JLab – LC2018 – 18 May 2018

Entanglement on the Light Front

Piet Mulders

European Research Council

Netherlands Organisation for Scientific Research

Nikhef

VU UNIVERSITY AMSTERDAM

p.j.g.mulders@vu.nl

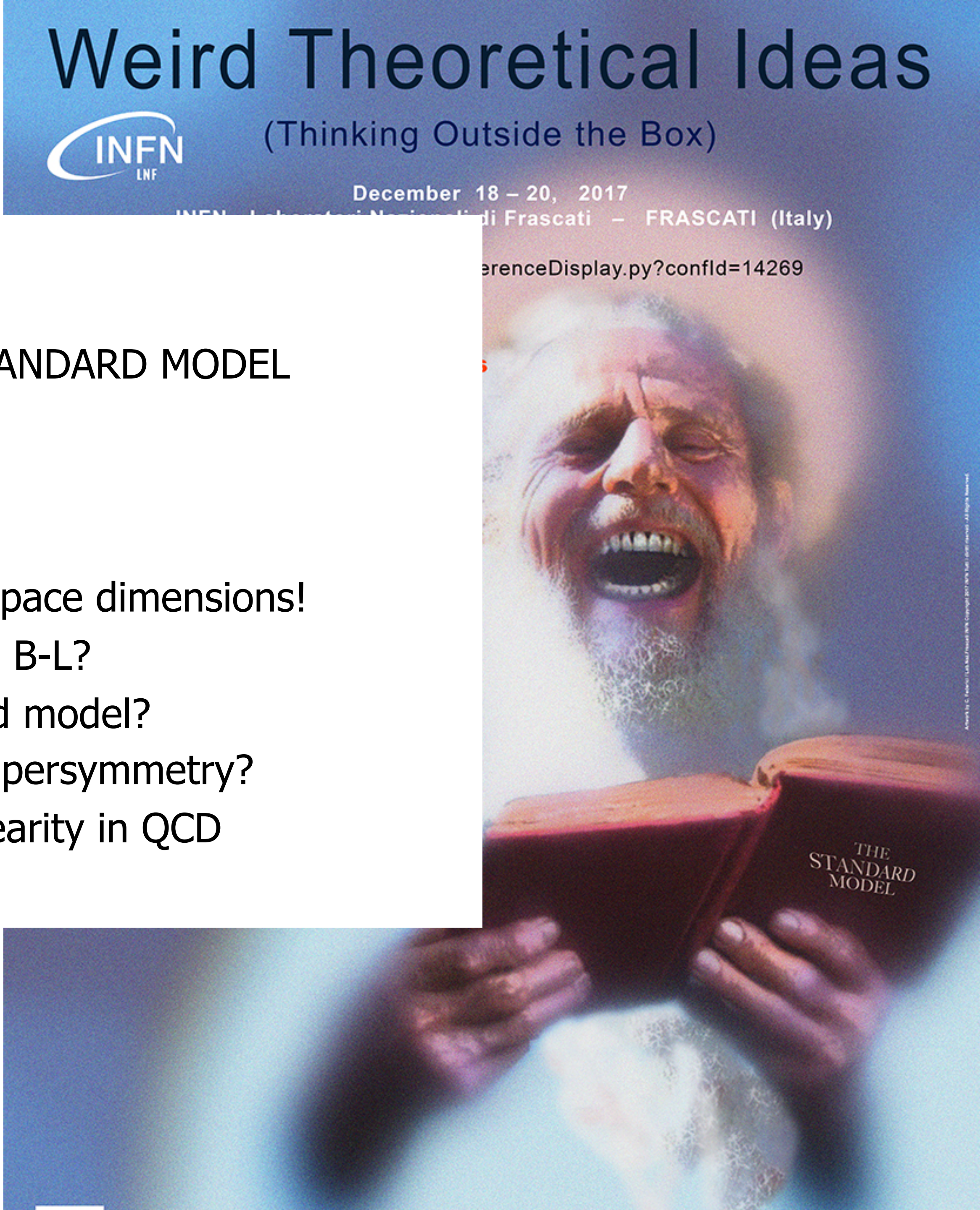
abstract

LC2018 (JLab, 18 May 2018):

Entanglement on the Light Front

P.J. Mulders

The importance and success of collinearity in QCD at high energies and its simple quantum mechanical interpretation on the light front in 3D can be combined in a larger picture of emergent symmetries for the standard model in which both leptons and quarks originate as non-equivalent maximally entangled states living in a tripartite space of 1D states. Even if at this stage it does not upset the present successful phenomenology or make striking predictions , the hope is that it could simplify the complexity and apparent arbitrariness of the standard model.



- (NOT) HAPPY WITH STANDARD MODEL

- In spite of its success!

- Three families, colors, space dimensions!
- Left-right (a)symmetry? B-L?
- Mass ranges in standard model?
- Naturalness? Missing supersymmetry?
- Confinement and collinearity in QCD (jets, PDFs, TMDs)?

MULTIPARTITE STATES

QIT – entangled states and QCD

- Parton-hadron duality in hard QCD scattering: PDFs x FFs
  - nucleon is pure state → ensemble of partons [Kharzeev & Levin 1702.03489]
  - Parton physics in essence good collinear d.o.f. $\frac{1}{2}\gamma^-\gamma^+\psi \ \text{ and } \ g_T^{\alpha\mu}A_\mu^a$
  - hard (short distance) process: partons → partons
  - emerging partons are pure state(s) → ensemble of hadron states

- Entangled (pure) states $|\Phi>$ in $\mathcal{H}^A \otimes \mathcal{H}^B$ with a density matrix $\rho = |\Phi><\Phi|$ lead to ensembles (non-pure state) in the reduced spaces.
  - EPR bipartite pure state leads to a 50% - 50% ensemble in both subspaces.

- Conclusion: both hadrons and partons live in a multipartite Hilbert space !
- Possibly combined with a principle of maximal entanglement (MaxEnt), such as hinted at in Cervera-Lierta, Latorre, Rojo & Rottoli (1703.02989): maximally entangled chiral left/right two-particle states are consistent with QED (g_A=0) & electroweak (g_V=0), at least if sin Θ_W = ½

QIT – pure and entangled states

- A pure quantum state: density matrix ρ = |Φ><Φ| of rank 1 which is a projection operator $\rho^2 = \rho$ and $\mathrm{Tr}(\rho^2) = \mathrm{Tr}(\rho) = 1$.
- An ensemble of states is characterized by a (hermitean) density matrix

$$\rho = \sum_a |n\rangle\, p_n \langle n| \text{ with } \mathrm{Tr}(\rho) = \sum_n p_n = 1$$

(for the next points |n> will be choosen orthogonal)

- Measures for purity are among others the concurrence

$$\text{pure} = \; 0 \leq \frac{C^2(\rho)}{2} = 1 - \mathrm{Tr}(\rho^2) = 1 - \sum_n^N p_n^2 = \sum_{m\neq n}^N p_m p_n \leq \frac{N-1}{N} \; = \text{non-pure}$$

- or the von Neumann entropy (in case of set of probabilites Shannon entropy)

$$\text{pure} = 0 \leq S = -\mathrm{Tr}(\rho \ln \rho) = -\sum_n^N p_n \ln p_n < \ln N = \text{non-pure}$$

- Entangled states live in a multipartite Hilbert space, e.g. $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$ and cannot be written as a direct product state $|\Phi\rangle \neq |a\rangle \otimes |b\rangle \otimes |c\rangle = |abc\rangle$

Essence of multipartite spaces is presence of decoupled bases for spaces: Alice-Bob-Eve, multi-particle, flavor-spin, ...

QIT – signal of entangled states

- For entangled pure states $|\Phi\rangle$ in $\mathcal{H}^A \otimes \mathcal{H}^B$ the density matrix $\rho = |\Phi\rangle\langle\Phi|$ leads to a non-pure reduced density matrix $\rho_A \equiv \mathrm{Tr}_B(\rho)$ and $\rho_B \equiv \mathrm{Tr}_A(\rho)$
  - Example: an entangled pure bipartite state can be Schmidt decomposed $|\Phi\rangle = \sum_n \sqrt{p_n}|a_n b_n\rangle$ with $|a_n\rangle$ and $|b_n\rangle$ state-specific orthogonal bases.
  - The reduced density matrix is $\rho_A \equiv \mathrm{Tr}_B(\rho) = \sum_n |a_n\rangle\, p_n \langle a_n|$
  - The state is entangled if matrix ρ_A is non-pure (has rank larger than one)
  - purity in subspace gives a measure for the entanglement, maximized for an equal partition of probabilities $p_n = 1/N$.
- For a general bipartite state built from qubit bases $|R\rangle$ and $|L\rangle$ given by w.f. $|\Phi\rangle = a|RR\rangle + b|RL\rangle + c|LR\rangle + d|LL\rangle$ one has $\Delta = 2|ad - bc| \to 2\sqrt{p_1 p_2}$.
- Entangled multipartite states separate in equivalence classes (defined via Stochastic Local Operations and Classical Communication, SLOCC), where local refers to the subspace.
- For bipartite states there is just a single class of maximally entangled Bell states represented by $|\mathrm{Bell}\rangle = \frac{1}{\sqrt{2}}(|RR\rangle + |LL\rangle)$ (equivalently $|RR\rangle \pm |LL\rangle$ or $|RL\rangle \pm |LR\rangle$)

  Note that $\rho = |\mathrm{Bell}\rangle\langle\mathrm{Bell}| \implies \rho_A = \frac{1}{2}(|R\rangle\langle R| + |L\rangle\langle L|)$

Maximizing entanglement for bipartite states

- How can states become entangled?
- Theorem (new) in simple two-state example: scattering increases entanglement
  - Assume scattering states $|RL\rangle \pm |LR\rangle$ are MaxEnt with $2\delta = \delta_+ \pm \delta_-$

$$S = \begin{bmatrix} e^{i(\delta_+ + \delta_-)} & 0 \\ 0 & e^{i(\delta_+ - \delta_-)} \end{bmatrix} \longrightarrow S = e^{i\delta_+} \begin{bmatrix} \cos\delta_- & i\sin\delta_- \\ i\sin\delta_- & \cos\delta_- \end{bmatrix}$$

on basis |RL> and |LR>

  - Then scattering increases entanglement!

$$|in\rangle = \sqrt{p_1}|RL\rangle + \sqrt{p_2}|LR\rangle \longrightarrow |out\rangle = S^\dagger |in\rangle$$

$$\Delta = 2\sqrt{p_1 p_2} \longrightarrow \Delta = 2|\sqrt{p_1 p_2}\cos\delta_- - i(p_1 + p_2)\sin\delta_-| \geq 2\sqrt{p_1 p_2}$$

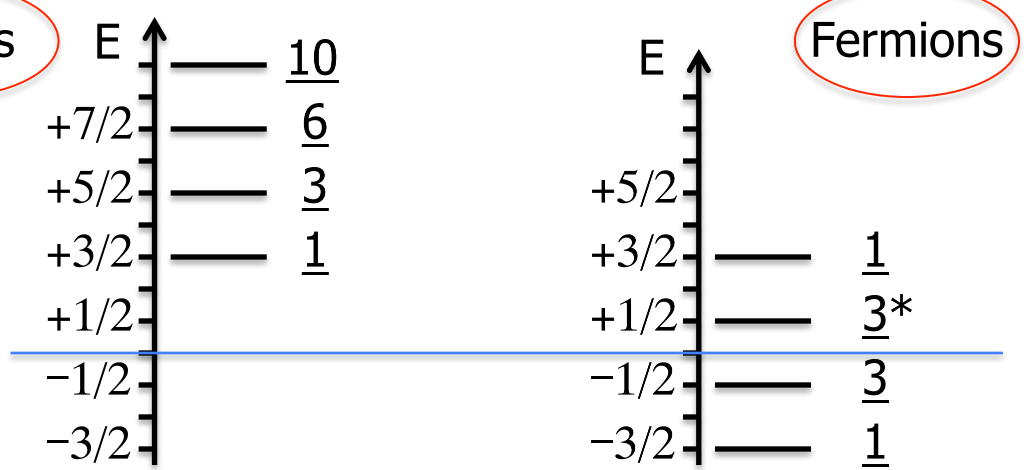
- This generalizes the result in Cervera-Lierta, Latorre, Rojo & Rottoli (1703.02989) and may lend support to their principle of maximal entanglement (MaxEnt)

QIT – example of multipartite states

- Look at multipartite spaces $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$ with basis of (pure) quantum states
  - Example: a 3D harmonic oscillator: states $|n_x n_y n_z>$ or $|n_r lm>$ or $|\underline{n}>$

level	degeneracy	(n_x, n_y, n_z)	SO(3) $(n_r\ell)$	SU(3) $(\underline{n})$
0	1	(0,0,0)	0s	$\underline{1}$
1	3	(1,0,0), . . .	0p	$\underline{3}$
2	6	(2,0,0), (1,1,0), . . .	1s $\oplus$ 0d	$\underline{6}$
3	10	(3,0,0), (2,1,0), (1,1,1), . . .	1p $\oplus$ 0f	$\underline{10}$
4	15	. . .	2s $\oplus$ 1d $\oplus$ 0g	$\underline{15}_s$

  - 3D HO is separable, has rotational [SO(3)] and more [SU(3)] symmetry. Symmetry eigenstates (involving ABC) are in general entangled states.
  - Bosons



Leptons and partons as multipartite states

- For entangled bipartite states there is just one class of entangled (Bell) states
  - $|\mathrm{Bell}\rangle = \frac{1}{\sqrt{2}}(|RL\rangle + |LR\rangle)$ or $\frac{1}{\sqrt{2}}(|RR\rangle + |LL\rangle)$

Class: equivalence through local operators and classical communication (SLOCC)

- For entangled tripartite states there are two classes (Dur, Vidal, Cirac 2000)
  - $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$ (fragile)
  - $|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$ (robust)

Class: for equivalence existence of local unitary transformations suffices $U \otimes U \otimes \ldots$

- Multipartites and R/L basis states relevant for symmetries of standard model:
  - 1D field theory: $\mathcal{H} = \mathcal{H}^{\otimes x}$ with right- and left-movers (chirality states)
  - Tripartite states: $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$
    - space (3D): leptons, electroweak (GHZ-class)
    - color: quarks in 1D, strong (W-class)

A paradigm shift?

- Start with less dimensions (1+3 → 1+1) advantageous
  - Convergence: $d[\phi] = (d-2)/2 \rightarrow 0$, $d[\xi] = (d-1)/2 \rightarrow ½$, naturalness, …
    [see Stojkovic – 1406.2696]
- Appropriate quantum states in multipartite space: ontological basis
  [see 't Hooft – 1405.1548]
  - 3D HO with states $|n_x n_y n_z\rangle$ or $|n_r l m\rangle$ or $|\underline{n}\rangle$; natural basis for spectroscopy of hadrons in quark model in multipartite space is the ($n_r lm$) basis.

N	configuration	SU(6) × O(3) multiplets
0	$(0s)^3$	$[56, 0^+]$
1	$(0s)^2(1p)$	$(56, 1^-)$ $[70, 1^-]$
2	$(0s)^2(2s)$	$(56, 0^+)$ $[70, 0^+]$
	$(0s)^2(2d)$	$(56, 2^+)$ $[70, 2^+]$
	$(0s)(1p)^2$	$[56, 0^+]$ $[56, 2^+]$ $(70, 0^+)$ $(70, 1^+)$ $(70, 2^+)$ $[20, 1^+]$

- Chirality (R/L) corresponding in 1D to right- and left-movers, P^+, P^- eigenstates can lead to emergence of chiral structure of quarks and leptons in standard model and the full $SU(3)_C \times [SU(2) \times U(1)]_{EW}$ symmetry (and even family structure).

A paradigm shift?

- Tripartite space for quarks naturally has color dual to space/electroweak.
  - Can explain why color naturally is decoupled from electroweak interactions
  - Color invisible in 3D: local gauge invariance! No free quarks or gluons!
  - Global color visible in 3D via valence quarks, N vs 1/N, f x D (distribution x fragmentation), color flow (future and past pointing gauge links), …
  - has a role for Wilson loops generating (gluon) TMDs (1805.05219)
- Natural arena for light-front approach with a 'preferred' space direction: quantization of good fields, dominating the OPE at high energies, these are asymptotic (free) fields (Kogut & Soper): $\frac{1}{2}\gamma^-\gamma^+\psi \quad \text{and} \quad g_T^{\alpha\mu}A_\mu^a$
- New ways to look at color-kinematic duality, soft collinear effective theory (SCET), ...

EMERGENCE OF SPACE-TIME DEPENDENCE

All Possible Symmetries of the S Matrix*

SIDNEY COLEMAN† AND JEFFREY MANDULA‡
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 16 March 1967)

We prove a new theorem on the impossibility of combining space-time and internal symmetries in any but a trivial way. The theorem is an improvement on known results in that it is applicable to infinite-parameter groups, instead of just to Lie groups. This improvement is gained by using information about the S matrix; previous investigations used only information about the single-particle spectrum. We define a symmetry group of the S matrix as a group of unitary operators which turn one-particle states into one-particle states, transform many-particle states as if they were tensor products, and commute with the S matrix. Let G be a connected symmetry group of the S matrix, and let the following five conditions hold: (1) G contains a subgroup locally isomorphic to the Poincaré group. (2) For any $M>0$, there are only a finite number of one-particle states with mass less than M. (3) Elastic scattering amplitudes are analytic functions of s and t, in some neighborhood of the physical region. (4) The S matrix is nontrivial in the sense that any two one-particle momentum eigenstates scatter (into something), except perhaps at isolated values of s. (5) The generators of G, written as integral operators in momentum space, have distributions for their kernels. Then, we show that G is necessarily locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.

I. INTRODUCTION

UNTIL a few years ago, most physicists believed that the exact or approximate symmetry groups of the world were (locally) isomorphic to direct products of the Poincaré group and compact Lie groups. This world-view changed drastically with the publication of the first papers on $SU(6)$[1]; these raised the dazzling possibility of a relativistic symmetry group which was not simply such a direct product. Unfortunately, all attempts to find such a group came to disastrous ends, and the situation was finally settled by the discovery of

symmetry group of the S matrix, which contains the Poincaré group and which puts a finite number of particles in a supermultiplet. Let the S matrix be nontrivial and let elastic scattering amplitudes be analytic functions of s and t in some neighborhood of the physical region. Finally, let the generators of G be representable as integral operators in momentum space, with kernels that are distributions. Then G is locally isomorphic to the direct product of the Poincaré group and an internal symmetry group. (This is a loose statement of the theorem; a more precise one follows below.)

Basic symmetries including SUSY

- Hilbert space

$$\{(a^\dagger)^n|0\rangle, b^\dagger|0\rangle\} \qquad [a,a^\dagger]=1,\ \{b,b^\dagger\}=1$$

- Supercharges

$$Q^\dagger_{ik} = b_i\, a^\dagger_k \text{ and } Q_{ik} = b^\dagger_i a_k \qquad \{Q^\dagger_{ik}, Q_{jl}\} = \tfrac{1}{2}\,\delta_{ij}\{a^\dagger_l, a_k\} + \tfrac{1}{2}\,\delta_{kl}[b^\dagger_i, b_j]$$

$$a^\dagger_k \xrightarrow{Q_{ik}} b^\dagger_i \qquad a^\dagger_k \xleftarrow{Q^\dagger_{ik}} b^\dagger_i$$

hamiltonian/number operators (i=j, k=l)
& unitary rotations

- For boson and fermion fields

$$\varphi = \frac{1}{\sqrt{2\omega}}(a+a^\dagger) \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}}(b+b^\dagger)$$

$$Q = \sqrt{\omega}(a^\dagger b - b^\dagger a)$$

$$[Q,\varphi] = \xi \qquad \{Q,\xi\} = \{Q,[Q,\varphi]\} = F = iD\varphi$$

$$[Q,F] = [Q,\{Q,\xi\}] = iD\xi$$

Single (free) field

$$F = [\varphi, H] = iD\varphi = i\dot{\varphi}$$

$$iD = i\partial + gA$$

unitary rotations

- Implement symmetries via constraints F

... and a nontrivial vacuum (not everything is for free!)

$$\phi(t) = \mathcal{T}\exp\left(-i\int_0^t ds\cdot D\right)\phi$$

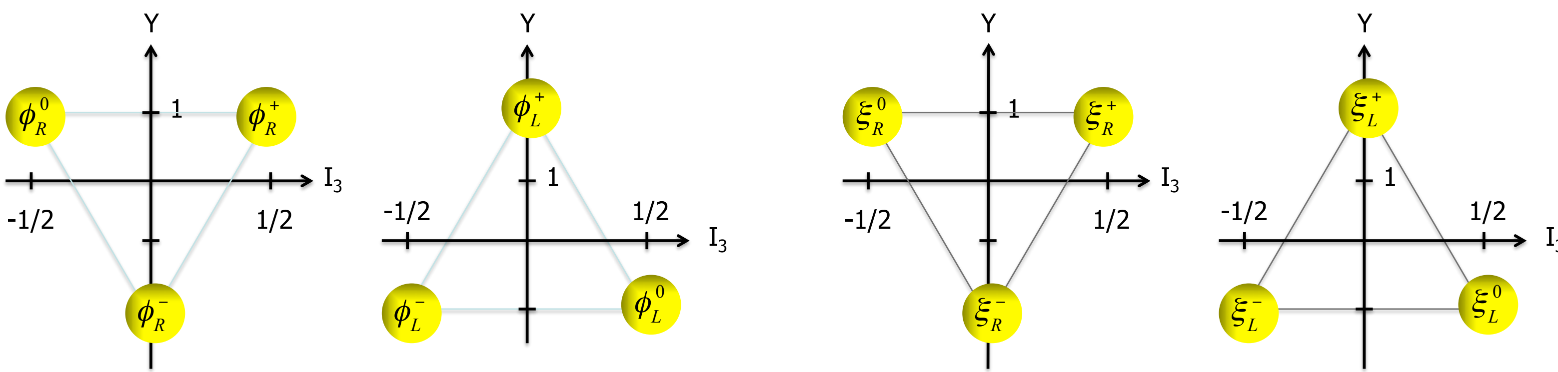
Basic starting point

- Right-Left symmetry
- Supersymmetry (Wess-Zumino structure)

  - Bosons: $$\phi\sqrt{2} = e^{i\pi/4}\phi_R + e^{-i\pi/4}\phi_L = \phi_S + i\phi_P$$

  - Fermions: $$\xi\sqrt{2} = \begin{bmatrix} \xi_R \\ -i\,\xi_L \end{bmatrix}$$

- Space-time structure via covariant derivatives and supercharges
- Different 'kinds' of particles corresponding to SLOCC equivalence classes where the fun starts in tripartite space with the following basis:



Emerging symmetries & space-time

Fields	Generators Space-time & Internal
Real/Majorana: $\phi \quad \xi$ and $\langle\phi\rangle = 1$	H
$\phi_{R/L} \quad \xi_{R/L}$ and $\langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino)	P^+, P^- K, SU(3)

$U(1)_R \times U(1)_L \times SU(3)$

Emerging symmetries & space-time

Fields	Generators Space-time	&	Internal
Real/Majorana: $\phi \quad \xi \quad \text{and} \ \langle\phi\rangle = 1$	H		
$\phi_{R/L} \quad \xi_{R/L} \quad \text{and} \ \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino → gauge theory)	P+, P–		K, <u>SU(3)</u>
			Z(2)
1D: $\phi_S \quad \phi_P \longrightarrow A^a_3 \quad \psi$	H, P, K		<u>SU(3)</u>

$$iD_\sigma\phi^i = i\partial_\sigma\phi^i + g_0 \sum_{a=1,\dots 8} A^a_\sigma (T_a)^i_j \phi^j$$

P(1,1) x SU(3)

Emerging symmetries & space-time

Fields	Generators Space-time & Internal
Real/Majorana: $\phi \quad \xi$ and $\langle\phi\rangle = 1$	H
$\phi_{R/L} \quad \xi_{R/L}$ and $\langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino → gauge theory)	P^+, P^- K, SU(3) Z(2)
1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$	H, P, K SU(3) = [SO(3),SU(2)xU(1)] Z(3)
3D: $\phi_S \quad A_k^a \quad \psi$	H, **P**, **K**, **J** SU(2)xU(1)

$$iD_\sigma\phi^i = i\partial_\sigma\phi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \phi^j$$

$$iD_\mu\phi^i = i\partial_\mu\phi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)^i_j \phi^j$$

P(3,1) x SU(2) x U(1)

Emerging symmetries & space-time

Fields	Generators Space-time & Internal

- Real/Majorana: $\phi \quad \xi \quad \text{and } \langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L} \quad \text{and } \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$
  (Wess-Zumino → gauge theory)
- 1D: $\phi_S \quad \phi_P \longrightarrow A^a_3 \quad \psi$

$$iD_\sigma\phi^i = i\partial_\sigma\phi^i + g_0 \sum_{a=1,\dots 8} A^a_\sigma (T_a)^i_j \phi^j$$

- 3D: $\phi_S \quad A^a_k \quad \psi$

$$iD_\mu\phi^i = i\partial_\mu\phi^i + g \sum_{a=1,2,3,8} A^a_\mu (T_a)^i_j \phi^j$$

- and ....

$$n^\sigma_\pm \longrightarrow n^\mu_\alpha \quad \gamma^\sigma = \begin{bmatrix} 0 & n^\sigma_- \\ n^\sigma_+ & 0 \end{bmatrix} \longrightarrow \gamma^\mu = \begin{bmatrix} 0 & \bar\sigma^\mu \\ \sigma^\mu & 0 \end{bmatrix}$$

in order to match space-time and field symmetries and respect Coleman-Mandula

Generators (Space-time & Internal):

- H
- P+, P– K, SU(3)

Z(2)

- H, P, K SU(3) = [SO(3),SU(2)xU(1)]

Z(3)

- H, **P**, **K**, **J** SU(2)xU(1)

P(3,1) x SU(2) x U(1)

A(4)

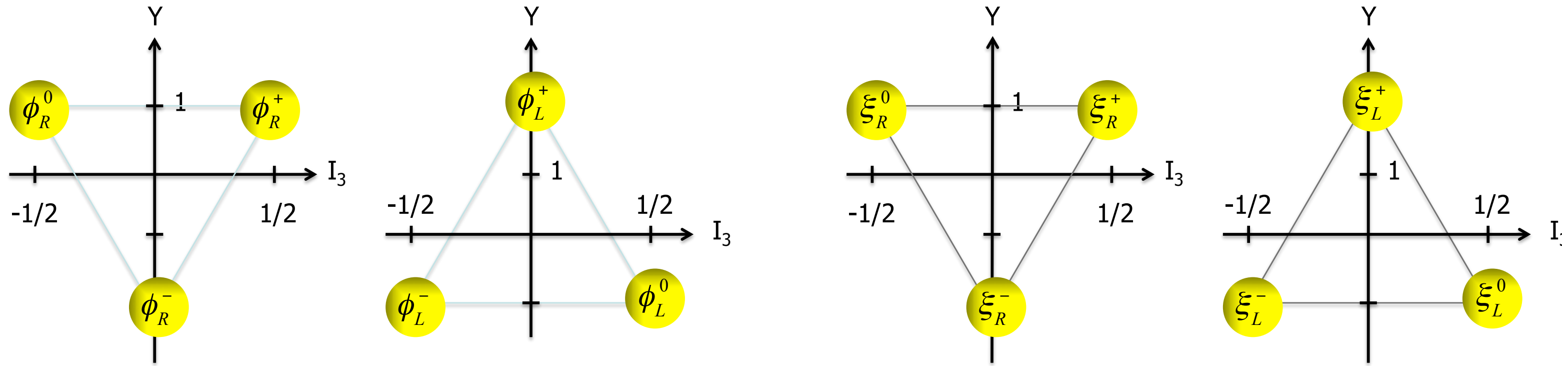
skip

MULTIPARTITE BOSONS IN THE STANDARD MODEL

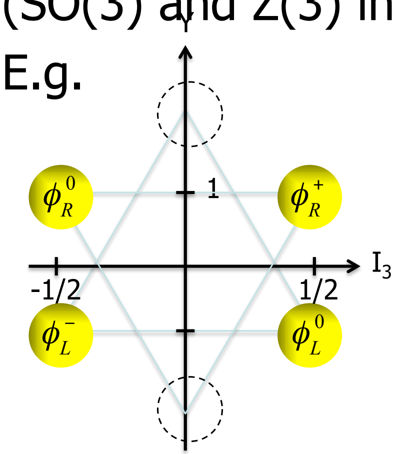
1D bosonic basis states for standard model

skip

- Basis of each 1D spaces: P(1,1) x SU(3): $\vec{\phi} = (\phi^1, \phi^2, \phi^3), \vec{\xi} = ((\xi^1, \xi^2, \xi^3)$
- Assign Y-I_3 using the SU(3) symmetry.



- built on vacuum |0,0,0>
  (SO(3) and Z(3) invariant, with nonzero vev)
  E.g.



$$\phi_R = \frac{1}{\sqrt{2}} \exp\left(+\tfrac{i}{2} \sum_{a=1,2,3,8} \theta^a \lambda_a\right) \begin{bmatrix} 1+\varphi_H \\ 0 \\ 0 \end{bmatrix}$$

$$\phi_L = \frac{1}{\sqrt{2}} \exp\left(-\tfrac{i}{2} \sum_{a=1,2,3,8} \theta^a \lambda_a\right) \begin{bmatrix} 0 \\ 1+\varphi_H \\ 0 \end{bmatrix}$$

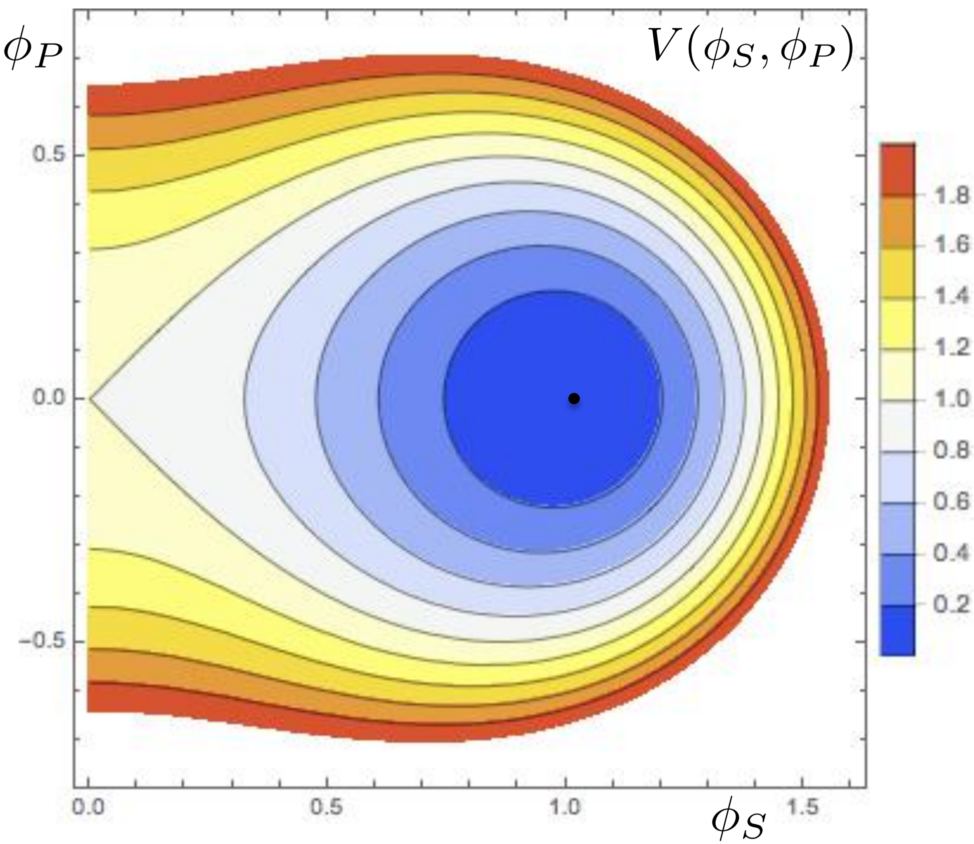
Bosonic field potential

skip

- Nonzero vacuum expectation value with SO(3) x Z(3) invariant g.s.

$$\phi\sqrt{2} = e^{i\pi/4}\phi_R + e^{-i\pi/4}\phi_L$$
$$= \phi_S + i\phi_P$$

$$\langle\phi\rangle = 1/\sqrt{2}$$
$$\langle\phi_S\rangle = 1,\ \langle\phi_P\rangle = 0$$
$$\langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$$



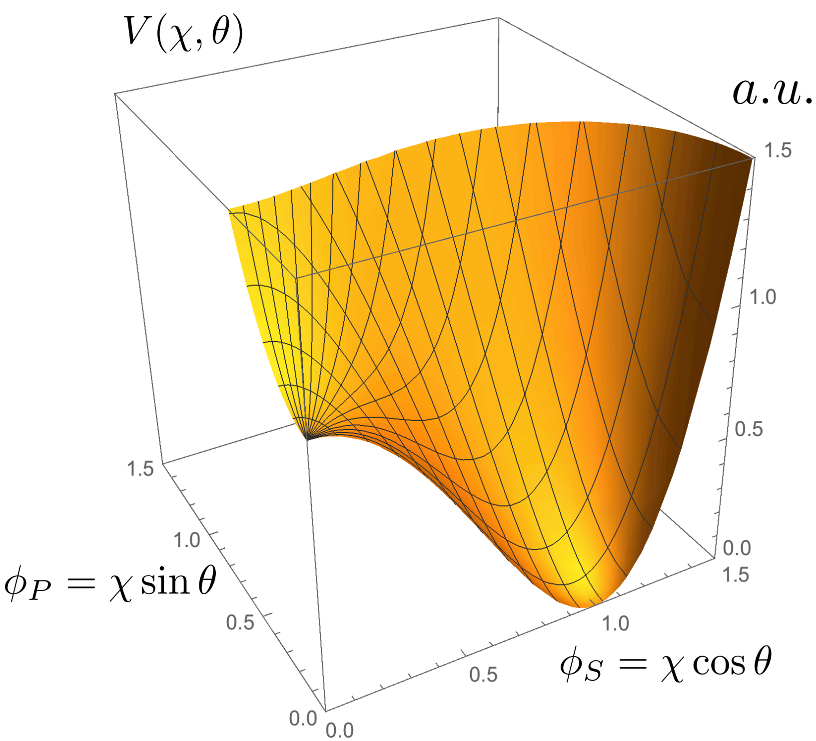
- Wess-Zumino

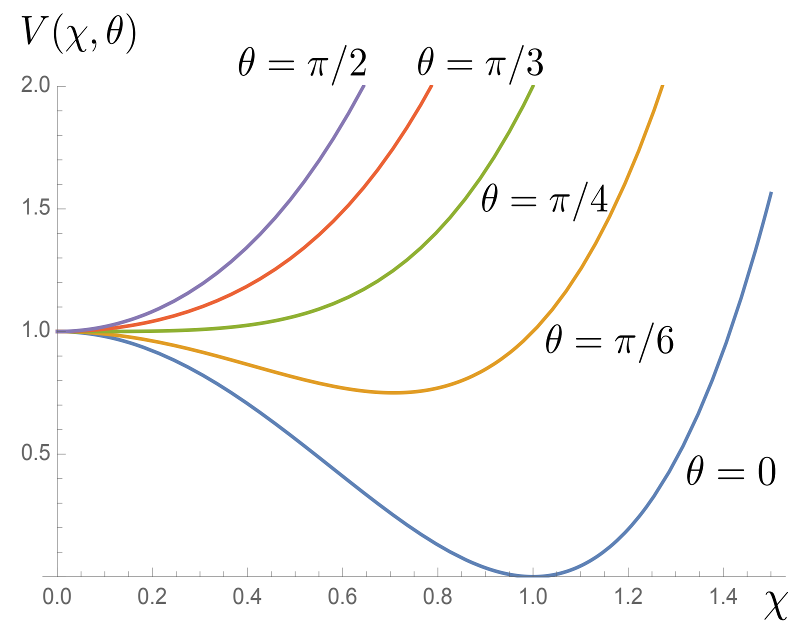
$$V = \tfrac{1}{8}M^2\left((\phi^2 + \phi^{*2} - 1)^2 - (\phi^2 - \phi^{*2})^2\right)$$
$$= \tfrac{1}{8}M^2\left(4\phi_S^2\phi_P^2 + (1 - \phi_S^2 + \phi_P^2)^2\right)$$
$$= \tfrac{1}{8}M^2\left((\phi_R^2 - \phi_L^2)^2 + (1 - 2\phi_R\phi_L)^2\right)$$

Wess-Zumino potential

skip

$$V = \tfrac{1}{8}M^2\left((\phi^2+\phi^{*2}-1)^2-(\phi^2-\phi^{*2})^2\right)$$
$$= \tfrac{1}{8}M^2\left(\chi^4\sin^2(2\theta)+(\chi^2\cos(2\theta)-1)^2\right)$$





Bosonic excitations: electroweak and strong

Skip

- 3D Electroweak symmetry breaking is SU(2) x U(1) → $U(1)_{QED}$

$$
\begin{aligned}
iD_\mu\phi &= i\partial_\mu\phi + \frac{g}{2}\left(\sum_{i=1}^{3} W_\mu^i\lambda_i + B_\mu\lambda_8\right)\phi \\
&= i\partial_\mu\phi + \frac{g}{\sqrt{2}}(W_\mu^+ I_- + W_\mu^- I_+)\phi + \left(g\,W_\mu^0 I_3 + \frac{g}{2\sqrt{3}}B_\mu Y\right)\phi
\end{aligned}
$$

- SU(3) embedding for electroweak gives embarrasingly good 'zeroth order' results:
  - Implies weak mixing angle $\sin\theta_W = 1/2$ (Weinberg 1972)
  - gives $M_H^2 = M^2/2$, $M_W^2 = 3M_Z^2/4$, $M_Z = M/2$

- 1D strong sector: $\mathcal{L} = \frac{1}{2}\partial^\mu\varphi_S\partial_\mu\varphi_S - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \overline{\psi}(i\not{D} - M - g_0\,\varphi_S)\psi$

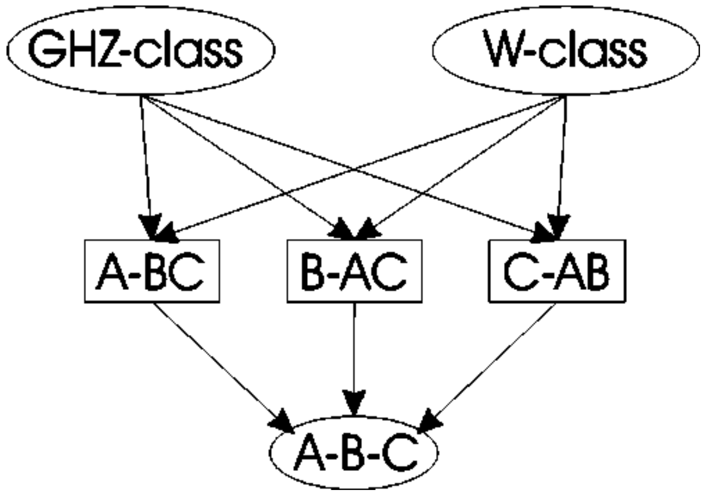
  8 instantaneous gluons and a scalar field, resembling $XQCD_{1+1}$ (Kaplan 1306.5818) and dynamics governed by via Wilson loop.

$$
W[C] = \exp\left(-ig\oint_C ds^\mu A_\mu(s)\right) \qquad\qquad gF_{\tau\sigma} = \delta W[C]/\delta\sigma^{\tau\sigma}
$$

- Wilson loops may be the portal towards 3D tomography of nucleon (see 1805.05219 and refs therein)

MULTIPARTITE FERMIONS IN THE STANDARD MODEL

Leptons and quarks as fermionic tripartite classes



tripartite entangled

bipartite entangled

product states

Different local classes of tripartite pure states. The direction ot the arrows indicates which noninvertible transformations between classes are possible (taken from Dur, Vidal, Cirac 2000)

Note: this states that maximally entangled GHZ- and W-class states can also be bipartite entangled, but note the difference in reduced spaces

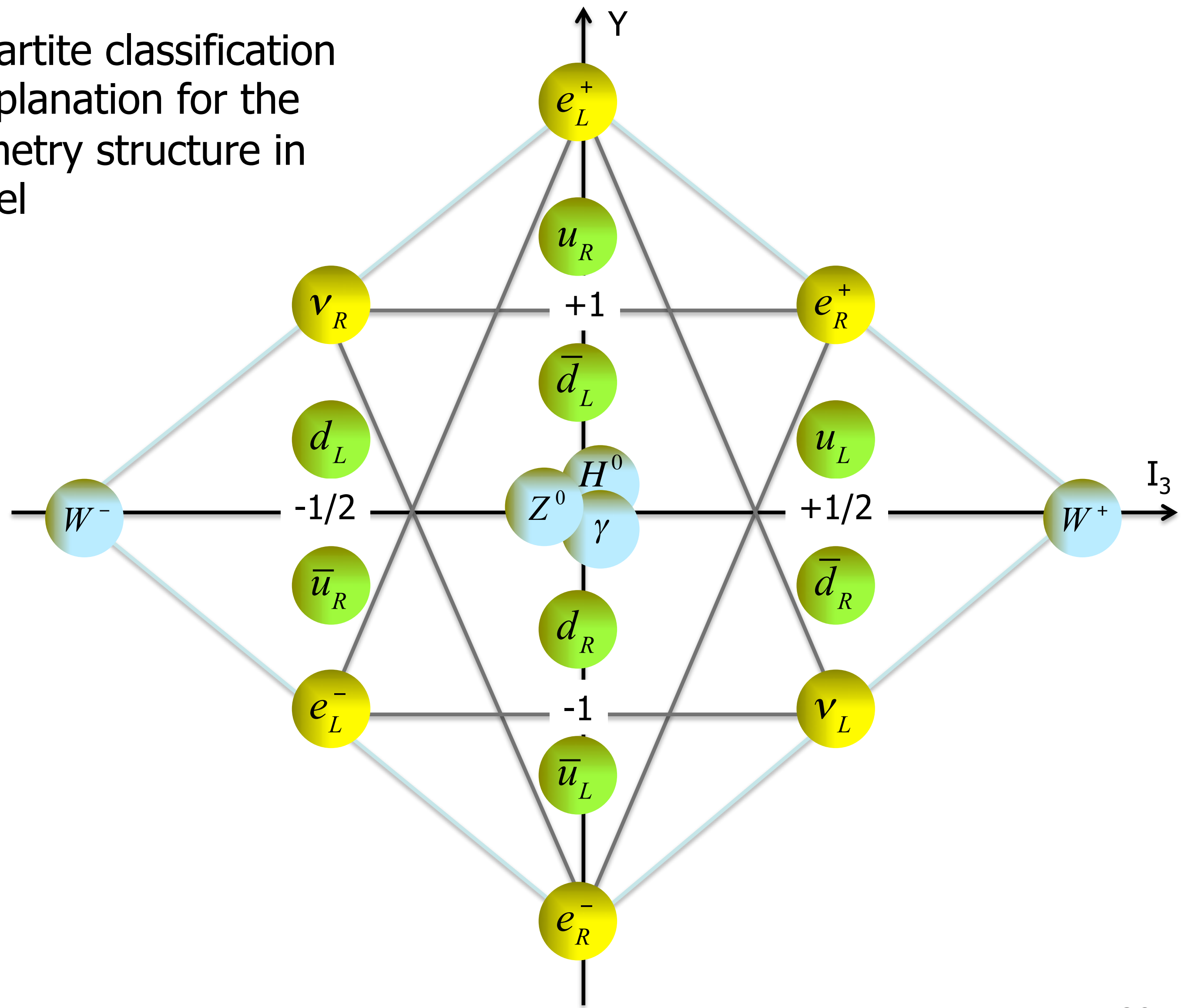
$$|\mathrm{Bell}\rangle \implies \rho_A = \tfrac{1}{2}\left(|R\rangle\langle R| + |L\rangle\langle L|\right)$$

$$|\mathrm{GHZ}\rangle \implies \rho_{AB} = \tfrac{1}{2}\left(|RR\rangle\langle RR| + |LL\rangle\langle LL|\right)$$

$$|\mathrm{W}\rangle \implies \rho_{AB} = \tfrac{2}{3}|\mathrm{Bell}\rangle\langle \mathrm{Bell}| + \tfrac{1}{3}|RR\rangle\langle RR|$$

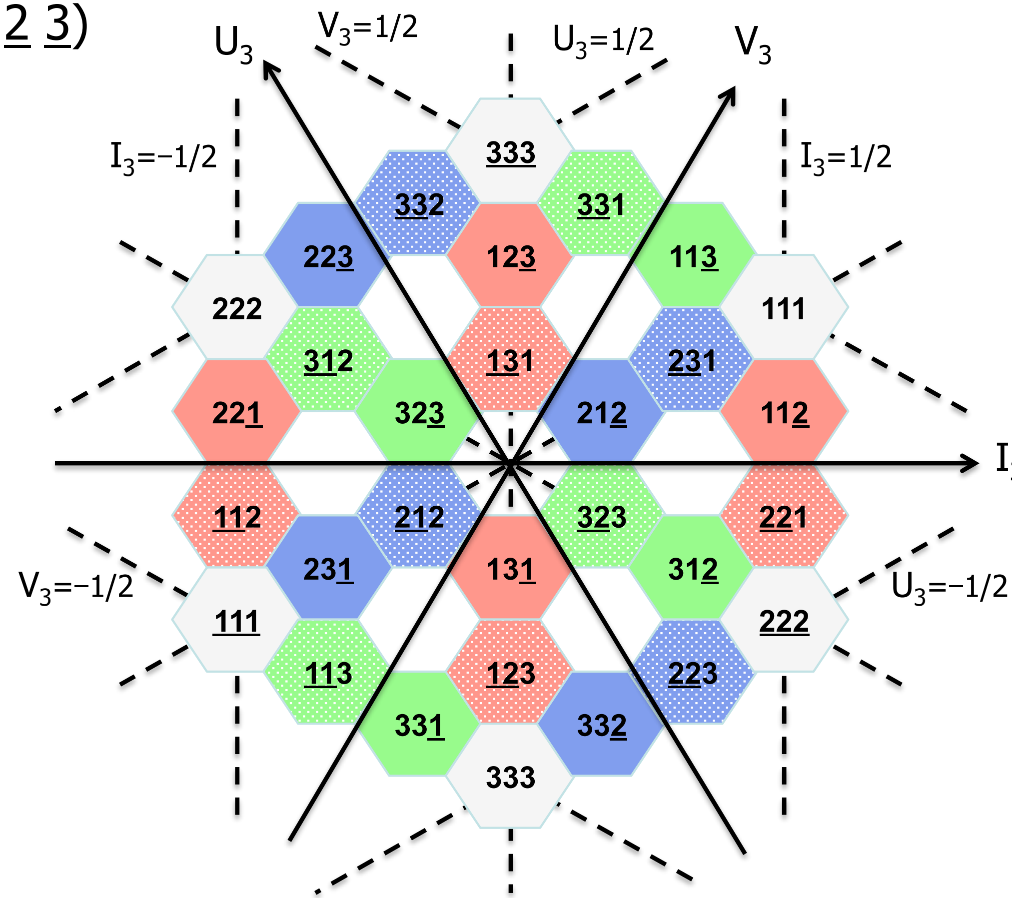
Standard model particle content

Important: multipartite classification gives a natural explanation for the electroweak symmetry structure in the standard model



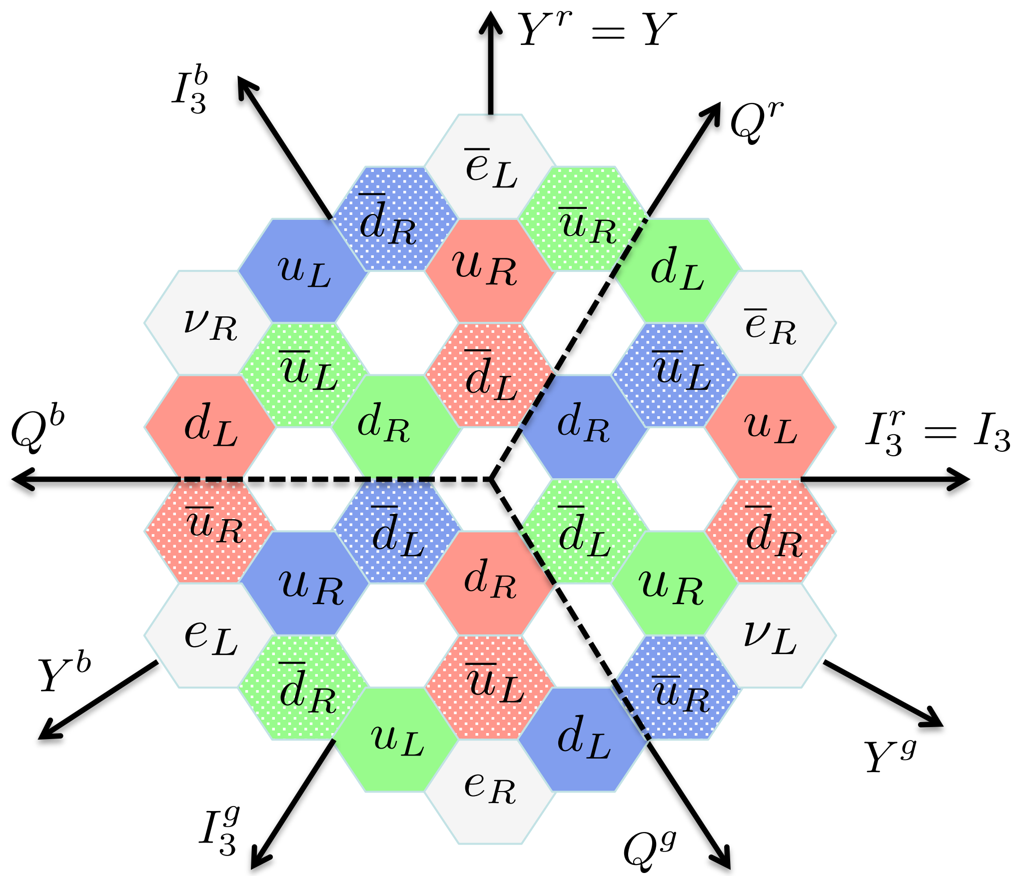
Fermionic excitations: leptons and quarks

- Tripartite states (R: 1 2 3 & L: $\underline{1}$ $\underline{2}$ $\underline{3}$)
- Aligned (RRR, LLL) GHZ states
  - SO(3) → asymptotic/space
    I, U, *and* V allowed
  - Three A(4) singlets
- Mingled (RRL, RLL) W-states
  - non-asymptotic
    I, U, *or* V allowed
  - Three A(4) triplets



Fermionic excitations: electroweak identification

- LEPTONS
  Aligned (RRR, LLL)
  - SO(3) → asymptotic/space
    I, U, *and* V allowed
  - Three A(4) singlets (families)
  - Family mixing is
    tri-bimaximal
- QUARKS
  Mingled (RRL, RLL)
  - non-asymptotic
    I, U, *or* V allowed
  - Three A(4) triplets (families)
  - Just one heavy quark!



- Resembles the rishon model
  (Harari & Seiberg 1982)

Concluding remarks

- Can a multipartite Hilbert space provide a simpler basis of the degrees of freedom in the standard model, still having the right basic ingredients and symmetries?
- It looks promising and it may provides a new view for many phenomena in QCD (Confinement, quark-hadron duality, importance of SCET for PDFs, FFs including TMDs, multitude of effective models for QCD)
- However, there are still many open ends!