

# **Large transverse momentum in semi-inclusive deeply inelastic scattering beyond the lowest order**

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Nobuo Sato

University of Connecticut  
Light Cone 18  
JLab, 2018

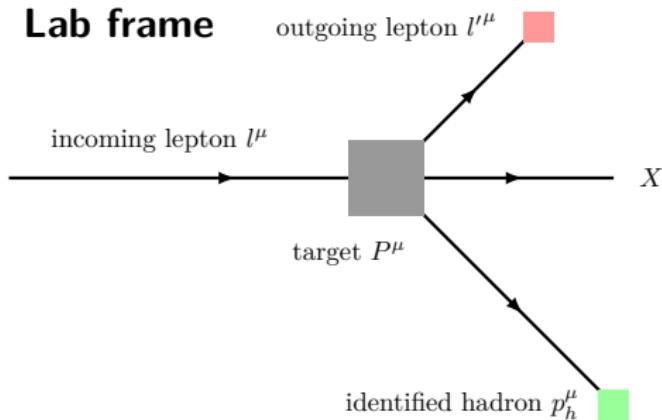
In collaboration with:

- O. Gonzalez
- T. Rogers
- B. Wang

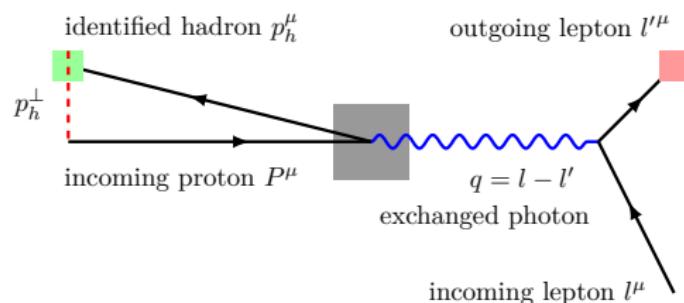


# Semi inclusive deep inelastic scattering (SIDIS)

## Lab frame



## Breit frame



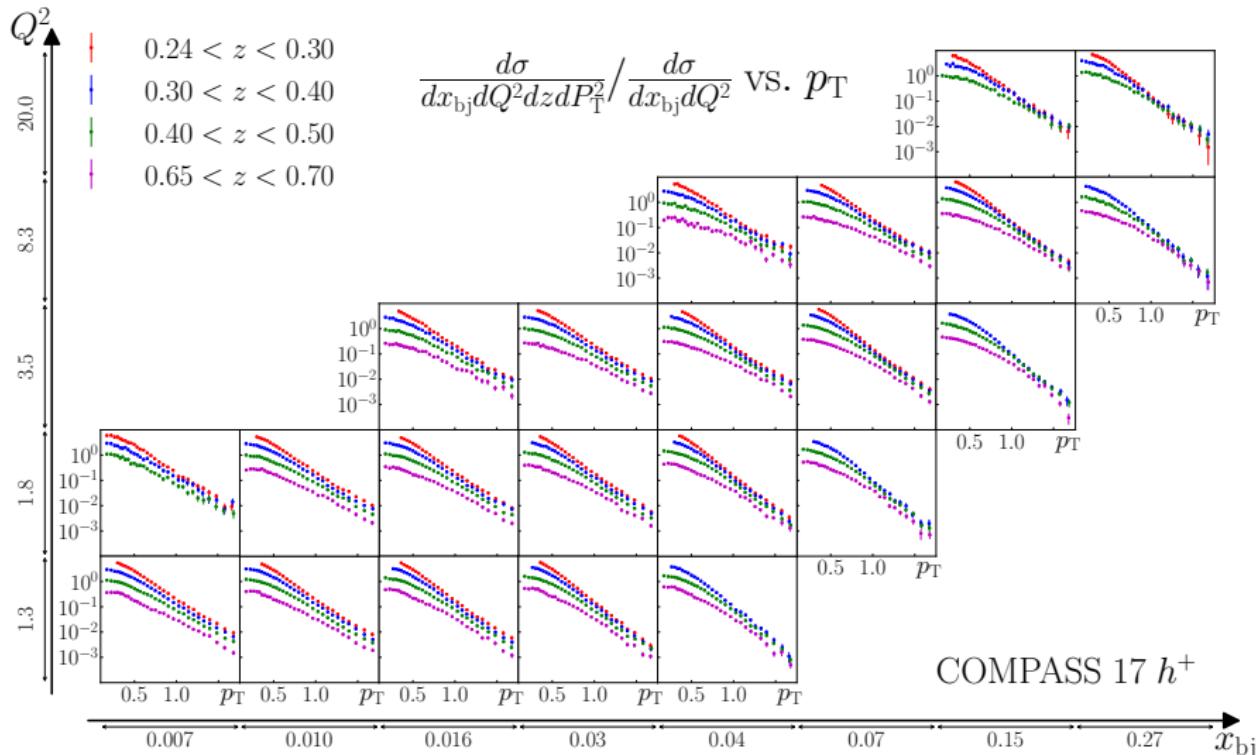
- Process is dominated by one photon exchange with large virtuality  $Q^2 \gg \lambda_{\text{QCD}}$

- In the Breit frame the proton and the photon has zero transverse momentum.

- The detected hadron has  $p_h^\perp$  relative to proton-photon axis

- Key question :** How is  $p_h^\perp$  generated at short distances?

# SIDIS data: $l + d \rightarrow l' + h^+ + X$



- JLab 12 data have measurements up to  $p_T \sim 1.4$

# Kinematic regions

- Detected hadron's rapidity

$$y_h = \frac{1}{2} \ln \left( \frac{p_h^+}{p_h^-} \right)$$

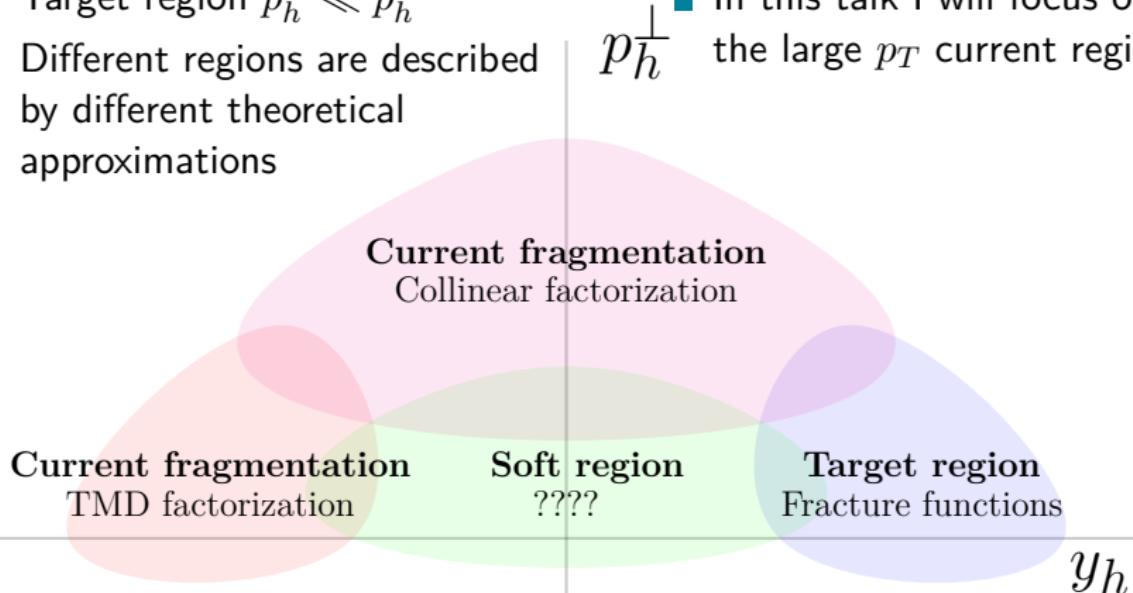
- Current region  $p_h^- \gg p_h^+$

- Target region  $p_h^- \ll p_h^+$

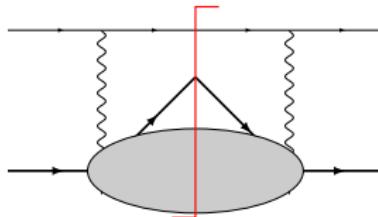
- Different regions are described by different theoretical approximations

- The higher the c.o.m energy, the larger the separation among the regions

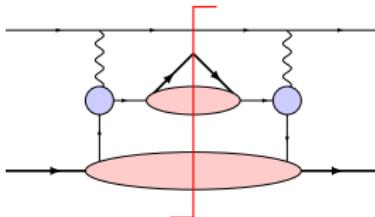
- In this talk I will focus on the large  $p_T$  current region



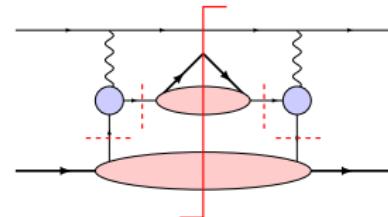
# Factorization in the current region



Full contribution

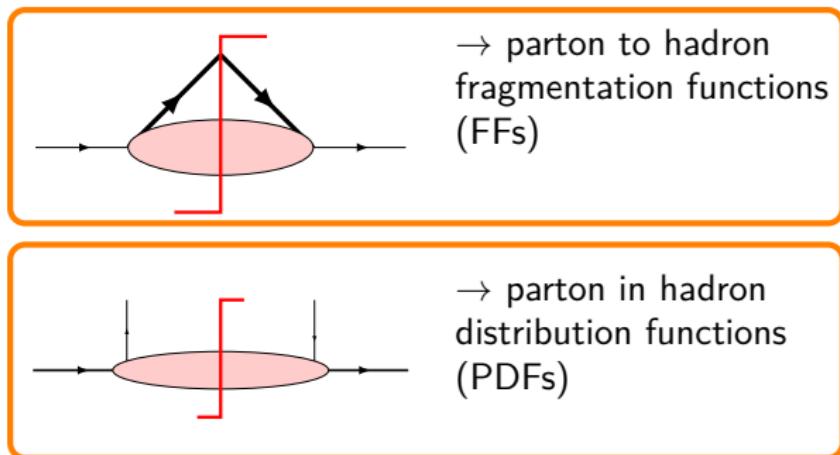


Leading regions: Hard  
and soft blobs

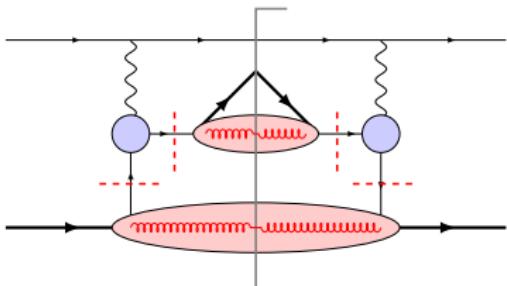
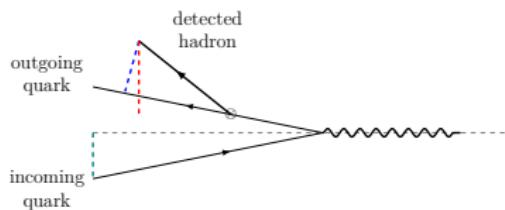
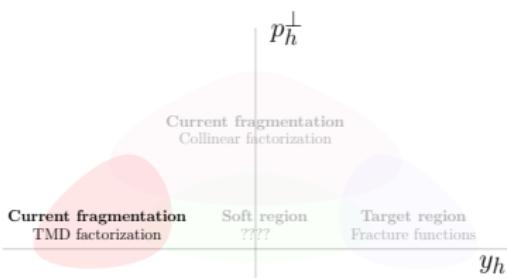


Kinematic approximations to  
separate the soft blobs from  
hard blob

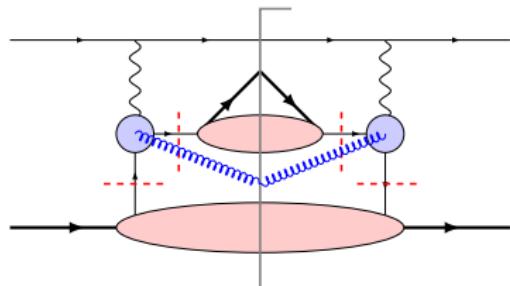
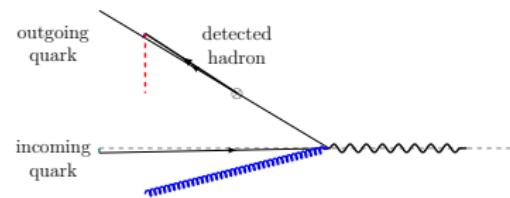
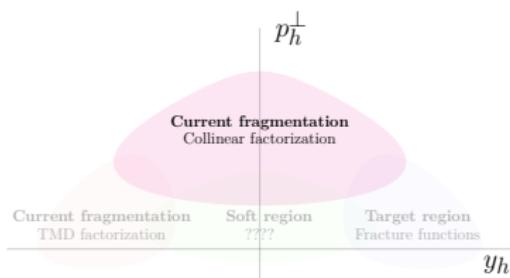
- Each blob represents all possible Feynman graphs with restricted power counting for their internal propagators
- The soft parts can have a physical interpretation



## Small transverse momentum



## Large transverse momentum



# Combining large and small $p_h^\perp$ approximation

- SIDIS reaction

$$l + P \rightarrow l' + p_h + X$$

- SIDIS invariants

$$q = l - l' \quad Q^2 = -q^2$$

$$x = \frac{Q^2}{2P \cdot q} \quad z = \frac{P \cdot p_h}{P \cdot q}$$

$$q_T = p_h^\perp / z$$

- SIDIS cross section

$$\Gamma \equiv \frac{d\sigma}{dx dQ^2 dz dq_T}$$

- The W+Y construction

$$\begin{aligned}\Gamma &= \Gamma \\ &= \mathbf{T}_{\text{TMD}} \Gamma + [\Gamma - \mathbf{T}_{\text{TMD}} \Gamma] \\ &= \underbrace{\mathbf{T}_{\text{TMD}} \Gamma}_{\mathbf{W}} + \underbrace{[\Gamma - \mathbf{T}_{\text{TMD}} \Gamma]}_{\mathbf{Y}} \\ &\quad + \mathcal{O}(m^2/Q^2) \Gamma\end{aligned}$$

- Nomenclature

$$W \equiv \mathbf{T}_{\text{TMD}} \Gamma$$

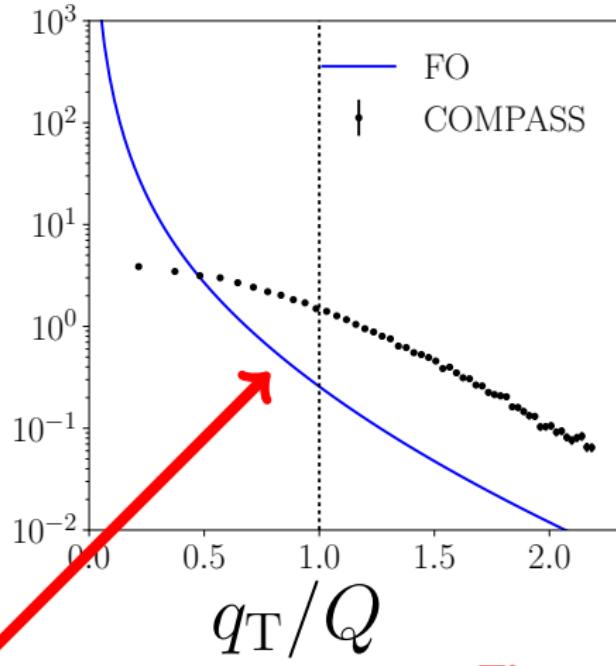
$$FO \equiv \mathbf{T}_{\text{coll}} \Gamma$$

$$ASY \equiv \mathbf{T}_{\text{coll}} \mathbf{T}_{\text{TMD}} \Gamma$$

$$Y \equiv FO - ASY$$

# Does it work?

$$\frac{\frac{d\sigma}{dxdz dQ^2 dp_T^2}}{\frac{d\sigma}{dxdQ^2}}$$



- Need order  $\alpha_S^2$  or beyond?
- Issues with fragmentation functions?
- Subleading power corrections?

The unpolarized SIDIS cross sections needs to be ready to interpret upcoming TMD data from JLab 12

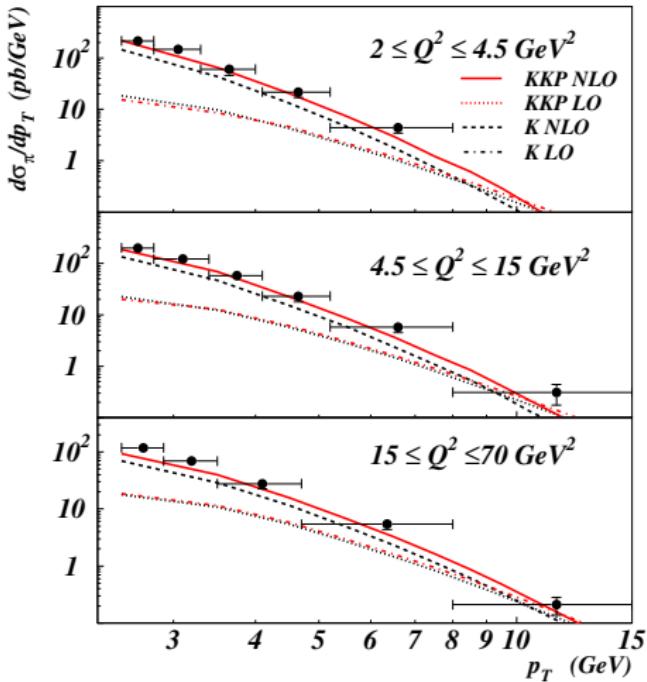
■ Kinematics

$$Q^2 = 1.92 \text{ GeV}^2$$

$$x = 0.0318$$

$$z = 0.375$$

# order $\alpha_S^2$ corrections to FO



- There are strong indications that order  $\alpha_S^2$  corrections are very important
- An order of magnitude of corrections at small  $p_T$ .
- As a sanity check, we need to have an independent calculation

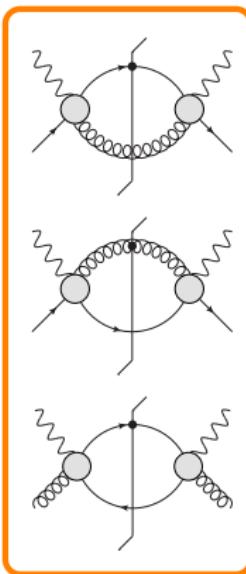
## Our setup for $O(\alpha_S^2)$ contribution

$$W^{\mu\nu}(P, q, P_H) = \int_{x-}^{1+} \frac{d\xi}{\xi} \int_{z-}^{1+} \frac{d\zeta}{\zeta^2} \hat{W}_{ij}^{\mu\nu}(q, x/\xi, z/\zeta) f_{i/P}(\xi) d_{H/j}(\zeta)$$

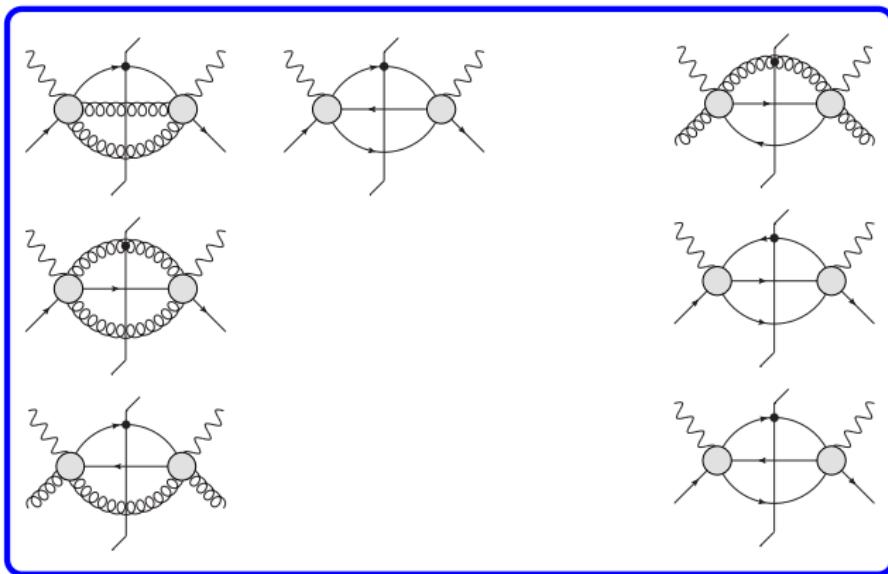
$$\begin{aligned} \{P_g^{\mu\nu} \hat{W}_{\mu\nu}^{(N)}; P_{PP}^{\mu\nu} \hat{W}_{\mu\nu}^{(N)}\} &\equiv \frac{1}{(2\pi)^4} \int \{|M_g^{2 \rightarrow N}|^2; |M_{pp}^{2 \rightarrow N}|^2\} d\Pi^{(N)} - \text{Subtractions} \\ &\equiv \{P_g^{\mu\nu} \hat{W}_{\mu\nu}^{(N)}; P_{PP}^{\mu\nu} \hat{W}_{\mu\nu}^{(N)}\}_{\text{unsub}} - \text{Subtractions} \end{aligned}$$

- ✓ Compute  $2 \rightarrow 2$  virtual graphs (Passarino-Veltman)
- ✓ Compute  $2 \rightarrow 3$  real graphs
- ✓ Integrate 3-body PS analytically using dim reg
- ✓ Cancel double and single IR poles

# Our setup for $O(\alpha_S^2)$ contribution



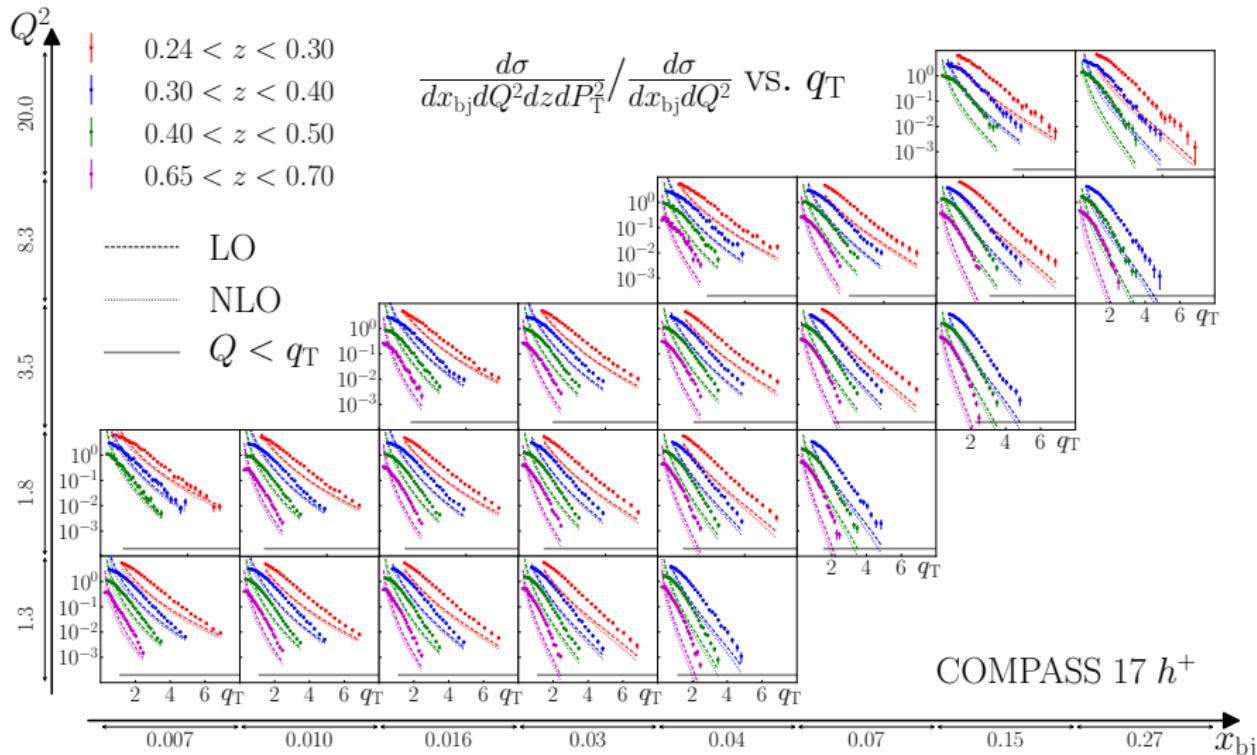
Born/Virtual



Real

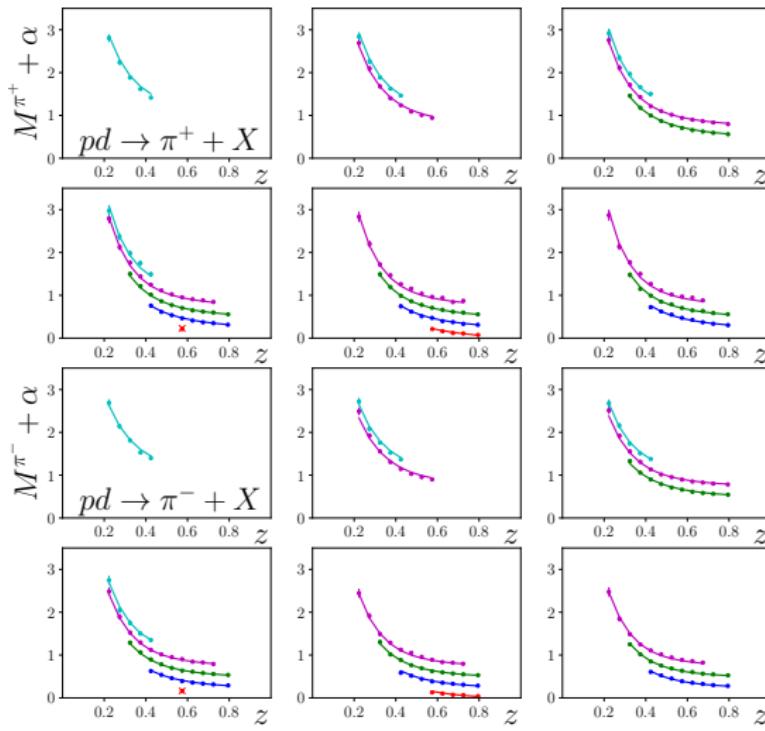
- Dots indicates the fragmenting parton

# Preliminary results



- The  $O(\alpha_S^2)$  seems to not improve the theory prediction

# $q_T$ integrated SIDIS data



- $\alpha$  is used to offset the data
- The  $q_T$  integrated spectrum is successfully described in collinear factorization at NLO

# Summary and outlook

$$\frac{d\sigma}{dx \ dy \ d\Psi \ dz \ d\phi_h \ dP_{hT}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \sum_{i=1}^{18} F_i(x, z, Q^2, P_{hT}^2) \beta_i$$

$F_i$	Standard label	$\beta_i$
$F_1$	$F_{UU,T}$	1
$F_2$	$F_{UU,L}$	$\varepsilon$
$F_3$	$F_{LL}$	$S_{  } \lambda_e \sqrt{1 - \varepsilon^2}$
$F_4$	$F_{UT}^{\sin(\phi_h + \phi_S)}$	$ \vec{S}_\perp  \varepsilon \sin(\phi_h + \phi_S)$
$F_5$	$F_{UT,T}^{\sin(\phi_h - \phi_S)}$	$ \vec{S}_\perp  \sin(\phi_h - \phi_S)$
$F_6$	$F_{UT,L}^{\sin(\phi_h - \phi_S)}$	$ \vec{S}_\perp  \varepsilon \sin(\phi_h - \phi_S)$
$F_7$	$F_{UU}^{\cos 2\phi_h}$	$\varepsilon \cos(2\phi_h)$
$F_8$	$F_{UT}^{\sin(3\phi_h - \psi_S)}$	$ \vec{S}_\perp  \varepsilon \sin(3\phi_h - \phi_S)$
$F_9$	$F_{LT}^{\cos(\phi_h - \phi_S)}$	$ \vec{S}_\perp  \lambda_e \sqrt{1 - \varepsilon^2} \cos(\phi_h - \phi_S)$
$F_{10}$	$F_{UL}^{\sin 2\phi_h}$	$S_{  } \varepsilon \sin(2\phi_h)$
$F_{11}$	$F_{LT}^{\cos \phi_S}$	$ \vec{S}_\perp  \lambda_e \sqrt{2\varepsilon(1 - \varepsilon)} \cos \phi_S$
$F_{12}$	$F_{LL}^{\cos \phi_h}$	$S_{  } \lambda_e \sqrt{2\varepsilon(1 - \varepsilon)} \cos \phi_h$
$F_{13}$	$F_{LT}^{\cos(2\phi_h - \phi_S)}$	$ \vec{S}_\perp  \lambda_e \sqrt{2\varepsilon(1 - \varepsilon)} \cos(2\phi_h - \phi_S)$
$F_{14}$	$F_{UL}^{\sin \phi_h}$	$S_{  } \sqrt{2\varepsilon(1 + \varepsilon)} \sin \phi_h$
$F_{15}$	$F_{LU}^{\sin \phi_h}$	$\lambda_e \sqrt{2\varepsilon(1 - \varepsilon)} \sin \phi_h$
$F_{16}$	$F_{UU}^{\cos \phi_h}$	$\sqrt{2\varepsilon(1 + \varepsilon)} \cos \phi_h$
$F_{17}$	$F_{UT}^{\sin \phi_S}$	$ \vec{S}_\perp  \sqrt{2\varepsilon(1 + \varepsilon)} \sin \phi_S$
$F_{18}$	$F_{UT}^{\sin(2\phi_h - \phi_S)}$	$ \vec{S}_\perp  \sqrt{2\varepsilon(1 + \varepsilon)} \sin(2\phi_h - \phi_S)$

- I discussed the current status of  $F_{UU}$
- At present, there is no successful description of data using W+Y
- There are many results where the success is shown by restricting the  $q_T$  range and avoiding the inclusion of Y
- The key question: how  $p_T$  is generated at low energies is still a challenging question