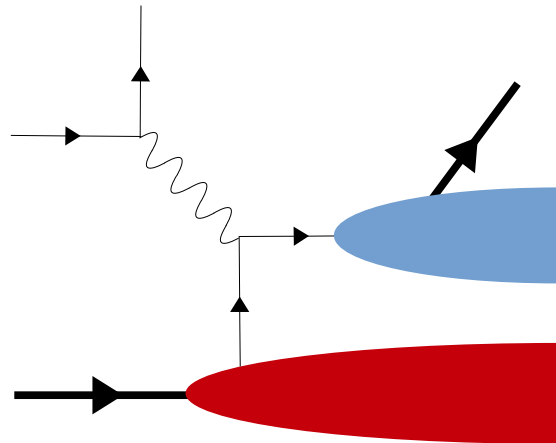


SIDIS Phenomenology

J. Osvaldo Gonzalez-Hernandez

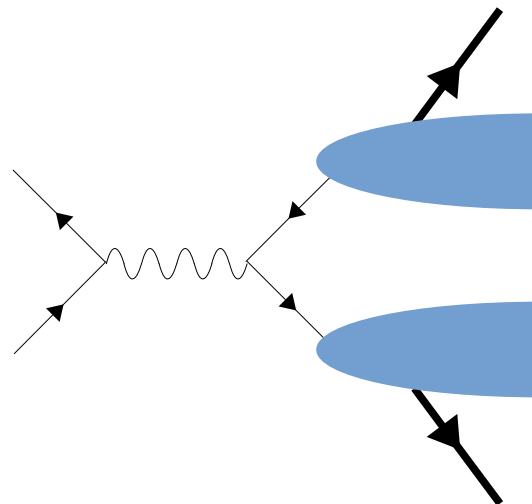
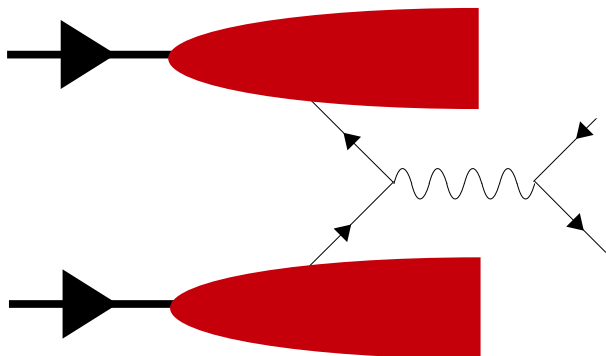
University of Turin



SIDIS

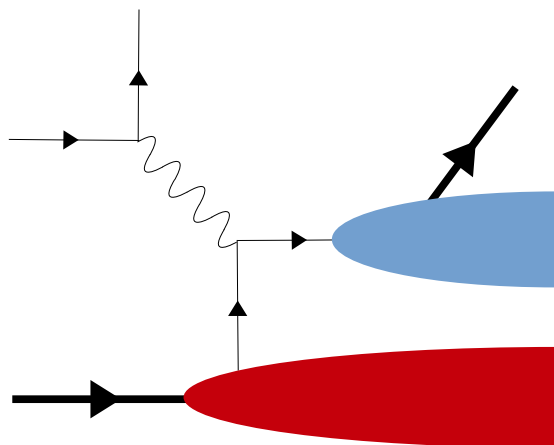
Phenomenology

Drell Yan



e^+e^-

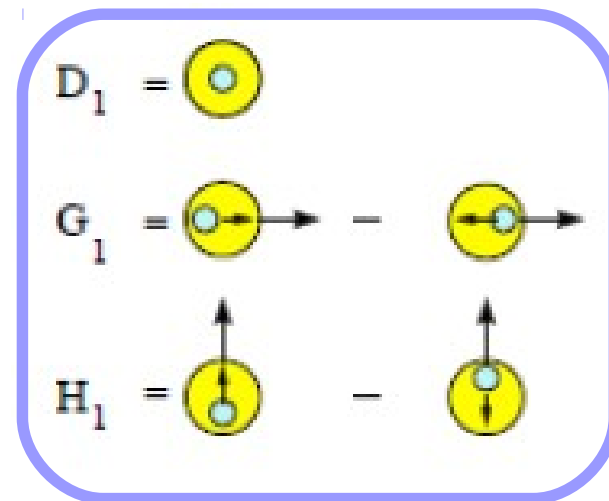
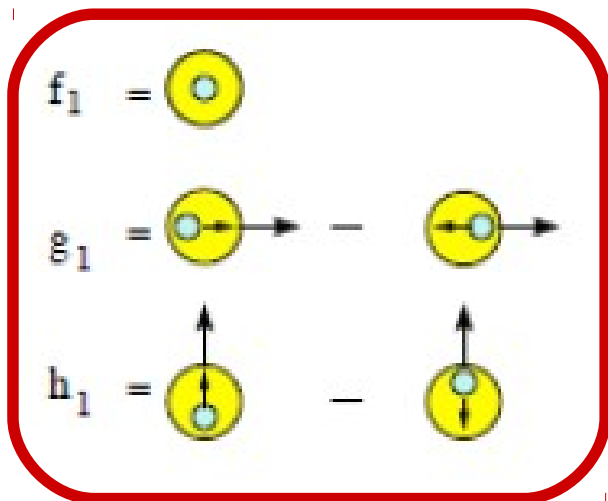
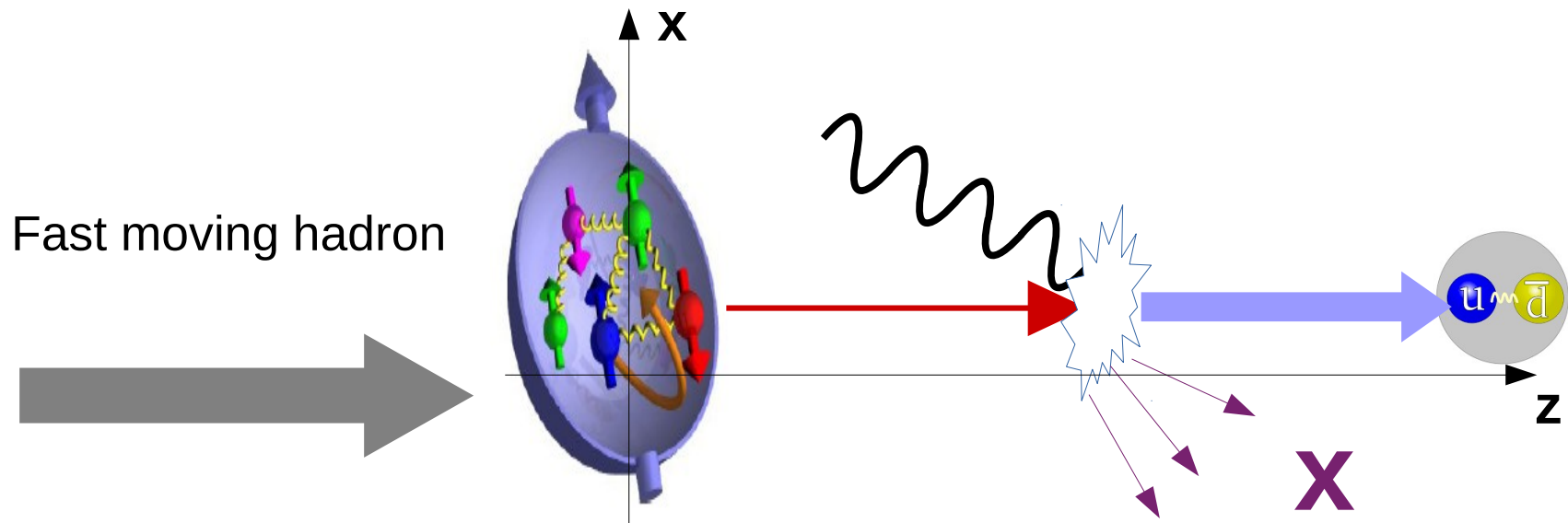
PDFs



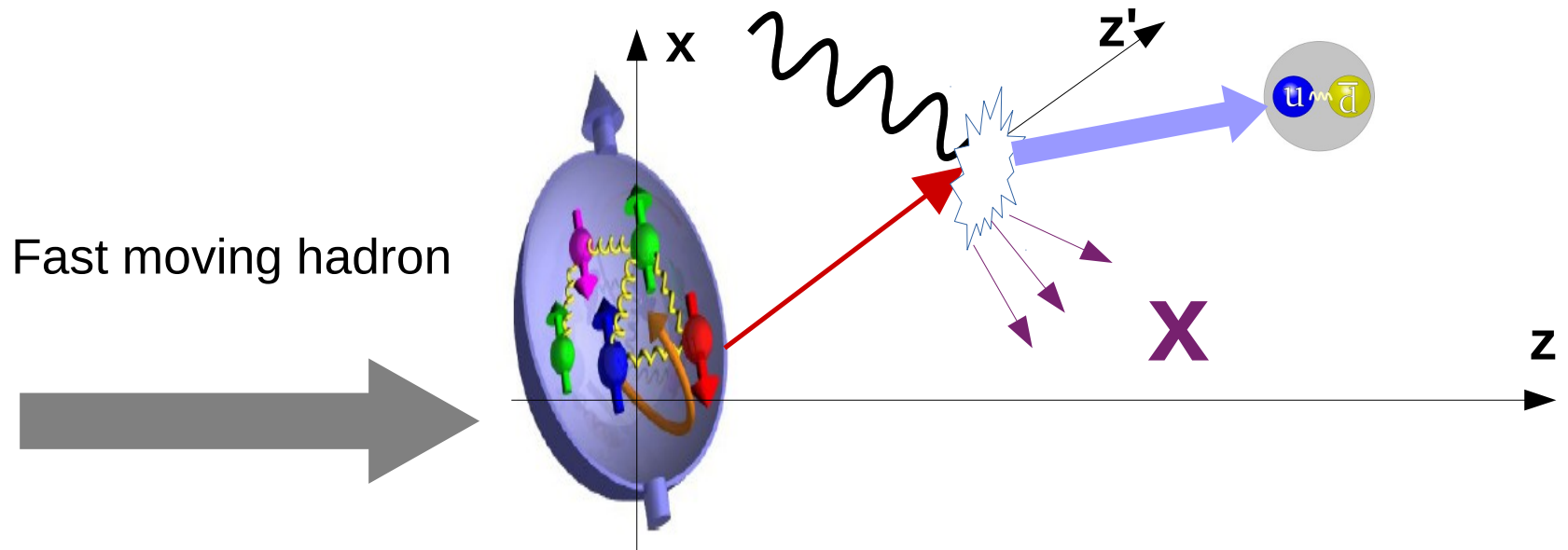
Fragmentation Functions

SIDIS

SIDIS: Collinear case



SIDIS: TMD case



Mulders & Tangerman, NPB 461 (1996) 197

Distribution Functions		Fragmentation Functions	
$f_1 = \odot$		$D_1 = \odot$	
$g_1 = \odot \rightarrow - \odot \rightarrow$	$g_{1T} = \odot \uparrow - \odot \uparrow$	$G_1 = \odot \rightarrow - \odot \rightarrow$	$G_{1T} = \odot \uparrow - \odot \uparrow$
$h_1 = \odot \uparrow - \odot \downarrow$		$H_1 = \odot \uparrow - \odot \downarrow$	
$f_{1T}^\perp = \odot \uparrow - \odot \downarrow$		$D_{1T}^\perp = \odot \uparrow - \odot \downarrow$	
$h_1^\perp = \odot \uparrow - \odot \downarrow$		$H_1^\perp = \odot \uparrow - \odot \downarrow$	
$h_{1L}^\perp = \odot \rightarrow - \odot \rightarrow$	$h_{1T}^\perp = \odot \uparrow - \odot \uparrow$	$H_{1L}^\perp = \odot \rightarrow - \odot \rightarrow$	$H_{1T}^\perp = \odot \uparrow - \odot \uparrow$

Ultimately, we want to extract TMDs

Fourier Transform of:

$$\begin{aligned} \tilde{F}_j(x, b_T, Q, \zeta_F) &= \left(\frac{\sqrt{\zeta_F}}{\mu_b} \right)^{\tilde{K}(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{in}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b) \\ &\times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left(\gamma_F(\mu; 1) - \ln \left(\frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\} \\ &\times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\}, \end{aligned}$$

Unpolarized TMD PDF

$$\begin{aligned} \tilde{D}_j(z, b_T, Q, \zeta_D) &= \left(\frac{\sqrt{\zeta_D}}{\mu_b} \right)^{\tilde{K}(b_*, \mu_b)} \sum_k \int_z^1 \frac{d\hat{z}}{\hat{z}^3} \tilde{C}_{kj}^{out}(z/\hat{z}, b_*, \mu_b, \mu_b^2) D_j(\hat{z}, \mu_b) \\ &\times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left(\gamma_D(\mu; 1) - \ln \left(\frac{\sqrt{\zeta_D}}{\mu} \right) \gamma_K(\mu) \right) \right\} \\ &\times \exp \left\{ -g_H(z, b_T) - g_K(b_T) \ln \left(\frac{\sqrt{\zeta_D}}{\sqrt{\zeta_{D0}}} \right) \right\}. \end{aligned}$$

Unpolarized TMD FF

Ultimately, we want to extract TMDs

Often times, simple models are considered (no-evolution)

$$f_{q/p}(x, k_{\perp}) = f_{q/p}(x) \frac{e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$
$$D_{h/q}(z, p_{\perp}) = D_{h/q}(z) \frac{e^{-p_{\perp}^2 / \langle p_{\perp}^2 \rangle}}{\pi \langle p_{\perp}^2 \rangle} .$$

Useful to map the shape of TMDs and to quantify kinematical dependencies on data. (but careful, limited validity)

May be seen as a “snapshot” of the a TMD at some kinematics.

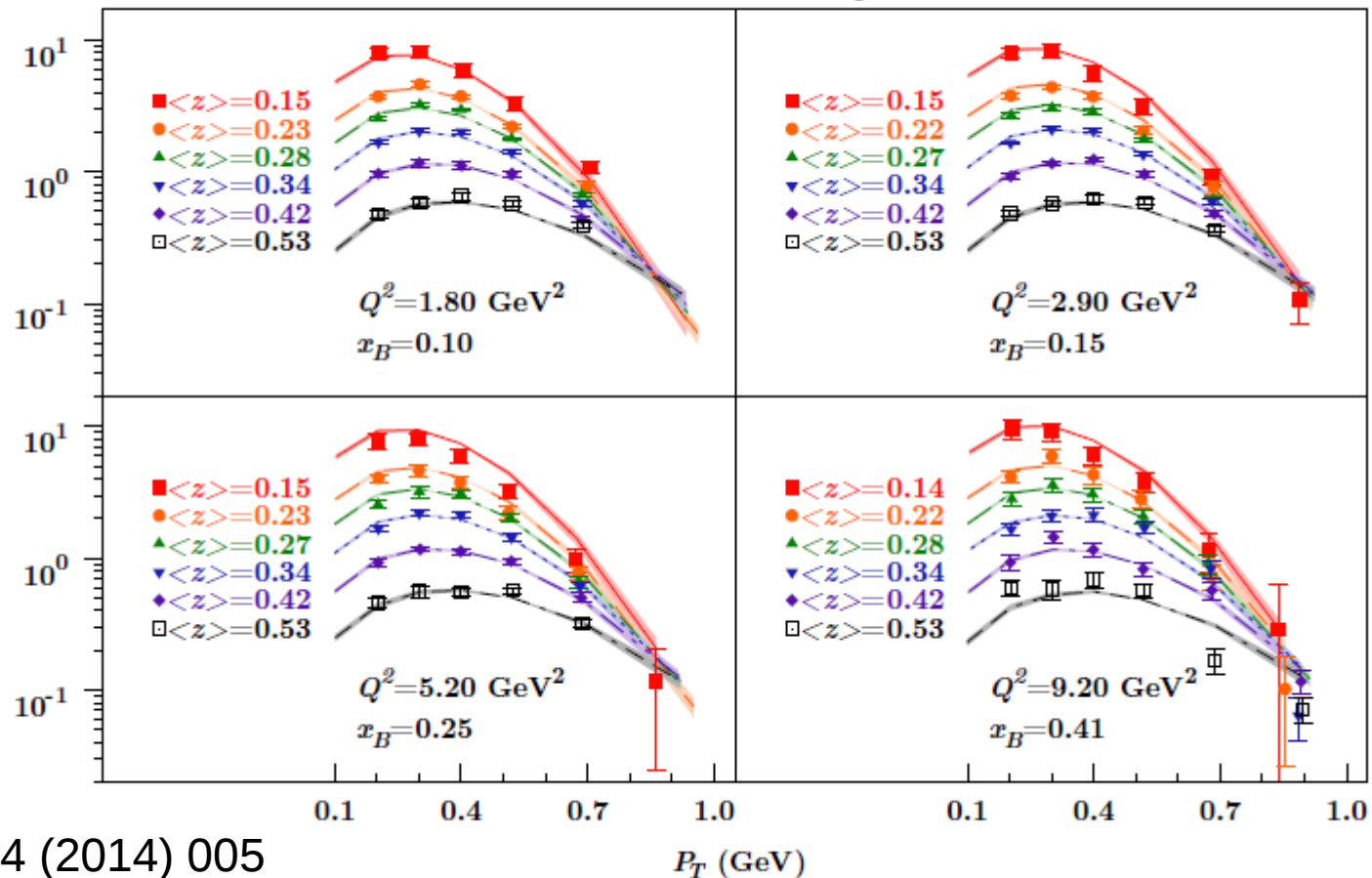
Example of simple model analysis

$$f_{q/p}(x, k_{\perp}) = f_{q/p}(x) \frac{e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$

$$D_{h/q}(z, p_{\perp}) = D_{h/q}(z) \frac{e^{-p_{\perp}^2 / \langle p_{\perp}^2 \rangle}}{\pi \langle p_{\perp}^2 \rangle} .$$

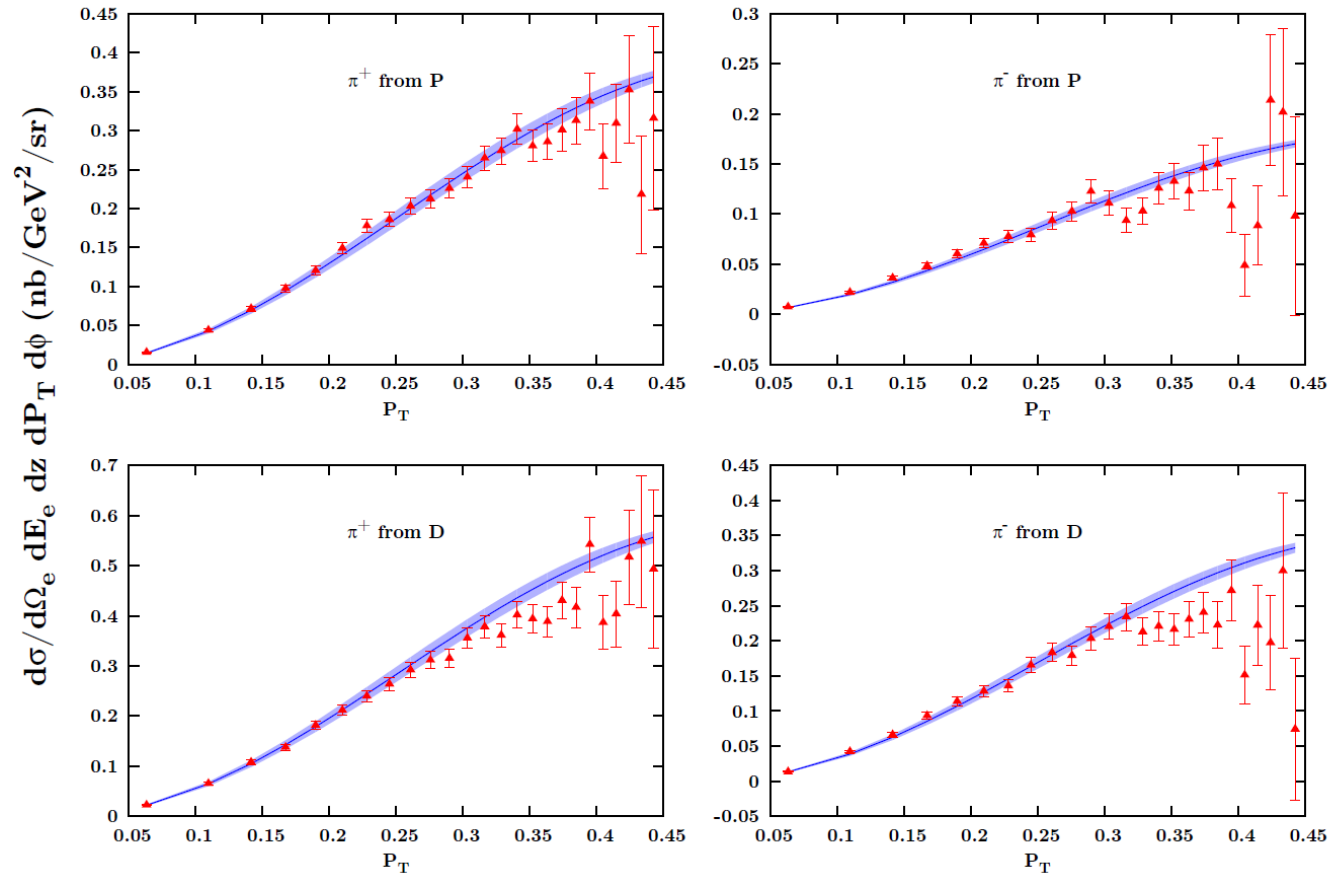
$$\langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle .$$

HERMES $M_p^{\pi^+}$



JHEP 1404 (2014) 005

Anselmino, Boglione, Melis, JOGH, Prokudin

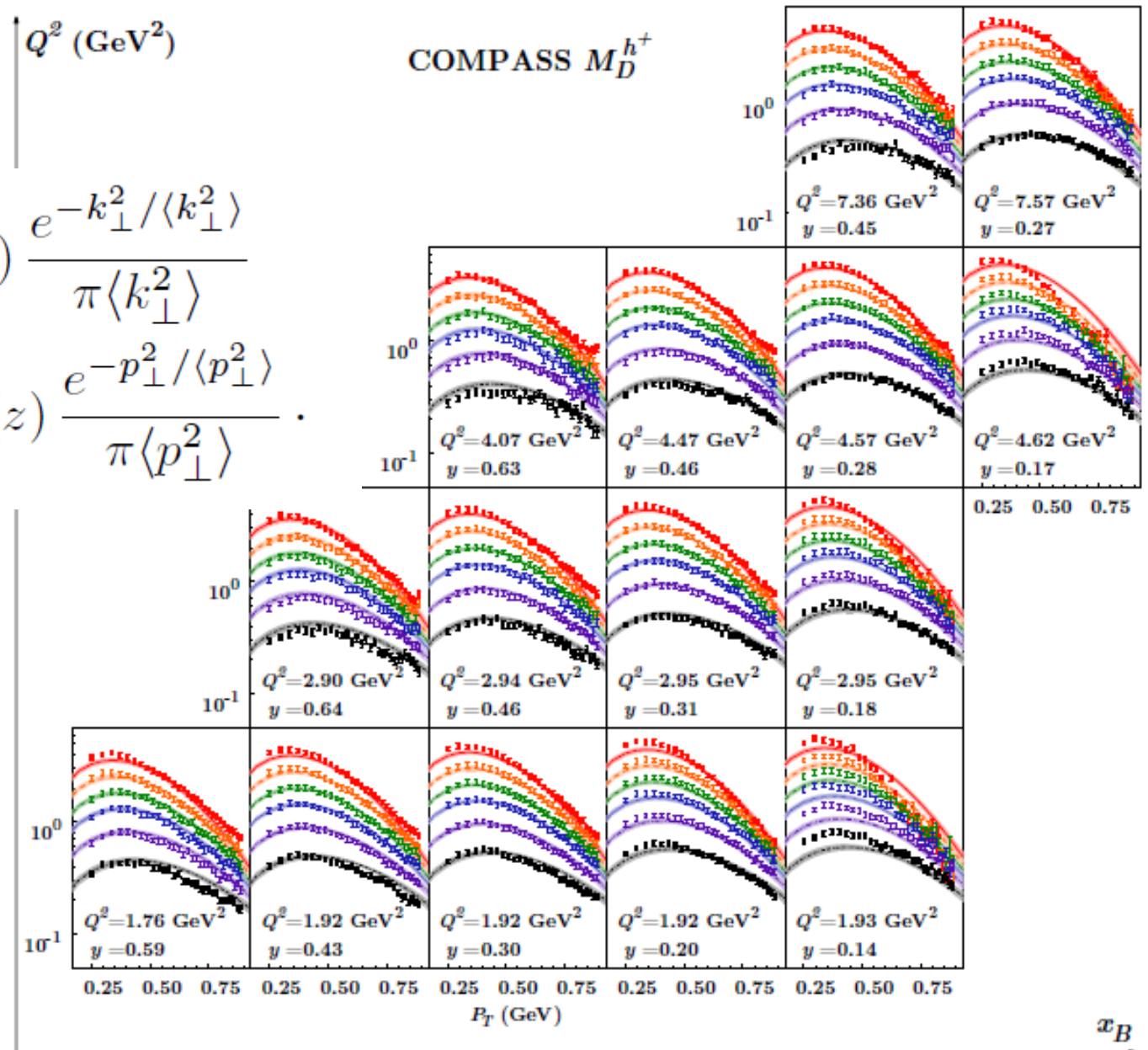


Constant widths: Comparison to Jlab data

Simple model

$$f_{q/p}(x, k_{\perp}) = f_{q/p}(x) \frac{e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$

$$D_{h/q}(z, p_{\perp}) = D_{h/q}(z) \frac{e^{-p_{\perp}^2 / \langle p_{\perp}^2 \rangle}}{\pi \langle p_{\perp}^2 \rangle}.$$

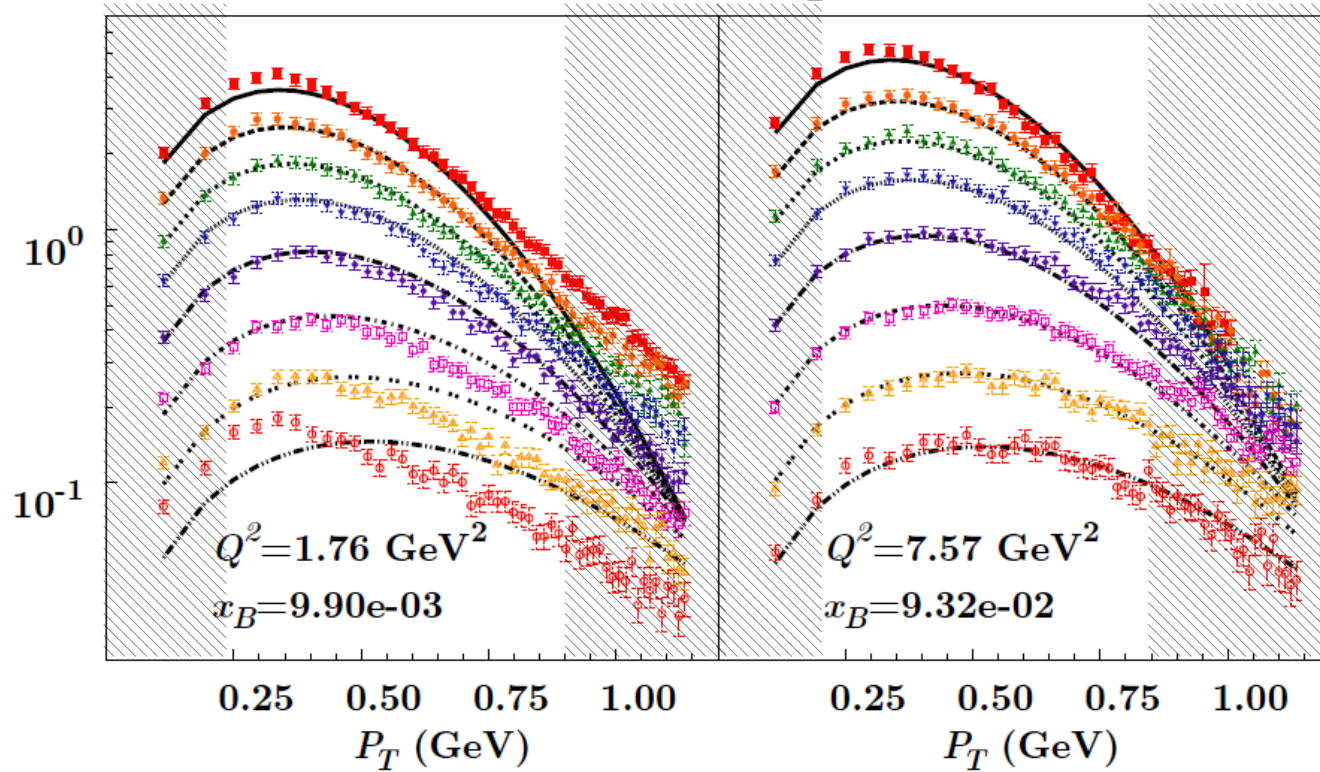


JHEP 1404 (2014) 005

Anselmino, Boglione, Melis, JOGH, Prokudin

COMPASS $M_D^{h^+}$

- $\langle z \rangle = 0.23$
- $\langle z \rangle = 0.28$
- ▲ $\langle z \rangle = 0.33$
- ▼ $\langle z \rangle = 0.38$
- ◆ $\langle z \rangle = 0.45$
- $\langle z \rangle = 0.55$
- △ $\langle z \rangle = 0.65$
- $\langle z \rangle = 0.75$

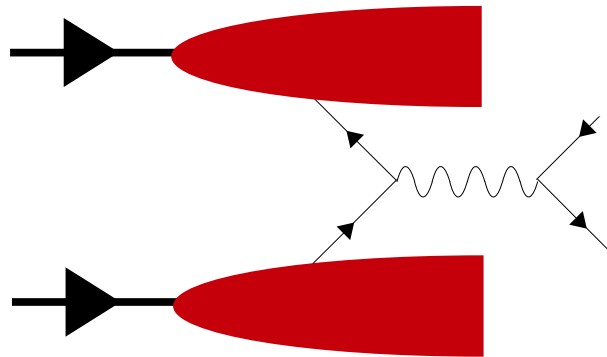


Need to go beyond simple model picture

What about the full (actual) definitions of the TMDs?

Some very interesting implementations in the market.

Drell Yan

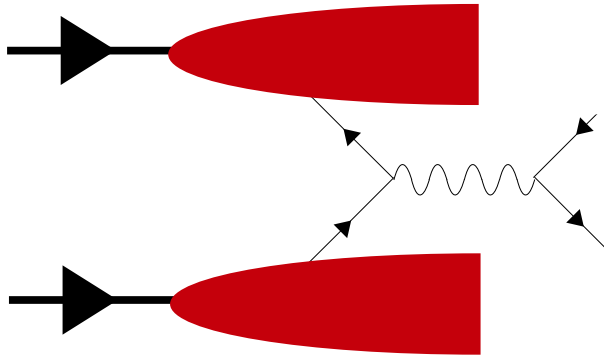


**Under control, high
precision phenomenology:**

See a recent example:
Eur.Phys.J. C78 (2018) no.2, 89
Ignazio Scimemi, Alexey Vladimirov

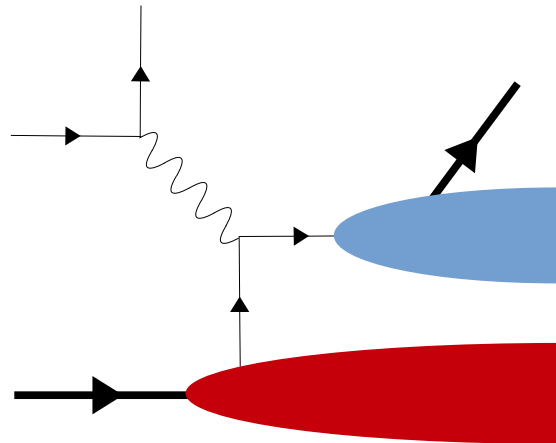
Global Fits? First attempts

Drell Yan



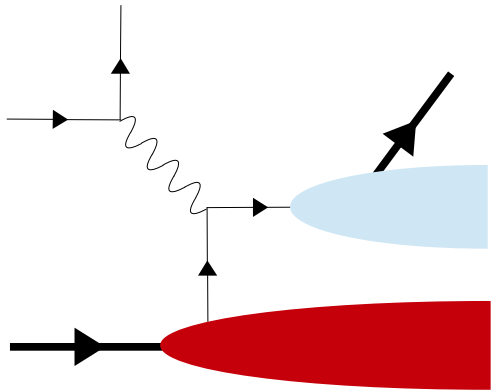
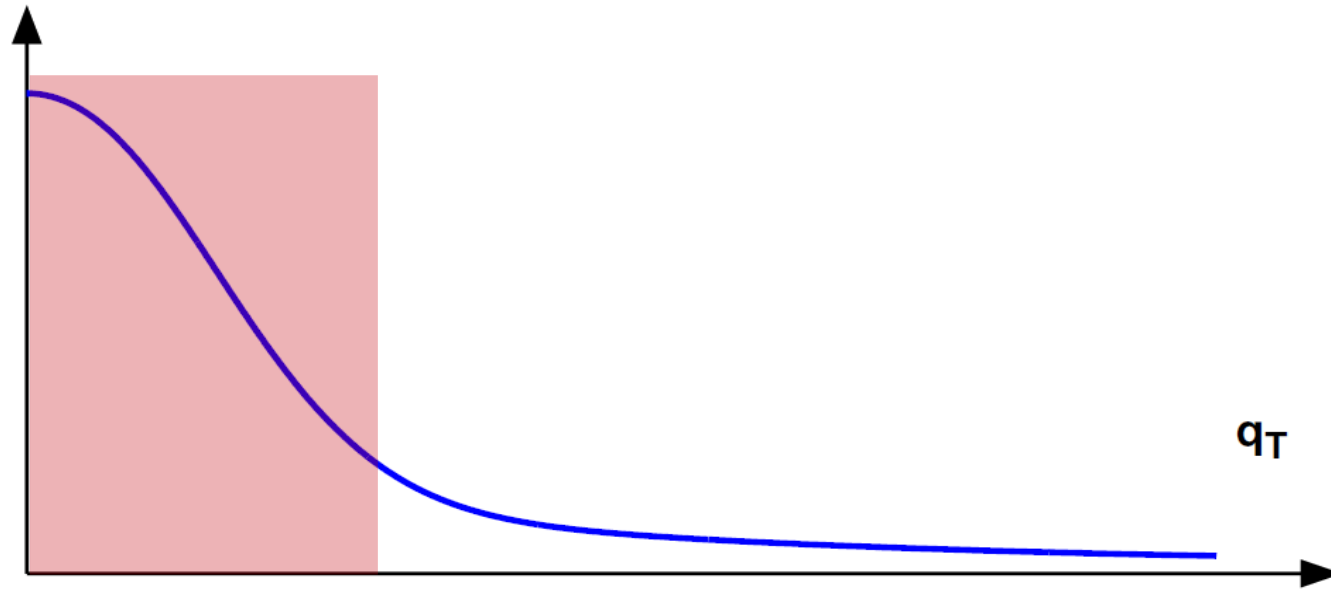
A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
JHEP 1706 (2017) 081

PDFs

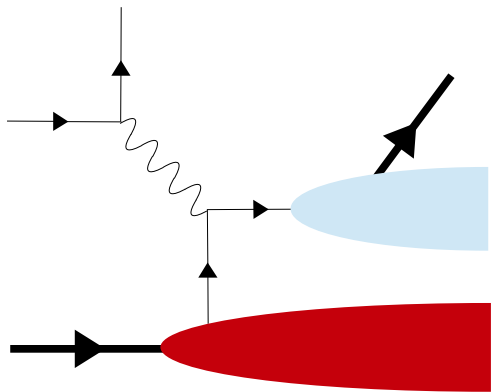
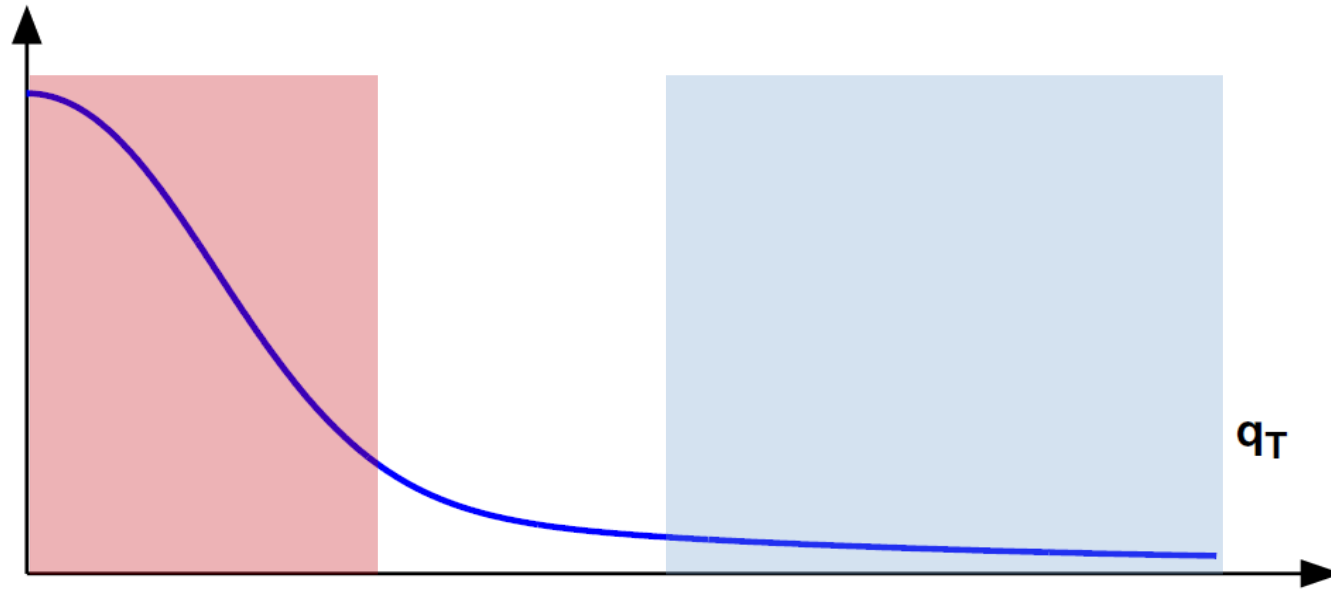


Fragmentation
Functions

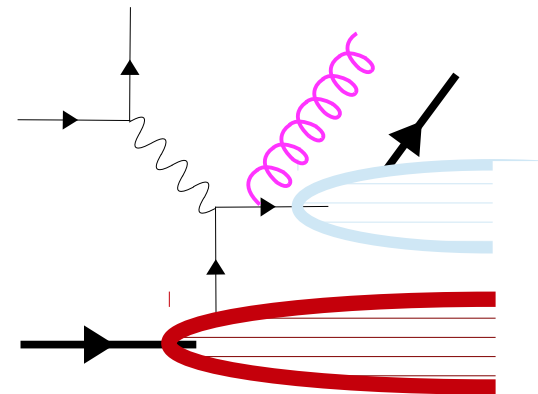
SIDIS



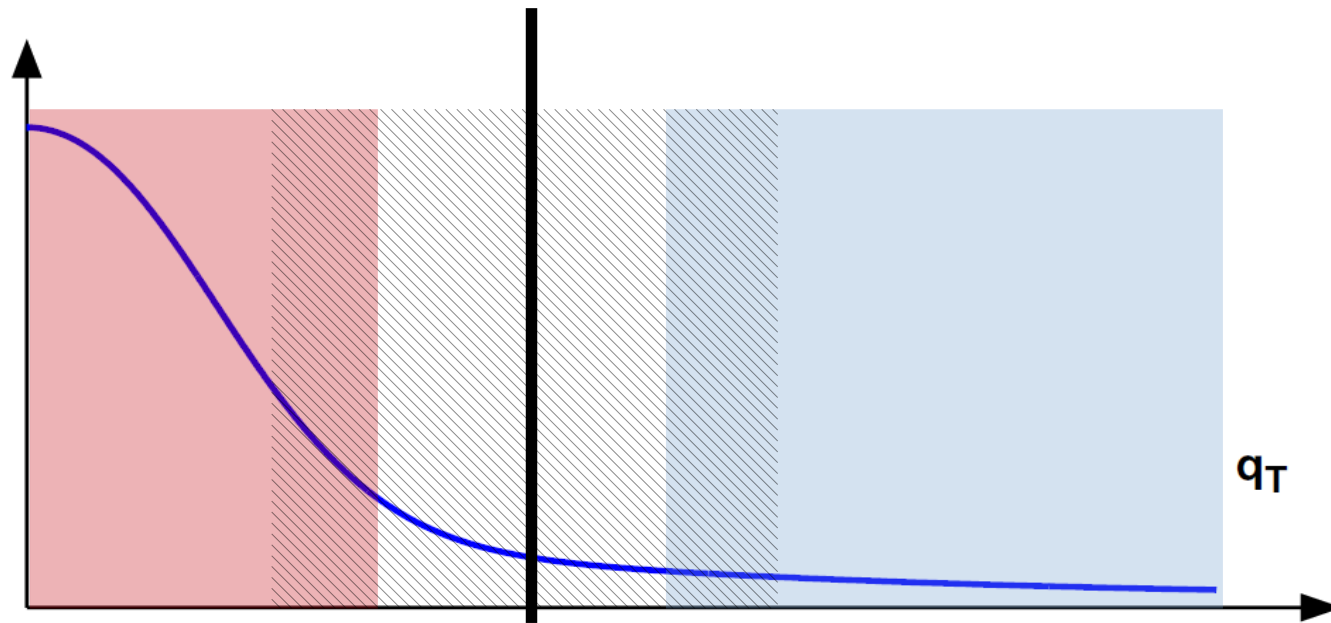
Non-perturbative
Transverse Momentum



Non-perturbative
Transverse Momentum

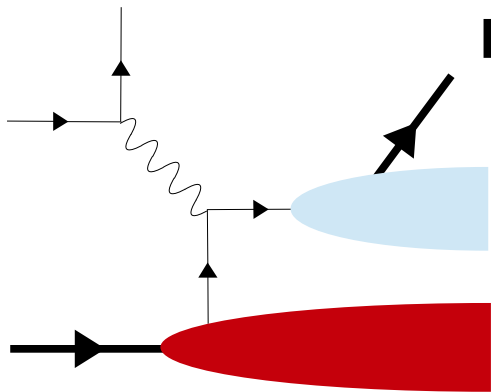


Hard gluon radiation

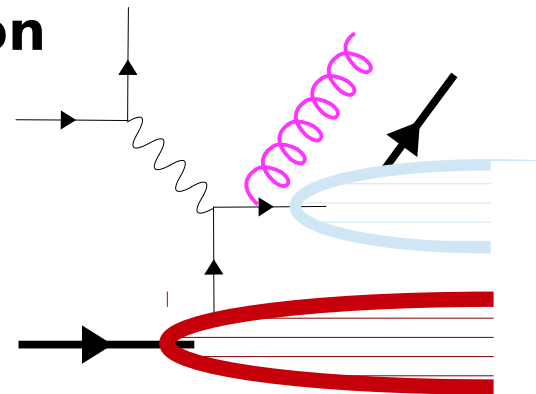


$q_T \sim Q$

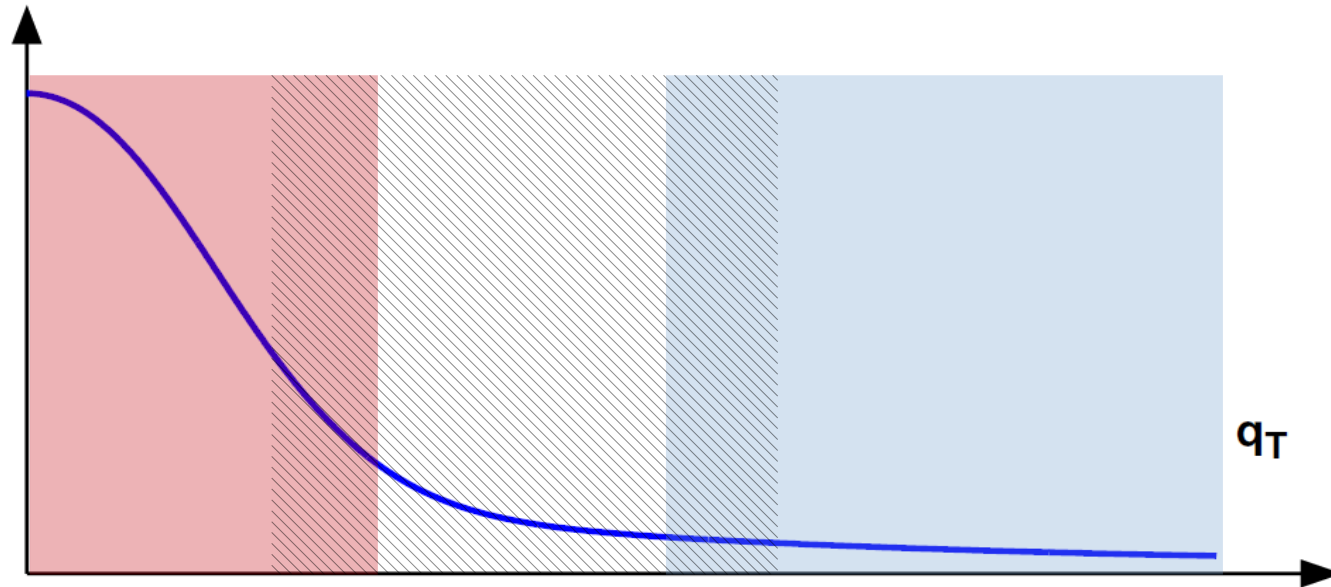
Matching region



Non-perturbative
Transverse Momentum



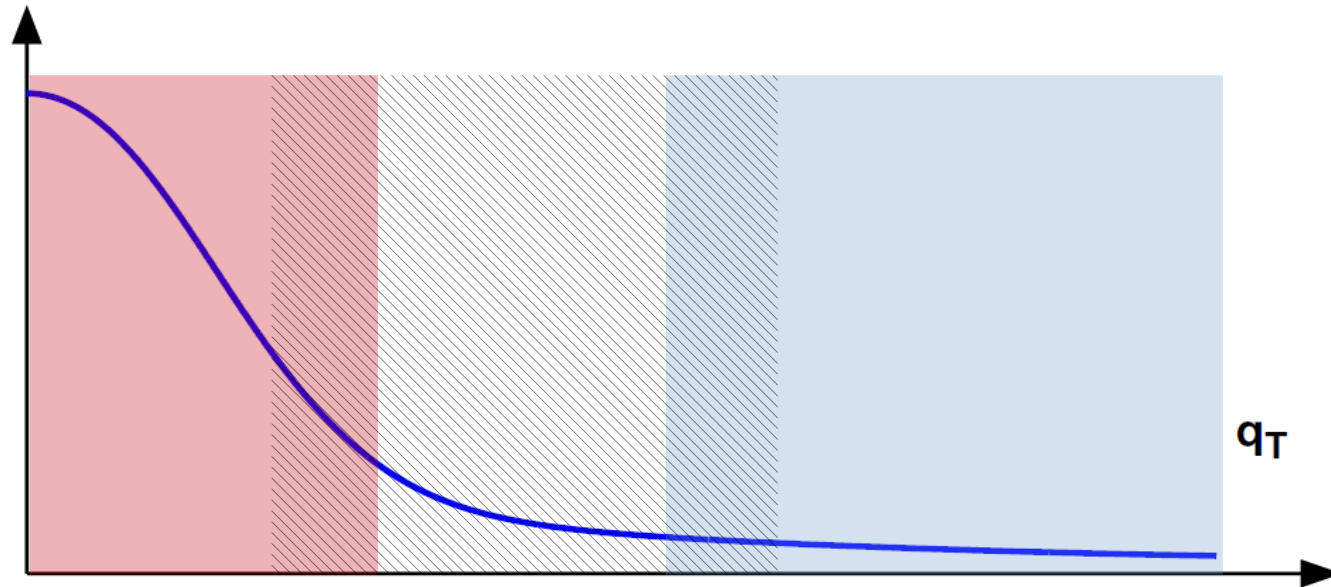
Hard gluon radiation



Collins-Soper-Sterman (CSS) formalism provides a matching scheme (other schemes available).

Successful implementations In Drell-Yan.

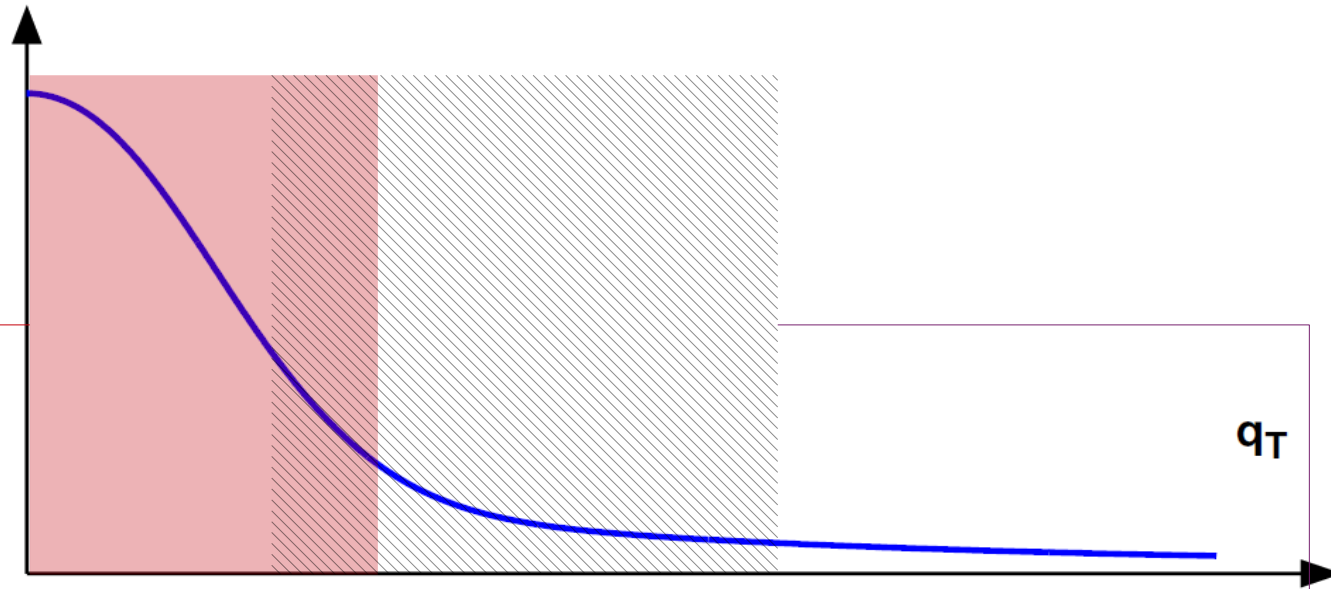
Several challenges in SIDIS.



Collins-Soper-Sterman (CSS) formalism provides a matching scheme (other schemes available).

Successful implementations In Drell-Yan.

Several challenges in SIDIS.

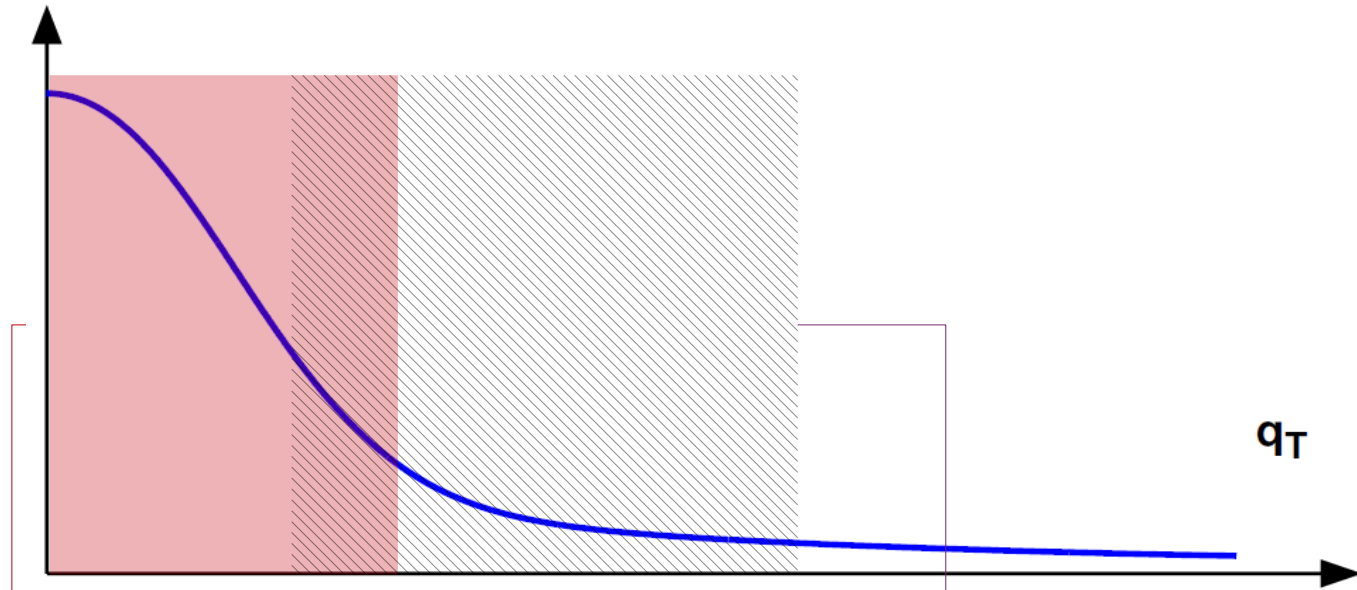


Unpolarized SIDIS cross section (current region)

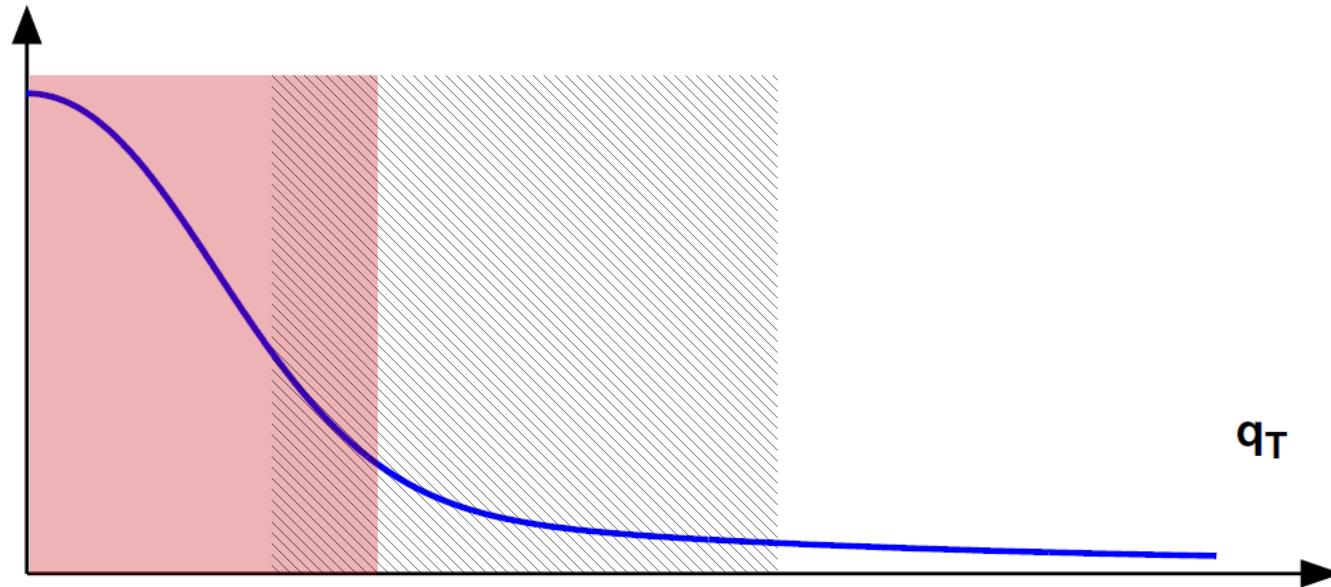
$$\frac{d\sigma^{\ell+p \rightarrow \ell' h X}}{dx_B dQ^2 dz_h dP_T^2} = \frac{2\pi^2\alpha^2}{(x_B s)^2} \frac{[1 + (1-y)^2]}{y^2} F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q \text{ F.T. } \left\{ \tilde{D}_{h/q}(z, z \mathbf{b}_\perp; Q) \tilde{f}_{q/P}(x, \mathbf{b}_\perp; Q) \right\} + \text{large } q_T \text{ corrections}$$

+ power suppressed terms



$$W + Y$$



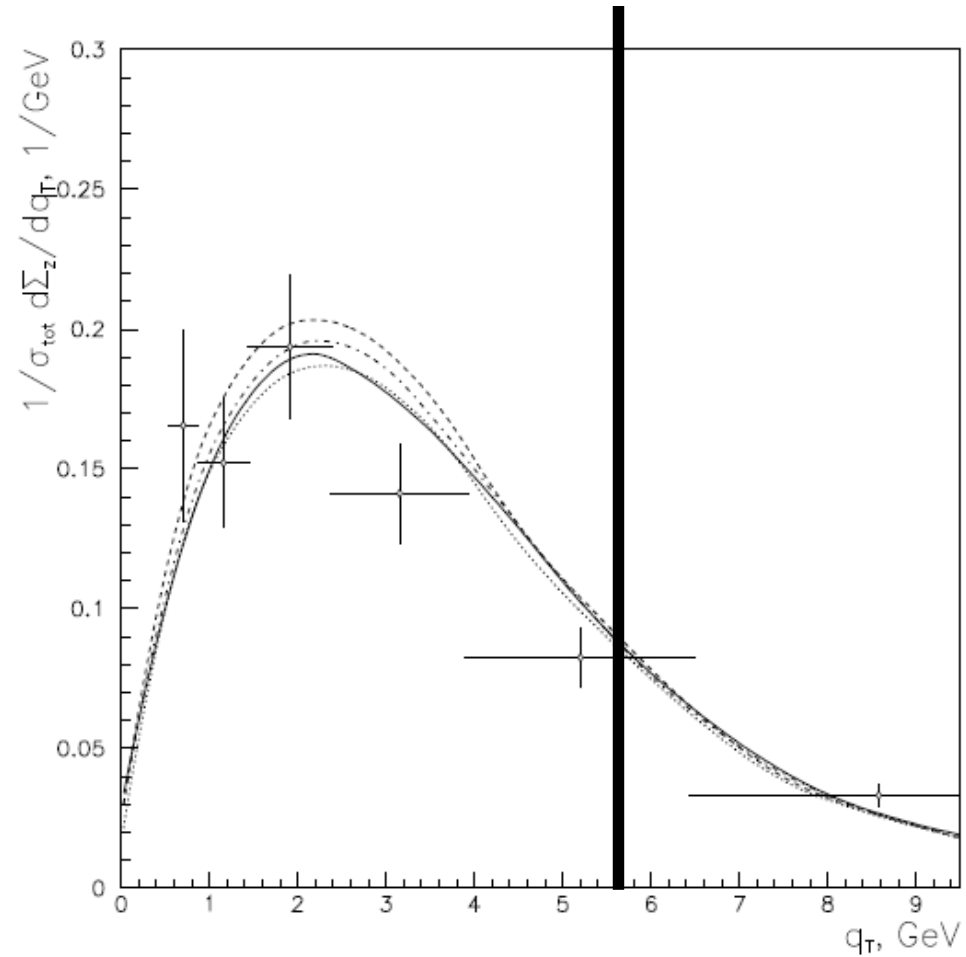
No sharp transitions between regions.

Large q_T -corrections cannot be ignored in TMD phenomenology.

$$\boxed{W} + \boxed{Y}$$

$$\langle x \rangle = 0.0049, \quad \langle Q^2 \rangle = 32.6 \text{ GeV}^2$$

HERA data



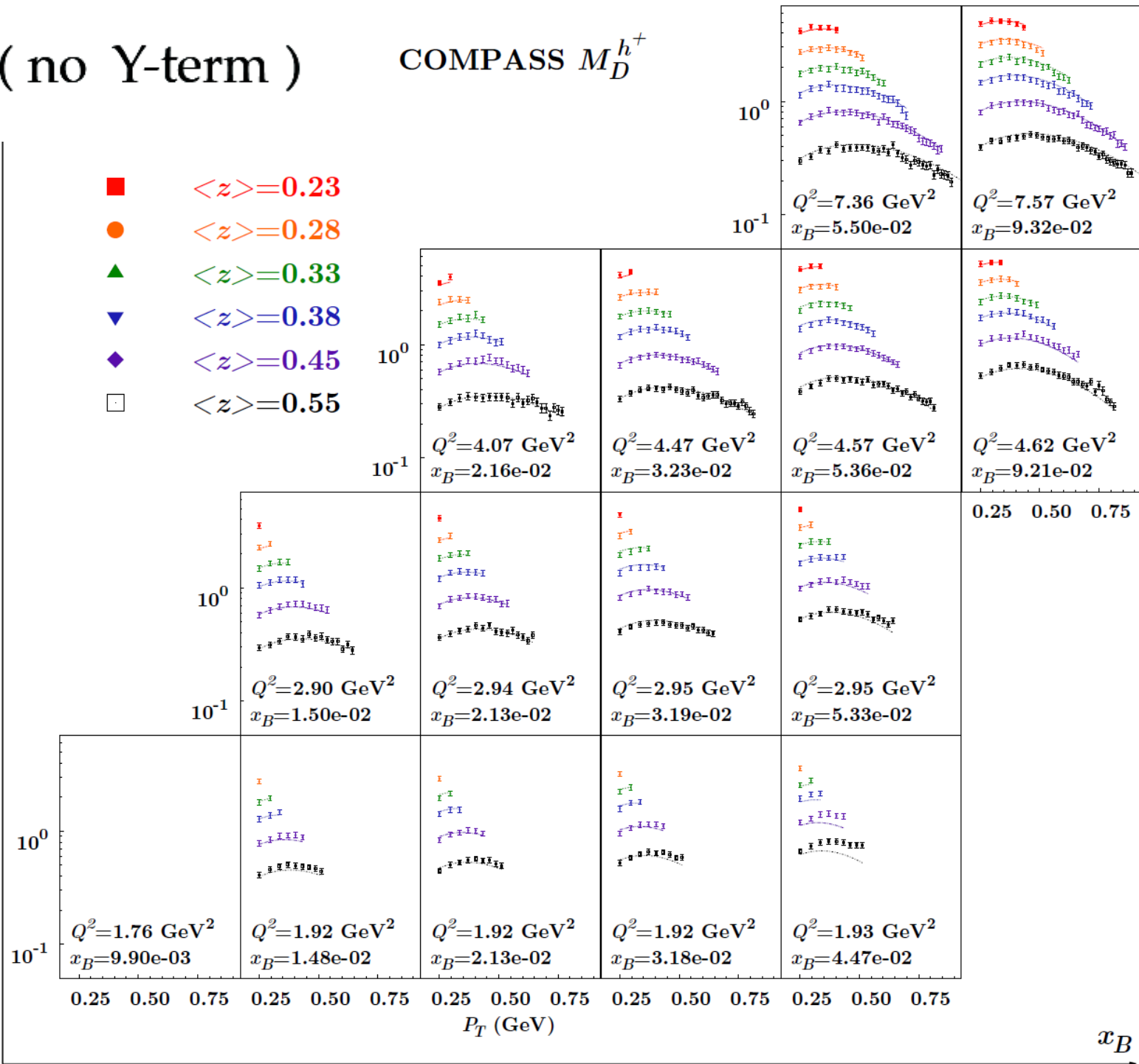
Nadolsky, Stump, Yuan
 Phys.Rev.D61:014003,2000

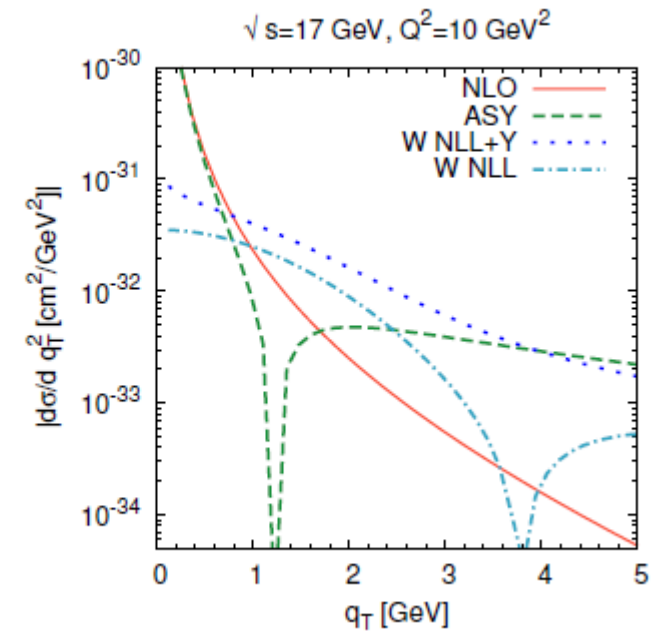
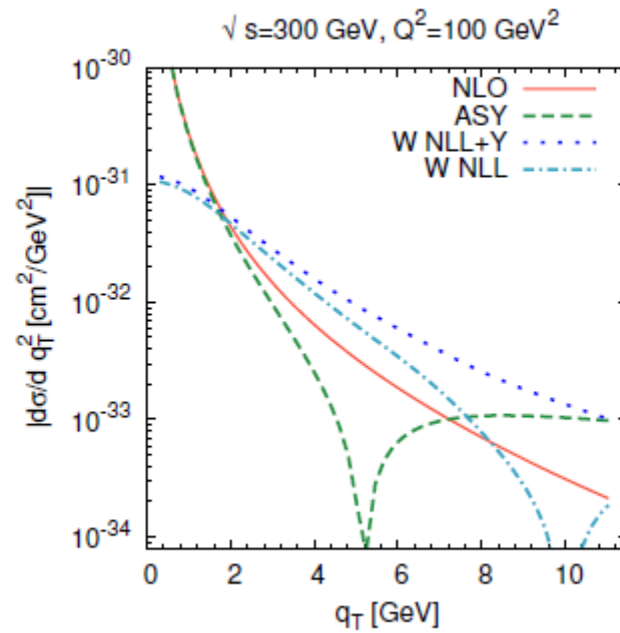
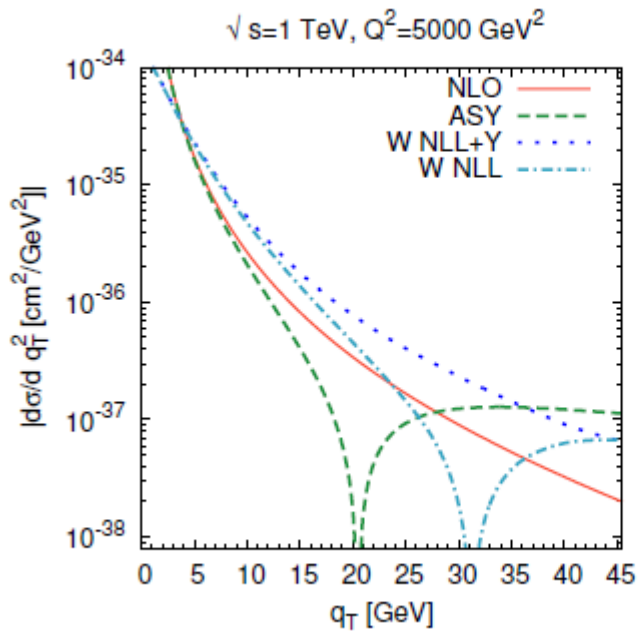
$$\langle E_T \rangle_{\Phi_B} = \frac{1}{\sigma_{tot}} \sum_B \int_{\Phi_B} d\Phi_B E_T \frac{d\sigma(e + A \rightarrow e + B + X)}{d\Phi_B}$$



(no Y-term)

COMPASS $M_D^{h^+}$





Matching is crucial, cannot afford to miss any constraint.

Final remarks...

Any other TMD extraction relies on Unpolarized functions

$e^+e^- \rightarrow \pi\pi X$
(Belle and BaBar)

SIDIS
(HERMES and COMPASS)

Unpolarized TMDFF

Collins TMDFF

$$\frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d^2\mathbf{P}_{1T} d\cos\theta_2} = \frac{3\pi\alpha^2}{2s} \left\{ \boxed{D_{h_1 h_2}} + \boxed{N_{h_1 h_2}} \cos 2\phi_1 \right\}$$

$$P_0^{U,L,C} = \frac{N^{U,L,C}}{D^{U,L,C}}$$

Ratio

$$D^U = D_{\pi^+\pi^-} + D_{\pi^-\pi^+}$$

$$N^U = N_{\pi^+\pi^-} + N_{\pi^-\pi^+}$$

$$D^L = D_{\pi^+\pi^+} + D_{\pi^-\pi^-}$$

$$N^L = N_{\pi^+\pi^+} + N_{\pi^-\pi^-}$$

$$D^C = D^U + D^L$$

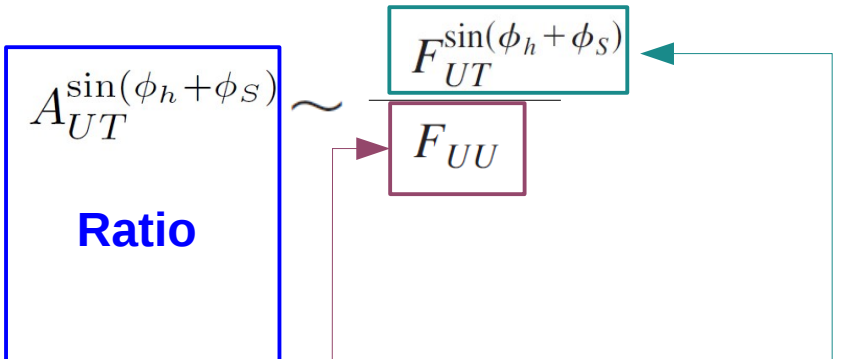
$$N^C = N^U + N^L,$$

$$\frac{A_0^U}{A_0^{L(C)}} \equiv 1 + \cos(2\phi_1) \boxed{A_0^{UL(C)}} \quad \text{Double Ratio}$$

$$\frac{d\sigma^{\ell(S_e)+p(S) \rightarrow \ell' h X}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} =$$

$$\frac{2\alpha^2}{Q^4} \left\{ \frac{1 + (1-y)^2}{2} F_{UU} + \dots \right.$$

$$\left. + S_T(1-y)(\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)}) \right\}$$



Unpolarized
TMDFF
& TMDPDF

TMD Transversity
& Collins function

Thank you.