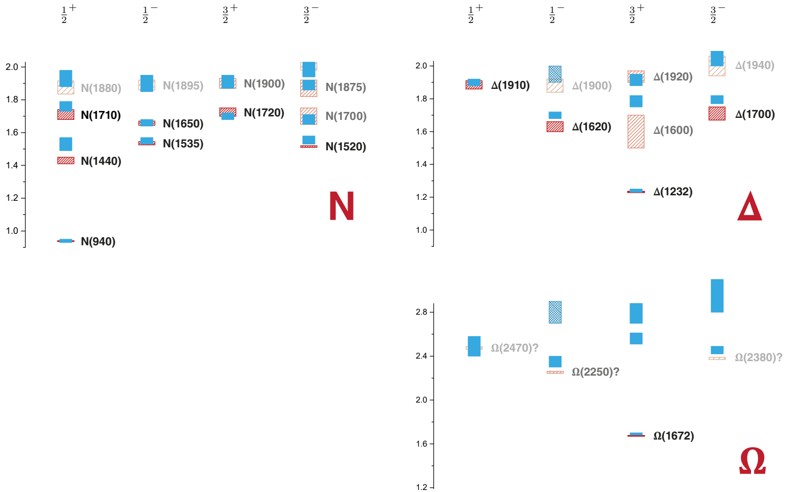
Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



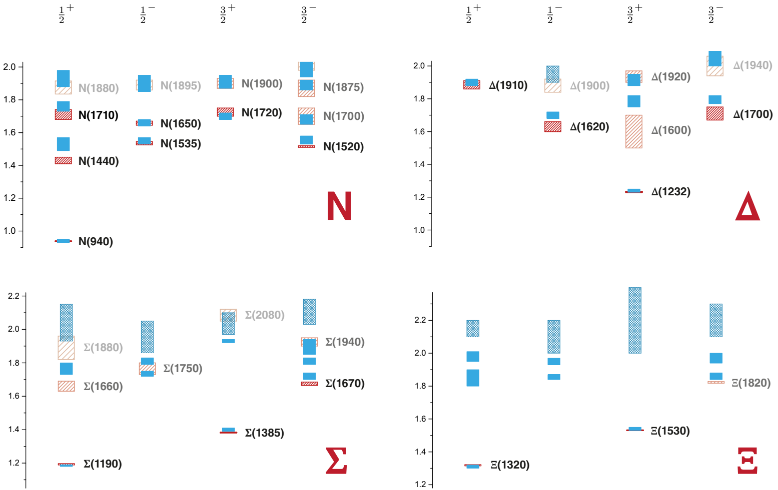


- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

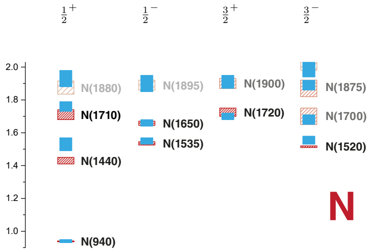
Strange baryons

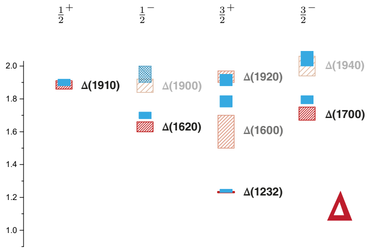


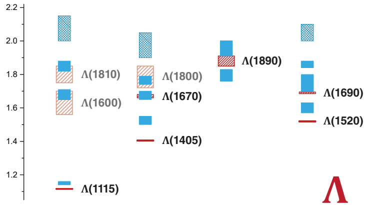
Strange baryons



Strange baryons







- Strange baryons similar to **light baryons:**

  $\mathbf{\Omega} \rightarrow \mathbf{\Delta}$
  $\mathbf{\Sigma}, \mathbf{\Xi} \rightarrow \mathbf{N} + \mathbf{\Delta}$ $\rightarrow$ rich spectrum!
  $\mathbf{\Lambda} \rightarrow \mathbf{N}$ + singlets

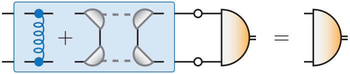
- Roper, $\mathbf{\Delta}$(1600), $\mathbf{\Lambda}$(1405), $\mathbf{\Lambda}$(1520): additional dynamics?

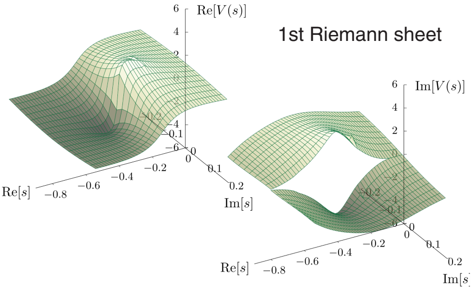
GE, Fischer, in preparation

Resonances!

ρ meson as a dynamical resonance

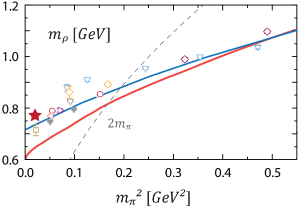
Williams, 1804.11161

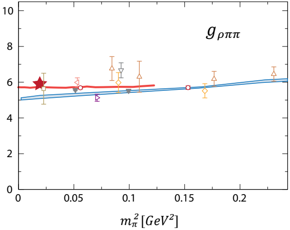




RL

RL + $\pi\pi$





Lattice references: GE et al., PPNP 91 (2016) 1606.09602

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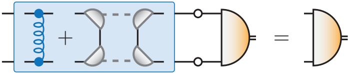

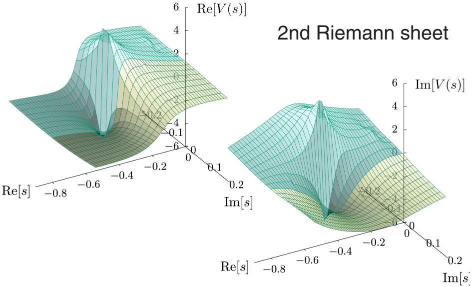

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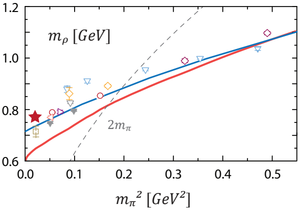

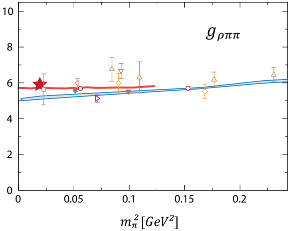

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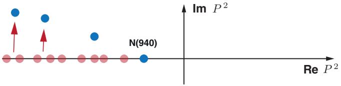
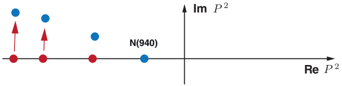
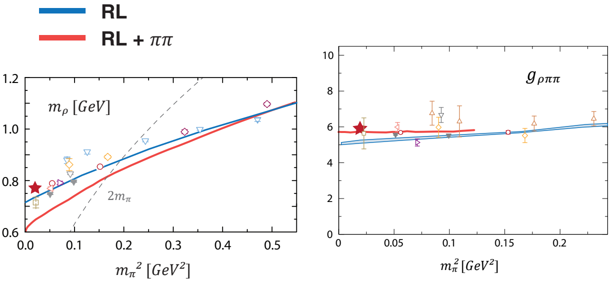
Resonances!

Lattice:
Proper treatment of
resonances essential

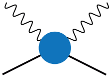


DSE / BSE:
Resonance dynamics
“on top of” quark-gluon dynamics

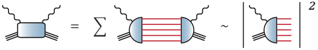




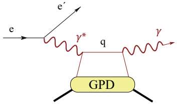
Compton scattering



Structure functions
& PDFs in forward limit

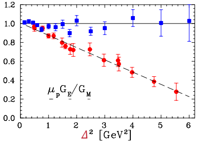


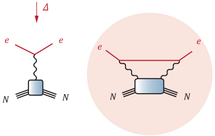
Handbag dominance
& GPDs in DVCS



TPE corrections to form factors

Guichon, Vanderhaeghen, PRL 91 (2003)



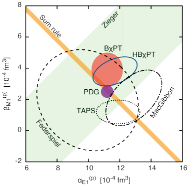


Proton radius puzzle?

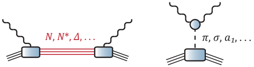
Antonigni et al., 2013, Pohl et al. 2013,
Birse, McGovern 2012, Carlson 2015

Nucleon polarizabilities

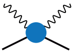
Hagelstein, Miskimen, Pascalutsa,
Prog. Part. Nucl. Phys. 88 (2016)



Resonances!



Kinematics



$$= \sum_{i=1}^{18} c_i(\eta_+, \eta_-, \omega, \lambda)\, \bar{u}(p_f)\, X_i^{\mu\nu}(p, Q, Q')\, u(p_i)$$

18 CFFs

4 kinematic variables:

$$\eta_+ = \frac{Q^2+Q'^2}{2m^2}$$
$$\eta_- = \frac{Q\cdot Q'}{m^2}$$
$$\omega = \frac{Q^2-Q'^2}{2m^2}$$
$$\lambda = -\frac{p\cdot Q}{m^2}$$

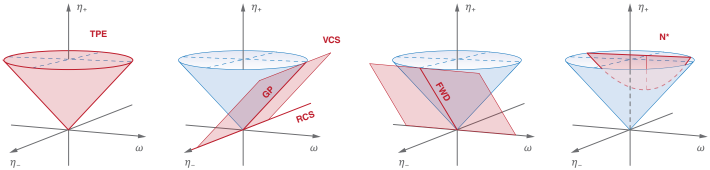
18 Compton tensors,
form minimal basis

- systematic derivation
- similar to Tarrach basis
  Tarrach, Nuovo Cim. A28 (1975)

$$X_i' = U_{ij}\, X_j\,, \qquad \det U = const.$$

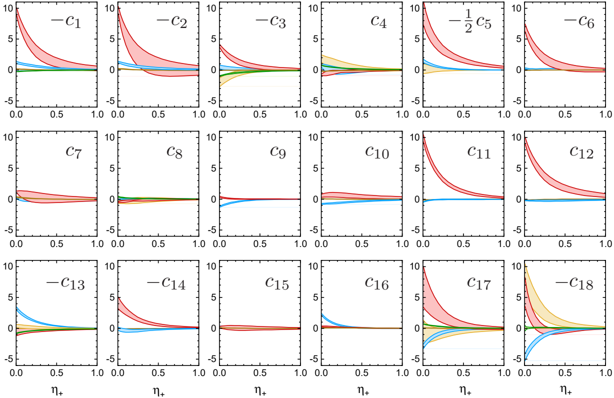
- CFFs free of kinematics

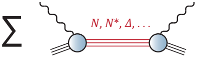
$$X_1^{\mu\nu} = \frac{1}{m^4}\, t^{\mu\alpha}_{Q'p}\, t^{\alpha\nu}_{pQ}\,,$$
$$X_2^{\mu\nu} = \frac{1}{m^2}\, t^{\mu\nu}_{Q'Q}\,,$$
$$X_3^{\mu\nu} = \frac{1}{m^4}\, t^{\mu\alpha}_{Q'Q'}\, t^{\alpha\nu}_{QQ}\,,$$
$$X_4^{\mu\nu} = \frac{1}{m^6}\, t^{\mu\alpha}_{Q'Q'}\, p^\alpha p^\beta\, t^{\beta\nu}_{QQ}\,,$$
$$X_5^{\mu\nu} = \frac{\lambda}{m^4}\left(t^{\mu\alpha}_{Q'Q'}\, t^{\alpha\nu}_{pQ} + t^{\mu\alpha}_{Q'p}\, t^{\alpha\nu}_{QQ}\right),$$
$$X_6^{\mu\nu} = \frac{1}{m^2}\, \varepsilon^{\mu\nu}_{Q'Q}\,,$$
$$X_7^{\mu\nu} = \frac{1}{im^3}\left(t^{\mu\alpha}_{Q'Q'}\, \varepsilon^{\alpha\nu}_{\gamma Q} - \varepsilon^{\mu\alpha}_{Q'\gamma}\, t^{\alpha\nu}_{QQ}\right),$$
$$X_8^{\mu\nu} = \frac{\omega}{im^3}\left(t^{\mu\alpha}_{Q'Q'}\, \varepsilon^{\alpha\nu}_{\gamma Q} + \varepsilon^{\mu\alpha}_{Q'\gamma}\, t^{\alpha\nu}_{QQ}\right),$$
$$\vdots$$

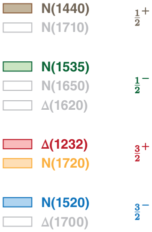


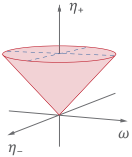
GE, Ramalho, in preparation

Compton form factors



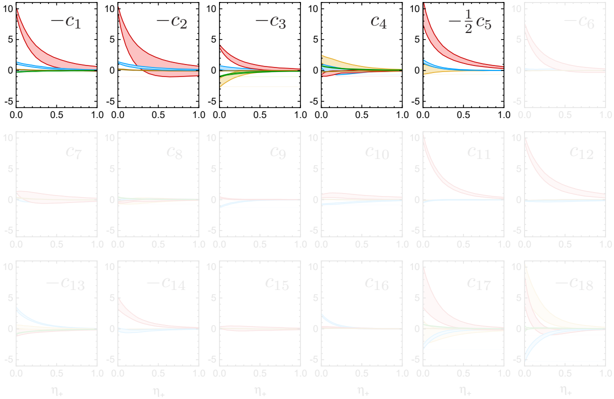






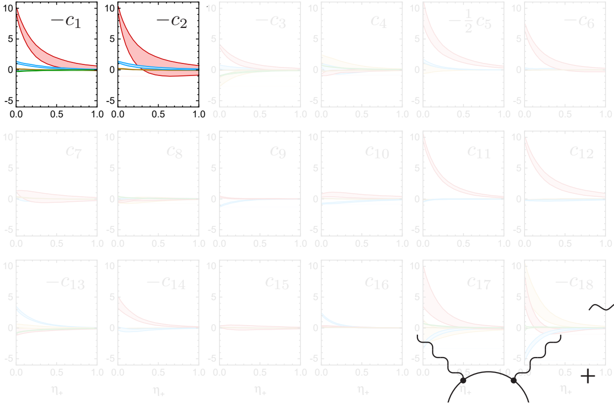
To be multiplied with $\frac{(m_R^2-m^2)^2}{(s-m_R^2)(u-m_R^2)} = \frac{\delta^2}{(\eta_-+\delta)^2-4\lambda^2}$

Compton form factors

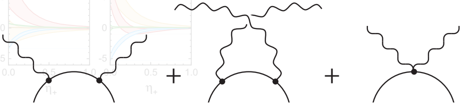


- CS on **scalar particle**

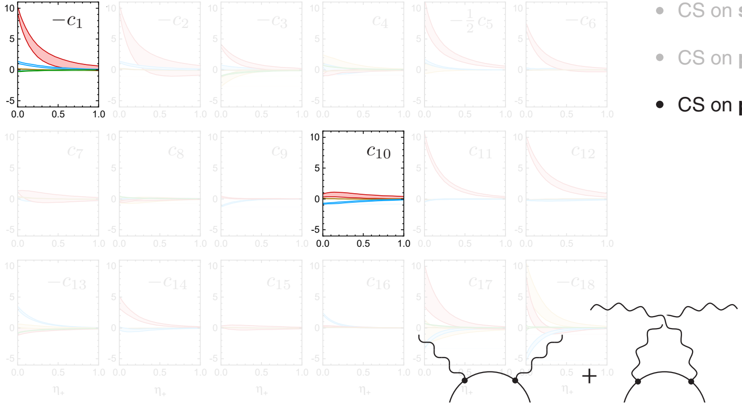
Compton form factors



- CS on **scalar particle**
- CS on **pointlike scalar**

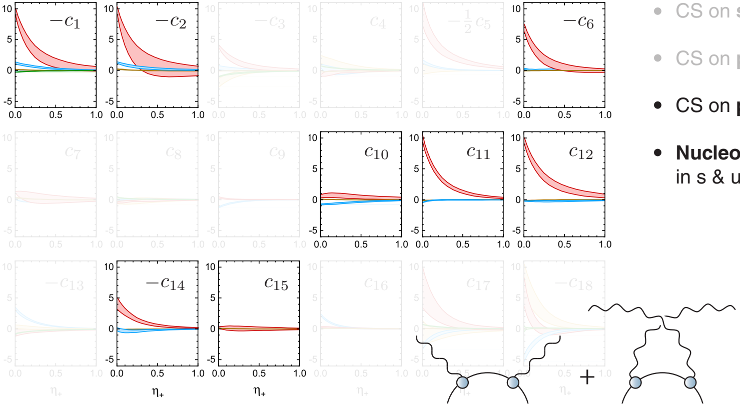


Compton form factors



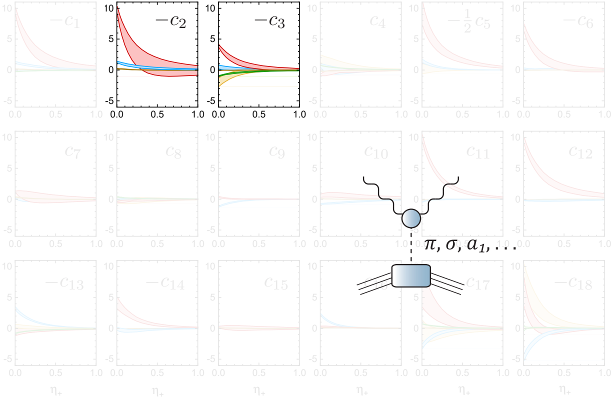
- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**

Compton form factors



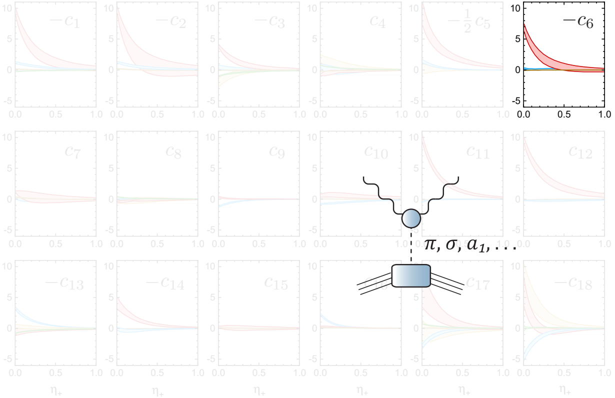
- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**
- **Nucleon Born poles** in s & u channel

Compton form factors



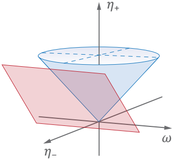
- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**
- **Nucleon Born poles** in s & u channel
- **Scalar pole** in t channel

Compton form factors

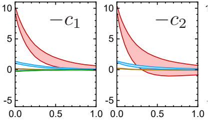


GE, Fischer, Weil, Williams,
PLB 774 (2017)

- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**
- **Nucleon Born poles** in s & u channel
- **Scalar pole** in t channel
- **Pion pole** in t channel ($\pi^0 \to \gamma^* \gamma^*$)



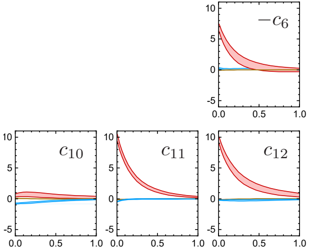
Polarizabilities

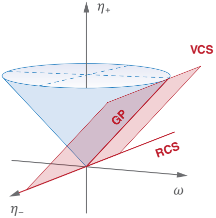


Scalar polarizabilities:

$$\begin{bmatrix} \alpha+\beta \\ \beta \end{bmatrix} = -\frac{\alpha_{\text{em}}}{m^3} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Large **Δ(1232)** contribution, also **N(1520)** non-negligible



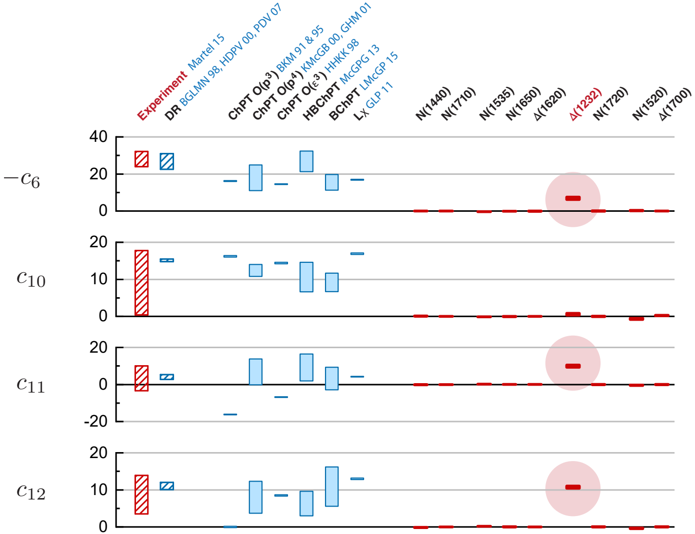


Spin polarizabilities:

$$\begin{bmatrix} \gamma_{E1E1} \\ \gamma_{M1M1} \\ \gamma_{E1M2} \\ \gamma_{M1E2} \end{bmatrix} = \frac{\alpha_{\text{em}}}{2m^4} \begin{bmatrix} c_6 + 4c_{11} - 4c_{12} \\ -c_6 - 2c_{10} + 4c_{12} \\ c_6 + 2c_{10} \\ -c_6 \end{bmatrix}$$

$$\begin{bmatrix} \gamma_0 \\ \gamma_\pi \end{bmatrix} = -\frac{2\alpha_{\text{em}}}{m^4} \begin{bmatrix} c_{11} \\ c_6 + c_{10} + c_{11} - 2c_{12} \end{bmatrix}$$

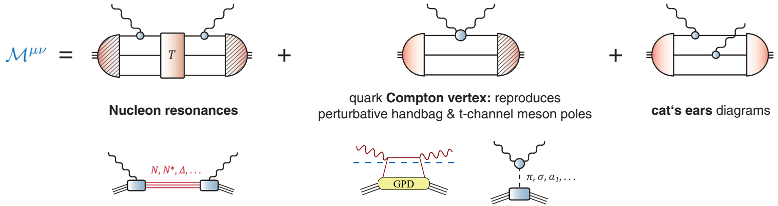
Spin polarizabilities



Only $\Delta(1232)$ important

Compton scattering

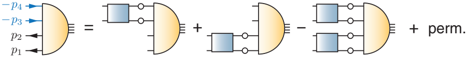
Scattering amplitude: GE, Fischer, PRD 85 (2012) & PRD 87 (2013)



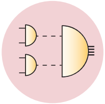
- **Poincaré covariance** and **crossing symmetry** automatic
- **em. gauge invariance** and **chiral symmetry** automatic
  (as long as all ingredients calculated within same truncation)
- **perturbative processes** included
- **s, t, u channel poles** dynamically generated,
  no need for "offshell hadrons"

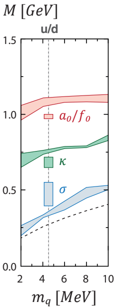
Towards multiquarks

- **Light scalar mesons** σ, κ, a_0, f_0 as **tetraquarks:** solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)



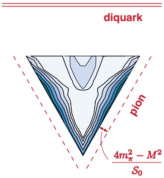
→





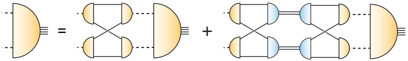
BSE dynamically generates **meson poles** in wave function:

$f_i(\mathcal{S}_0, ▽, △, ○) \to$ 1500 MeV
$f_i(\mathcal{S}_0, ▽, △, ○) \to$ 1500 MeV
$f_i(\mathcal{S}_0, ▽, △, \cdot) \to$ 1200 MeV
$f_i(\mathcal{S}_0, ▽, \cdot, ○) \to$ **350 MeV !!**



Four quarks rearrange to **"meson molecule"**

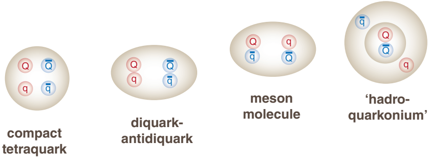
- Similar in **meson-meson / diquark-antidiquark** approximation (analogue of quark-diquark for baryons)
  Heupel, GE, Fischer, PLB 718 (2012)



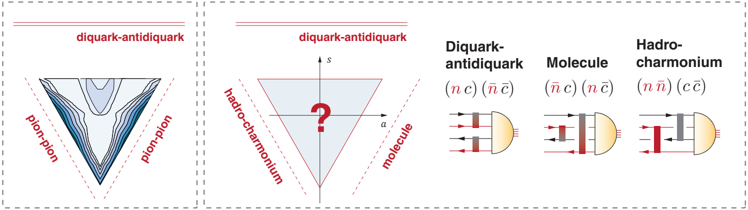
Backup slides

Tetraquarks in charm region?

- Can we **distinguish** different tetraquark configurations?

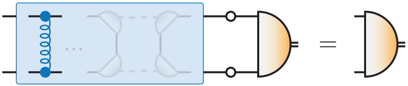


- **Four quarks** dynamically rearrange themselves into dq-$\overline{\text{dq}}$, molecule, hadroquarkonium; strengths determined by four-body BSE:

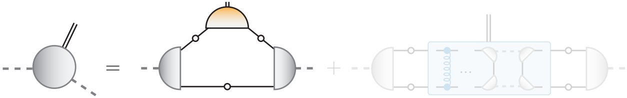


Resonances?

$\rho \to \pi\pi$: **resonance dynamics**
only beyond rainbow-ladder,
would shift ρ pole into complex plane
(above $\pi\pi$ threshold)

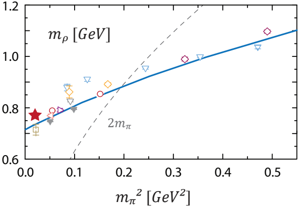


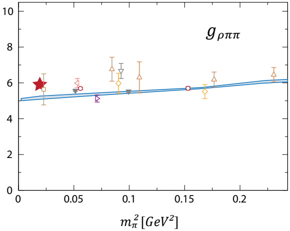
But ρ **decay width**
already calculable
in rainbow-ladder



Rainbow-ladder vs. lattice:

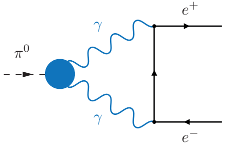
References: GE et al., PPNP 91 (2016) 1606.09602





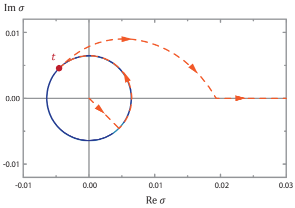
Developing numerical tools

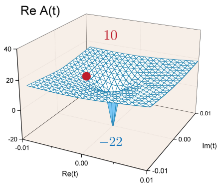
Rare pion decay $\pi^0 \to e^+ e^-$:

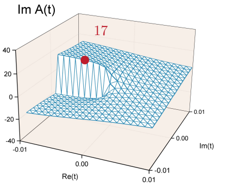


$$A(t) = \int d\sigma \int dz \, \cdots \, \frac{1}{k^2+m^2} \, \frac{1}{Q^2} \, \frac{1}{Q'^2}$$

Photon and lepton poles produce branch cuts in complex plane: **deform integration contour!**







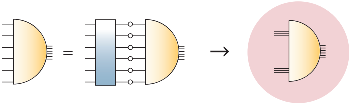
- Result agrees with dispersion relations
- Algorithm is stable & efficient
- Can be applied to any integral as long as **singularity locations** known

Weil, GE, Fischer, Williams, PRD 96 (2017)

→ talk by **Richard Williams**

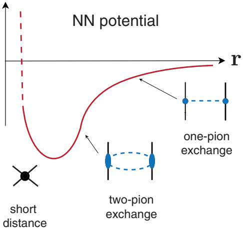
Towards multiquarks

Transition from **quark-gluon** to **nuclear degrees of freedom:**

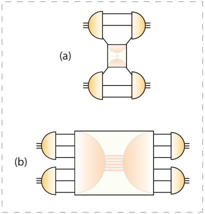


- 6 ground states, one of them **deuteron**
  Dyson, Xuong, PRL 13 (1964)
- Dibaryons vs. **hidden color?**
  Bashkanov, Brodsky, Clement, PLB 727 (2013)
- **Deuteron FFs** from quark level?

Microscopic origins of nuclear binding?



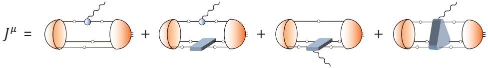
Weise, Nucl. Phys. A805 (2008)



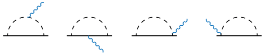
- only quarks and gluons
- **quark interchange** and **pion exchange** automatically included
- **dibaryon** exchanges

Form factors

Nucleon em. form factors
from three-quark equation
GE, PRD 84 (2011)

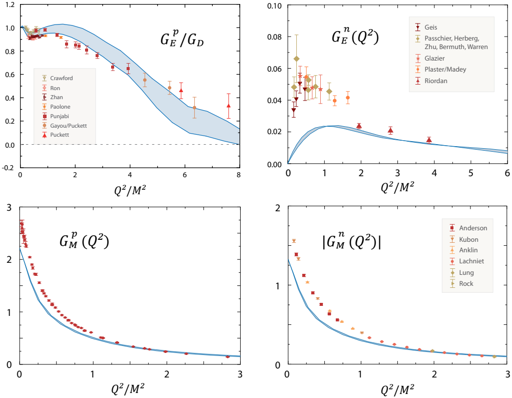


- **"Quark core without pion cloud"**



- **similar:** $N \to \Delta\gamma$ transition,
  axial & pseudoscalar FFs,
  octet & decuplet em. FFs

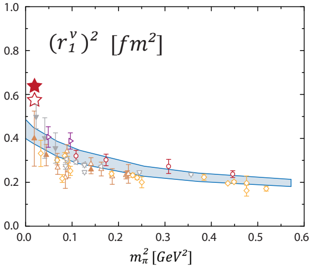
  Review: GE, Sanchis-Alepuz, Williams,
  Fischer, Alkofer, PPNP 91 (2016), 1606.09602



Form factors

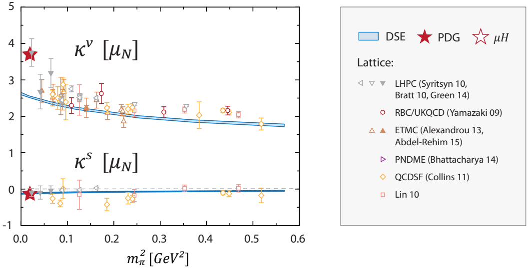
Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

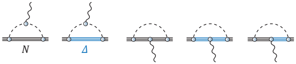


Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

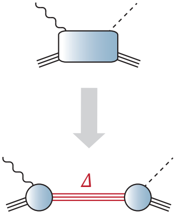
Exp: $\kappa^s = -0.12$

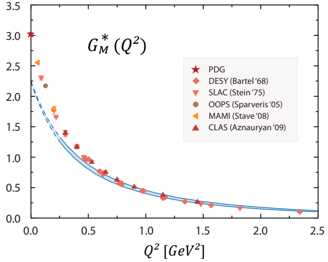
Calc: $\kappa^s = -0.12(1)$

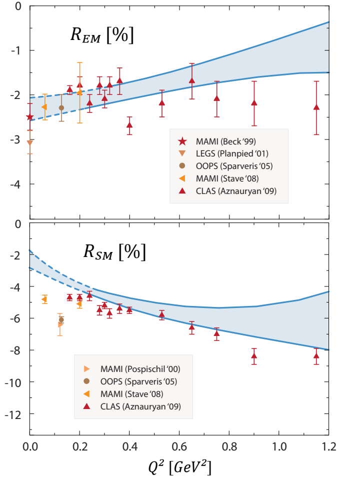
!!

GE, PRD 84 (2011)

Nucleon-Δ-γ transition



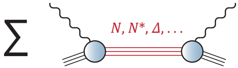




- **Magnetic dipole transition (G_M^*)** dominant:
  quark spin flip (s wave). "Core + 25% pion cloud"

- **Electric & Coulomb quadrupole ratios**
  small & negative, encode deformation.
  Reproduced without pion cloud: **OAM** from **p waves!**
  GE, Nicmorus, PRD 85 (2012)

Nucleon resonances



$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
N(940)	**N(1720)**	**N(1535)**	**N(1520)**
N(1440)	N(1900)	**N(1650)**	N(1700)
N(1710)		N(1895)	N(1875)
N(1880)			
Δ(1910)	**Δ(1232)**	**Δ(1620)**	**Δ(1700)**
	Δ(1600)	Δ(1900)	Δ(1940)
	Δ(1920)		

Need **em. transition FFs**

But vertices are half offshell:
need 'consistent couplings'

Pascalutsa, Timmermans, PRC 60 (1999)

- **em gauge invariance:** $Q^\mu \, \Gamma^{\alpha\mu} = 0$
- **spin-3/2 gauge invariance:** $k^\alpha \, \Gamma^{\alpha\mu} = 0$
- invariance under **point transformations:** $\gamma^\alpha \, \Gamma^{\alpha\mu} = 0$
- no kinematic dependencies, **"minimal" basis**

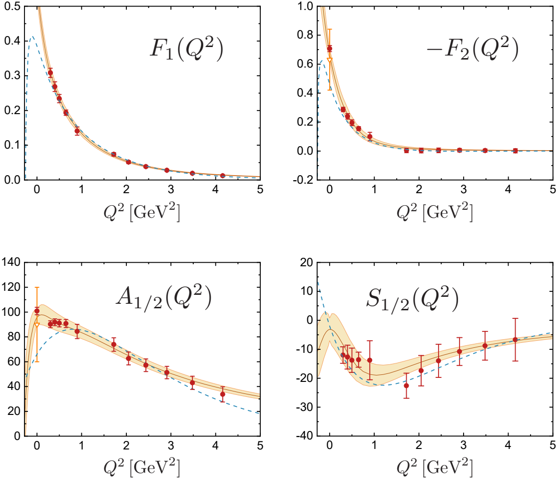
Most general **offshell vertices** satisfying these constraints:

GE, Ramalho, in preparation

$$\frac{1}{2}^+ \to \frac{1}{2}^\pm : \quad \Gamma^\mu = \begin{bmatrix} \mathbf{1} \\ \gamma_5 \end{bmatrix} \sum_{i=1}^{8} F_i \, T_i^\mu \qquad \begin{cases} t^{\mu\nu}_{QQ}\gamma^\nu \\ [\gamma^\mu, \not{Q}] \\ \dots \end{cases}$$

$$\frac{1}{2}^+ \to \frac{3}{2}^\pm : \quad \Gamma^{\alpha\mu} = \begin{bmatrix} \gamma_5 \\ \mathbf{1} \end{bmatrix} \sum_{i=1}^{12} F_i \, T_i^{\alpha\mu} \qquad \begin{cases} \varepsilon^{\alpha\mu}_{kQ} \\ t^{\alpha\mu}_{kQ} \\ i t^{\alpha\beta}_{k\gamma} t^{\beta\mu}_{QQ} \\ \dots \end{cases}$$

Nucleon resonances



N(1535) transition FFs:
no kinematic constraints

Fit

Example:
N(1535) helicity amplitudes

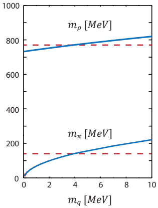
PDG

CLAS data
userweb.jlab.org/~mokeev/resonance_electrocouplings

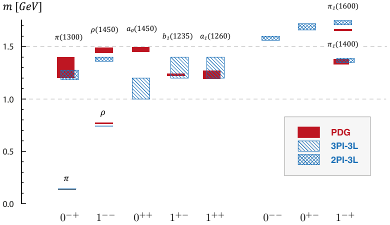
MAID
Tiator, Drechsel, Kamalov, Vanderhaeghen, EPJ 198 (2011)

Mesons

- Pion is **Goldstone boson:** $m_\pi^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder



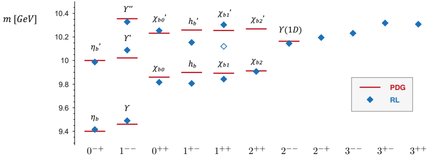
Williams, Fischer, Heupel, PRD 93 (2016)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

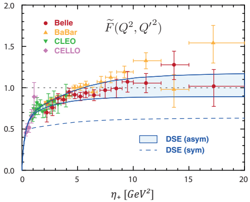
see also Chang, Roberts, PRL 103 (2009), PRC 85 (2012)

- **Charmonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



- **Pion transition form factor**

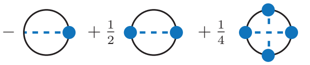


GE, Fischer, Weil, Williams, PLB 774 (2017)

nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
3PI at 3-loop: **all two- and three-point functions are dressed;** 4, 5, ... do not appear.

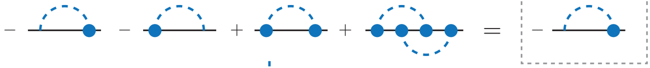
$\Gamma_2 =$



see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

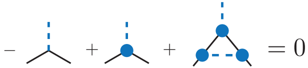
Self-energy:

$\Sigma = \frac{\delta \Gamma_2}{\delta D} =$



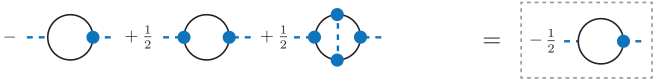
Vertex:

$\frac{\delta \Gamma_2}{\delta V} = 0 \quad \Rightarrow$



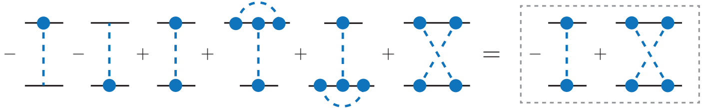
Vacuum polarization:

$\Sigma' = \frac{\delta \Gamma_2}{\delta D'} =$



BSE kernel:

$-K = \frac{\delta \Sigma}{\delta D} =$



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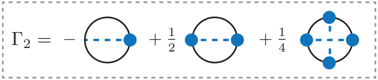

see: Sanchis-Alepuz & Williams,
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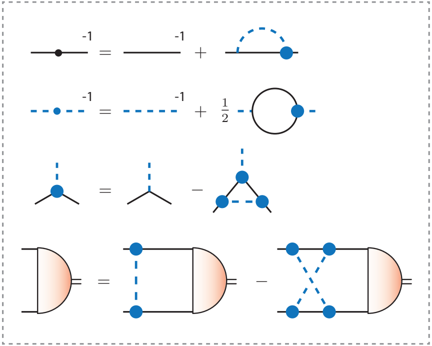
So we arrive at a closed system of equations:



- Crossed ladder cannot be added by hand, requires **vertex correction!**

nPI effective action

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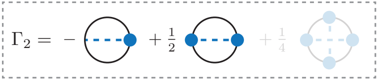

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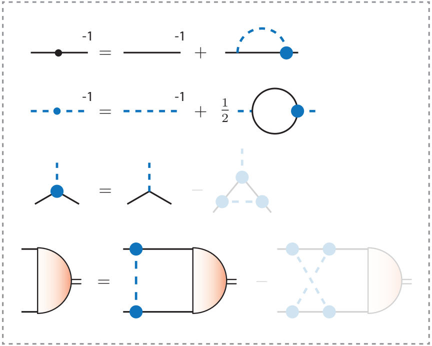
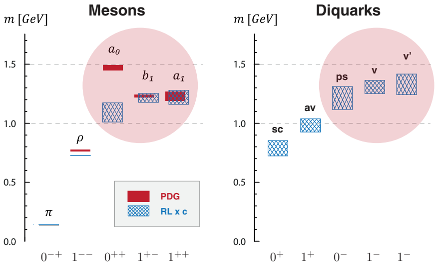
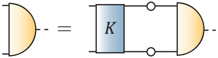


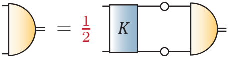
- Crossed ladder
  cannot be added by hand,
  requires **vertex correction!**
- without 3-loop term:
  rainbow-ladder
  with tree-level vertex ⇒ 2PI
- but still requires **DSE solutions**
  for propagators!
- Similar in QCD. nPI truncation
  guarantees chiral symmetry,
  massless pion in chiral limit, etc.

The role of diquarks

Mesons and 'diquarks' closely related:
after taking traces, only factor 1/2 remains
⇒ **diquarks 'less bound' than mesons**





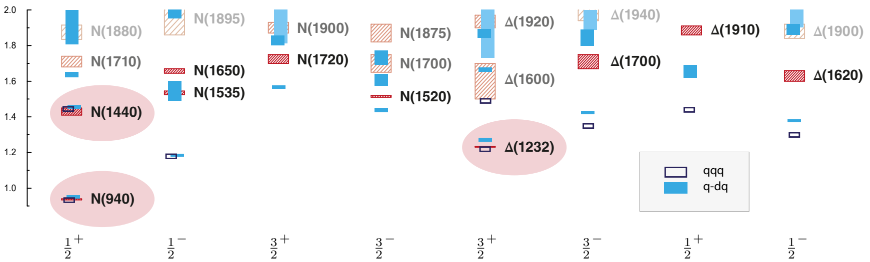


Pseudoscalar & vector mesons ⇔ **Scalar & axialvector diquarks**
already good in rainbow-ladder — sufficient for nucleon and Δ

Scalar & axialvector mesons ⇔ **Pseudoscalar & vector diquarks**
too light, repulsion beyond RL — important for remaining channels

Baryon spectrum I

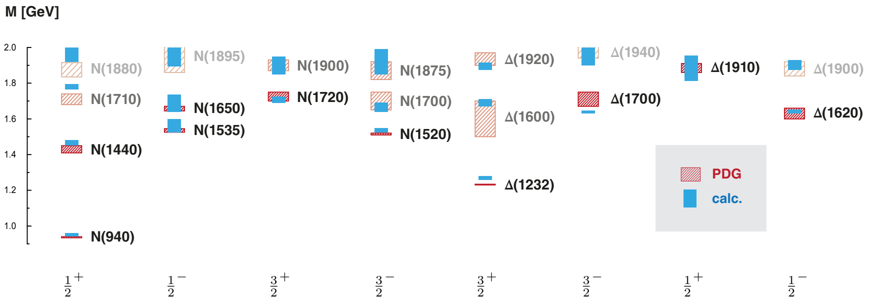
Three-quark vs. quark-diquark in rainbow-ladder: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

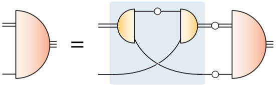


- **qqq** and **q-dq** agrees: N, Δ, Roper, N(1535)
- # levels compatible with experiment: **no states missing**
- N, Δ and their 1st excitations (including **Roper**) agree with experiment
- But remaining states too low ⇒ wrong level ordering between Roper and N(1535)

Baryon spectrum

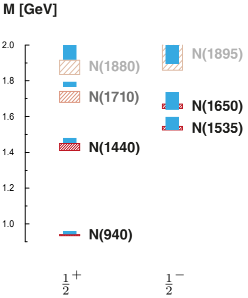
Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)





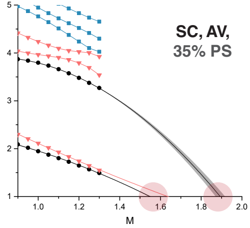
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

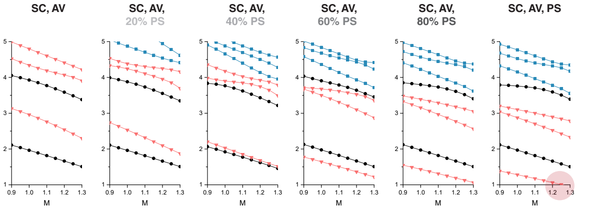
Baryon spectrum



Level ordering between
Roper and N(1535):

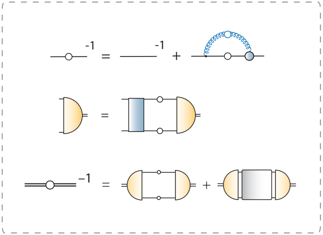
dynamics of ps diquark produces
2 nearby states: **N(1535), N(1650)**

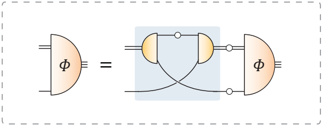




Strange baryons

	[nn]	{nn}	[ns]	{ns}	{ss}
N	●	●			
Δ		●			
Λ	●		●	●	
Σ		●	●	●	
Ξ			●	●	●
Ω					●



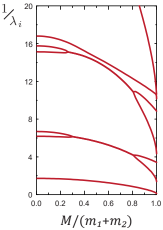


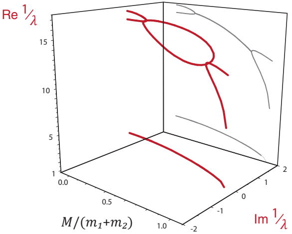
Complex eigenvalues?

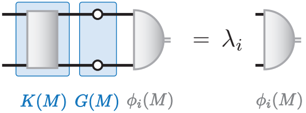
Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with "**anomalous**" **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)







K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

$$K\, L^\dagger L\, \phi_i = \lambda_i\, \phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

$\Rightarrow$ Hermitian problem with same EVs!

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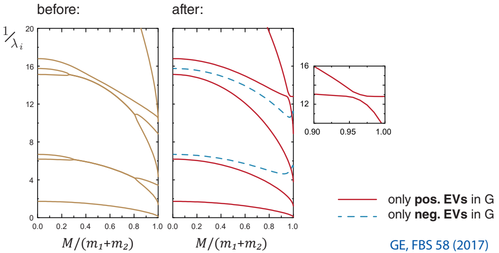
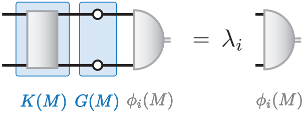


GE, FBS 58 (2017)



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⇒ Hermitian problem with same EVs!

⇒ all EVs strictly **real**
⇒ level repulsion
⇒ "anomalous states" removed?

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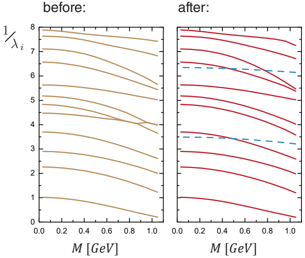
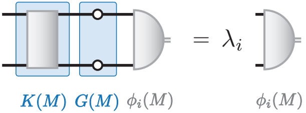


Eigenvalue spectrum for pion channel
GE, FBS 58 (2017)



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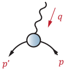
⇒ level repulsion
⇒ "anomalous states" removed?

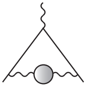
Muon g-2

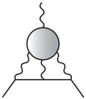
- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ



$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



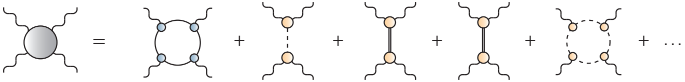


a_μ $[10^{-10}]$

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

- **LbL amplitude:** ENJL & MD model results
  Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	-1	2	-2	$(\times 10^{-10})$

Muon g-2

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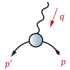
  total SM prediction deviates from exp. by ~3σ



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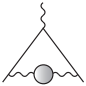
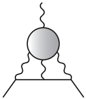


Hadronic vacuum polarization



Hadronic light-by-light scattering

$a_\mu\ [10^{-10}]$ — Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

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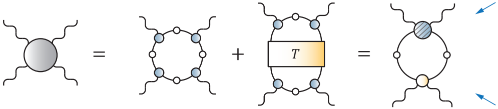
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



quark Compton vertex

Born terms

- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE, Fischer, Heupel, PRD 92 (2015)