Recent developments in nucleon spin decomposition

Jaffe-Manohar

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The proton spin problem

The proton has spin ½.

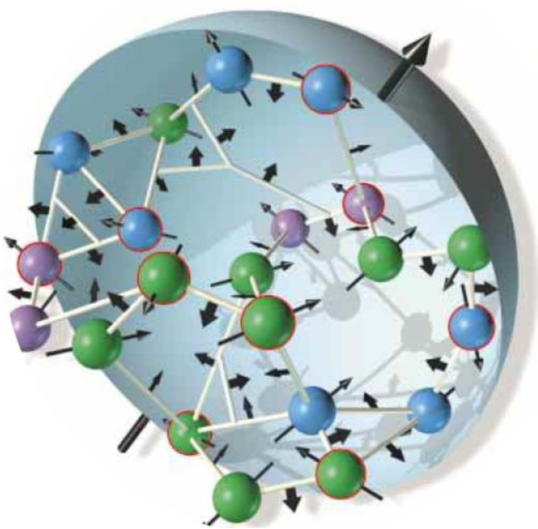
The proton is not an elementary particle.

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity

Gluons' helicity

Orbital angular Momentum (OAM)



In the quark model, $\Delta\Sigma = 1$

with relativistic effects, $\Delta\Sigma \approx 0.7$

EMC result (1987) $\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14$

Current value $\Delta\Sigma = 0.25 \sim 0.3$

Global analysis for ΔG

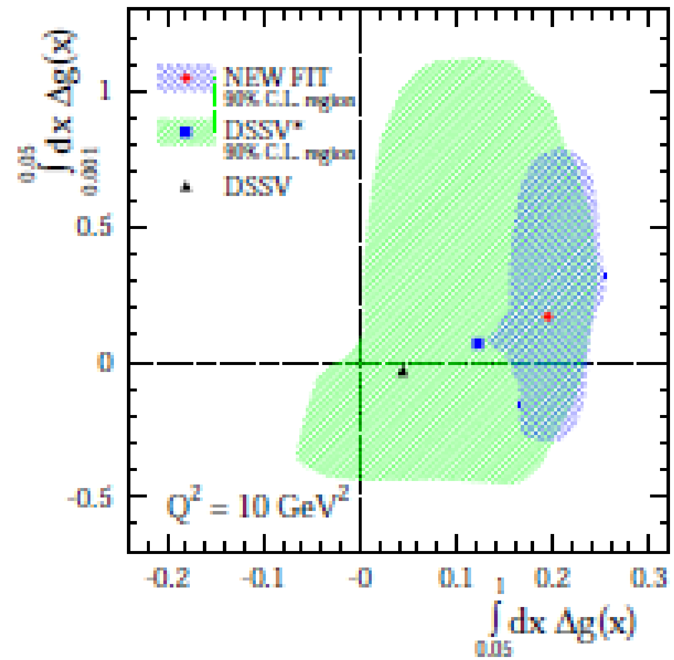
$$\int_{0.05}^{1} dx\Delta g(x,Q^2=10\mathrm{GeV}^2) = 0.20^{+.06}_{-.07} \qquad \text{DSSV++}$$

$$\int_{0.05}^{0.2} dx\Delta g(x,Q^2=10\mathrm{GeV}^2) = 0.17 \pm 0.06 \qquad \text{NNPDFpol1.1}$$

$$\int_{0.001}^{0.8} dx\Delta g(x,Q^2 = 1\ \mathrm{GeV}^2) = 0.5 \pm 0.4 \qquad \text{JAM15}$$

H. Gao, talk at DIS2018

HUGE uncertainty from the small-x region
→ RHIC 510GeV, EIC



How does $\Delta G(x)$ behave at small-x?

What is the role of the OAM?

Complete gauge invariant decomposition

Chen, Lu, Sun, Wang, Goldman (2008)
YH (2011)

Gauge invariant completion of Jaffe-Manohar

$$\langle PS|\epsilon^{ij}F^{i+}A^j_{phys}|PS\rangle = 2S^+\Delta G$$

$$\lim_{\Delta\to 0}\langle P'S|\bar{\psi}\gamma^+ i\overleftrightarrow{D}^i_{pure}\psi|PS\rangle = iS^+\epsilon^{ij}\Delta_{\perp j}L^q_{can}$$

$$\lim_{\Delta\to 0}\langle P'S|F^{+\alpha}\overleftrightarrow{D}^i_{pure}A^{phys}_\alpha|PS\rangle = -i\epsilon^{ij}\Delta_{\perp j}S^+L^g_{can}$$

where $A^\mu_{phys} = \dfrac{1}{D^+}F^{+\mu}$ $\qquad D^\mu_{pure} = D^\mu - igA^\mu_{phys}$

cf. Ji's kinetic OAM $\quad \lim_{\Delta\to 0}\langle P'S|\bar{\psi}\gamma^+ i\overleftrightarrow{D}^i\psi|PS\rangle = iS^+\epsilon^{ij}\Delta_{\perp j}L^q_{Ji}$

OAM from the Wigner distribution

Lorce, Pasquini (2011);
YH (2011);

Define

$$L^q = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \underbrace{W^q(x, \vec{b}_\perp, \vec{k}_\perp)}_{\text{Wigner distribution}}$$

Go to the momentum space $b_\perp \to \Delta_\perp$ and look for the component

$$W^{q,g} = i\frac{S^+}{P^+}\epsilon^{ij}k_\perp^i \Delta_\perp^j f^{q,g}(x, k_\perp) + \cdots$$

Then

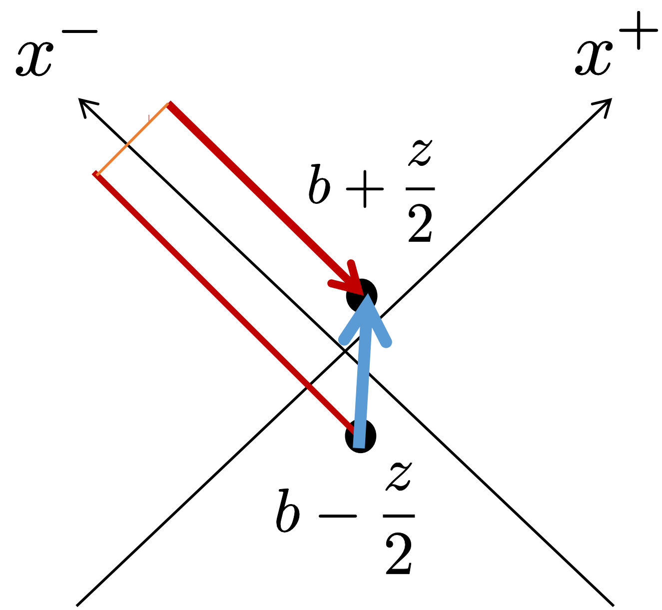
$$L^{q,g} = \int dx \int d^2k_\perp f^{q,g}(x, k_\perp)$$

Nice, but which OAM is this??

Canonical (Jaffe-Manohar) OAM from the light-cone Wilson line

YH (2011)

$$\int \vec{b}\times\vec{k}W_{light-cone}(\vec{b},\vec{k}) = \langle \bar{\psi}\vec{b}\times i\overleftrightarrow{D}_{pure}\psi\rangle$$



Kinetic (Ji) OAM from the straight Wilson line

Ji, Xiong, Yuan (2012)

$$\int \vec{b}\times\vec{k}W_{straight}(\vec{b},\vec{k})$$

$$= \langle \bar{\psi}\vec{b}\times i\overleftrightarrow{D}\psi\rangle$$

`Potential' OAM

$$L_{pot} \equiv L^q_{\mathrm{Ji}} - L^q_{can} = \int dx^-\langle \vec{b}\times\vec{F}\rangle$$

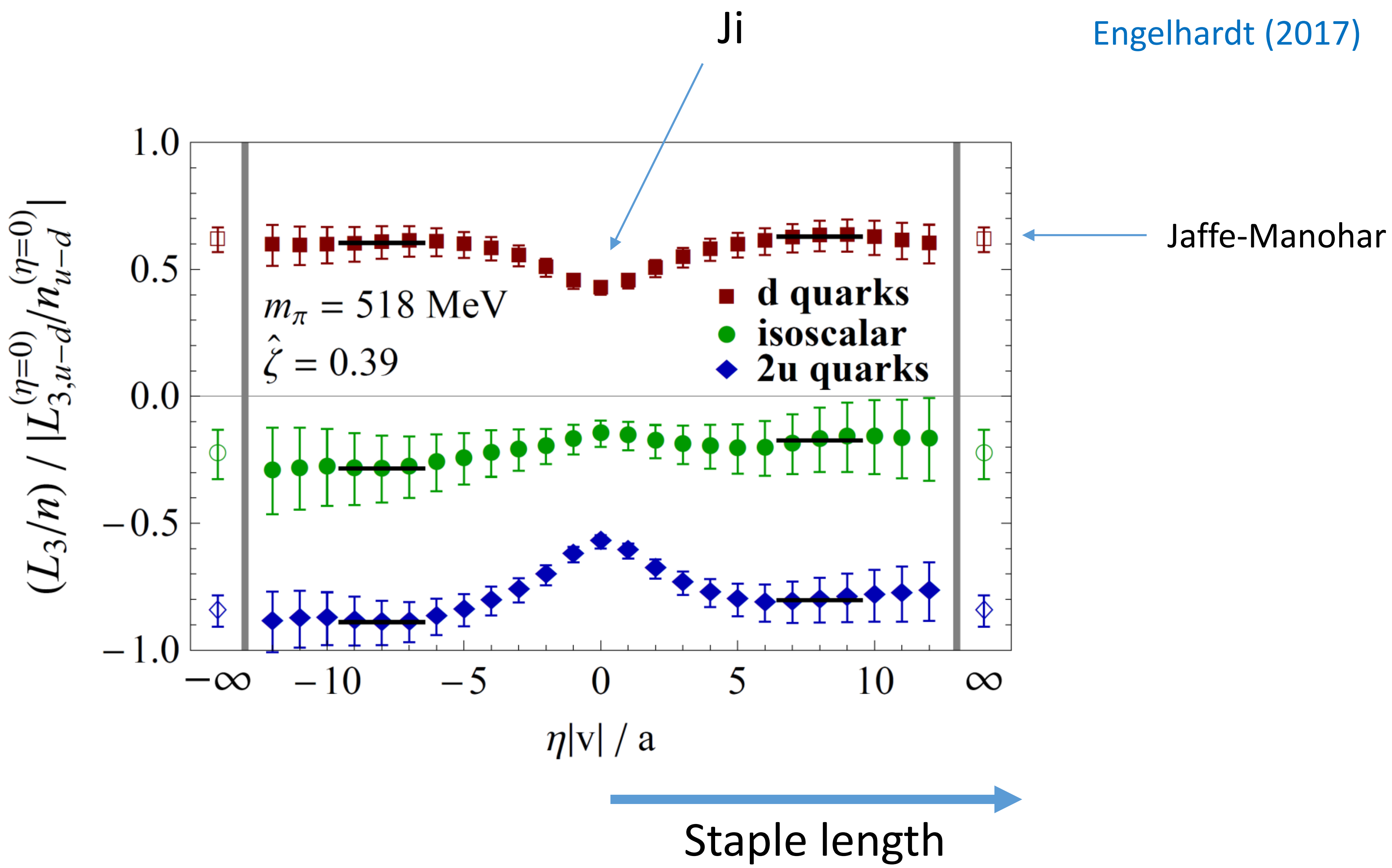
Lorentz force

Torque acting on a quark

Burkardt (2012)

Jaffe-Manohar vs. Ji First lattice result

Engelhardt (2017)



`PDF’ for OAM

Take the staple-shaped Wilson line and define

$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$L^{q,g}(x) = \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

Agrees with Harindranath, Kundu (1998); Hagler, Schafer (1998)
in the light-cone gauge.

How does $L_{q,g}(x)$ behave as $x \to 0$?

Warning: This is NOT the usual (twist-two) PDF.

Twist structure of OAM distributions

YH, Yoshida (2012)

Wandzura-Wilczek part

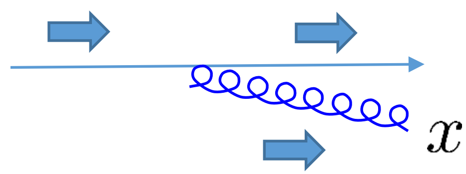
$$L^q_{can}(x) = x\int_x^{\epsilon(x)}\frac{dx'}{x'}(H_q(x')+E_q(x')) - x\int_x^{\epsilon(x)}\frac{dx'}{x'^2}\tilde{H}_q(x')$$
$$-x\int_x^{\epsilon(x)}dx_1\int_{-1}^{1}dx_2\Phi_F(x_1,x_2)\mathcal{P}\frac{3x_1-x_2}{x_1^2(x_1-x_2)^2}$$
$$-x\int_x^{\epsilon(x)}dx_1\int_{-1}^{1}dx_2\tilde{\Phi}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^2(x_1-x_2)}\,.$$

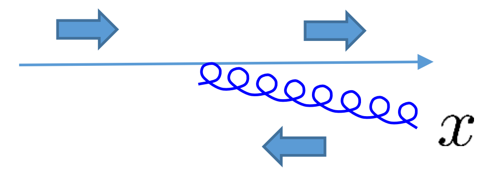
Genuine twist-three part

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)}\frac{dx'}{x'^2}(H_g(x')+E_g(x')) - x\int_x^{\epsilon(x)}\frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)}\frac{dx'}{x'^3}\int dX\,\Phi_F(X,x') + 2x\int_x^{\epsilon(x)}dx_1\int_{-1}^{1}dx_2\tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$
$$+2x\int_x^{\epsilon(x)}dx_1\int_{-1}^{1}dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

Spin at small-x?

q→ qg splitting





$$P_{gq}(x) = C_F \left(\frac{1}{x} + \frac{(1-x)^2}{x} \right) \qquad \Delta P_{gq}(x) = C_F \left(\frac{1}{x} - \frac{(1-x)^2}{x} \right)$$

No $1/x$ enhancement! Spin effects are always suppressed by $x \sim (\text{energy})^{-1}$

HOWEVER, they can be enhanced by double logarithms $(\alpha_s \ln^2 1/x)^n$

Bartels, Ermolaev, Ryskin (1996),
Kovchegov, Pitonyak, Sievert (2015~)

The large uncertainty in $\Delta G(x)$ in the small-x region is the current major problem.

I give two arguments that $L_g(x) \approx -\Delta G(x)$ at small-x.

Gluon Wigner and OAM at small-x

Gluon Wigner distribution

YH, Xiao, Yuan (2016)

$$xW(x,\vec{k}_\perp,\vec{b}_\perp)$$
$$= \int_{\Delta_\perp} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int_{z^-,z_\perp} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P-\Delta/2|\mathrm{Tr}F^{+i}(-z/2)U^{[+]}F^{+}_{i}(z/2)U^{[-]}|P+\Delta/2\rangle$$

Approximate $e^{ixP^+z^-} \approx 1$

$$\boxed{xW(x,\vec{k}_\perp,\vec{b}_\perp) \approx \frac{2N_c}{\alpha_s}\int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) S(\vec{r}_\perp,\vec{b}_\perp)}$$

``Dipole S-matrix”

$$S(\vec{r}_\perp,\vec{b}_\perp) = \left\langle \frac{1}{N_c}\mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger\left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right)\right\rangle_x$$

→ Balitsky-Kovchegov-JIMWLK equation

$$S(x,\Delta_\perp,q_\perp) \equiv \int d^2x_\perp d^2y_\perp e^{iq_\perp\cdot(x_\perp-y_\perp)+i(x_\perp+y_\perp)\cdot\frac{\Delta_\perp}{2}} \left\langle P+\frac{\Delta}{2}\right| \frac{1}{N_c}\mathrm{Tr}\left[U(x_\perp)U^\dagger(y_\perp)\right] \left|P-\frac{\Delta}{2}\right\rangle$$

$$= P(x,\Delta_\perp,q_\perp) + iq_\perp\cdot\Delta_\perp O(x,|q_\perp|) \;\; + q_\perp\times S_\perp O_s(x,q_\perp)$$

"Pomeron" "odderon" "spin-dependent odderon"

The dipole S-matrix cannot depend on the longitudinal spin
← forbidden by PT symmetry

Lesson: All information about (longitudinal) spin is lost in the eikonal approximation.

$$e^{ixP^+z^-} \approx 1$$

OAM as a next-to-eikonal effect

YH, Nakagawa, Xiao, Yuan, Zhao (2017)

Go to next-to-eikonal

$$e^{ixP^+z^-} \approx 1 + ixP^+z^- \quad \Rightarrow \quad W = W_0 + \delta W$$

$$\begin{aligned}\delta W(x,\Delta_\perp,q_\perp,S) = \frac{4P^+}{g^2(2\pi)^3}\int d^2x_\perp d^2y_\perp e^{i(q_\perp+\frac{\Delta_\perp}{2})\cdot x_\perp + i(-q_\perp+\frac{\Delta_\perp}{2})\cdot y_\perp} \\ \times\left\{\int_{-T}^{T} dz^-\left(q_\perp^i - \frac{\Delta_\perp^i}{2}\right)\left\langle \mathrm{Tr}\left[U_{Tz^-}(x_\perp)\overleftarrow{D}_i U_{z^- -T}(x_\perp)U^\dagger(y_\perp)\right]\right\rangle\right. \\ \left. + \int_{-T}^{T} dz^-\left(q_\perp^i + \frac{\Delta_\perp^i}{2}\right)\langle \mathrm{Tr}\left[U(x_\perp)U_{-Tz^-}(y_\perp)D_i U_{z^-T}(y_\perp)\right]\rangle\right\}.\end{aligned}$$

Can have spin-dependent matrix element. Involves half-infinite Wilson lines

TMD polarized gluon distribution

$$ix\Delta G(x,q_\perp)\frac{S^+}{P^+} \equiv 2\int \frac{d^2z_\perp dz^-}{(2\pi)^3P^+}e^{-ixP^+z^-+iq_\perp\cdot z_\perp}\left\langle PS\left|\epsilon_{ij}F^{+i}\left(\tfrac{z}{2}\right)U_-F^{+j}\left(-\tfrac{z}{2}\right)U_+\right|PS\right\rangle$$

$$\approx \frac{4P^+}{g^2(2\pi)^3}\int d^2x_\perp d^2y_\perp e^{i(q_\perp+\frac{\Delta_\perp}{2})\cdot x_\perp+i(-q_\perp+\frac{\Delta_\perp}{2})\cdot y_\perp}$$

$$\times\epsilon_{ij}\left\{q_\perp^j\int_{-\infty}^{\infty}dz^-\left\langle\mathrm{Tr}\left[U_{\infty z^-}(x_\perp)\overleftarrow{D}_iU_{z^--\infty}(x_\perp)U^\dagger(y_\perp)\right]\right\rangle\right.$$

$$\left.+q_\perp^i\int_{-\infty}^{\infty}dz^-\langle\mathrm{Tr}\left[U(x_\perp)U_{-\infty z^-}(y_\perp)D_jU_{z^-\infty}(y_\perp)\right]\rangle\right\}$$

Exactly the same matrix element appears.
→ Linear relation between $\Delta G(x)$ and $L_g(x)$

$$L_g(x)\approx-\Delta G(x)+\cdots$$

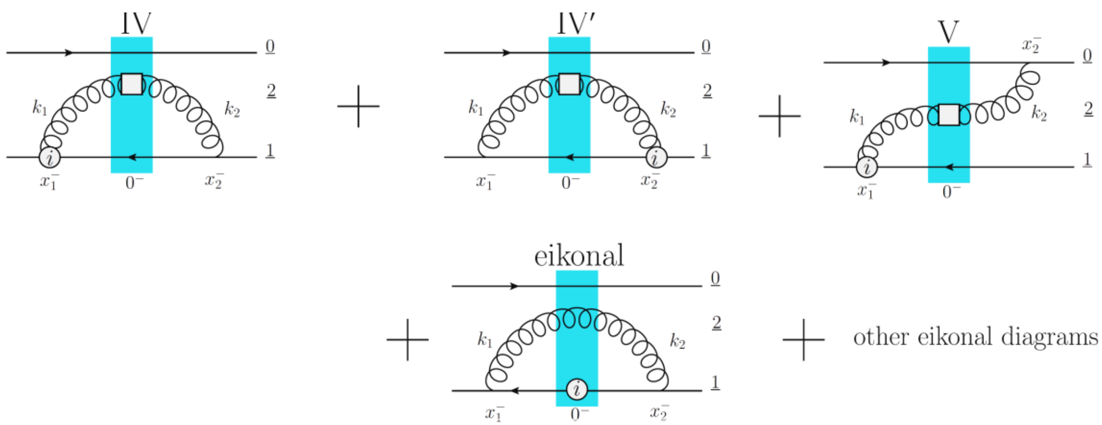
YH, Nakagawa, Xiao, Yuan, Zhao (2017)

Helicity evolution at small-x

Kovchegov, Pitonyak, Sievert (2015~)

Small-x evolution of the polarized dipole operator

$$U_{\infty,-\infty}(x_\perp)U_{-\infty,z}D_\perp U_{z,\infty}(y_\perp)$$



Resummation of double logarithms $\alpha_s \ln^2 \frac{1}{x}$ $\Delta f(x) \sim \left(\frac{1}{x}\right)^{\alpha_h}$

$$\boxed{\alpha_h^{q,S} = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}}} \qquad \boxed{\alpha_h^{q,NS} = \sqrt{2}\sqrt{\frac{\alpha_s N_c}{2\pi}}} \qquad \boxed{\alpha_h^{G} = \frac{13}{4\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

`DGLAP' equation for OAM PDF

cf. Hagler, Schafer (1998)

$$\frac{d}{d\ln Q^2}\begin{pmatrix} L_q(x) \\ L_g(x) \end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix} \hat{P}_{qq}(z) & \hat{P}_{qg}(z) & \Delta\hat{P}_{qq}(z) & \Delta\hat{P}_{qg}(z) \\ \hat{P}_{gq}(z) & \hat{P}_{gg}(z) & \Delta\hat{P}_{gq}(z) & \Delta\hat{P}_{gg}(z) \end{pmatrix}\begin{pmatrix} L_q(x/z) \\ L_g(x/z) \\ \Delta q(x/z) \\ \Delta G(x/z) \end{pmatrix},$$

$$\begin{aligned}
\hat{P}_{qq}(z) &= C_F\left(\frac{z(1+z^2)}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right), \\
\hat{P}_{qg}(z) &= n_f z(z^2+(1-z)^2), \\
\hat{P}_{gq}(z) &= C_F(1+(1-z)^2), \\
\hat{P}_{gg}(z) &= 6\frac{(z^2-z+1)^2}{(1-z)_+} + \frac{\beta_0}{2}\delta(z-1), \\
\Delta\hat{P}_{qq}(z) &= C_F(z^2-1), \\
\Delta\hat{P}_{qg}(z) &= n_f(1-3z+4z^2-2z^3), \\
\Delta\hat{P}_{gq}(z) &= C_F(-z^2+3z-2), \\
\Delta\hat{P}_{gg}(z) &= 6(z-1)(z^2-z+2),
\end{aligned}$$

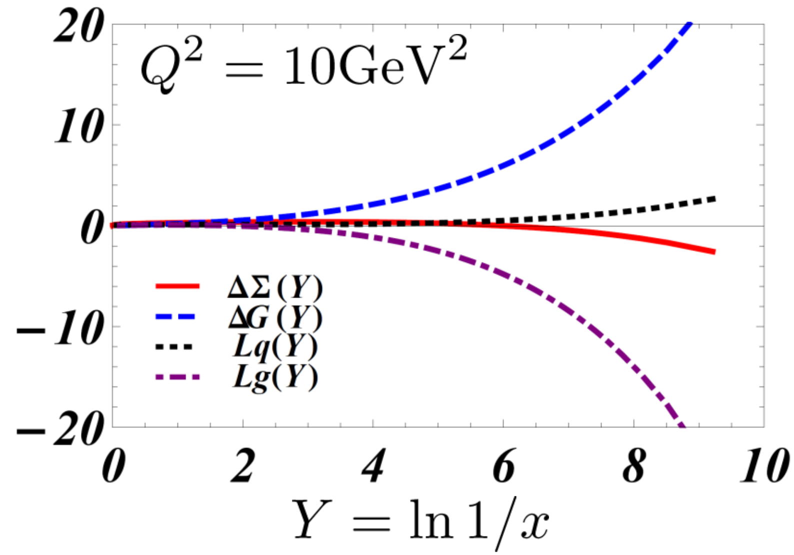
Numerical results

YH, Yang (2018)

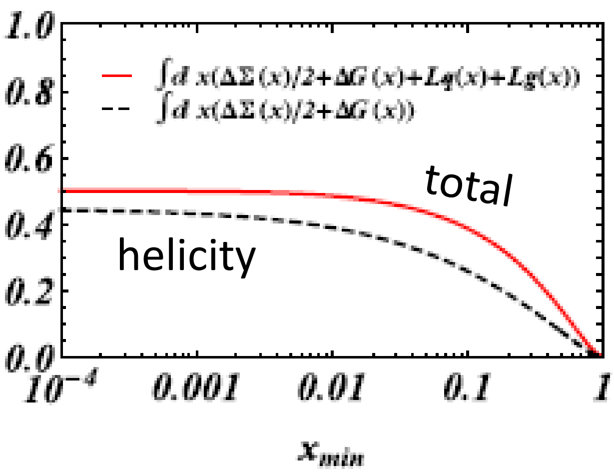
`Democratic model'

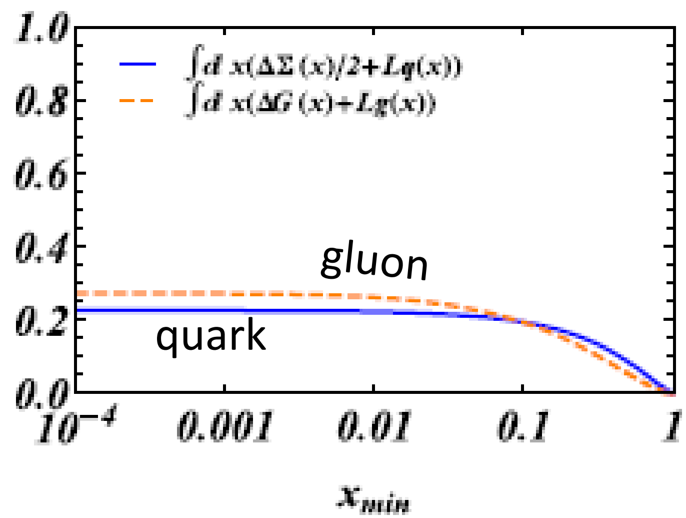
$$\Delta\Sigma(x,Q_0^2)=\frac{1}{4},\quad \Delta G(x,Q_0^2)=\frac{1}{8},$$

$$L_q(x,Q_0^2)=\frac{1}{8},\quad L_g(x,Q_0^2)=\frac{1}{8},$$



$$\int_{x_{min}}^{1} dx\left(\frac{1}{2}\Delta\Sigma(x)+\Delta G(x)+L_q(x)+L_g(x)\right)$$



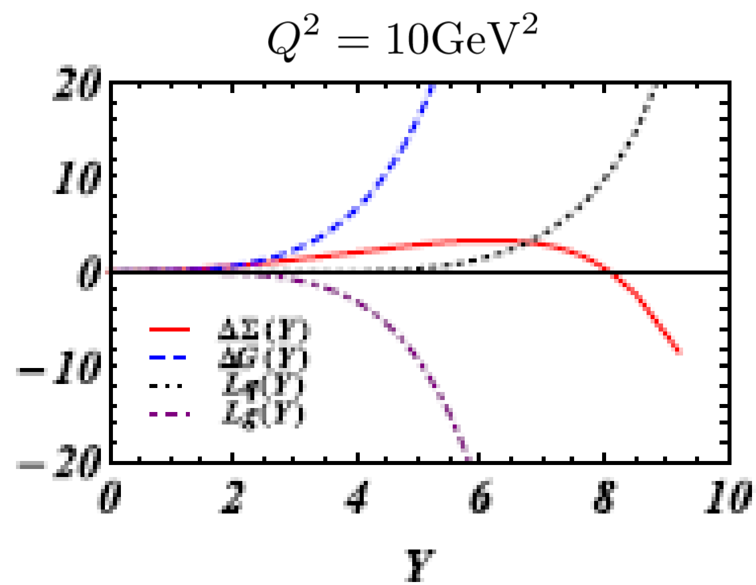


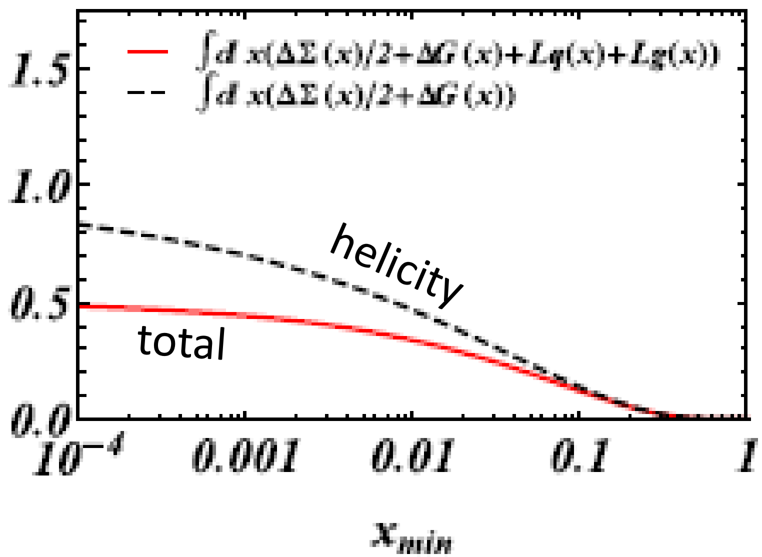
`Helicity dominance model'

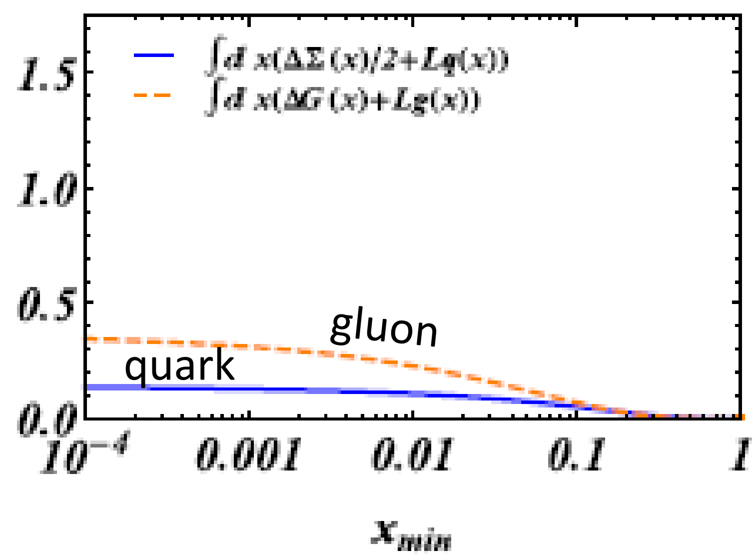
$$\Delta\Sigma(x,Q_0^2) = A_q x^{-0.3}(1-x)^3$$

$$\Delta G(x,Q_0^2) = A_g x^{-0.6}(1-x)^3$$

$$L_q(x,Q_0^2) = L_g(x,Q_0^2) = 0$$







Analytical insights

Use the ansatz $L_g(x, Q^2) = A(Q^2)\frac{1}{x^c}$ $\Delta G(x, Q^2) \approx B(Q^2)\frac{1}{x^c}$

and approximately solve the DGLAP equation analytically

$$\frac{L_g(x, Q^2)}{\Delta G(x, Q^2)} \approx -\frac{2}{c+1}$$

$$\frac{\Delta\Sigma(x)}{\Delta G(x)} \approx -n_f \frac{1-c}{c(1+c)\left[6\left(-H_{c-1} + \frac{1}{c} - \frac{1}{1+c}\right)\frac{\beta_0}{2}\right]}$$

$$L_q(x) \approx -\frac{\Delta\Sigma(x)}{1+c}$$

At small-x, helicity and OAM distributions are related to each other by the Regge parameter c .

Observables for OAM

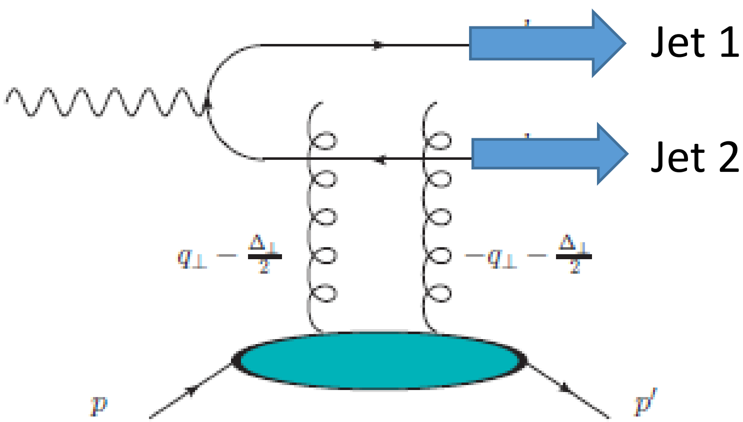
→ talk by S. Bhattacharya

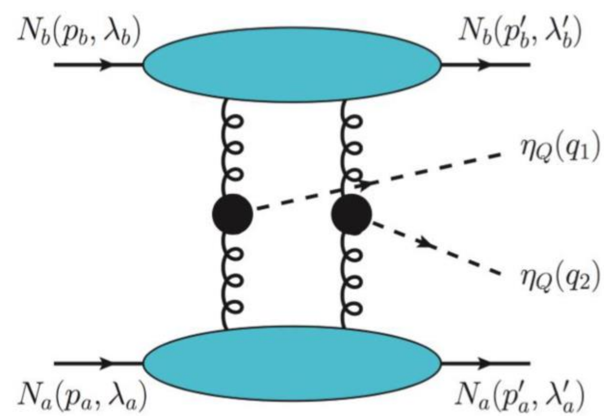
Essentially the measurement of Wigner/GTMD .
Tag two hadrons (jets) in the final state, together with the recoiling proton

Relative momentum between the two hadrons

$$W(x, k_\perp, \Delta_\perp)$$

Recoiling proton momentum





Ji, Yuan, Zhao (2017); YH, Nakagawa, Xiao, Yuan, Zhao (2017),
Bhattacharya, Metz, Zhou (2017); Bhattacharya, Metz, Ojha, Tsai, Zhou (2018)

Conclusion

- OAM is the holy grail of spin physics. A lot of recent progress in theory, still there's a long way to go.

- x-distribution for OAM well-defined. Significant cancelation between OAM and helicity at small-x.

- Resolution of the spin puzzle
  → OAM at large-to-moderate-x.
  A couple of observables have been recently proposed.