Energy-momentum tensor for unpolarized proton target

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The quantity of our interest is the energy momentum tensor (EMT) on unpolarized proton state,

$$\langle T^{\mu\nu} \rangle = \frac{1}{2} \sum_{s=\uparrow,\downarrow} \frac{\langle p', s | \hat{T}^{\mu\nu}(0) | p, s \rangle}{2\sqrt{p'^+ p^+}} \, ,$$

which Fourier transformation leads to the EMT in the position space,

$$\widetilde{\langle T^{\mu\nu} \rangle}(x) = \int \frac{d^2 \vec{\Delta}_\perp d\Delta^+}{(2\pi)^3} \, e^{i\Delta \cdot x} \, \langle T^{\mu\nu} \rangle \, ,$$

where $\Delta = p' - p$, $P = \frac{1}{2}(p' + p)$ and $t = \Delta^2$.

C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

The definition of $\langle T^{\mu\nu} \rangle$ originate from derivation based on the Wigner distribution. We can define the Wigner distribution of the proton state of momentum $(P^+, \vec{P}_\perp)$ at "space point" $(X^-, \vec{X}_\perp)$ following,

$$\rho(X^-, \vec{X}_\perp, P^+, \vec{P}_\perp) =$$
$$= \int \frac{d^2\vec{\Delta}_\perp d\Delta^+}{(2\pi)^3} \frac{e^{i\Delta^+ X^- - i\vec{\Delta}_\perp \cdot \vec{X}_\perp}}{2\sqrt{p'^+ \, p^+}} \left| P - \frac{\Delta}{2} \right\rangle \left\langle P + \frac{\Delta}{2} \right|.$$

Then,

$$\widetilde{\langle T^{\mu\nu} \rangle}(x) = \text{Tr}[\hat{T}^{\mu\nu}(x) \, \rho(0, \vec{0}_\perp, P^+, \vec{P}_\perp)] = \int \frac{d^2\vec{\Delta}_\perp d\Delta^+}{(2\pi)^3} \, e^{i\Delta \cdot x} \, \langle T^{\mu\nu} \rangle \, .$$

P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)
E. P. Wigner, Phys. Rev. 40, 749 (1932)

The matrix element of the general local asymmetric energy–momentum tensor for a spin-1/2 target reads

$$\langle p', s' | \hat{T}^{\mu\nu}(0) | p, s \rangle =$$

$$= \bar{u}(p', s') \left\{ \frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C(t) + M \eta^{\mu\nu} \bar{C}(t) \right.$$

$$+ \frac{P^\mu i\sigma^{\nu\lambda} \Delta_\lambda}{4M} \left[A(t) + B(t) + D(t) \right]$$

$$+ \left. \frac{P^\nu i\sigma^{\mu\lambda} \Delta_\lambda}{4M} \left[A(t) + B(t) - D(t) \right] \right\} u(p, s) .$$

X.-D. Ji, Phys. Rev. Lett., 78, 610 (1997)
C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

The study of the EMT became especially important after obtaining by Ji a relation between the EMT and GPDs

$$\int_{-1}^{1} dx \, H(x, \xi, t) = A(t) + 4\xi^2 C(t) \,,$$

$$\int_{-1}^{1} dx \, E(x, \xi, t) = B(t) - 4\xi^2 C(t) \,.$$

Besides this, it is know that $D(t) = -g_A(t)$, the axial form factor, and $\bar{C}(t)$ can be related to the scalar form factor.

X.-D. Ji, Phys. Rev. Lett., 78, 610 (1997)
X.-D. Ji, Phys. Rev. D55, 7114 (1997)
B.L.G. Bakker, E. Leader, T.L. Trueman, Phys. Rev. D 70, 114001 (2004)

The study of the EMT is important because:

- ▶ $T^{\mu\nu}$ is a fundamental quantity, which allows to access for example a spin decomposition.
- ▶ DVCS gives a way to experimentally measure $T^{\mu\nu}$, *e.g.* JLab.
- ▶ Its form factors have a clear interpretation as spatial densities ($\vec{\Delta}$ is related to $\vec{r}$).
- ▶ EMT form factors and GPDs constrains each other.
- ▶ Studding EMT form factors one has an access to the limit $t = \Delta^2 \to 0$, which is excluded experimentally.

E. Leader, C. Lorcé, Phys.Rept. 541 (2014) 163

To better understand these formal definitions we write the EMT in a position space x,

$$\widetilde{\langle T^{\mu\nu}\rangle}(x) = \int \frac{d^2\vec{\Delta}_\perp d\Delta^+}{(2\pi)^3} e^{+i\Delta^- x^+ + i\Delta^+ x^- - i\vec{\Delta}_\perp \cdot \vec{x}_\perp} \langle T^{\mu\nu}\rangle \ ,$$

where $\Delta^- = \dfrac{\vec{P}_\perp \cdot \vec{\Delta}_\perp - P^-\Delta^+}{P^+}$ and we consider it in the

Drell-Yan frame where $\Delta^+ = 0$ and $\vec{P}_\perp = \vec{0}$. Then

$$\widetilde{\langle T^{\mu\nu}\rangle}(x_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{x}_\perp} \langle T^{\mu\nu}\rangle$$

is averaged EMT over x^+ and x^-.

S.D. Drell, T.-M. Yan, Phys.Rev.Lett. 24 (1970) 181

The EMT in the DYF reads,

$$\langle T^{++} \rangle = P^+ A \,,$$

$$\langle T^{+-} \rangle = \frac{1}{2P^+} \left[M^2 A + 2 \left(M^2 \bar{C} + \vec{\Delta}_\perp^{\,2} C \right) - \frac{\vec{\Delta}_\perp^{\,2}}{4} (B+D) \right] \,,$$

$$\langle T^{-+} \rangle = \frac{1}{2P^+} \left[M^2 A + 2 \left(M^2 \bar{C} + \vec{\Delta}_\perp^{\,2} C \right) - \frac{\vec{\Delta}_\perp^{\,2}}{4} (B-D) \right] \,,$$

$$\langle T^{ij} \rangle = \frac{1}{P^+} \left[\Delta_\perp^i \Delta_\perp^j C + \left(M^2 \bar{C} + \vec{\Delta}_\perp^{\,2} C \right) \eta^{ij} \right] \quad \text{for } i,j \in \{1,2\} \,.$$

The EMT in the DYF is not symmetric,

$$\left\langle T^{[\mu\nu]} \right\rangle \Big|_{\text{DYF}} = \frac{\vec{\Delta}_\perp^2}{4P^+} D(t)\, \eta^{+[\mu}\eta^{\nu]-} = -i\Delta_\lambda \left\langle S^{\lambda\mu\nu} \right\rangle ,$$

where spin tensor reads

$$\left\langle S^{\lambda\mu\nu} \right\rangle = -i\, \frac{g_A(t)}{4(P^+)^2} \epsilon^{\lambda\mu\nu\sigma} \epsilon_{\sigma-\alpha\beta} P^\alpha \Delta^\beta .$$

Thus $D(t) = -g_A(t)$ and

$$S^i(\vec{r}_\perp) = -\frac{1}{4P^+} \epsilon^{3ij} \frac{r_j}{r} \frac{d\tilde{D}(r)}{dr} \quad \text{where } r = |\vec{r}_\perp| .$$

C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

In the position space $\langle T^{-+} \rangle$ and $\langle T^{ij} \rangle$ for $i, j \in \{1, 2\}$ read,

$$\widetilde{\langle T^{(-+)} \rangle}(\vec{r}_\perp) = \frac{1}{P^+} \left[M^2 \tilde{A}(r) + \frac{1}{4} \left(\frac{1}{r} \frac{d\tilde{B}(r)}{dr} + \frac{d^2\tilde{B}(r)}{dr^2} \right) \right]$$

$$+ \frac{2}{P^+} \left[M^2 \tilde{\bar{C}}(r) - \frac{1}{r} \frac{d\tilde{C}(r)}{dr} - \frac{d^2\tilde{C}(r)}{dr^2} \right]$$

$$\widetilde{\langle T^{ij} \rangle}(\vec{r}_\perp) = \frac{1}{P^+} \left[\frac{1}{r} \frac{d\tilde{C}(r)}{dr} - \frac{d^2\tilde{C}(r)}{dr^2} \right] \frac{r_i r_j}{r^2}$$

$$+ \frac{1}{P^+} \left[M^2 \tilde{\bar{C}}(r) - \frac{d^2\tilde{C}(r)}{dr^2} \right] \eta^{ij} ,$$

The symmetrized EMT in the position space has similar structure to the anisotropic fluid density in the IMF where $u_\perp = 0$, $u^+ \to \infty$ and $u^- \to 0$, while $u^+ u^- = \dfrac{1}{2}$.

Reminder: an anisotropic fluid density reads

$$\Theta^{\mu\nu} = (\epsilon + p_t)u^\mu u^\nu - p_t\, \eta^{\mu\nu} + (p_r - p_t)\, \chi^\mu \chi^\nu \,,$$

where $u \cdot u = 1$, $\chi \cdot \chi = -1$, $u \cdot \chi = 0$ and $(p_r - p_t)$ is a pressure anisotropy.

S.S. Bayin, Astrophys. J. 303, 101 (1986)

The Fourier transforms of the EMT leads to

$$p_r(r)P^+ = -M^2 \bar{\tilde{C}}(r) + \frac{1}{r}\frac{d\tilde{C}(r)}{dr} \,,$$

$$p_t(r)P^+ = -M^2 \bar{\tilde{C}}(r) + \left[\frac{2}{r}\frac{d\tilde{C}(r)}{dr} + \frac{d^2\tilde{C}(r)}{dr^2}\right] \,,$$

$$\epsilon(r)P^+ = M^2\left[\tilde{A}(r) + \bar{\tilde{C}}(r)\right] + \frac{1}{4}\left[\frac{1}{r}\frac{d\tilde{B}(r)}{dr} + \frac{d^2\tilde{B}(r)}{dr^2}\right] + \frac{1}{r}\frac{d\tilde{C}(r)}{dr} \,.$$

http://partons.cea.fr

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Main Page

What is PARTONS?

PARTONS is a C++ software framework dedicated to the phenomenology of Generalized Parton Distributions (GPDs). GPDs provide a comprehensive description of the partonic structure of the nucleon and contain a wealth of new information. In particular, GPDs provide a description of the nucleon as an extended object, referred to as 3-dimensional nucleon tomography, and give an access to the orbital angular momentum of quarks.

PARTONS provides a necessary bridge between models of GPDs and experimental data measured in various exclusive channels, like Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP). The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron Ion Collider (EIC).

PARTONS is useful to theorists to develop new models, phenomenologists to interpret existing measurements and to experimentalists to design new experiments.

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Get PARTONS

Here you can learn how to get your own version of PARTONS. We offer two ways:

- You can use our provided virtual machine with an out-of-the-box PARTONS runtime and development environment. This is the easiest way to start your experience with PARTONS.
- You can also build PARTONS by your own on GNU/Linux. This is useful if you want to have PARTONS on your computer without using the virtualization technology or if you want to use PARTONS on computing farms.

Study of the $\langle T^{\mu\nu} \rangle$ gives us a clear interpretation of the EMT form factors

▶ $A(t)$, $B(t)$, $C(t)$, $\bar{C}(t)$ are related to the energy and pressure density,

▶ $C(t)$ is related to the anisotropy pressure,

▶ $D(t)$ is related to the spin density.

M. V. Polyakov, Phys. Lett. B555, 57 (2003)

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