LightCone 2018

14-18 May 2018, JLab, USA

"Accessing dihadron fragmentation functions in e^+e^- and SIDIS."

P.R.D97, 074019 (2018); arXiv:1712.06384.

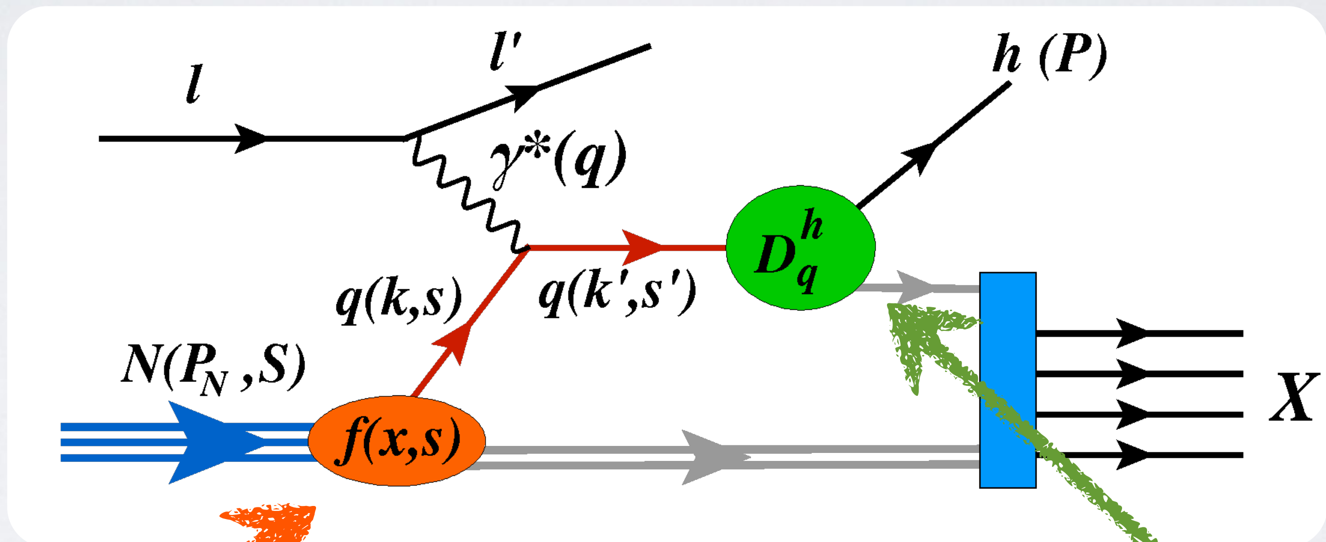
Hrayr Matevosyan

THE UNIVERSITY *of* ADELAIDE

SIDIS with <u>one</u> measured hadron

- ***Measurement of the <u>transverse momentum</u> of the produced hadron in SIDIS provides access to <u>TMD PDFs/FFs</u>.***

  - *SIDIS Process with TM of hadron measured.*



- *TMD PDFs*

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

- *TMD FFs*

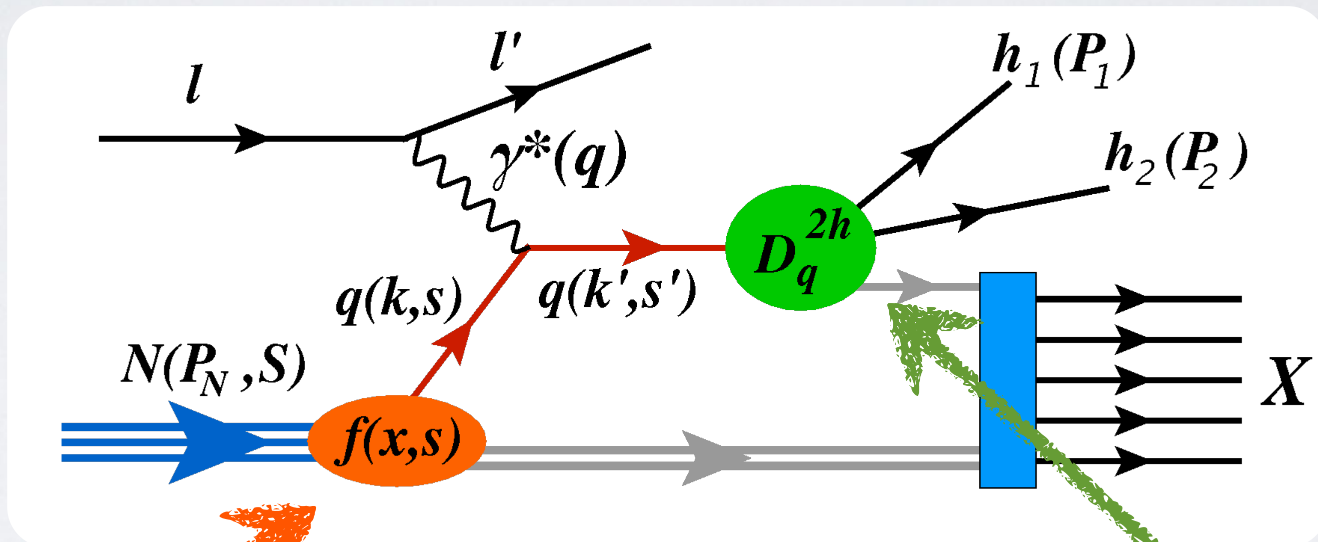
q/h	U
U	D_1
L	
T	$H_1^\perp$

*unpol/spinless h!

SIDIS with <u>two</u> measured hadrons

- ***Measuring <u>two-hadron</u> semi-inclusive DIS: an additional method for accessing TMD PDFs.***

  - *SIDIS Process with TM of hadrons measured.*



- *TMD PDFs*

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

- *TMD DiFFs*

q/h_1h_2	U
U	D_1
L	$G_1^\perp$
T	$H_1^\perp$ $H_1^\sphericalangle$

*unpol/spinless h!

SYSTEMATICS OF DIHADRON FRAGMENTATION FUNCTIONS

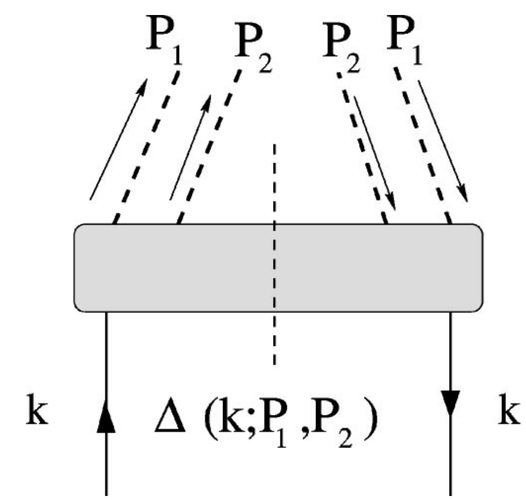
Two-Hadron Kinematics

A. Bianconi et al: PRD 62, 034008 (2000).

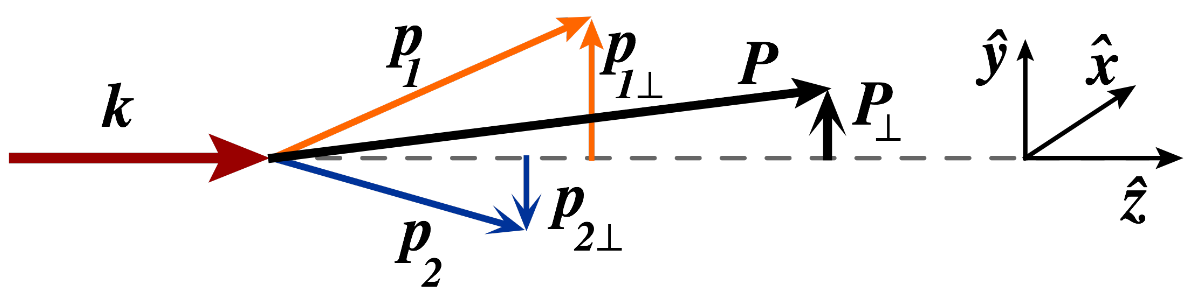
- **Total and Relative TM of hadron pair.**

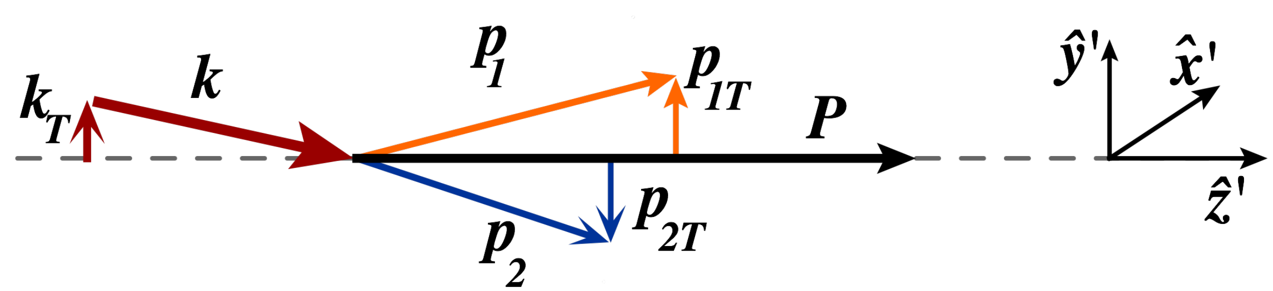
$$P = P_1 + P_2 \qquad z = z_1 + z_2$$

$$R = \frac{1}{2}(P_1 - P_2) \quad \xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}$$



- **Two Coordinate systems:**
  - $\perp$: modelling hadronization
  - T: field-theoretical definition of DiFFs





- **Lorentz Boost:**

$$\boldsymbol{P}_{1T} = \boldsymbol{P}_{1\perp} + z_1 \boldsymbol{k}_T$$

$$\boldsymbol{P}_{2T} = \boldsymbol{P}_{2\perp} + z_2 \boldsymbol{k}_T$$

$$\boldsymbol{k}_T = -\frac{\boldsymbol{P}_\perp}{z}$$

- **Relative TM in two systems**

$$\boldsymbol{R}_\perp = \frac{1}{2}(\boldsymbol{P}_{1\perp} - \boldsymbol{P}_{2\perp})$$

$$\boldsymbol{R}_T = \frac{z_2 \boldsymbol{P}_{1\perp} - z_1 \boldsymbol{P}_{2\perp}}{z}$$

Field-Theoretical Definitions

- *The quark-quark correlator.*

$$\Delta_{ij}(k;P_1,P_2) = \sum_X \int d^4\zeta e^{ik\cdot\zeta} \langle 0|\psi_i(\zeta)|P_1P_2,X\rangle\langle P_1P_2,X|\bar{\psi}_j(0)|0\rangle$$

- *The definitions of DiFFs from the correlator.*

Quark Polarization

$$\Delta^{[\gamma^-]} = D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Unpolarised

$$\Delta^{\gamma^-\gamma_5]} = \frac{\epsilon_T^{ij}R_{Ti}k_{Tj}}{M_h^2} G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Longitudinal

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij}R_{Tj}}{M_h} H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) + \frac{\epsilon_T^{ij}k_{Tj}}{M_h} H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Transverse

Field-Theoretical Definitions

- *The quark-quark correlator.*

$$\Delta_{ij}(k;P_1,P_2)=\sum_X\int d^4\zeta e^{ik\cdot\zeta}\langle 0|\psi_i(\zeta)|P_1P_2,X\rangle\langle P_1P_2,X|\bar{\psi}_j(0)|0\rangle$$

- *The definitions of DiFFs from the correlator.*

Quark Polarization

$$\Delta^{[\gamma^-]}=D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Unpolarised

related to "jet handedness"

$$\Delta^{[\gamma^-\gamma_5]}=\frac{\epsilon_T^{ij}R_{Ti}k_{Tj}}{M_h^2}G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Longitudinal

$$\Delta^{[i\sigma^{i-}\gamma_5]}=\frac{\epsilon_T^{ij}R_{Tj}}{M_h}H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)+\frac{\epsilon_T^{ij}k_{Tj}}{M_h}H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Transverse

Fourier Moments of DiFFs

- ***Expanded dependence on*** $\varphi_{RK} \equiv \varphi_R - \varphi_k$ ***in cos series***

$$D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{KR})) = \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(n\cdot\varphi_{KR})}{1+\delta_{0,n}}\; D_1^{[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|),$$

$$F^{[n]} = \int d\varphi_{KR}\;\cos(n\varphi_{KR})\;F\Big(\cos(\varphi_{KR})\Big)$$

- ***Integrated DiFFs and Fourier moments***

$$D_1^a(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\;D_1^{a,[0]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2)$$

$$G_1^{\perp a,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\left(\frac{\boldsymbol{k}_T^2}{2M_h^2}\right)\frac{|\boldsymbol{R}_T|}{M_h}\;G_1^{\perp a,[n]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2).$$

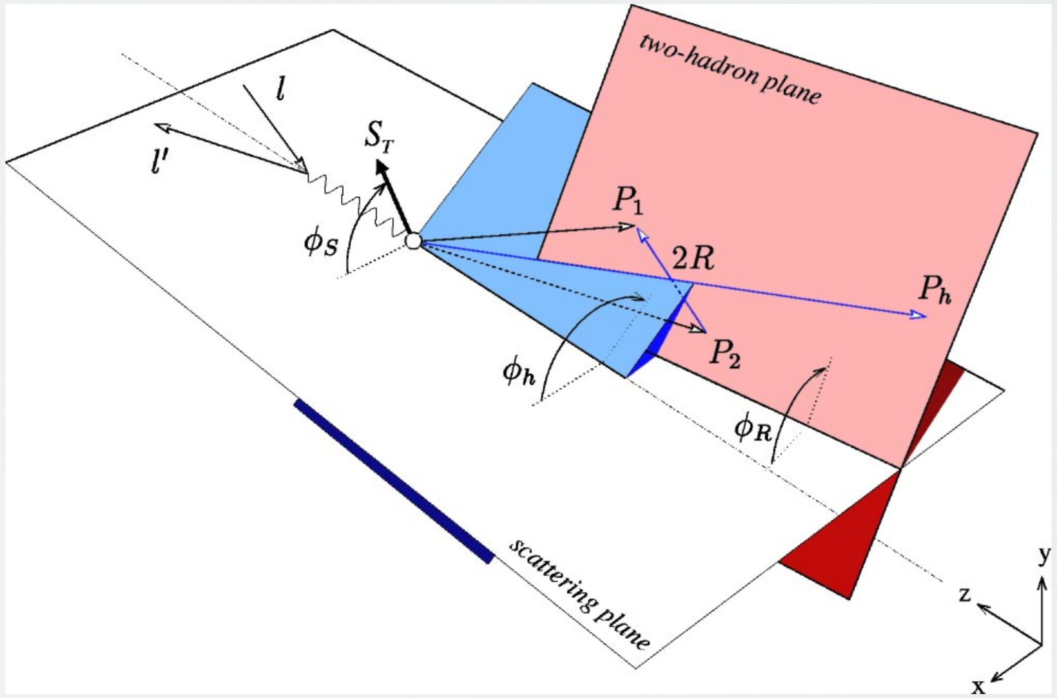
$$H_1^{\sphericalangle,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\;\frac{|\boldsymbol{R}_T|}{M_h}H_1^{\sphericalangle,[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

$$H_1^{\perp,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\;\frac{|\boldsymbol{k}_T|}{M_h}H_1^{\perp,[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

ACCESS TO TRANSVERSITY PDF From DiFF

M. Radici, et al: PRD 65, 074031 (2002).

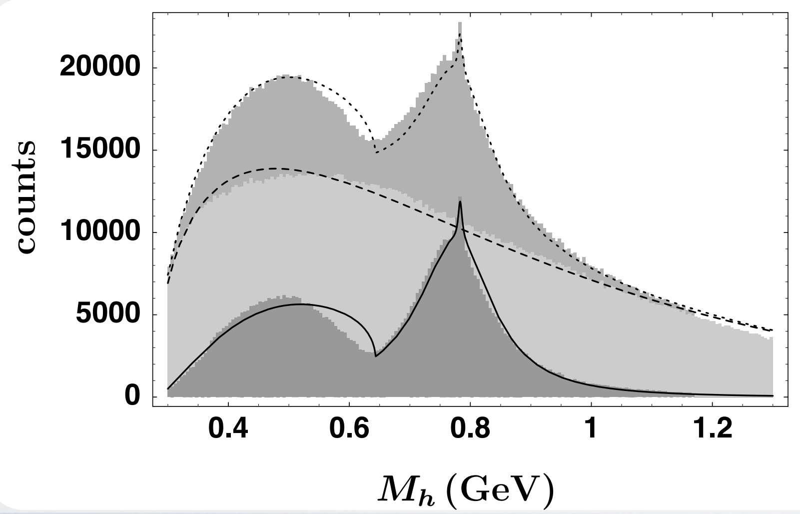
- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized* and *Interference* Dihadron FFs are needed!

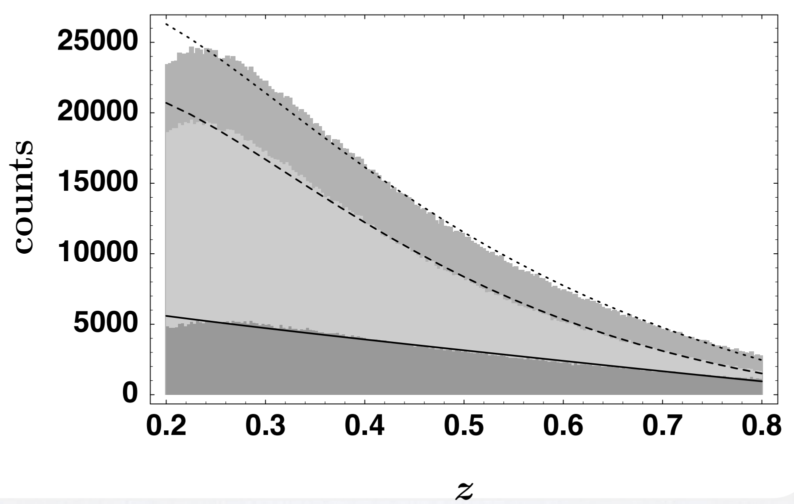


$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \;\; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \;\; D_1^q(z, M_h^2)}$$

- **Empirical Model for D_1^q has been fitted to PYTHIA simulations.**

A. Bacchetta and M. Radici, PRD 74, 114007 (2006).





Experiments

SSA: HERMES, COMPASS.

IFFs: BELLE.

Moments of DiFFs in SIDIS

A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

- *Here transversely polarised DiFFs are **admixture of cos Fourier moments** of both unintegrated DiFFs:*

$$H_{1,SIDIS}^{\sphericalangle}(z, M_H^2) = \left[H_1^{\sphericalangle[0]} + H_1^{\perp[1]}\right]$$

$$H_{1,SIDIS}^{\perp}(z, M_H^2) = \left[H_1^{\perp[0]} + H_1^{\sphericalangle[1]}\right]$$

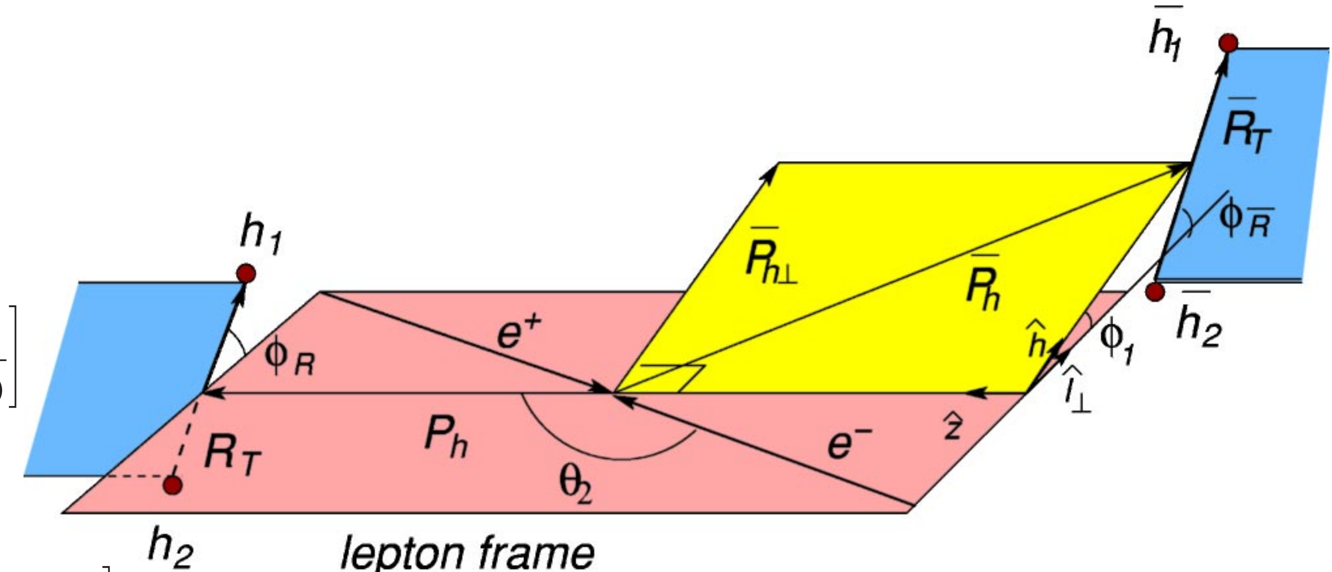
- *Generated by $\cos(\varphi_{RK})$ dependences of unintegrated DiFFs:*

$\varphi_{RK} \equiv \varphi_R - \varphi_k$

$$d\sigma_{UT} \sim \sin(\varphi_R + \varphi_S)\mathcal{C}\left[h_1^{\perp} H^{\sphericalangle}(\cos(\varphi_{RK}))\right] + \sin(\varphi_k + \varphi_S)\mathcal{C}\left[h_1^{\perp} H^{\perp}(\cos(\varphi_{RK}))\right] + ..$$

Back-to-back *two* hadron pairs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).



$$
\begin{aligned}
&\frac{d\sigma(e^+e^-\to(h_1h_2)(\bar{h}_1\bar{h}_2)X)}{d\boldsymbol{q}_T dz\,d\xi\,dM_h^2\,d\phi_R\,d\bar{z}\,d\bar{\xi}\,d\bar{M}_h^2\,d\phi_{\bar{R}}\,dy\,d\phi^l} \\
&= \sum_{a,\bar{a}} e_a^2 \frac{6\alpha^2}{Q^2} z^2\bar{z}^2 \Bigg\{ A(y)\mathcal{F}[D_1^a\bar{D}_1^a] + \cos(2\phi_1)B(y)\mathcal{F}\left[(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T - \boldsymbol{k}_T\cdot\bar{\boldsymbol{k}}_T)\frac{H_1^{\perp a}\bar{H}_1^{\perp a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] \\
&- \sin(2\phi_1)B(y)\mathcal{F}\left[(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{g}}\cdot\bar{\boldsymbol{k}}_T + \hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T)\frac{H_1^{\perp a}\bar{H}_1^{\perp a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] + \cos(\phi_R+\phi_{\bar{R}}-2\phi^l) \\
&\times B(y)|\boldsymbol{R}_T||\bar{\boldsymbol{R}}_T|\mathcal{F}\left[\frac{H_1^{\sphericalangle a}\bar{H}_1^{\sphericalangle a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] + \cos(\phi_1+\phi_R-\phi^l)B(y)|\boldsymbol{R}_T|\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T\frac{H_1^{\sphericalangle a}\bar{H}_1^{\perp a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] \\
&- \sin(\phi_1+\phi_R-\phi^l)B(y)|\boldsymbol{R}_T|\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\bar{\boldsymbol{k}}_T\frac{H_1^{\sphericalangle a}\bar{H}_1^{\perp a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] + \cos(\phi_1+\phi_{\bar{R}}-\phi^l)B(y)|\bar{\boldsymbol{R}}_T| \\
&\times \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{H_1^{\perp a}\bar{H}_1^{\sphericalangle a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] - \sin(\phi_1+\phi_{\bar{R}}-\phi^l)B(y)|\bar{\boldsymbol{R}}_T|\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\frac{H_1^{\perp a}\bar{H}_1^{\sphericalangle a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] + A(y)|\boldsymbol{R}_T||\bar{\boldsymbol{R}}_T| \\
&\times \Bigg(\sin(\phi_1-\phi_R+\phi^l)\sin(\phi_1-\phi_{\bar{R}}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T\frac{G_1^{\perp a}\bar{G}_1^{\perp a}}{M_1M_2\bar{M}_1\bar{M}_2}\right] + \sin(\phi_1-\phi_R+\phi^l)\cos(\phi_1-\phi_{\bar{R}}+\phi^l) \\
&\times \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{g}}\cdot\bar{\boldsymbol{k}}_T\frac{G_1^{\perp a}\bar{G}_1^{\perp a}}{M_1M_2\bar{M}_1\bar{M}_2}\right] + \cos(\phi_1-\phi_R+\phi^l)\sin(\phi_1-\phi_{\bar{R}}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T\frac{G_1^{\perp a}\bar{G}_1^{\perp a}}{M_1M_2\bar{M}_1\bar{M}_2}\right] + \cos(\phi_1-\phi_R+\phi^l) \\
&\times \cos(\phi_1-\phi_{\bar{R}}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{g}}\cdot\bar{\boldsymbol{k}}_T\frac{G_1^{\perp a}\bar{G}_1^{\perp a}}{M_1M_2\bar{M}_1\bar{M}_2}\right]\Bigg)\Bigg\},
\end{aligned}
\tag{19}
$$

- ***Can access both helicity and transverse pol. dependent DiFFs:***

$$
A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}
$$

$$
A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}
$$

Moments of DiFFs in e$^+$e$^-$

D. Boer et al: PRD 67, 094003 (2003).

- *In asymmetry: helicity-dependent DiFF in the.*

$\cos(\varphi_R - \varphi_k)$ **moment**

$$G_1^{\perp}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; \boxed{(\boldsymbol{k}_T \cdot \boldsymbol{R}_T)} \; G_1^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

- *In asymmetry: IFF.*

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; |\boldsymbol{R}_T| H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

- ***Differ from SIDIS !***

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = H_1^{\sphericalangle,[0]} \qquad H_{1,SIDIS}^{\sphericalangle}(z, M_H^2) = \left[H_1^{\sphericalangle[0]} + H_1^{\perp[1]}\right]$$

Moments of DiFFs in e+e-

D. Boer et al: PRD 67, 094003 (2003).

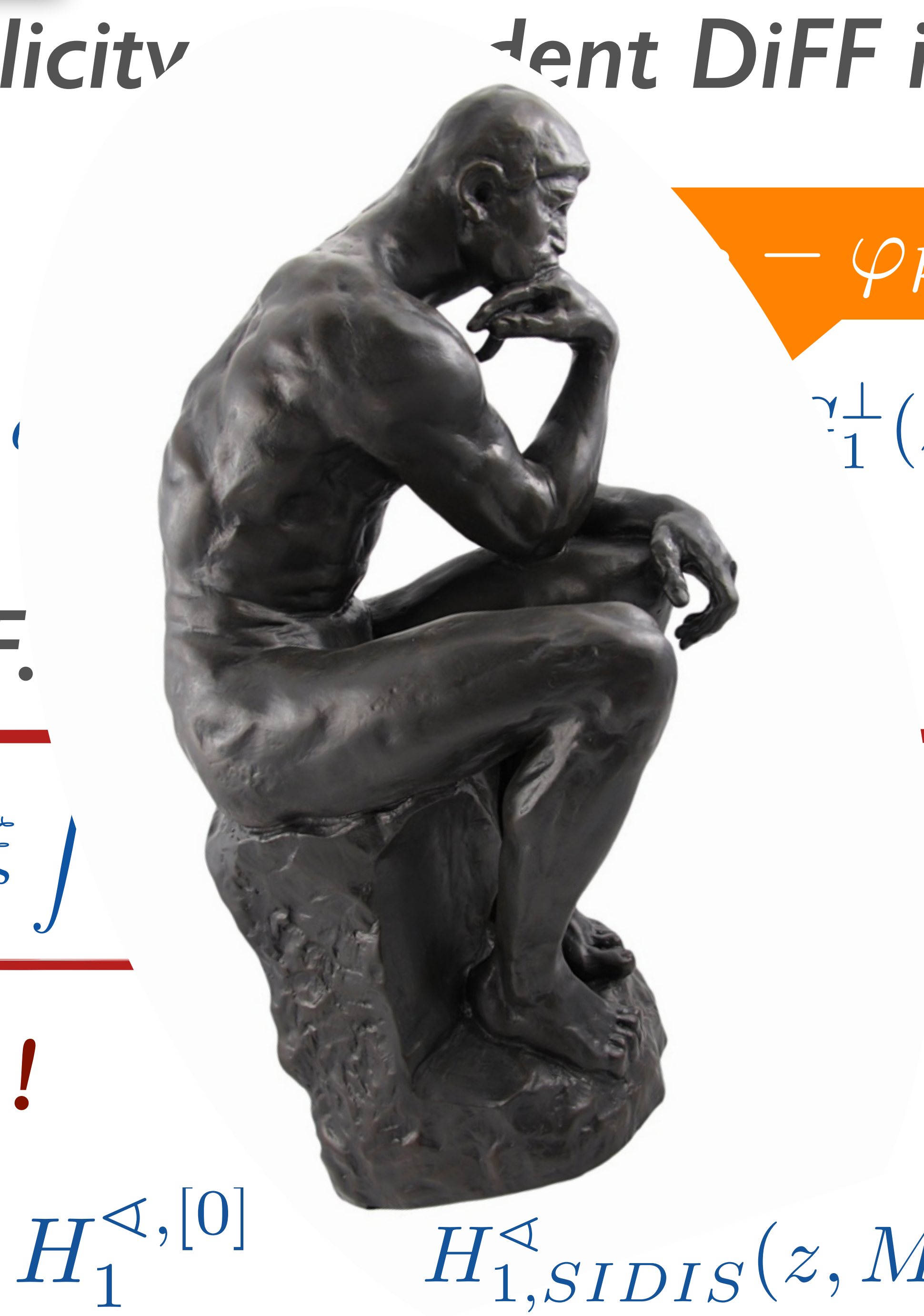
- *In asymmetry: helicity* … *dent DiFF in the.*

… $- \varphi_k)$ **moment**

$$G_1^{\perp}(z, M_h^2) = \int d\xi \int \ldots$$ $$\ldots{}_1^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

- *In asymmetry: IFF.*

$$\boxed{H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = \int d\xi \int \ldots \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)}$$



- ***Differ from SIDIS !***

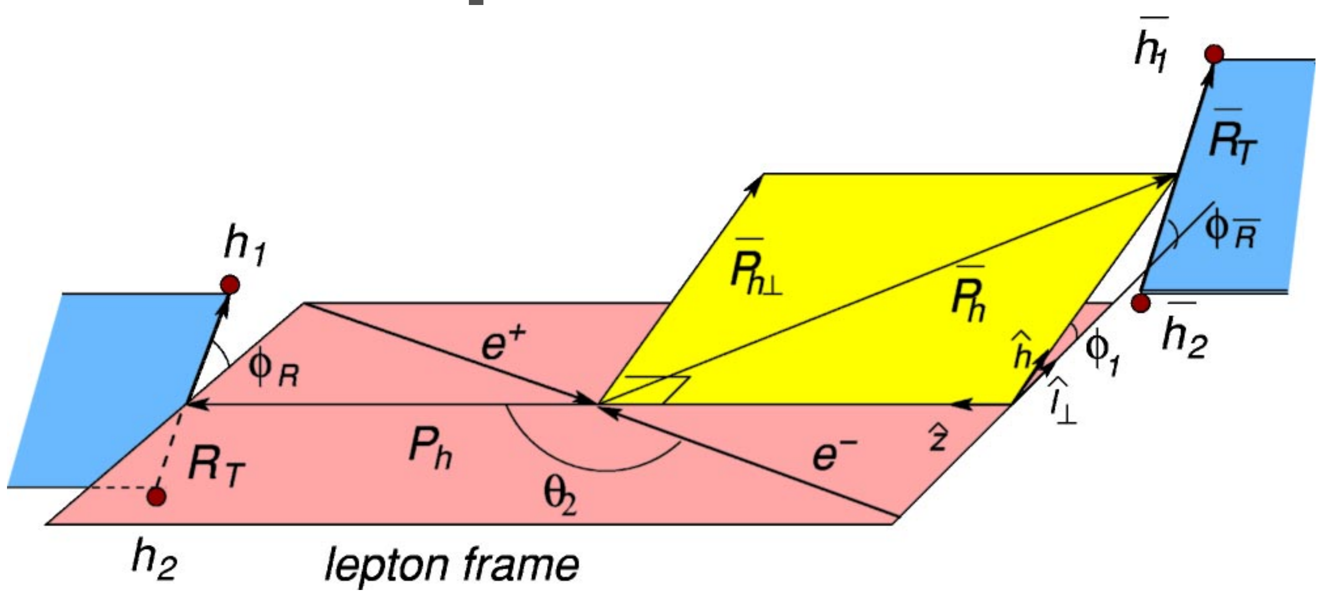
$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = H_1^{\sphericalangle,[0]} \qquad H_{1,SIDIS}^{\sphericalangle}(z, M_H^2) = \left[H_1^{\sphericalangle[0]} + H_1^{\perp[1]}\right]$$

- ***Might strongly affect combined analysis!***

Back-to-back *two* hadron pairs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).

- *Can access both helicity and transverse pol. dependent DiFFs:*

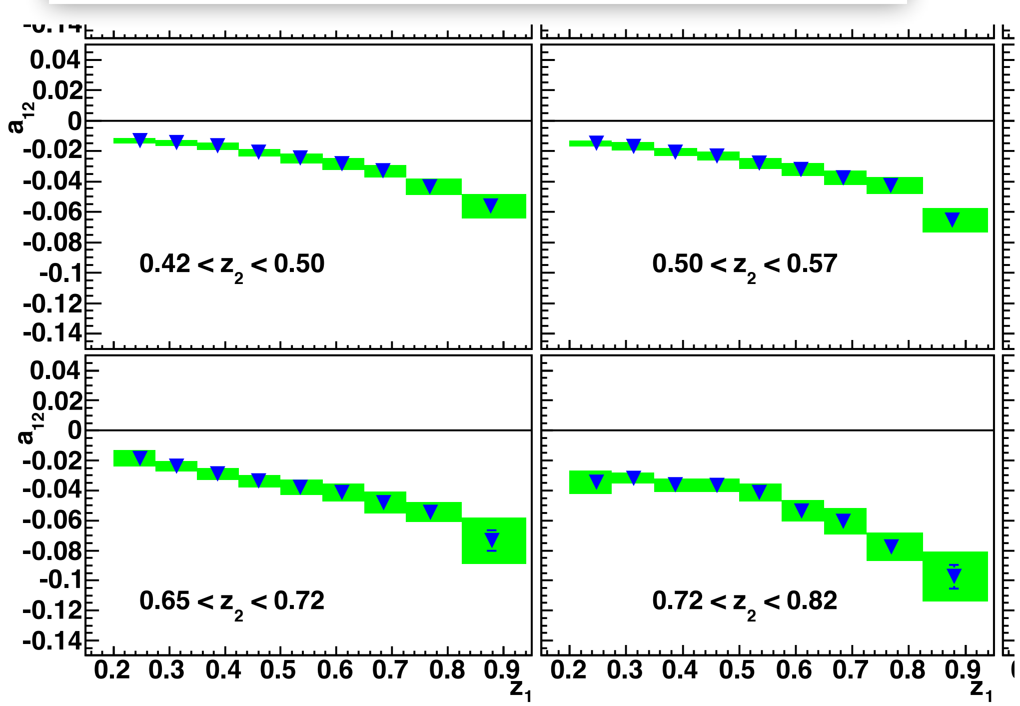


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

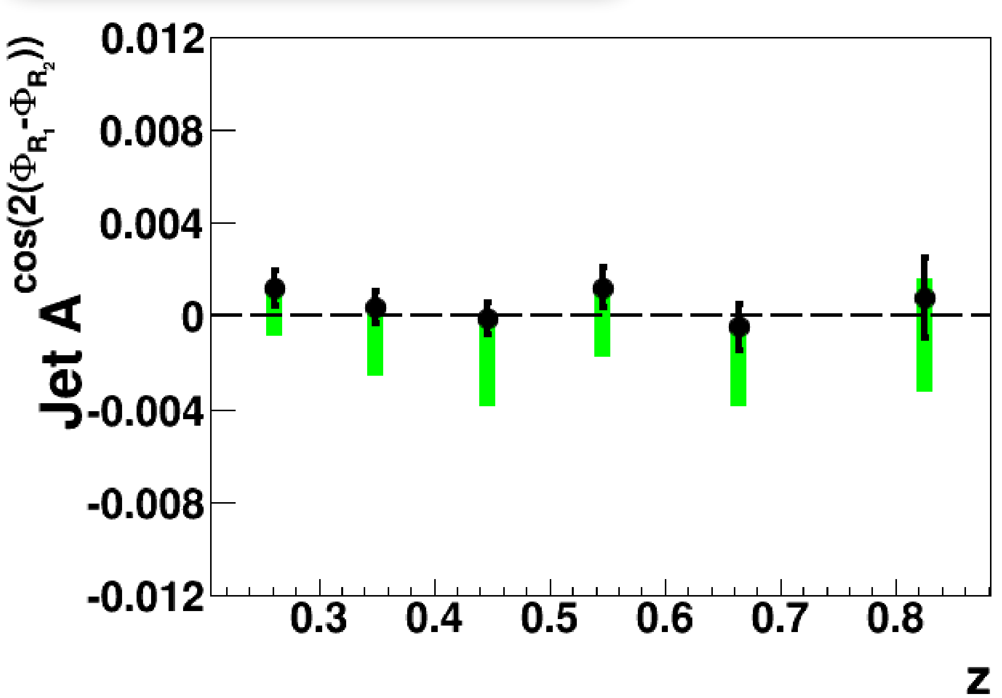
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



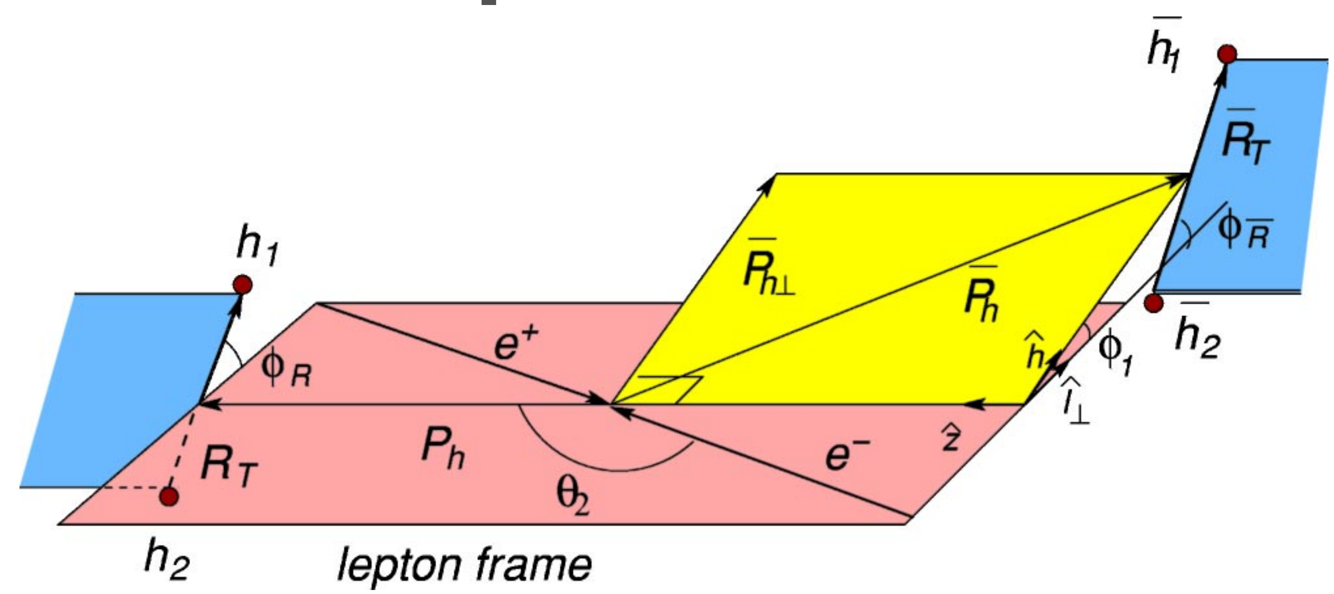
PoS DIS2015 (2015) 216



Back-to-back *two* hadron pairs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).

- ***Can access both helicity and transverse pol. dependent DiFFs:***

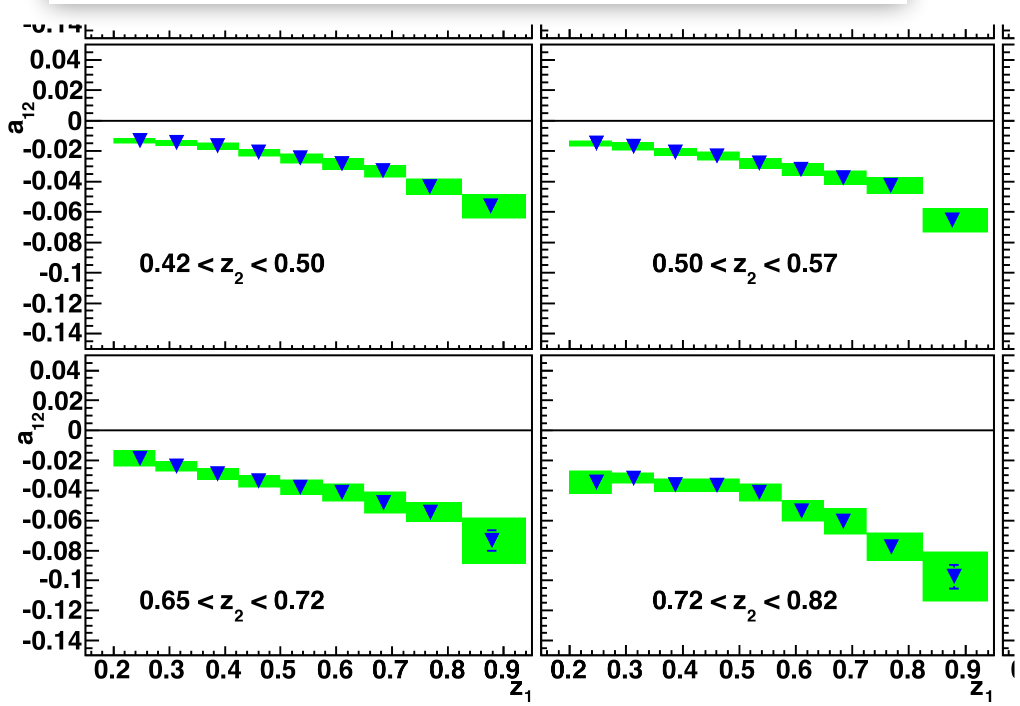


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z, M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z}, M_{\bar{h}}^2)}{D_1(z, M_h^2)\ \bar{D}_1(\bar{z}, M_{\bar{h}}^2)}$$

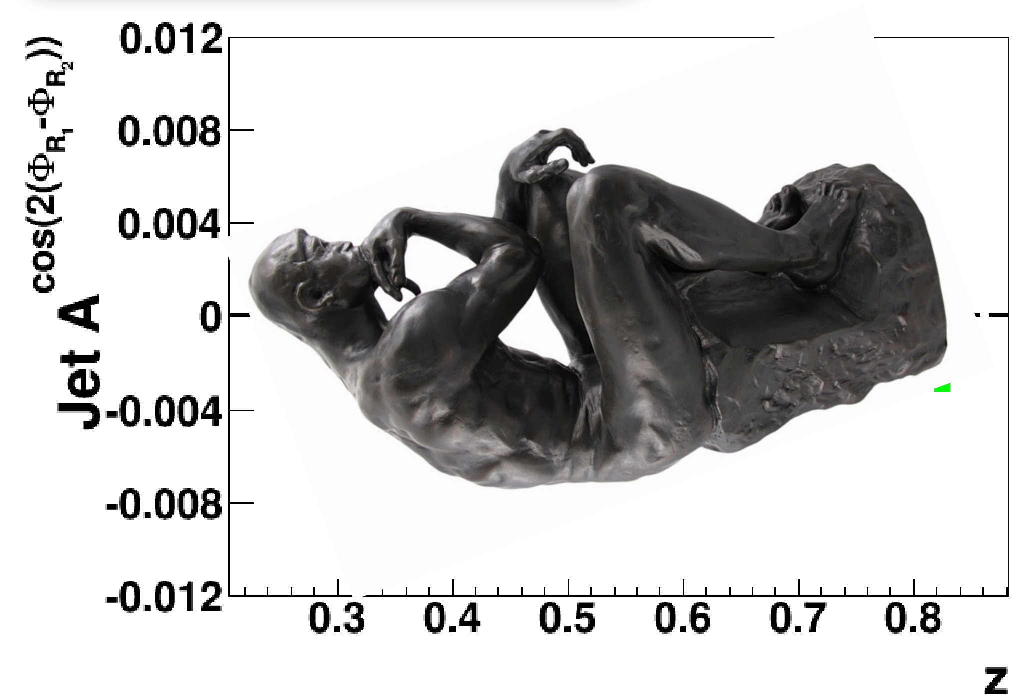
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z, M_h^2)\bar{G}_1^{\perp}(\bar{z}, M_{\bar{h}}^2)}{D_1(z, M_h^2)\ \bar{D}_1(\bar{z}, M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



PoS DIS2015 (2015) 216



How to resolve these?

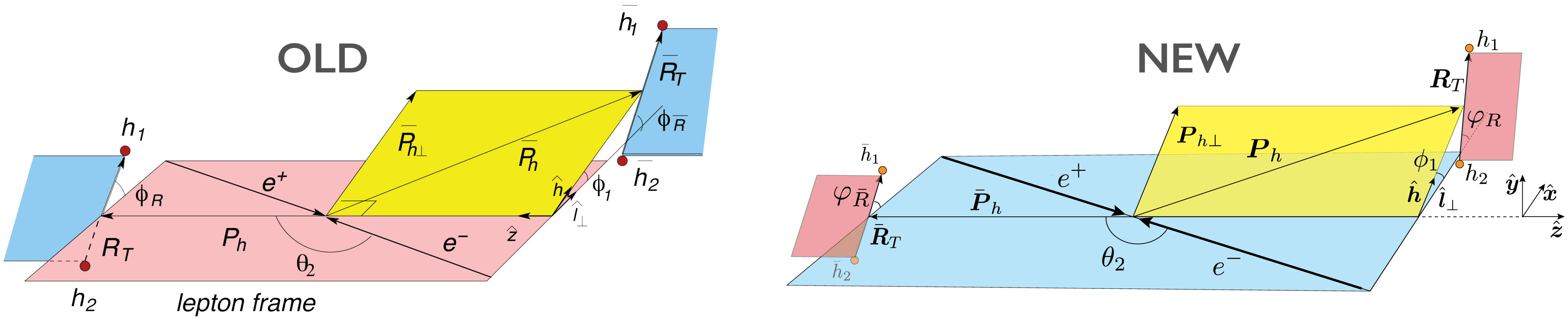
Quote from Anatoly Radyushkin:

‘‘I am old enough to know that if something is published, it is not necessarily correct”

Re-derived e⁺e⁻ Cross Section

H.M. , Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

- ***An error in kinematics was found:***



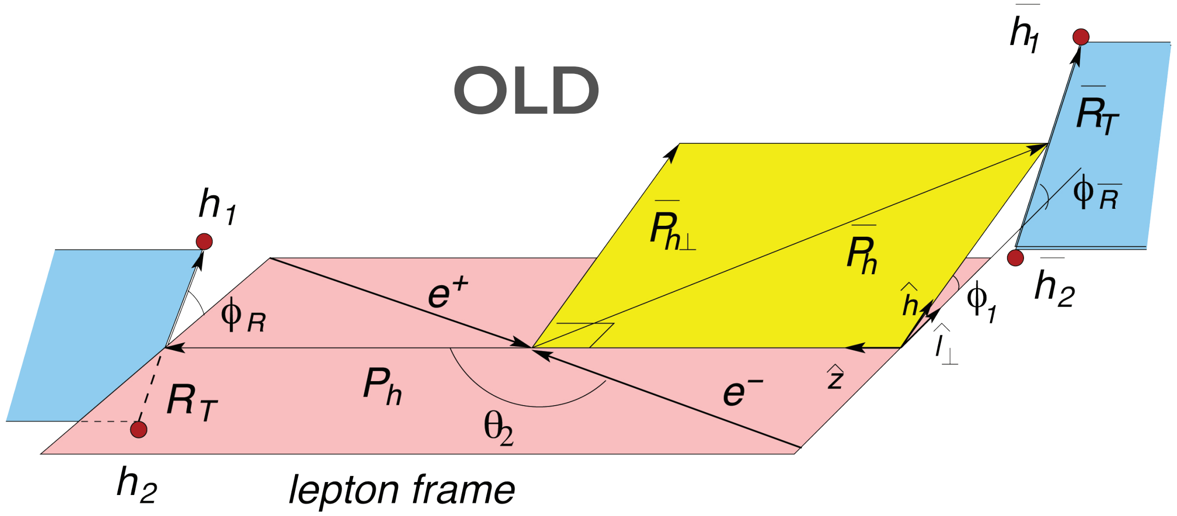
- ***The new fully differential cross-section expression:***

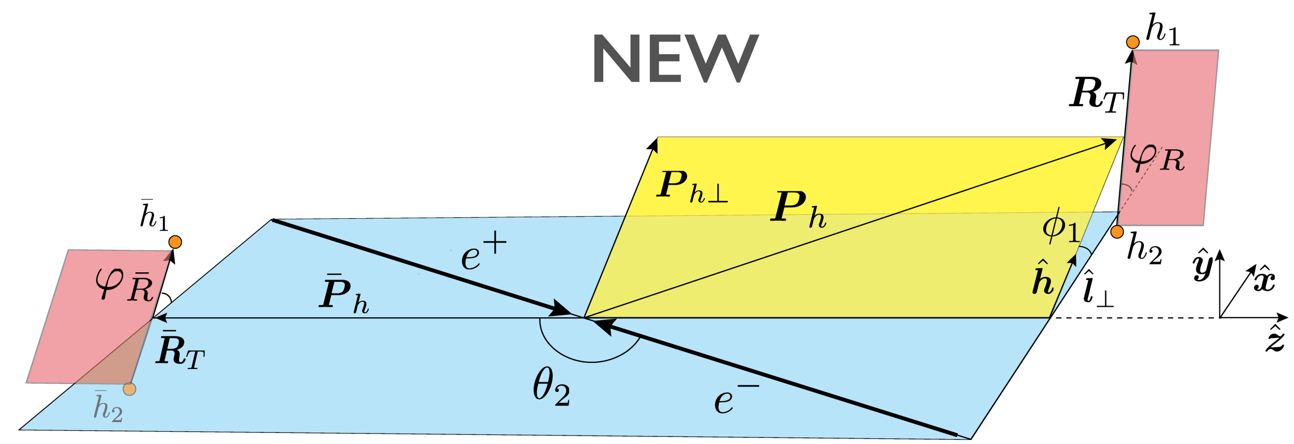
$$
\frac{d\sigma\Big(e^+e^-\to(h_1h_2)(\bar h_1\bar h_2)X\Big)}{d^2\boldsymbol{q}_T dz d\xi d\varphi_R dM_h^2 d\bar z d\bar\xi d\varphi_{\bar R} d\bar M_h^2 dy} = \frac{3\alpha^2}{\pi Q^2} z^2\bar z^2 \sum_{a,\bar a} e_a^2 \Bigg\{ A(y)\mathcal{F}\Big[D_1^a \bar D_1^{\bar a}\Big]
$$
$$
+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar M_h}\cos(\varphi_k+\varphi_{\bar k})H_1^{\perp a}\bar H_1^{\perp\bar a}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar M_h}\cos(\varphi_R+\varphi_{\bar R})H_1^{\sphericalangle a}\bar H_1^{\sphericalangle\bar a}\right]
$$
$$
+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar M_h}\cos(\varphi_k+\varphi_{\bar R})H_1^{\perp a}\bar H_1^{\sphericalangle\bar a}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar M_h}\cos(\varphi_R+\varphi_{\bar k})H_1^{\sphericalangle a}\bar H_1^{\perp\bar a}\right]
$$
$$
- A(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|\,|\boldsymbol{k}_T|}{M_h^2}\frac{|\bar{\boldsymbol{R}}_T|\,|\bar{\boldsymbol{k}}_T|}{\bar M_h^2}\sin(\varphi_k-\varphi_R)\sin(\varphi_{\bar k}-\varphi_{\bar R})G_1^{\perp a}\bar G_1^{\perp\bar a}\right]\Bigg\}.
$$

Re-derived e+e- Cross Section

H.M. , Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

- ***An error in kinematics was found:***





- ***The new fully differential cross-section expression:***

$$\frac{d\sigma\Big(e^+e^- \to (h_1 h_2)(\bar{h}_1\bar{h}_2)X\Big)}{d^2\boldsymbol{q}_T dz d\xi d\varphi_R dM_h^2 d\bar{z} d\bar{\xi} d\varphi_{\bar{R}} d\bar{M}_h^2 dy} = \frac{3\alpha^2}{\pi Q^2} z^2 \bar{z}^2 \sum_{a,\bar{a}} e_a^2 \Big\{ A(y)\mathcal{F}\Big[D_1^a \bar{D}_1^{\bar{a}}\Big]$$

$$\mathcal{F}[wD^a\bar{D}^{\bar{a}}] = \int d^2\boldsymbol{k}_T d^2\bar{\boldsymbol{k}}_T\ \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) w(\boldsymbol{k}_T, \bar{\boldsymbol{k}}_T, \boldsymbol{R}_T, \bar{\boldsymbol{R}}_T)\ D^a\ D^{\bar{a}}.$$

$$- A(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|\,|\boldsymbol{k}_T|}{M_h^2}\frac{|\bar{\boldsymbol{R}}_T|\,|\bar{\boldsymbol{k}}_T|}{\bar{M}_h^2}\sin(\varphi_k - \varphi_R)\sin(\varphi_{\bar{k}} - \varphi_{\bar{R}}) G_1^{\perp a}\bar{G}_1^{\perp\bar{a}}\right]\Big\}.$$

IFFs in e$^+$e$^-$ and SIDIS.

H.M. , Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

- *The asymmetry now involves **exactly the same** integrated IFF as in SIDIS!*

$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} = \frac{1}{2}\frac{B(y)}{A(y)}\frac{\sum_{a,\bar{a}} e_a^2 H_1^{\sphericalangle a}(z,M_h^2)\bar{H}_1^{\sphericalangle \bar{a}}(\bar{z},\bar{M}_h^2)}{\sum_{a,\bar{a}} e_a^2 D_1^a(z,M_h^2)\bar{D}_1^{\bar{a}}(\bar{z},\bar{M}_h^2)}$$

$$D_1(z,M_h^2) \equiv z^2 \int d^2\boldsymbol{k}_T \int d\xi\, D_1^{[0]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

$$H_{1,e^+e^-}^{\sphericalangle}(z,M_h^2) = H_1^{\sphericalangle,[0]} + H_1^{\perp,[1]} \equiv H_{1,SIDIS}^{\sphericalangle}(z,M_h^2)$$

✦ ***All the previous extractions of the transversity are valid !***

Helicity-dependent DiFF in e^+e^-

H.M., Kotzinian, Thomas: arXiv:1712.06384.

- *The relevant terms involving* $G_1^{\perp}$:

$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T \cdot \boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T \cdot \bar{\boldsymbol{k}}_T)\Big]$$

- ***Note:*** *any azimuthal moment involving only* $\varphi_R,\ \varphi_{\bar{R}}$ *is zero.*

Break-up the convolution: $\int d^2\boldsymbol{q}_T \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T)$

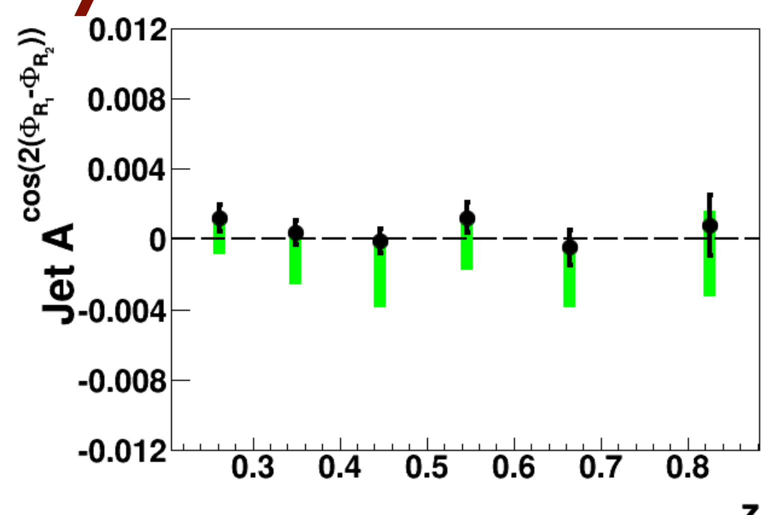
decouple $\boldsymbol{k_T}$ on both sides

Using: $\varphi_k \to \varphi_k' + \varphi_R$, $\int d^2\boldsymbol{k}_T \sin(\varphi_k)\cos(n\varphi_k) = 0$

$$\boxed{\langle f(\varphi_R, \varphi_{\bar{R}})\rangle_L = 0}$$

- ***The old asymmetry by Boer et. al. exactly vanishes!***
- ***Explains the BELLE results.***

$$\boxed{A^{\Rightarrow} = \frac{\langle\cos(2(\varphi_R - \varphi_{\bar{R}}))\rangle}{\langle 1\rangle} = 0!}$$



New way to access $G_1^{\perp}$ DiFF in e$^+$e$^-$

H.M. , Kotzinian, Thomas: arXiv:1712.06384.

- *The relevant terms involving $G_1^{\perp}$:*

$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T \cdot \boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T \cdot \bar{\boldsymbol{k}}_T)\Big]$$

- ***Need a q_T-weighted asymmetry to get non-zero result***

$$\left\langle \frac{q_T^2\left(3\sin(\varphi_q-\varphi_R)\sin(\varphi_q-\varphi_{\bar{R}})+\cos(\varphi_q-\varphi_R)\cos(\varphi_q-\varphi_{\bar{R}})\right)}{M_h \bar{M}_h}\right\rangle$$

$$=\frac{12\alpha^2 A(y)}{\pi Q^2}\sum_{a,\bar{a}} e_a^2\left(G_1^{\perp a,[0]}-G_1^{\perp a,[2]}\right)\left(\bar{G}_1^{\perp \bar{a},[0]}-G_1^{\perp \bar{a},[2]}\right),$$

- ***A new asymmetry to access*** $G_1^{\perp a} \equiv G_1^{\perp a,[0]} - G_1^{\perp a,[2]}$

$$\boxed{A^{\Rightarrow}_{e^+e^-}(z,\bar{z},M_h^2,\bar{M}_h^2)=4\frac{\sum_{a,\bar{a}} G_1^{\perp a}(z,M_h^2)\ G_1^{\perp \bar{a}}(\bar{z},\bar{M}_h^2)}{\sum_{a,\bar{a}} D_1^{a}(z,M_h^2)\ D_1^{\bar{a}}(\bar{z},\bar{M}_h^2)}}$$

New way to access $G_1^\perp$ DiFF in e$^+$e$^-$

H.M., Kotzinian, Thomas: arXiv:1712.06384.

- *The relevant terms involving* $G_1^\perp$ *:*

$$d\sigma_L \sim \mathcal{F}\left[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T\cdot\boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T\cdot\bar{\boldsymbol{k}}_T)\right]$$

- ***Need a q_T-weighted asymmetry to get non-zero result***

additional $\sin(\varphi_k - \varphi_R)$

$$\left\langle \frac{q_T^2\left(3\sin(\varphi_q-\varphi_R)\sin(\varphi_q-\varphi_{\bar{R}})+\cos(\varphi_q-\varphi_R)\cos(\varphi_q-\varphi_{\bar{R}})\right)}{M_h\bar{M}_h}\right\rangle$$

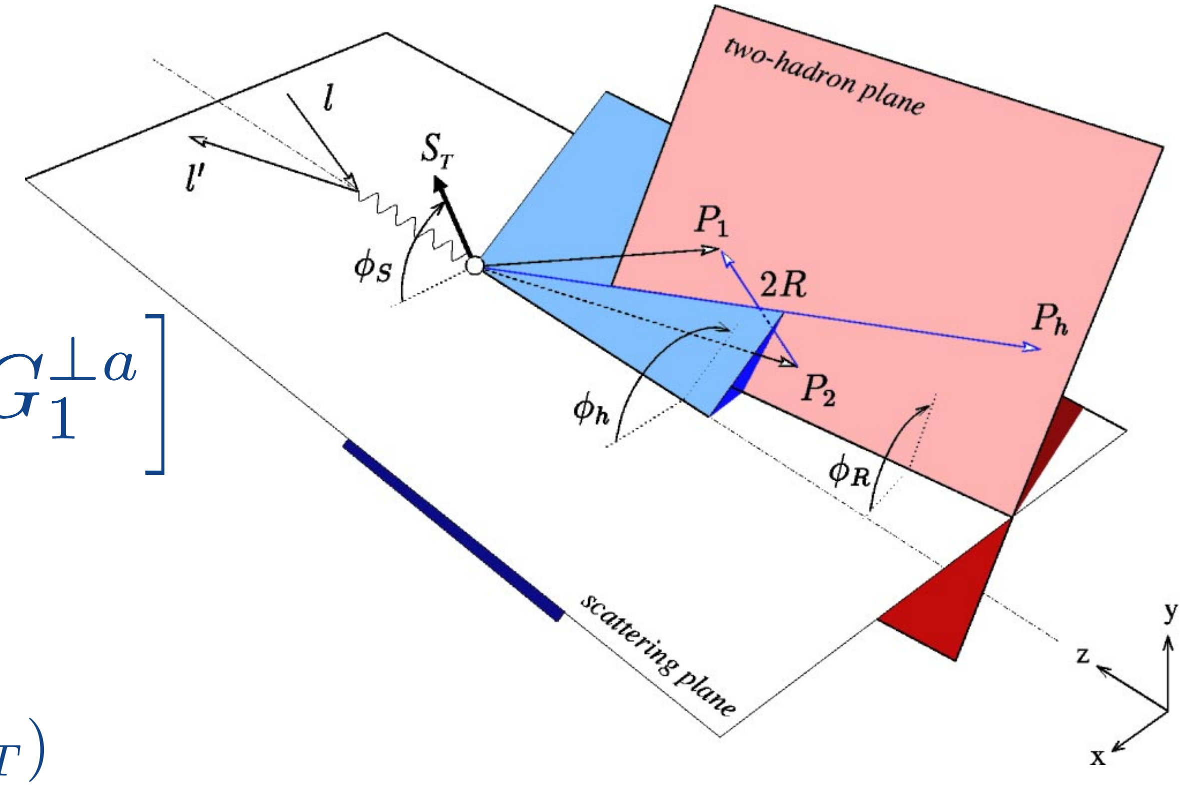
$$= \frac{12\alpha^2 A(y)}{\pi Q^2}\sum_{a,\bar{a}} e_a^2\left(G_1^{\perp a,[0]} - G_1^{\perp a,[2]}\right)\left(\bar{G}_1^{\perp \bar{a},[0]} - G_1^{\perp \bar{a},[2]}\right),$$

- ***A new asymmetry to access*** $G_1^{\perp a} \equiv G_1^{\perp a,[0]} - G_1^{\perp a,[2]}$

$$\boxed{A_{e^+e^-}^{\Rightarrow}(z,\bar{z},M_h^2,\bar{M}_h^2) = 4\frac{\sum_{a,\bar{a}} G_1^{\perp a}(z,M_h^2)\; G_1^{\perp \bar{a}}(\bar{z},\bar{M}_h^2)}{\sum_{a,\bar{a}} D_1^{a}(z,M_h^2)\; D_1^{\bar{a}}(\bar{z},\bar{M}_h^2)}}$$

New way to access $G_1^{\perp}$ DiFF in *SIDIS*

H.M., Kotzinian, Thomas: arXiv:1712.06384.



- *The relevant terms involving* $G_1^{\perp}$:

$$d\sigma_{UL} \sim S_L \mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\ g_{1L}^a\ G_1^{\perp a}\Big]$$

$$\mathcal{G}[w f^q D^q] \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T \delta^2\Big(\boldsymbol{k}_T - \boldsymbol{p}_T + \frac{\boldsymbol{P}_{h\perp}}{z}\Big)$$
$$\times\, w(\boldsymbol{p}_T, \boldsymbol{k}_T, \boldsymbol{R}_T) f^q(x, \boldsymbol{p}_T^2) D^q(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

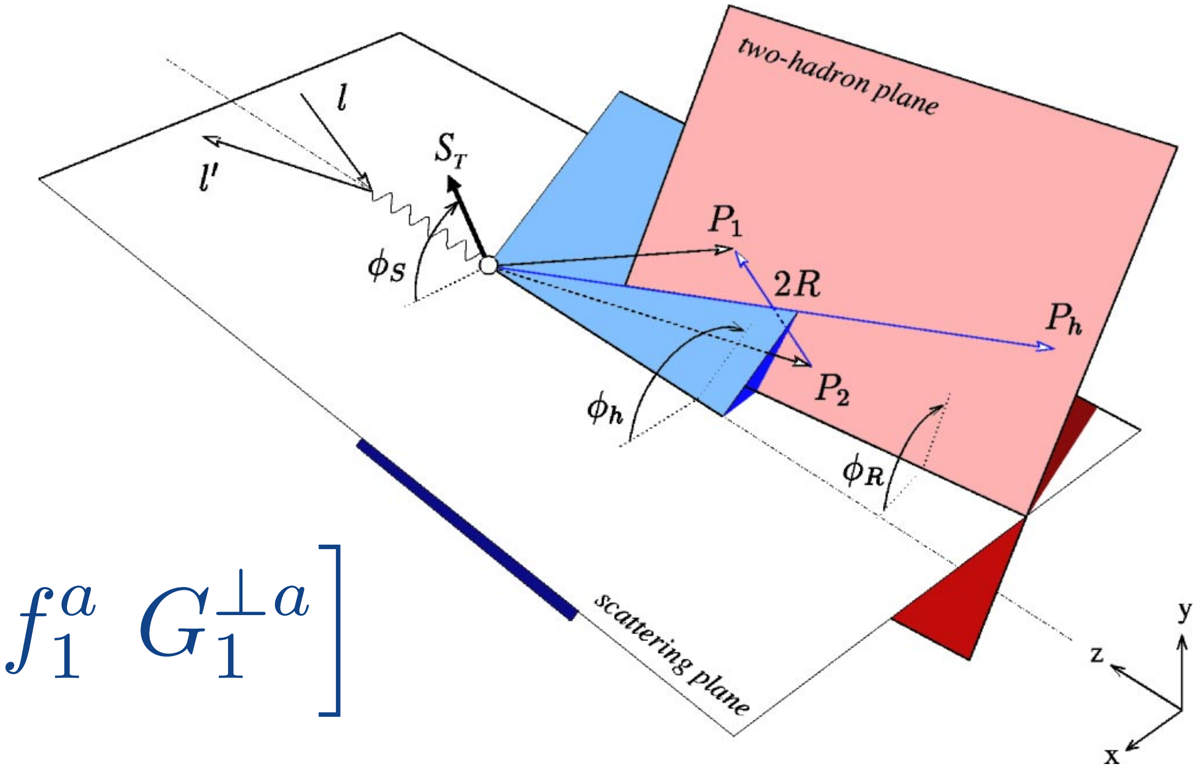
- *Weighted moment accesses same* $G_1^{\perp}$ *as in* **e^+e^-** .

$$\Big\langle \frac{P_{h\perp}\sin(\varphi_h - \varphi_R)}{M_h} \Big\rangle_{UL} \sim S_L \sum_a e_a^2\ g_{1L}^a(x)\ z\ G_1^{\perp a}(z, M_h^2)$$

$$\boxed{A_{SIDIS}^{\rightrightarrows}(x, z, M_h^2) = S_L \frac{\sum_a g_{1L}^a(x)\ z\ G_1^{\perp a}(z, M_h^2)}{\sum_a f_1^a(x)\ D_1^a(z, M_h^2)}.}$$

New way to access $G_1^{\perp}$ DiFF in *SIDIS: II*

- ***The relevant terms involving*** $G_1^{\perp}$ **:**



Consider a polarized beam.

$$d\sigma_{LU} \sim \lambda_e \mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2} \; f_1^a \; G_1^{\perp a}\Big]$$

- ***Weighted moment accesses same*** $G_1^{\perp}$ ***as in*** **e^+e^- .**

$$\left\langle \frac{P_{h\perp} \sin(\varphi_h - \varphi_R)}{M_h} \right\rangle_{LU} \sim \lambda_e \sum_a e_a^2 \; f_1^a(x) \; z \; G_1^{\perp a}(z, M_h^2)$$

$$\boxed{A_{SIDIS}^{\hookrightarrow}(x, z, M_h^2) \sim \lambda_e \frac{C'(y)}{A'(y)} \frac{\sum_a f_1^a(x) \; z \; G_1^{\perp a}(z, M_h^2)}{\sum_a f_1^a(x) \; D_1^a(z, M_h^2)} .}$$

Helicity DiFFs at COMPASS

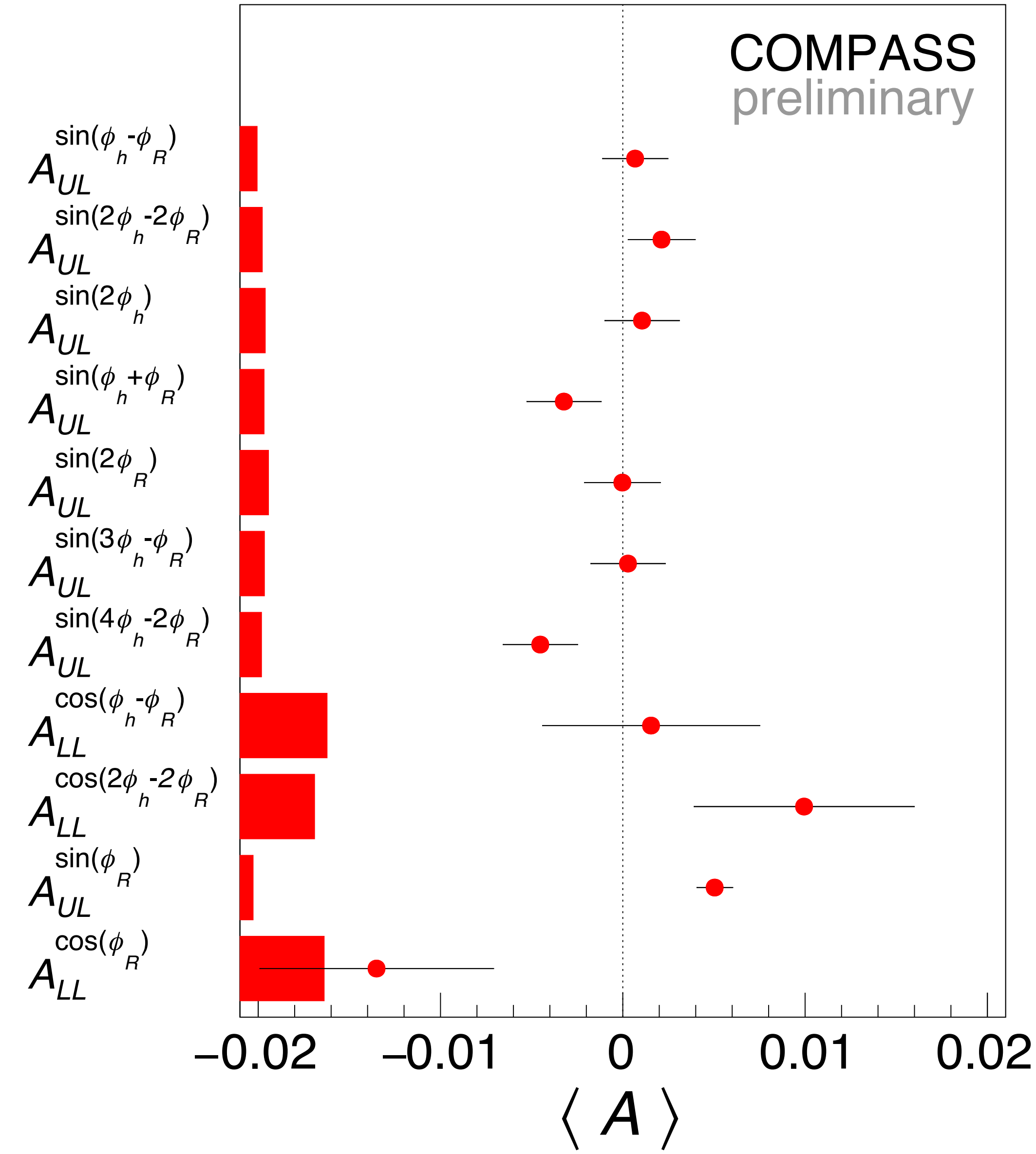
- *SIDIS extraction in COMPASS*

$$d\sigma_{UL} \sim -\,A(y)\mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k-\varphi_R)}{M_h^2}\; g_{1L}^a \; G_1^{\perp a}\Big]$$

$$+\,B(y)\mathcal{G}\Big[\frac{p_T k_T \sin(\varphi_p+\varphi_k)}{M M_h}\; h_{1L}^{\perp a} \; H_1^{\perp a}\Big]$$

$$+\,B(y)\mathcal{G}\Big[\frac{p_T R_T \sin(\varphi_p+\varphi_R)}{M M_h} h_{1L}^{\perp a} \; H_1^{\sphericalangle a}\Big]$$

$$\mathcal{G}[w f^q D^q] \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T \delta^2\Big(\boldsymbol{k}_T - \boldsymbol{p}_T + \frac{\boldsymbol{P}_{h\perp}}{z}\Big)$$
$$\times\, w(\boldsymbol{p}_T, \boldsymbol{k}_T, \boldsymbol{R}_T) f^q(x, \boldsymbol{p}_T^2) D^q(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$



- ✦ $A^{\sin(n(\varphi_h-\varphi_R))}$ **are <u>*convolutions*</u> of** g_{1L} **and** $G_1^{\perp}$ **!**
  - **Low <x> = 0.05 !**
  - **Limited statistics.**

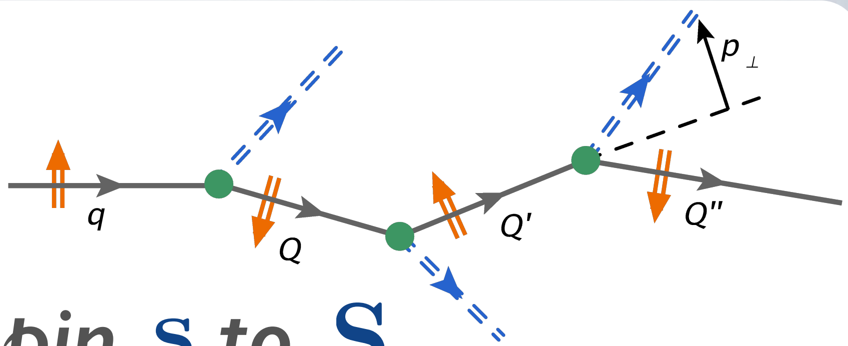
MODELLING

DIHADRON FRAGMENTATION FUNCTIONS

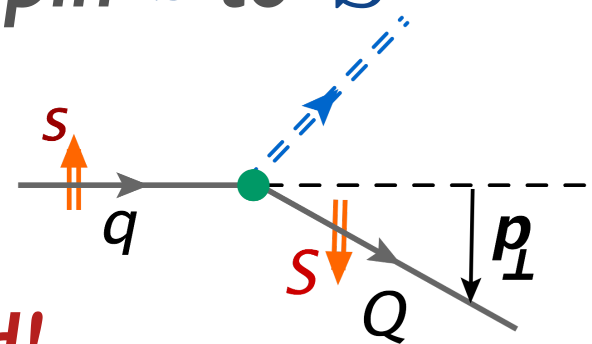
POLARIZATION IN QUARK-JET FRAMEWORK

✦ Extended quark-jet:

Bentz, Kotzinian, ***H.M***, Ninomiya, Thomas, Yazaki: PRD 94 034004 (2016).



- *The probability for the process* $q \to Q$, *initial spin* $\mathbf{s}$ *to* $\mathbf{S}$



$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) = \alpha_\mathbf{s} + \boldsymbol{\beta}_\mathbf{s} \cdot \mathbf{S}$$

- ***Intermediate*** *quarks in quark-jet are* ***unobserved!***

We need the induced final state spin $\mathbf{S}'$.

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) \sim \mathrm{Tr}[\rho^{\mathbf{S}'}\rho^{\mathbf{S}}] \sim 1 + \mathbf{S}' \cdot \mathbf{S}$$

- *Remnant quark's* $\mathbf{S}'$ ***uniquely determined by*** $z, \mathbf{p}_\perp$ ***and*** $\mathbf{s}$ ***!***

$$\boxed{\mathbf{S}' = \frac{\boldsymbol{\beta}_\mathbf{s}}{\alpha_\mathbf{s}}}$$

- *Process probability is* ***the same*** *as transition to* ***unpolarized state.***

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{0}) = \alpha_s$$

REMNANT QUARK'S POLARISATION

✦ We can express the spin of the remnant quark $\mathbf{S}' = \dfrac{\beta_{\mathbf{s}}}{\alpha_{\mathbf{s}}}$ in terms of *quark-to-quark TMD splittings*.

$$\alpha_q \equiv D(z, \boldsymbol{p}_\perp^2) + (\boldsymbol{p}_\perp \times \boldsymbol{s}_T) \cdot \hat{\boldsymbol{z}} \frac{1}{z\mathcal{M}} H^\perp(z, \boldsymbol{p}_\perp^2)$$

$$\beta_{q\|} \equiv s_L\, G_L(z, \boldsymbol{p}_\perp^2) - (\boldsymbol{p}_\perp \cdot \boldsymbol{s}_T) \frac{1}{z\mathcal{M}} H_L^\perp(z, \boldsymbol{p}_\perp^2)$$

$$\boldsymbol{\beta}_{q\perp} \equiv \boldsymbol{p}'_\perp \frac{1}{z\,\mathcal{M}} D_T^\perp(z, \boldsymbol{p}_\perp^2) - \boldsymbol{p}_\perp \frac{1}{z\mathcal{M}} s_L G_T(z, \boldsymbol{p}_\perp^2) + \boldsymbol{s}_T\, H_T(z, \boldsymbol{p}_\perp^2) + \boldsymbol{p}_\perp (\boldsymbol{p}_\perp \cdot \boldsymbol{s}_T) \frac{1}{z^2\mathcal{M}^2} H_T^\perp(z, \boldsymbol{p}_\perp^2)$$

✦ Input Elementary Splitting Functions: *Model or Parametrization!*

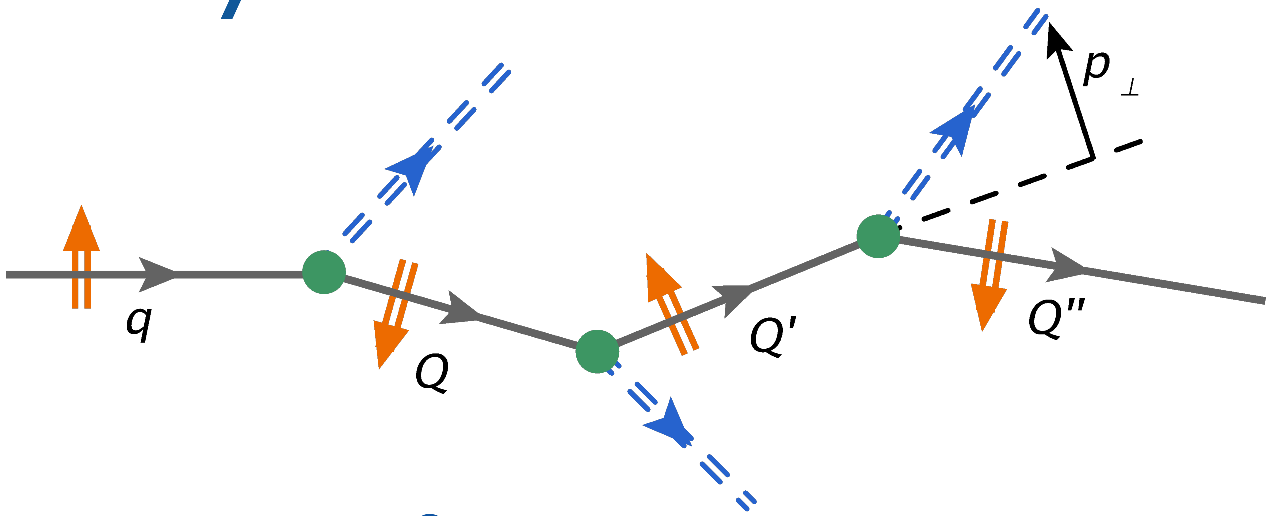
$$F^{q\to Q}(z, \boldsymbol{p}_\perp; \boldsymbol{s}, \boldsymbol{S})$$

Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

MC SIMULATION OF FULL HADRONIZATION

H.M., Kotzinian, Thomas: Phys. Rev. D95 04021, (2017)

✦ We can consider many hadron emissions.



✦ We can sample the $h, z, p_\perp^2, \varphi_h$ using

$$\boxed{f^{q\to h}(z, p_\perp^2, \varphi_h; \mathbf{S}_T)}$$

✦ Determine the momenta in the initial frame and calculate

$$\Delta N = \langle N_q^{h_1 h_2}(z, z+\Delta z, \varphi, \varphi+\Delta\varphi, ...)\rangle$$

✦ *Calculate the remnant quark's spin:* $\mathrm{S}' = \dfrac{\boldsymbol{\beta}_\mathbf{s}}{\alpha_\mathbf{s}}$

✦ We only need the "elementary" splittings.

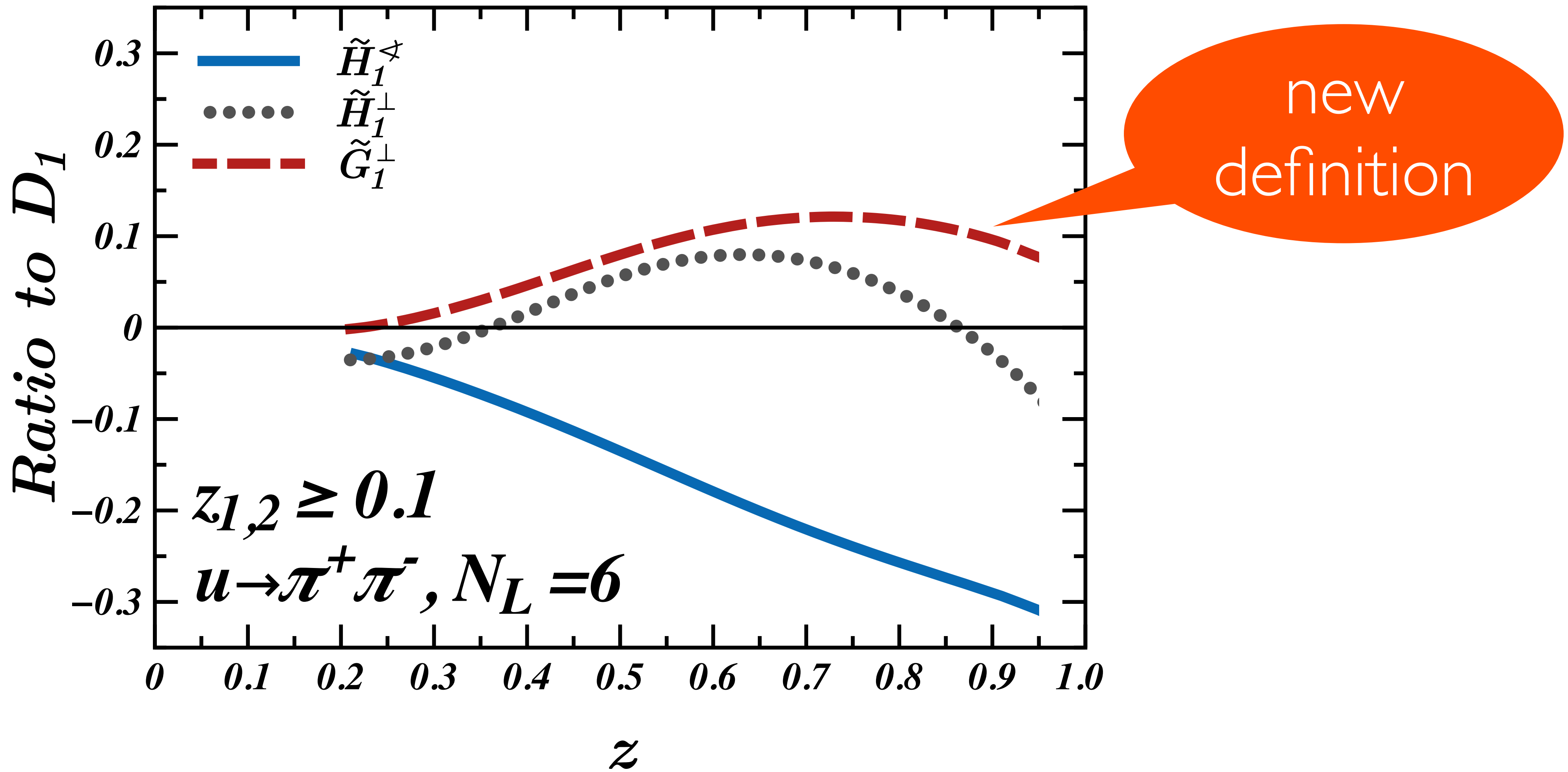
$$\boxed{f^{q\to h}} \qquad \boxed{f^{q\to Q}}$$

Feasibility of new measurements of $G_1^{\perp}$

✦ **The analysing powers of DiFFs from quark-jet framework.**

Phys. Rev. D96 074010, (2017); Phys. Rev. D97, 014019 (2018).

- $G_1^{\perp}$ *naturally smaller than* $H_1^{\sphericalangle}$*, but* ***should be measurable!***



✦ **Reanalyze BELLE and COMPASS data.**

✦ **Measure it at BELLE II and *JLab 12GeV*.**

CONCLUSIONS

❖ DiFFs provide information on the polarization of the fragmenting quark.

❖ Two problems appeared recently:

- *Inconsistency* of IFF definitions in *SIDIS* and **e^+e^-** asymmetries.
- *No signal* for the helicity-dependent DiFF from BELLE.

❖ *Re-derived* cross section for **e^+e^-** *resolved both issues*.

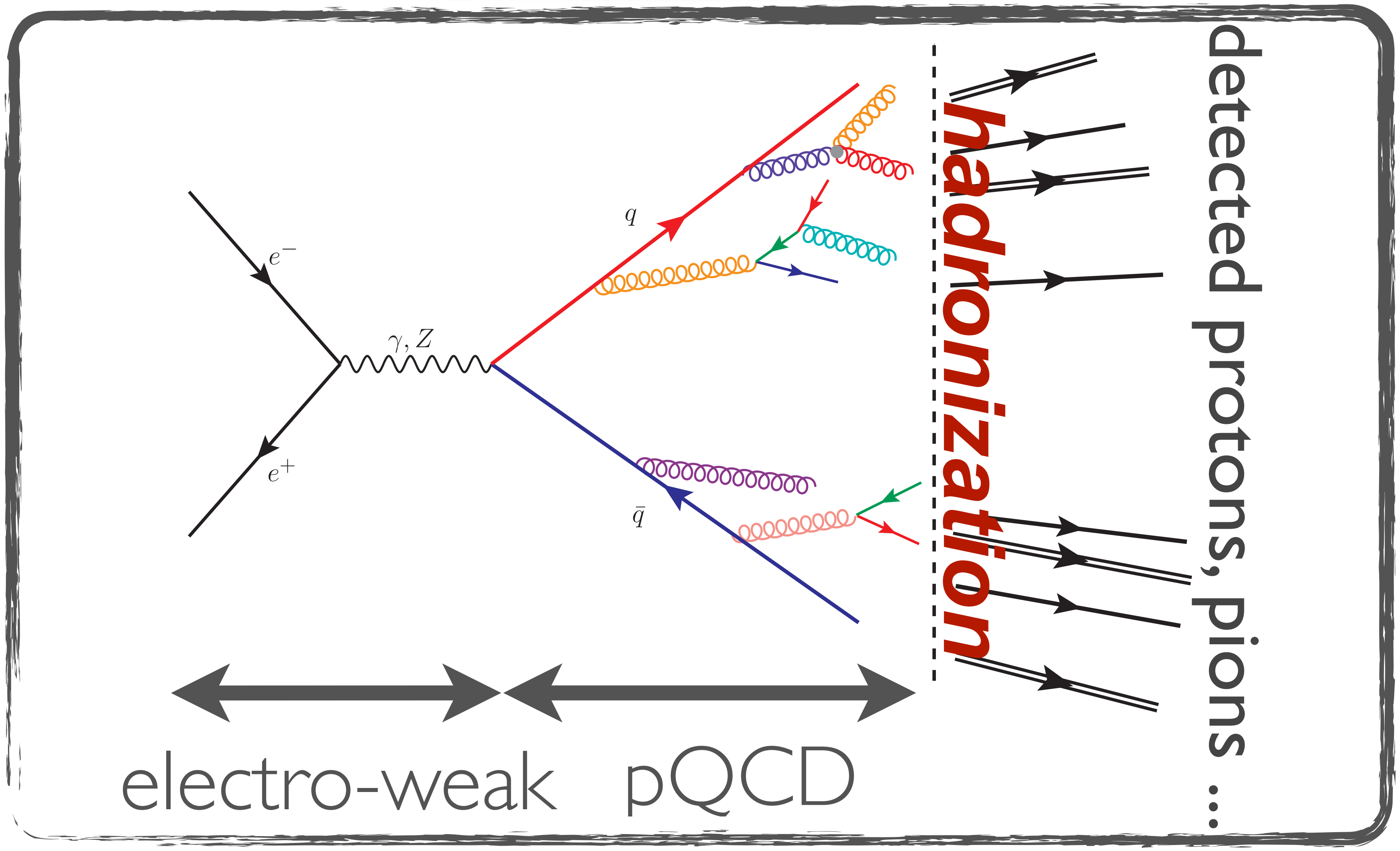
❖ *New asymmetries* to measure $G_1^\perp$ in *SIDIS* and **e^+e^-** : test *Universality*!

BACKUP SLIDES

Hadronization: $e^- e^+ \to hX$

- **The conjecture of *Confinement*:**

✦*NO free quarks or gluons have been directly observed:* *only* **HADRONS.**



✦ **Hadronization**: describes the process where colored quarks and gluons form colourless hadrons (in deep inelastic scattering).

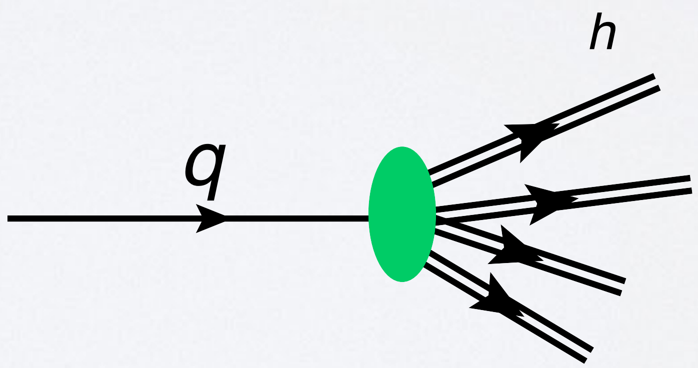
Fragmentation Functions

- The non-perturbative, universal functions encoding parton hadronization are the: ***Fragmentation Functions (FF)***.

$$\boxed{\frac{1}{\sigma}\frac{d}{dz}\sigma(e^-e^+ \to hX) = \sum_i \mathcal{C}_i(z,Q^2) \otimes D_i^h(z,Q^2)}$$

- **Unpolarized FF** is the **number density** for parton *i* to produce hadron *h* with LC momentum fraction **z**.

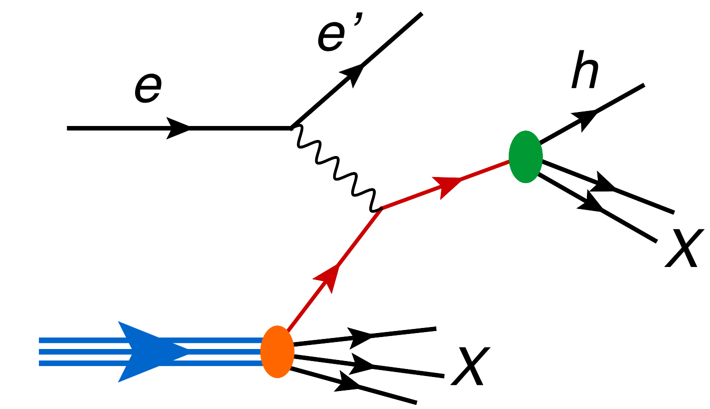
$$\boxed{D_i^h(z,Q^2)}$$



- **z** is the light-cone mom. fraction of the parton carried by the hadron

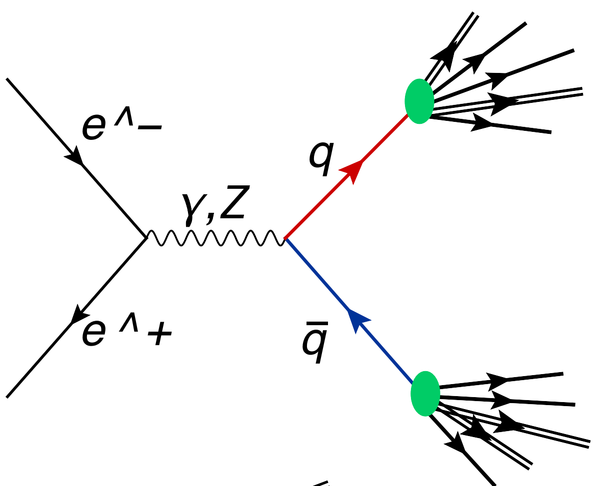
$$z = \frac{p^-}{k^-} \approx z_h = \frac{2E_h}{Q} \qquad a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$$

FACTORIZATION AND *UNIVERSALITY*



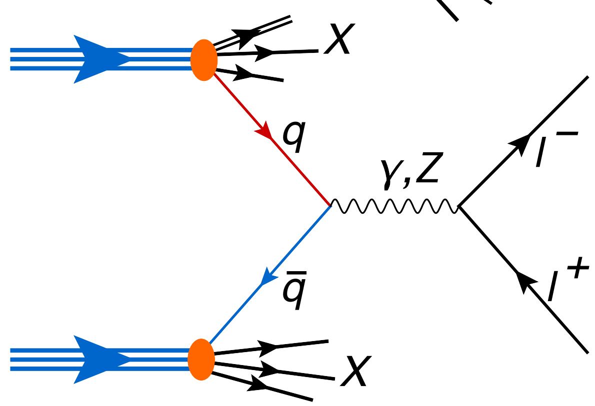
- SEMI INCLUSIVE DIS (SIDIS)

$$\boxed{\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h}$$



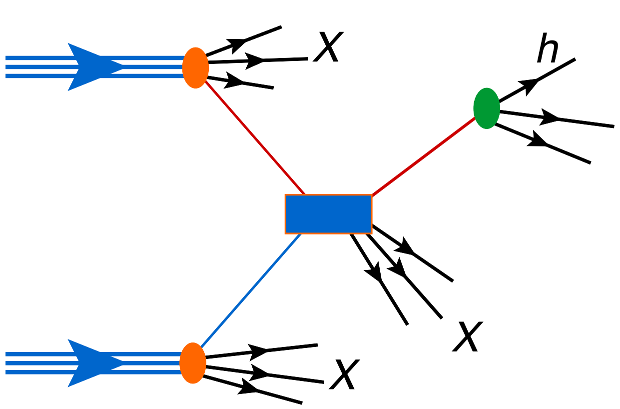
- e^+e^-

$$\boxed{\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)}$$



- DRELL-YAN (DY)

$$\boxed{\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}}$$



- Hadron Production

$$\boxed{\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h}$$

Number Densities

- ***The full number density:***

$$
\begin{aligned}
F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s}) = &\; D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) \\
&+ s_L\,\frac{(\boldsymbol{R}_T\times\boldsymbol{k}_T)\cdot\hat{\boldsymbol{z}}}{M_h^2}\,G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) \\
&+ \frac{(\boldsymbol{s}_T\times\boldsymbol{R}_T)\cdot\hat{\boldsymbol{z}}}{M_h}\,H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) \\
&+ \frac{(\boldsymbol{s}_T\times\boldsymbol{k}_T)\cdot\hat{\boldsymbol{z}}}{M_h}\,H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)
\end{aligned}
$$

- ***The differential number of hadron pairs:***

$$
dN_q^{h_1h_2} = F_q^{h_1h_2}(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s})\;dz\;d\xi\;d^2\boldsymbol{k}_T\;d^2\boldsymbol{R}_T
$$

DIFFS FROM THE NUMBER DENSITY

H.M., Kotzinian, Thomas: Phys. Rev. D96 074010, (2017)

- **Can only calculate number density form MC simulations.**
- **Extract DiFFs from specific angular modulations.**

$$F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L) = D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK})) - s_L\frac{R_T k_T \sin(\varphi_{RK})}{M_h^2} G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK}))$$

- **Unpolarized DiFF:** ***straight forward integration*** **of number density.**

$$\boxed{D_1(z,M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T\ F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L)}$$

- **Need** $\cot(\varphi_{RK})$ **to extract** ***helicity dependent DiFF!***

$$\tilde{G}_1^{\perp,[n]}(z,M_h^2) = \int d\xi \int d^2\boldsymbol{k}_T \frac{R_T k_T}{M_h^2} G_1^{\perp,[n]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2)$$

$$\boxed{\tilde{G}_1^{\perp,[n]}(z,M_h^2) = -\frac{1}{s_L}\int d\xi \int d^2\boldsymbol{k}_T \int d\varphi_R \frac{\cos(n\ \varphi_{RK})}{\sin(\varphi_{RK})} F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T)}$$

Note: here we use the definition by Boer et. al.

$$\tilde{G}_1^{\perp} \equiv \tilde{G}_1^{\perp,[1]} = -\frac{1}{s_L}\int d\xi \int d^2\boldsymbol{k}_T \int d\varphi_R \cot(\varphi_{RK}) F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T)$$

LONGITUDINAL POLARISATION

- **DiFF for longitudinally polarized quark:** $s_L\ (\boldsymbol{k}_T \times \boldsymbol{R}_T)\cdot\hat{z}$

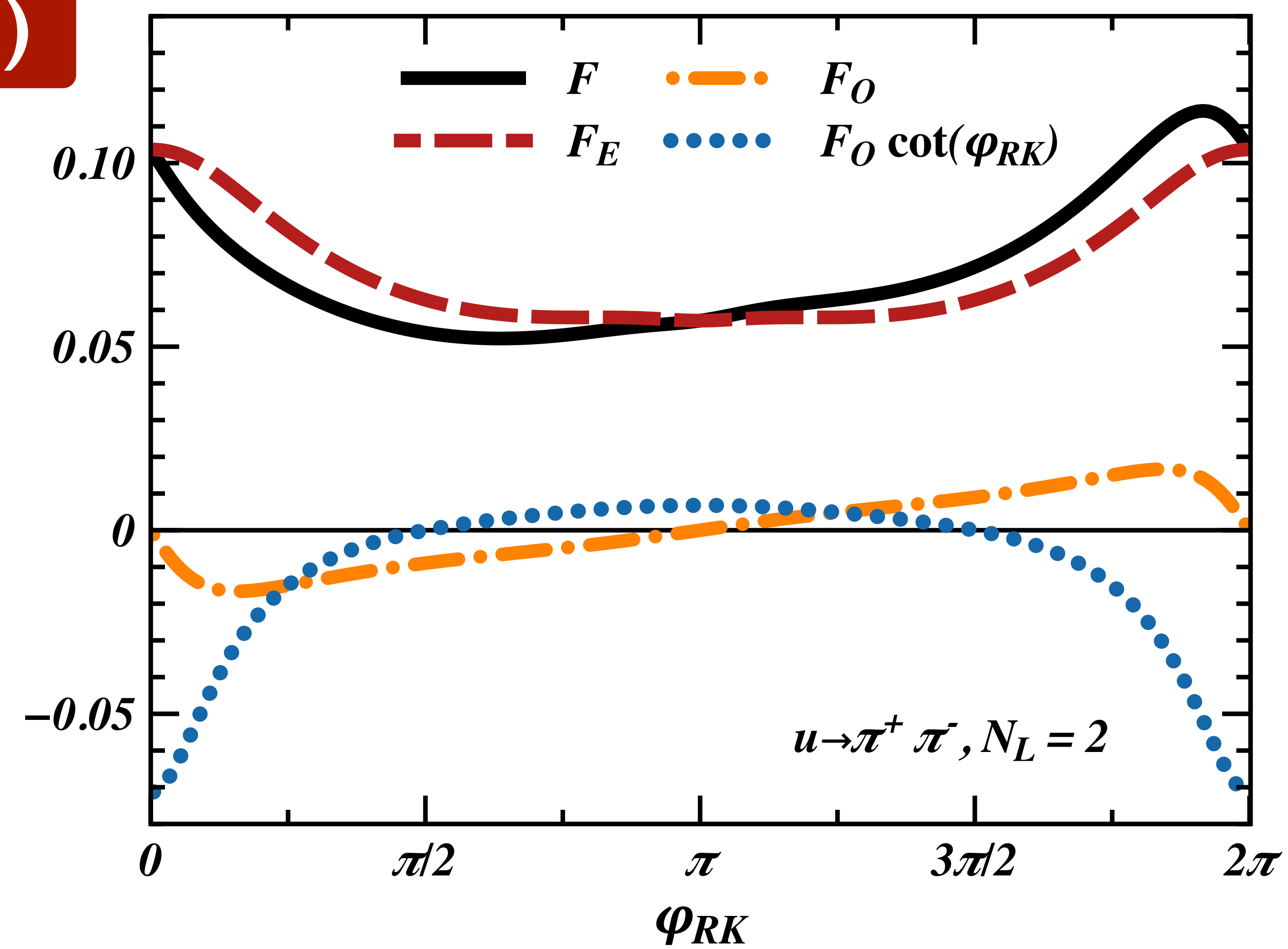
$$\tilde{G}_1^{\perp}(z) = -\frac{1}{s_L}\int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T\ \cot(\varphi_{RK}) F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L).$$

- **The extraction method works: the angular dependence for N$_L$=2.**

(given *large enough* statistics!)

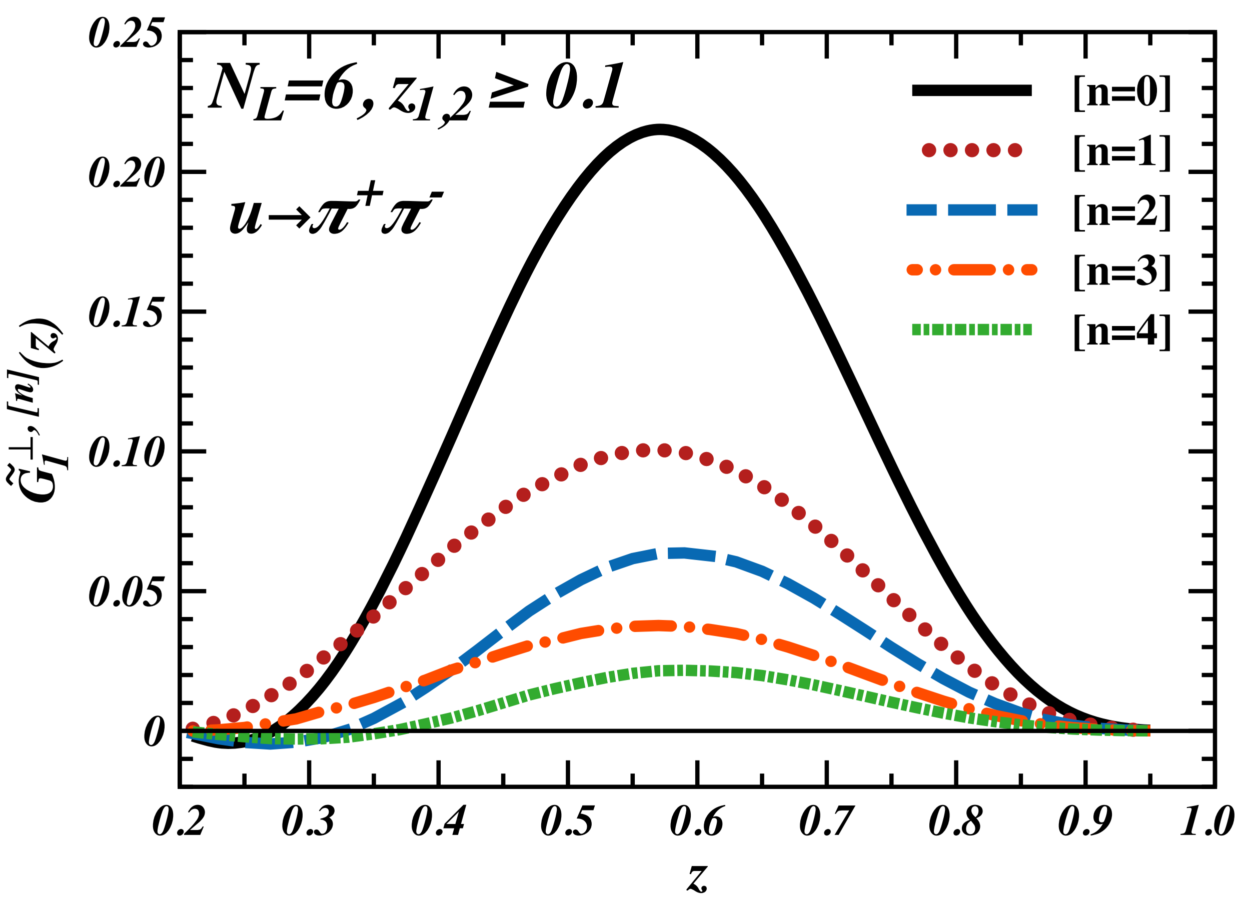
$$F_E(\varphi_{RK}) = \frac{F(\varphi_{RK}) + F(2\pi - \varphi_{RK})}{2}$$

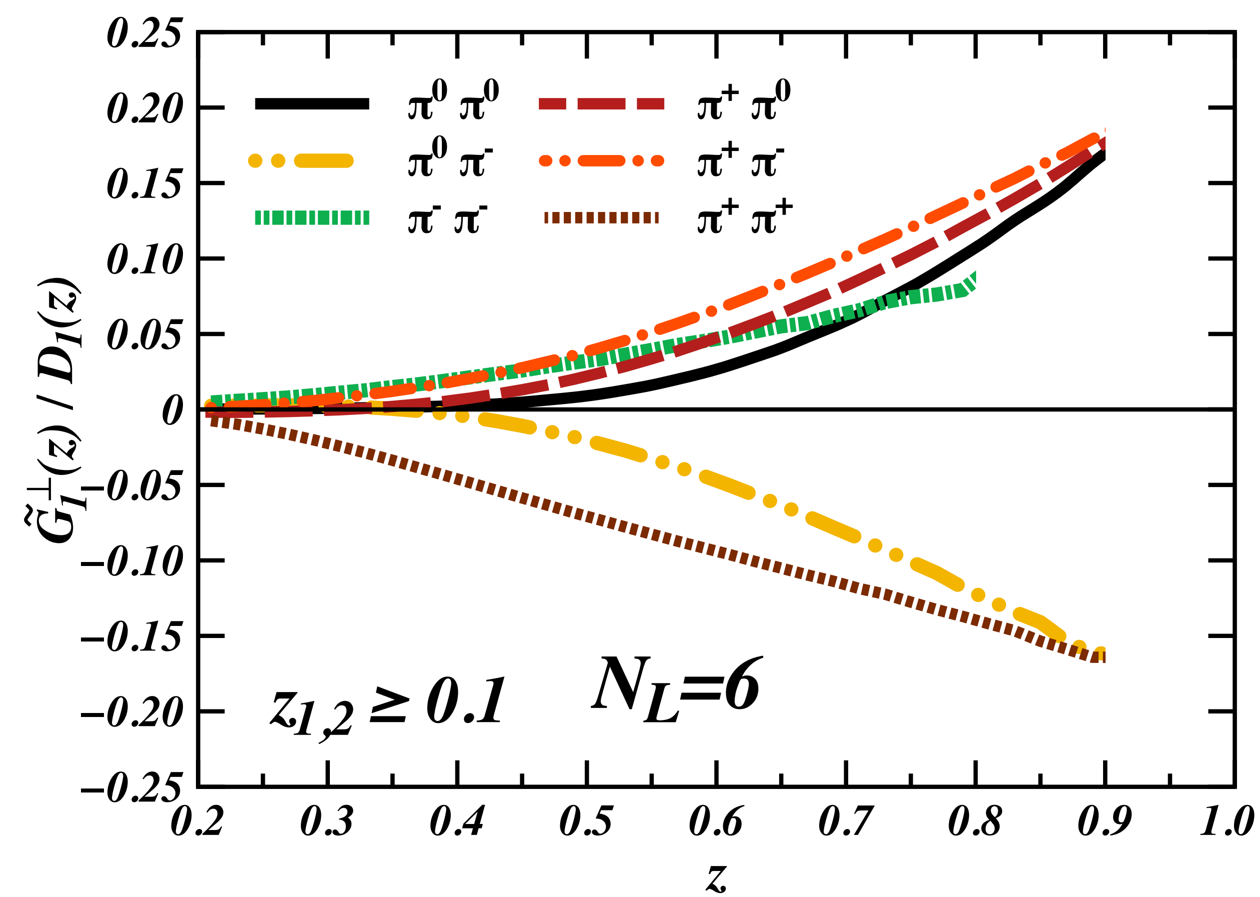
$$F_O(\varphi_{RK}) = \frac{F(\varphi_{RK}) - F(2\pi - \varphi_{RK})}{2}$$



Results for $G_1^\perp$

✦ Results for helicity DiFFs, ***several*** moments, various pairs. Cuts: $z_{1,2} \geq 0.1$





✦ Non-zero signal for various channels, ***sign change for*** $\pi^+\pi^+$ ***pairs***!

✦ $z_{1,2} \geq 0.1$ ***cut enhances the analysing power at high-z for larger N$_L$***!

TRANSVERSELY POL. DIFFS FROM NUMBER DENSITY

H.M., Kotzinian, Thomas, Phys. Rev. D 97, 014019 (2018).

✦ **Slightly more complicated procedure:**

$$F(\varphi_R,\varphi_k;\boldsymbol{s}_T) = D_1(\cos(\varphi_{RK})) + a_R \sin(\varphi_R - \varphi_s) H_1^{\sphericalangle}(\cos(\varphi_{RK})) + a_K \sin(\varphi_k - \varphi_s) H_1^{\perp}(\cos(\varphi_{RK}))$$

✦ **n-th moment of DiFFs:**

$$H_1^{\sphericalangle,[n]} = \frac{2}{s_T}\Big\langle \cos(\varphi_k - \varphi_s)\frac{\cos(n\cdot\varphi_{RK})}{\sin(\varphi_{RK})}F\Big\rangle$$

$$H_1^{\perp,[n]} = -\frac{2}{s_T}\Big\langle \cos(\varphi_R - \varphi_s)\frac{\cos(n\cdot\varphi_{RK})}{\sin(\varphi_{RK})}F\Big\rangle$$

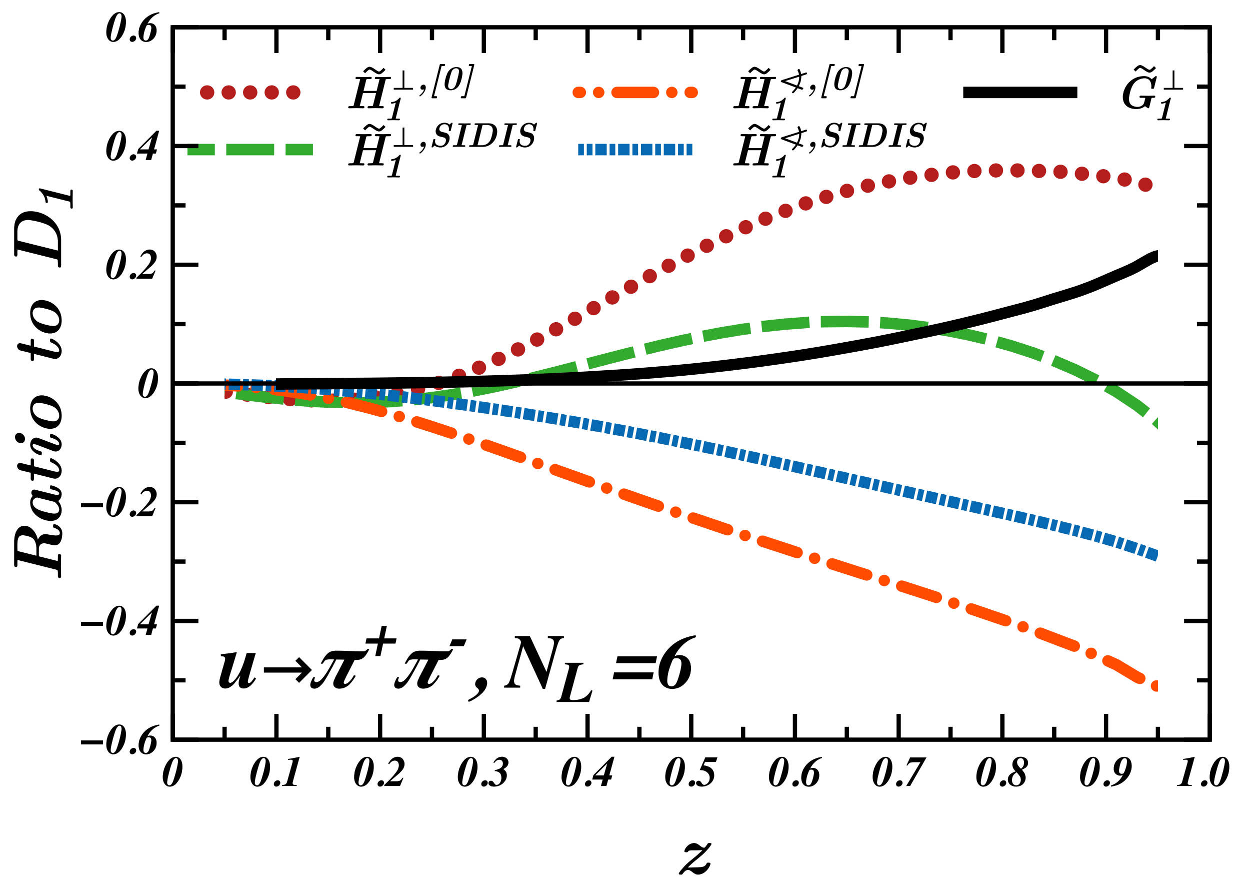
✦ **SIDIS DiFFs:**

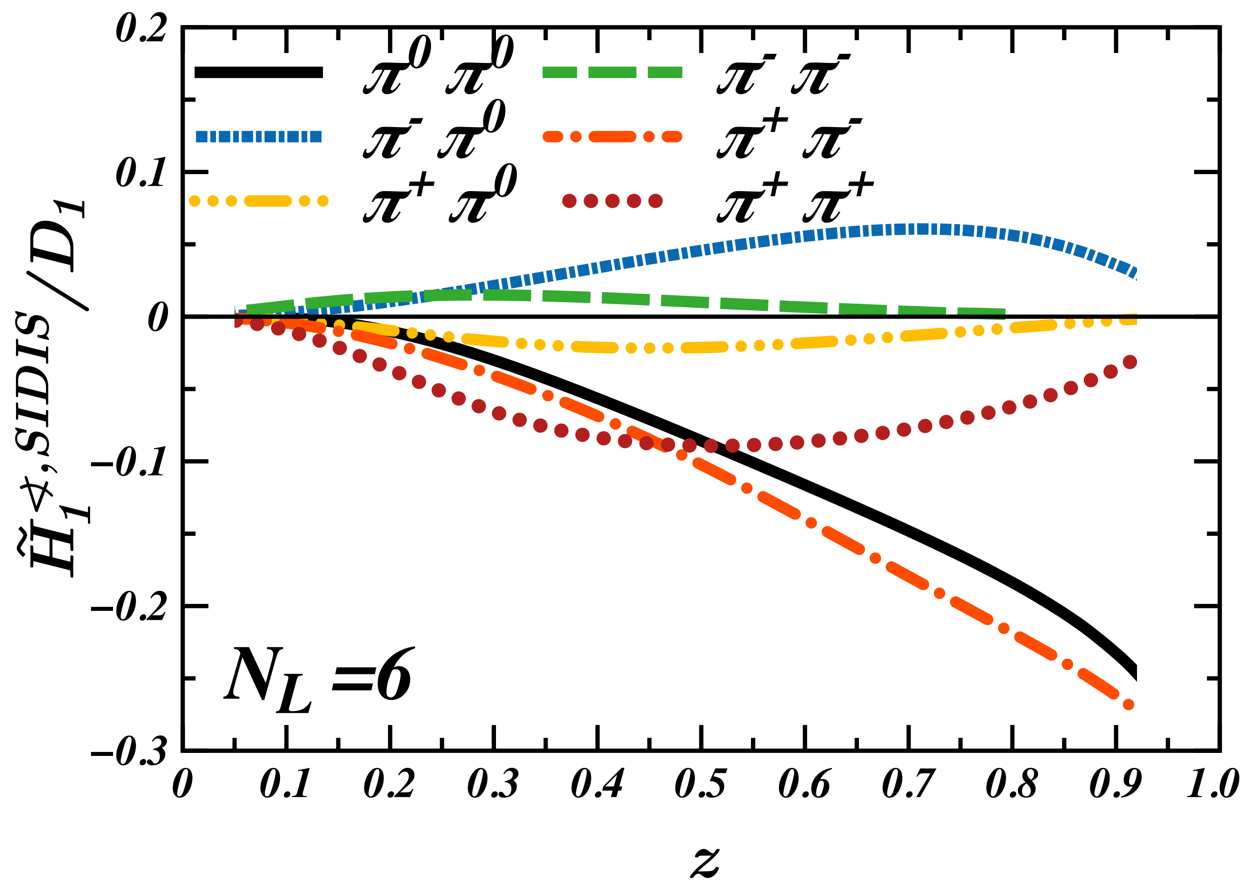
$$H_1^{\sphericalangle,SIDIS}(z) = \frac{2}{s_T}\Big\langle \sin(\varphi_R - \varphi_s)F\Big\rangle$$

$$H_1^{\perp,SIDIS}(z) = \frac{2}{s_T}\Big\langle \sin(\varphi_k - \varphi_s)F\Big\rangle$$

Analysing Power for Transverse Spin

✦ Comparing the analysing powers for all polarized DiFFs.





✦ Alternate signs for the two DiFFs.

✦ Significant differences between SIDIS and 0-th moments!

✦ *Signals for all possible hadron pairs.*